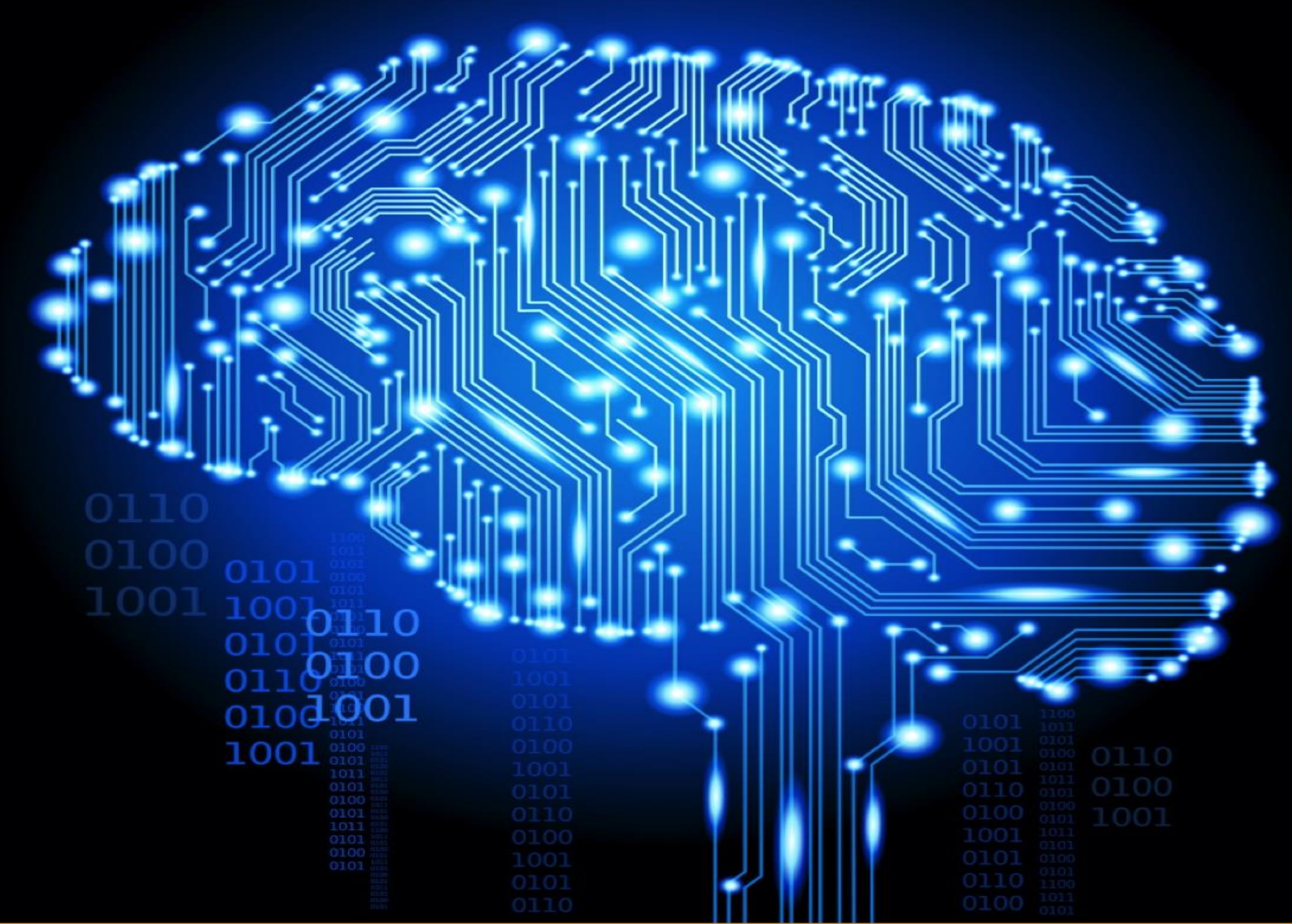

Lecture 13: Deep Sequential Models

Efstratios Gavves

Lecture overview

- Autoregressive Models
- PixelCNN, PixelCNN++, PixelRNN
- WaveNet
- Time-Aligned DenseNets

Autoregressive Models

- Let's assume we have signal modelled by an input random variable x
  - Can be an image, video, text, music, temperature measurements

- Is there an order in all these signals?

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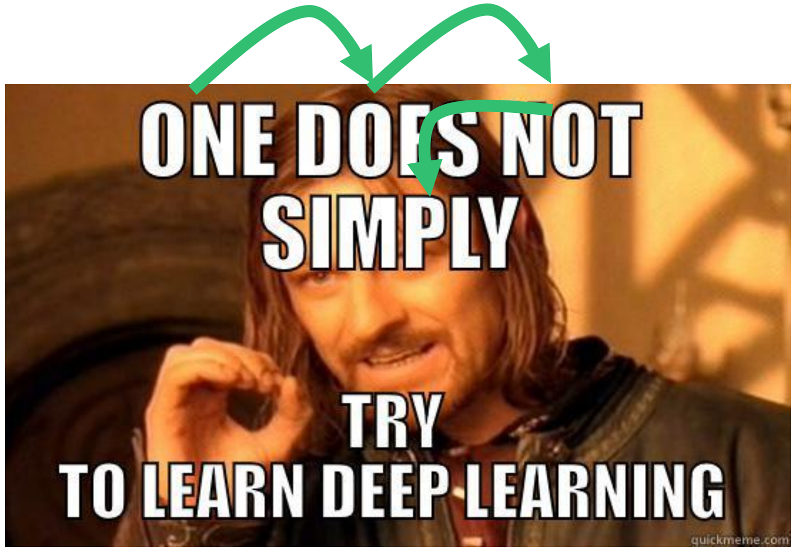
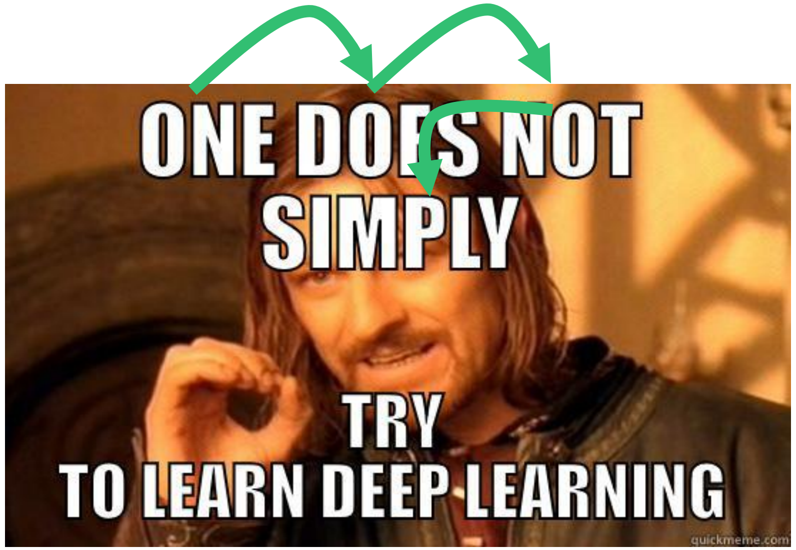


