Numerical Linear Algebra Problem Answer

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Notice: this is the answer based on my own understanding, which is for your reference only.

1 Fundamentals

1.1 Matrix-Vector Multiplication

(1.1)

1.1. Let B be a 4×4 matrix to which we apply the following operations:

1. double column 1,
2. halve row 3,
3. add row 3 to row 1,
4. interchange columns 1 and 4,
5. subtract row 2 from each of the other rows,
6. replace column 4 by column 3,
7. delete column 1 (so that the column dimension is reduced by 1).

(a) Write the result as a product of eight matrices.

(b) Write it again as a product ABC (same B) of three matrices.

Ans: (a). 1.

$$B\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.

$$B\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.

$$B\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

4.

$$B\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}$$

5.

$$B\begin{pmatrix}1 & -1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & -1 & 0 & 1\end{pmatrix}$$

6.

$$B\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}$$

7.

$$B\begin{pmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

(b).
TODO
(1.2)
TODO
(1.3)
(1.4)

1.3. Generalizing Example 1.3, we say that a square or rectangular matrix R with entries r_{ij} is *upper-triangular* if $r_{ij} = 0$ for $i > j$. By considering what space is spanned by the first n columns of R and using (1.8), show that if R is a nonsingular $m \times m$ upper-triangular matrix, then R^{-1} is also upper-triangular. (The analogous result also holds for lower-triangular matrices.)

1.4. Let $f_1, \ldots, f_8$ be a set of functions defined on the interval $[1, 8]$ with the property that for any numbers $d_1, \ldots, d_8$, there exists a set of coefficients $c_1, \ldots, c_8$ such that

$$\sum_{j=1}^{8} c_j f_j(i) = d_i, \qquad i = 1, \ldots, 8.$$

(a) Show by appealing to the theorems of this lecture that $d_1, \ldots, d_8$ determine $c_1, \ldots, c_8$ uniquely.

(b) Let A be the 8×8 matrix representing the linear mapping from data $d_1, \ldots, d_8$ to coefficients $c_1, \ldots, c_8$. What is the i, j entry of A^{-1}?