

# Manifold Learning and Sparse Representation Solution

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课后作业190. Use image Lena and K-SVD to learn a 256-atom dictionary for  $8 \times 8$  patches. Order the dictionary from “low frequency” to “high frequency” and rearrange in 2D in the zigzag manner.

课后作业194. Prove that the overlapped group Lasso  $\Omega_{overlap}^g(\mathbf{w})$  is a norm.

课后作业195. Prove that

$$\Omega_{overlap}^g(\mathbf{w}) = \sup\{\boldsymbol{\alpha}^T \mathbf{w} | \boldsymbol{\alpha} \in \mathbb{R}^k, \|\boldsymbol{\alpha}_g\| \leq 1, \forall g \in \mathcal{G}\}.$$

**课后作业196.** Consider 2D DCT transform. We know that for image patches, their high frequencies are more likely to be zeros than low frequencies are. So if we want to recover an image patch, we want the high frequencies to be zeros **before** the low frequencies. Then how to design a group sparsity regularizer on such a prior? Please refer to Figure 7.8(b). The entries on the  $i$ -th anti-diagonal are called of frequency  $i$ . Then this prior means that if  $i < j$  and entries of frequency  $i$  are zeros, then entries of frequency  $j$  are also zeros.

**课后作业197.** Explain why the unit balls of  $\Omega_{\text{group}}^{\mathcal{G}}(\cdot)$  and  $\Omega_{\text{overlap}}^{\mathcal{G}}(\cdot)$  for the groups  $\mathcal{G} = \{\{1, 2\}, \{2, 3\}\}$  are as the left and the right figures in Figure 8.10, respectively.