Autoregressive Models

- Let’s assume we have signal modelled by an input random variable x
  - Can be an image, video, text, music, temperature measurements

- Is there an order in all these signals? Other signals and orders?



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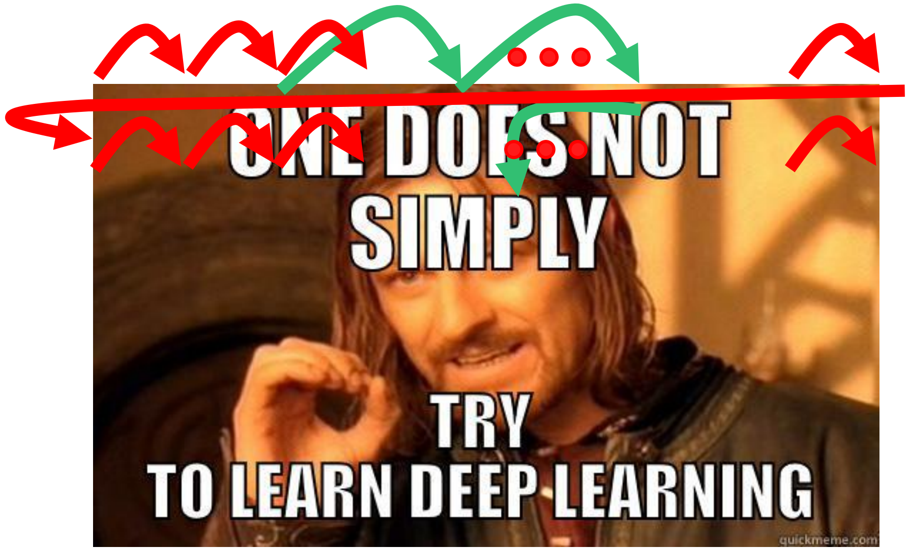


Autoregressive Models

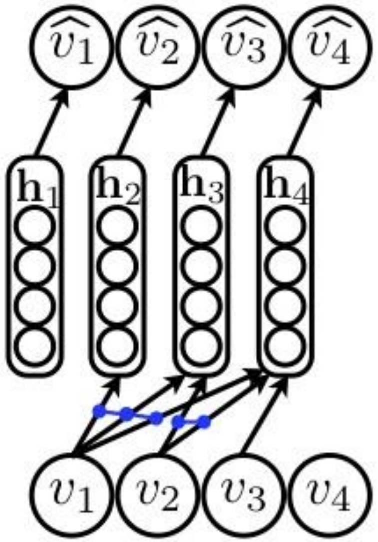
- If x is sequential, there is an order: $x = [x_1, \dots, x_k]$
  - E.g., the order of words in a sentence
- If x is *not* sequential, we can create an artificial order $x = [x_{r(1)}, \dots, x_{r(k)}]$
  - E.g., the order with which pixels make (generate) an image
- Then, the marginal likelihood is a product of conditionals

$$p(x) = \prod_{k=1}^{D} p(x_k | x_{j<k})$$

- Different from Recurrent Neural Networks
  (a) no parameter sharing
  (b) chains are not infinite in length

Autoregressive Models

- Pros: because of the product decomposition, $p(x)$ is tractable
- Cons: because the $p(x)$ is sequential, training is slower
  - To generate every new word/frame/pixel, the previous words/frames/pixels in the order must be generated first → no parallelism

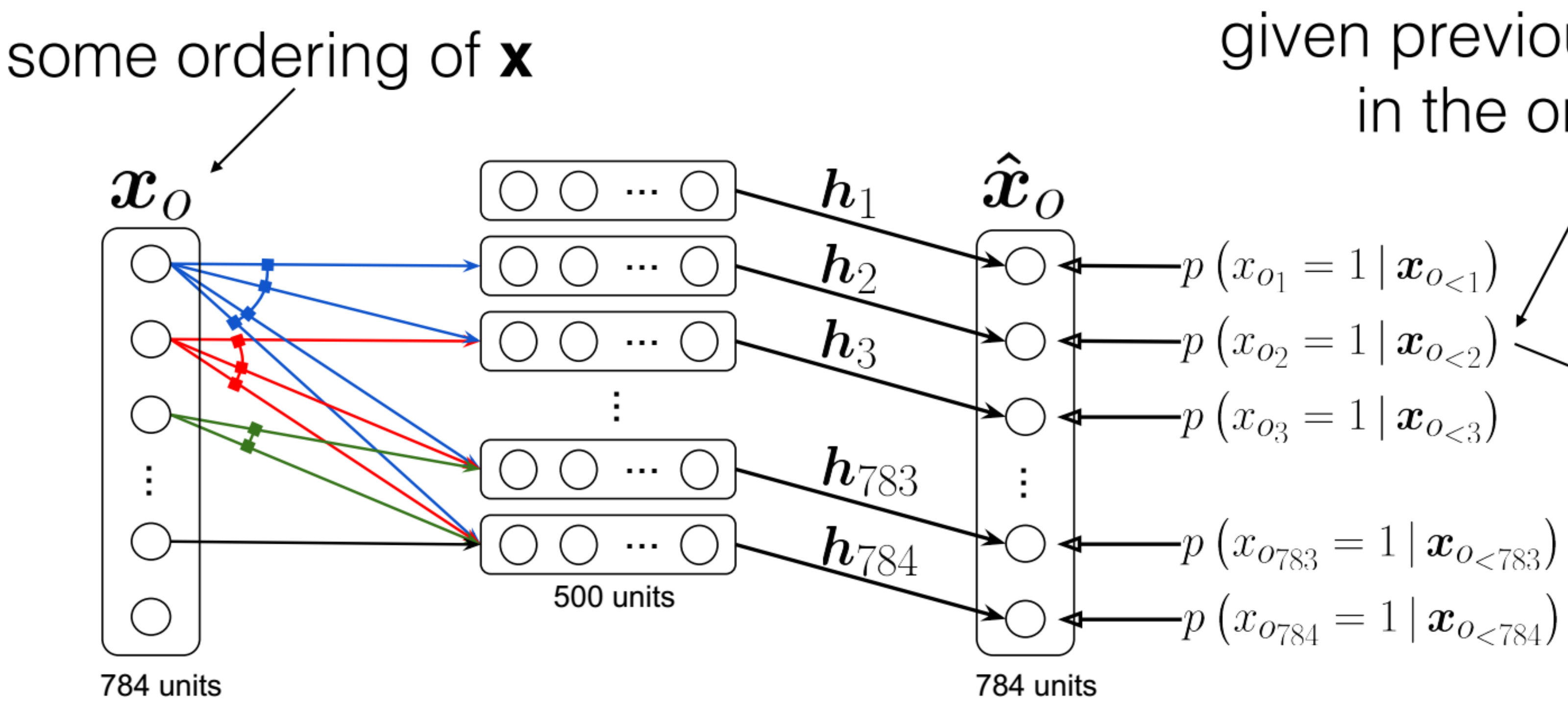


The where and now of Deep Autoregressive Models

- Sequential data is a natural fit
  - Language modelling, time series, etc
  - For non-sequential data they are ok, although arguably artificial
- **Question:** How to model the conditionals $p(x_k|x_{j<k})$
  - Logistic regression (Frey et al., 1996)
  - Neural networks (Bengio and Bengio, 2000)

- Modern deep autoregressors
  - NADE, MADE, PixelCNN, PixelCNN++, PixelRNN, WaveNet

NADE



$$p(x_{o_d} = 1 \mid x_{o_{<d}}) = \sigma(V_{o_d,:}\mathbf{h}_d + b_{o_d})$$
$$\mathbf{h}_d = \sigma(W_{:,o_d}\mathbf{x}_{o_d} + \mathbf{c})$$

Neural Autoregressive Distribution Estimation, Larochelle and Murray, AISTATS 2011

NADE

- Minimizing negative log-likelihood as usual

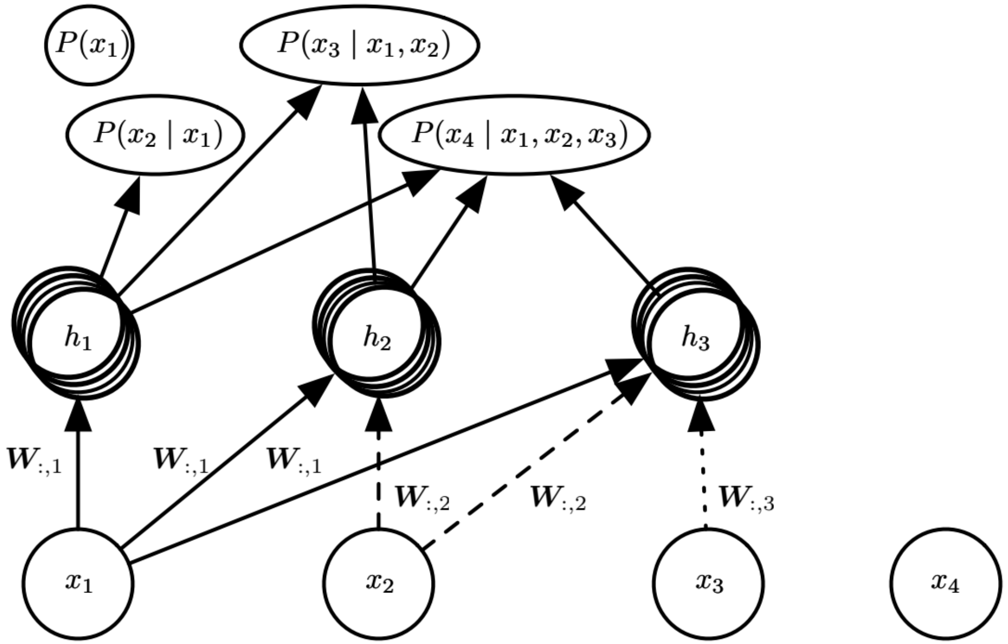
$$\mathcal{L} = -\log p(x) = -\sum_{k=1}^{D} p(x_k | x_{<k})$$

- Then, we model the conditional as

$$p(x_d | x_{<d}) = \sigma(\mathrm{V}_{\mathrm{d},:} \cdot h_d + \mathrm{b}_\mathrm{d})$$

where the latent variable h_d is defined as

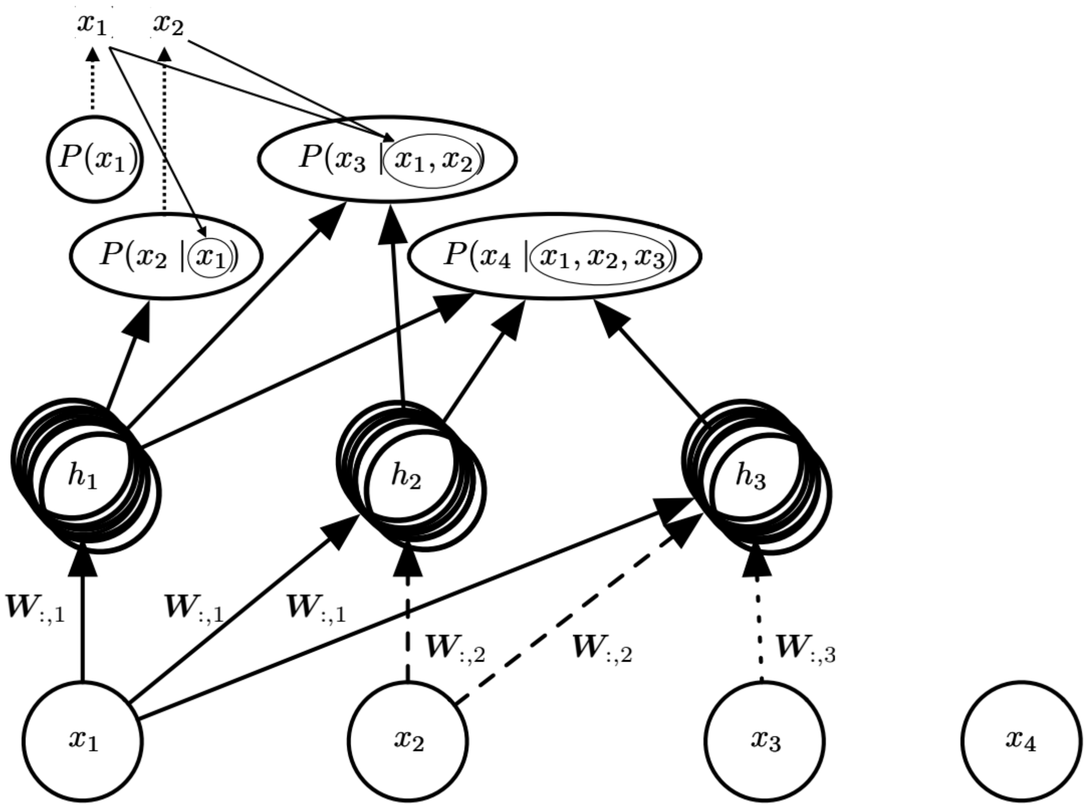
$$h_d = \sigma(W_{:,d} \cdot x_d + c)$$



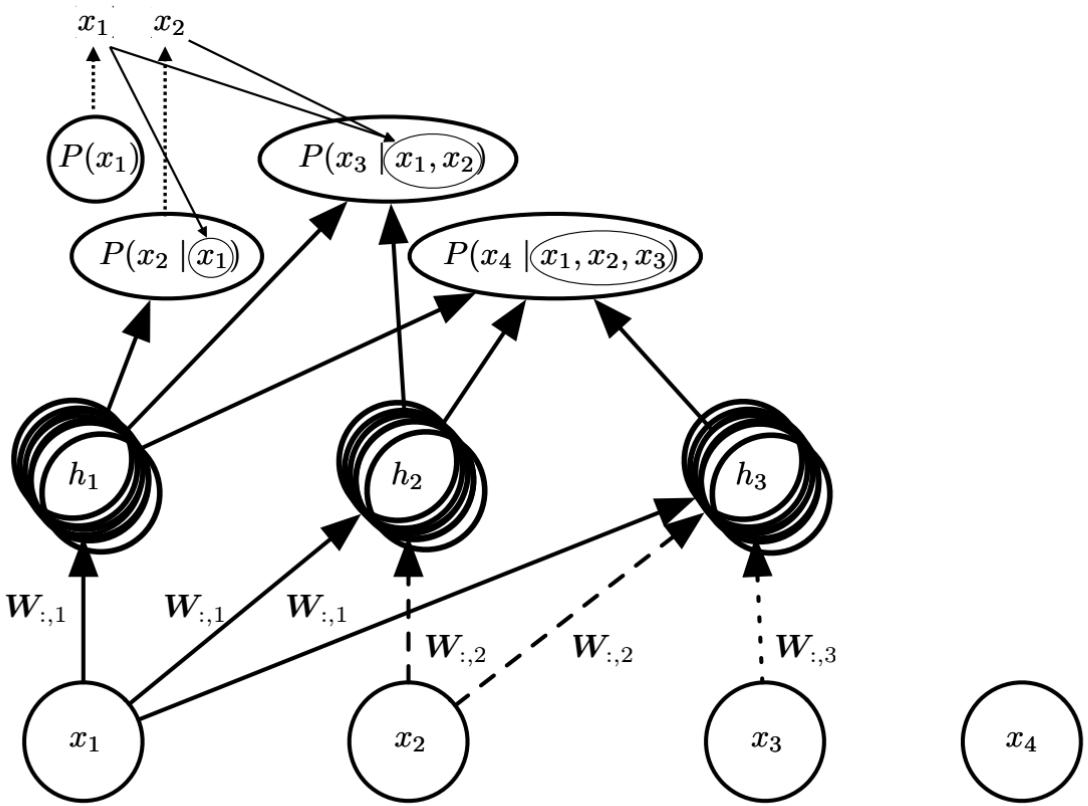
NADE: Training & Testing

- “Teacher forcing” training

Training: Use ground truth values (e.g. of pixels)



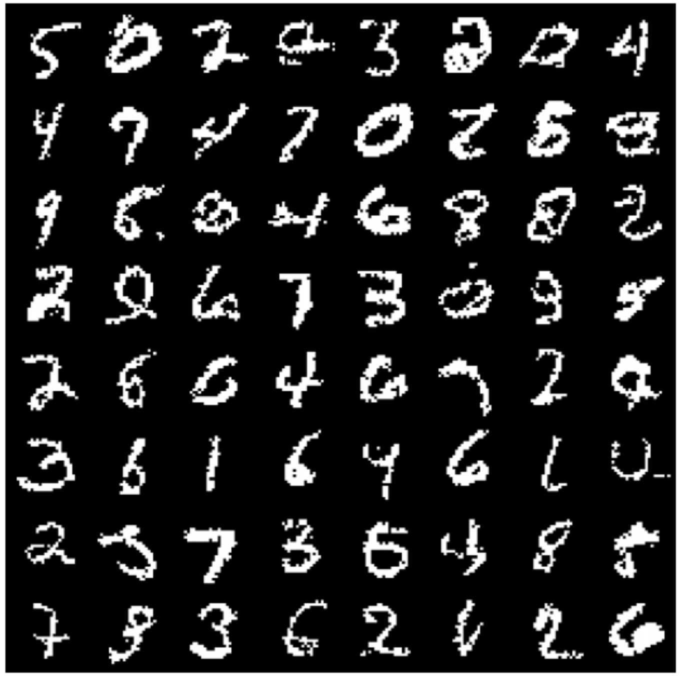
Testing: Use predicted values in previous order



NADE Visualizations

Binarized MNIST samples (NADE)

Binarized MNIST samples (DeepNADE)



Binarized MNIST samples (ConvNADE)

MADE

- **Question:** How could we construct an autoregressive autoencoder?
- **To rephrase:** How to modify an autoencoder such that each output x_k depends <u>only</u> on the previous outputs $x_{<k}$ (autoregressive property)?
  - Namely, the <u>present</u> k-th output $\tilde{x}_k$ must not depend on a computational path from future inputs $x_{k+1}, \dots, x_D$
  - Autoregressive: $p(x|\theta) = \prod_{k=1}^{D} p(x_k | x_{j<k}, \theta)$
  - Autoencoder: $p(\tilde{x}|x, \theta) = \prod_{k=1}^{D} p(\tilde{x}_k | x_k, \theta)$

Masked Autoencoder for Distribution Estimation, Germain, Mathieu et al., ICML 2015

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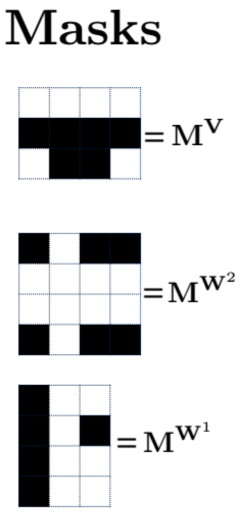
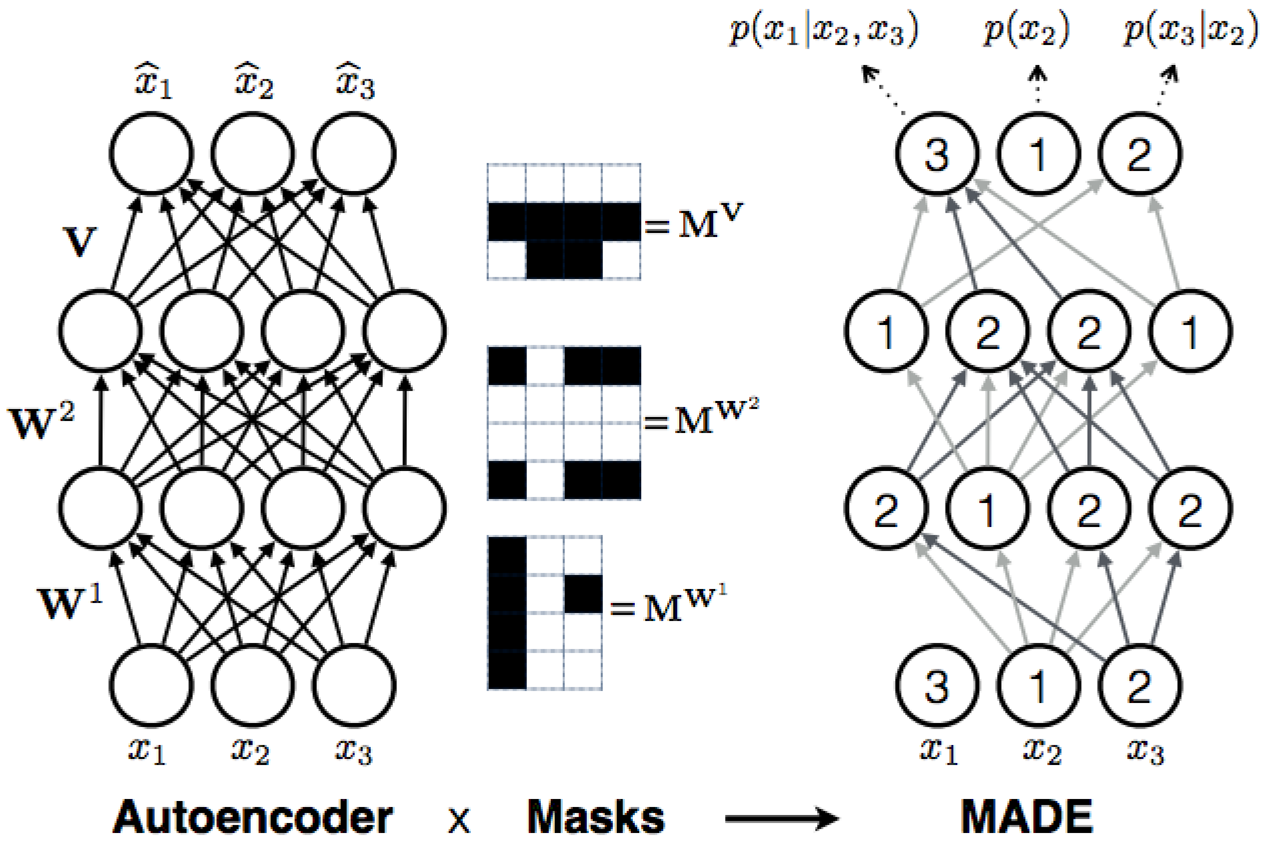
  - Autoencoder: $p(\tilde{x}|x, \theta) = \prod_{k=1}^{D} p(\tilde{x}_k|x_k, \theta)$
- Answer: Masked convolutions!

$$h(x) = g(b + (W \odot M^W) \cdot x)$$
$$\tilde{x} = \sigma(c + (V \odot M^V) \cdot h(x))$$



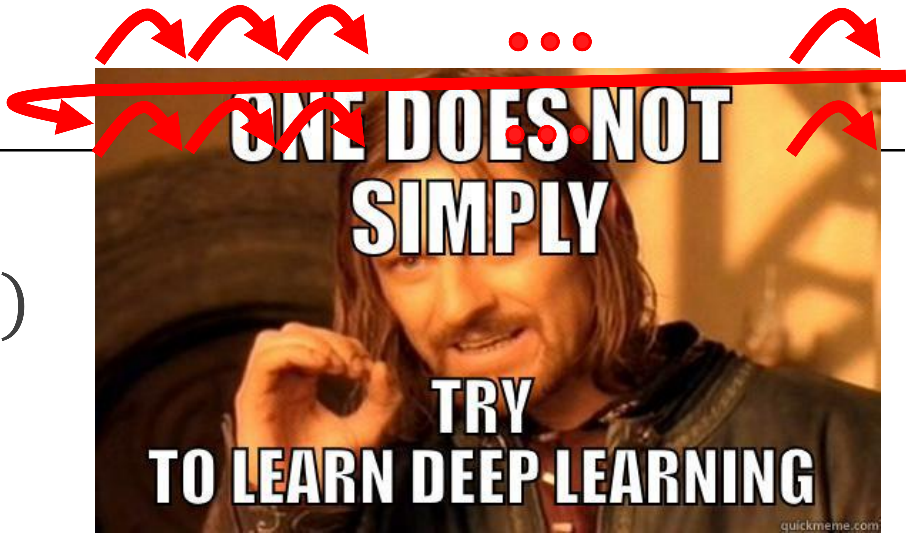
Masked Autoencoder for Distribution Estimation, Germain, Mathieu et al., ICML 2015

MADE



Masked Autoencoder for Distribution Estimation, Germain, Mathieu et al., ICML 2015

PixelRNN



- Unsupervised learning: learn how to model $p(x)$
- Decompose the marginal

$$p(x) = \prod_{i=1}^{n^2} p(x_i | x_1, \dots, x_{i-1})$$

- Assume row-wise pixel by pixel generation and sequential colors R→G→B
  - Each color conditioned on all colors from previous pixels and specific colors in the same pixel

$$p(x_{i,R} | x_{<i}) \cdot p(x_{i,G} | x_{<i}, x_{i,R}) \cdot p(x_{i,B} | x_{<i}, x_{i,R}, x_{i,G})$$

- Final output is 256-way softmax

Pixel Recurrent Neural Networks, van den Oord, Kalchbrenner and Kavukcuoglu, arXiv 2016

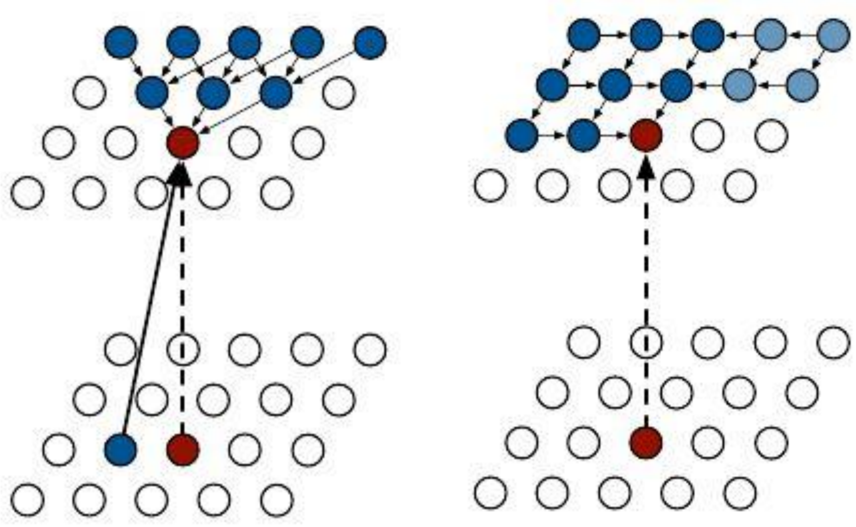
PixelRNN

- How to model the conditionals?

$$p(x_{i,R}|x_{<i}), p(x_{i,G}|x_{<i}, x_{i,R}), p(x_{i,B}|x_{<i}, x_{i,R}, x_{i,G})$$

- LSTM variants
  - 12 layers

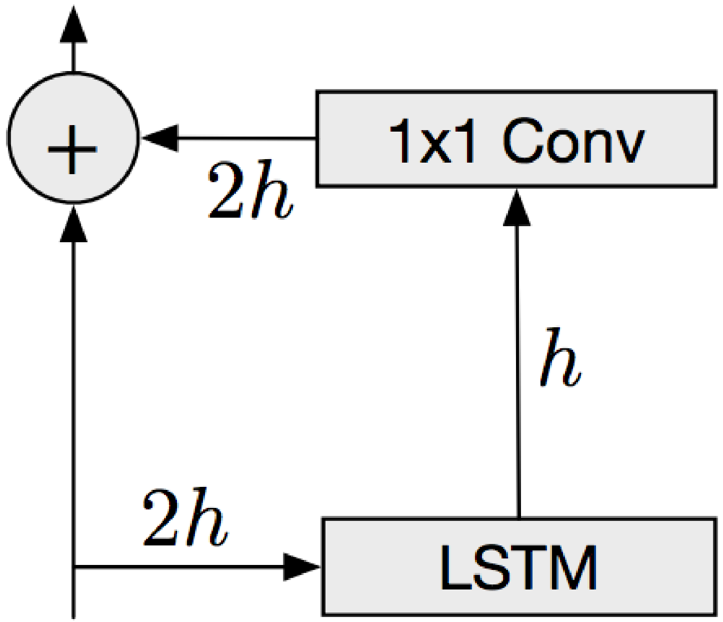
- Row LSTM
- Diagonal Bi-LSTM

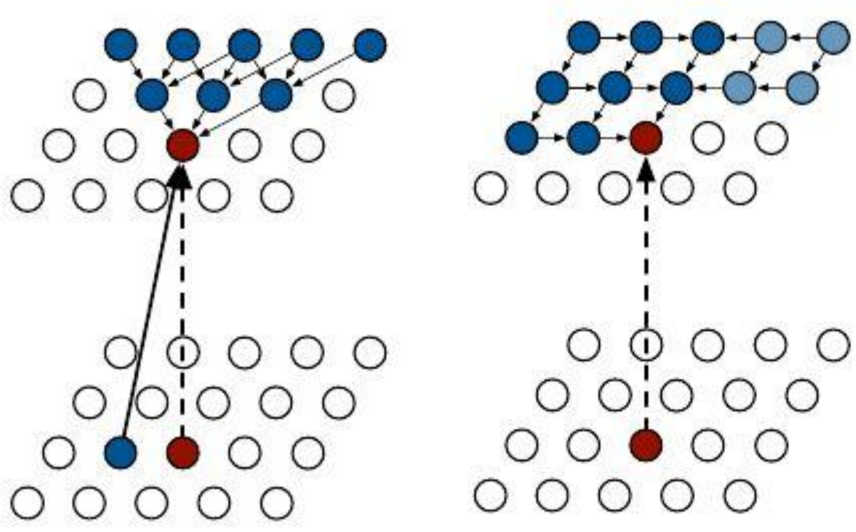


Row LSTM   Diagonal Bi-LSTM

PixelRNN

- Residual connections also to speed up convergence
- Pros: good modelling of $p(x)$ → nice image generation
- Cons: slow training, slow generation





Row LSTM

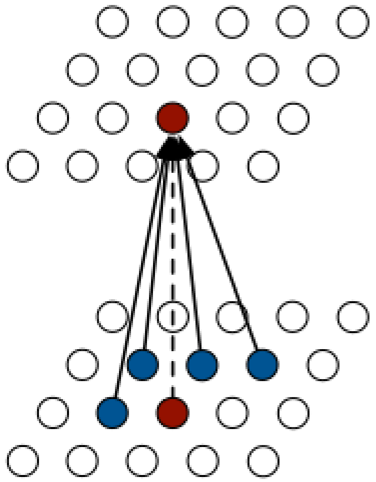
Diagonal Bi-LSTM

PixelRNN - Generations

Figure 1. Image completions sampled from a PixelRNN.

PixelCNN

- Unfortunately, PixelRNN is too slow
- Solution: replace recurrent connections with convolutions
  - Multiple convolutional layers to preserve spatial resolution
- Training is much faster because all true pixels are known in advance, so we can parallelize
  - Generation still sequential (pixels must be generated) → still slow



PixelCNN

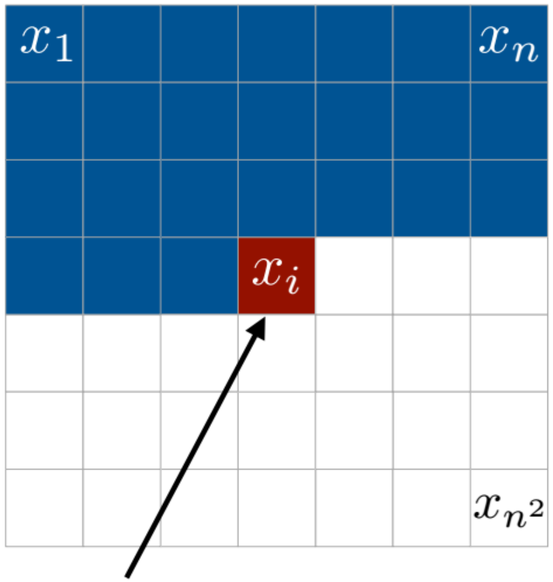
Stack of masked convolutions

1	1	1	1	1
1	1	1	1	1
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0

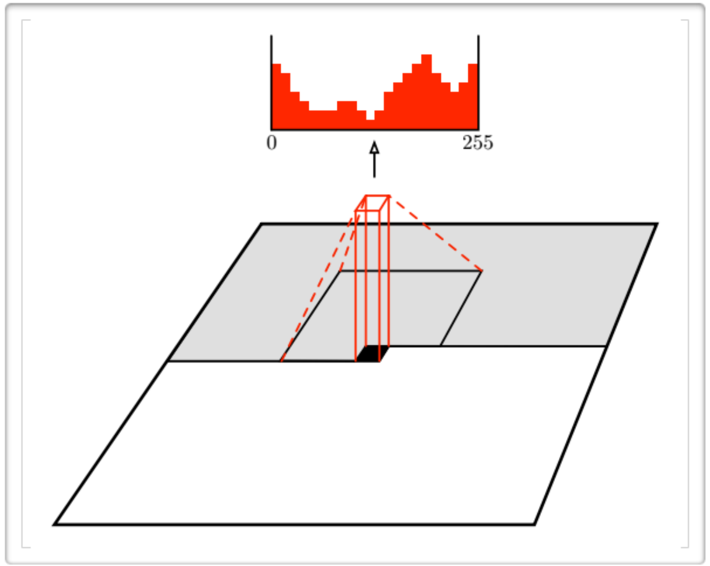
Pixel Recurrent Neural Networks, van den Oord, Kalchbrenner and Kavukcuoglu, arXiv 2016

PixelCNN

- Use masked convolutions again to enforce autoregressive relationships



$$p(x_i \mid \mathbf{x}_{<i})$$



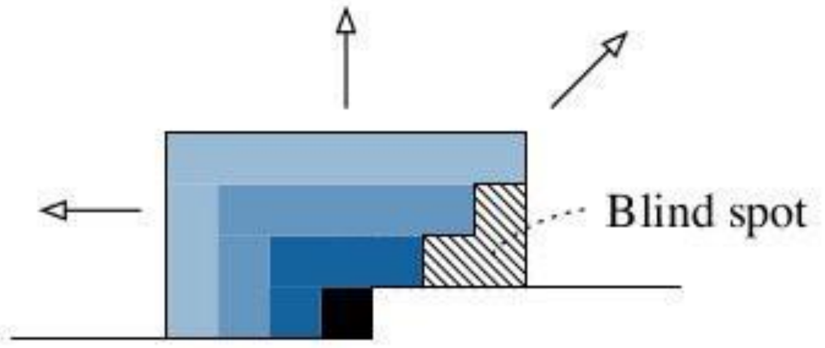
PixelCNN – Pros and Cons

- Cons: Performance is worse than PixelRNN
  - Why?

- **New problem:** the cascaded convolutions create a **"blind spot"**

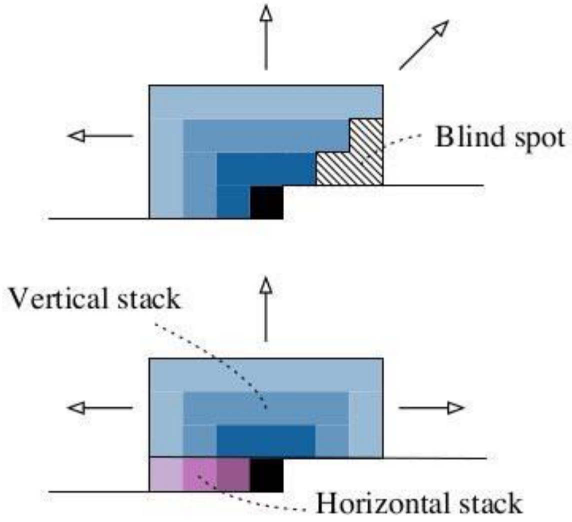
Blind spot

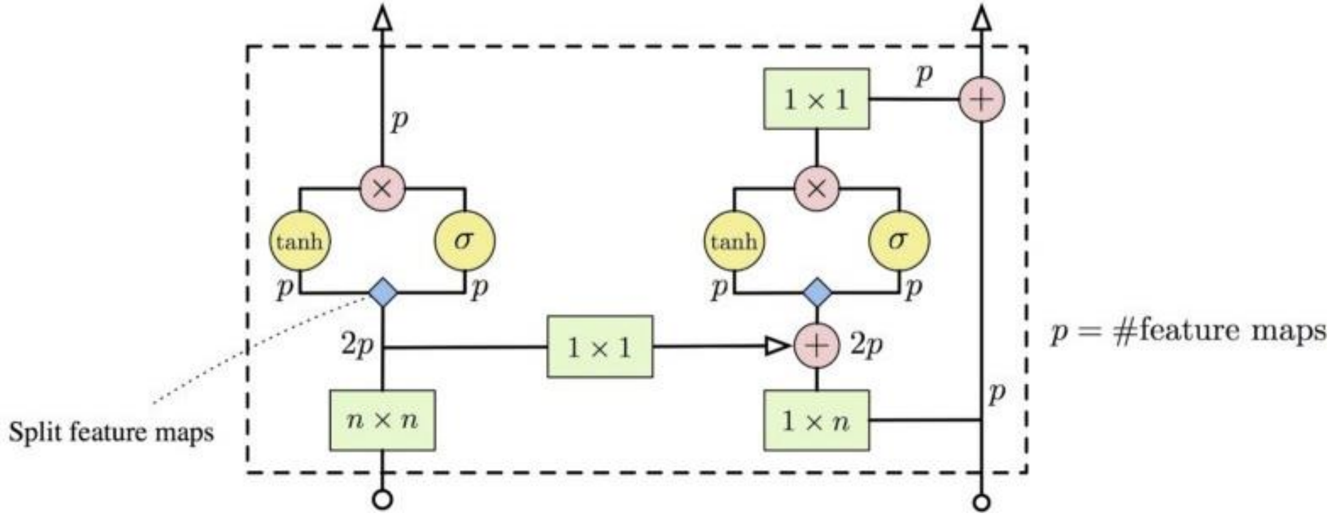
- Because of
  (a) the limited receptive field of convolutions and
  (b) computing all features at once (not sequentially)
  → cascading convolutions makes current pixel not depend on <u>all</u> previous
  → blind spot



Fixing the blind spot: Gated PixelCNN

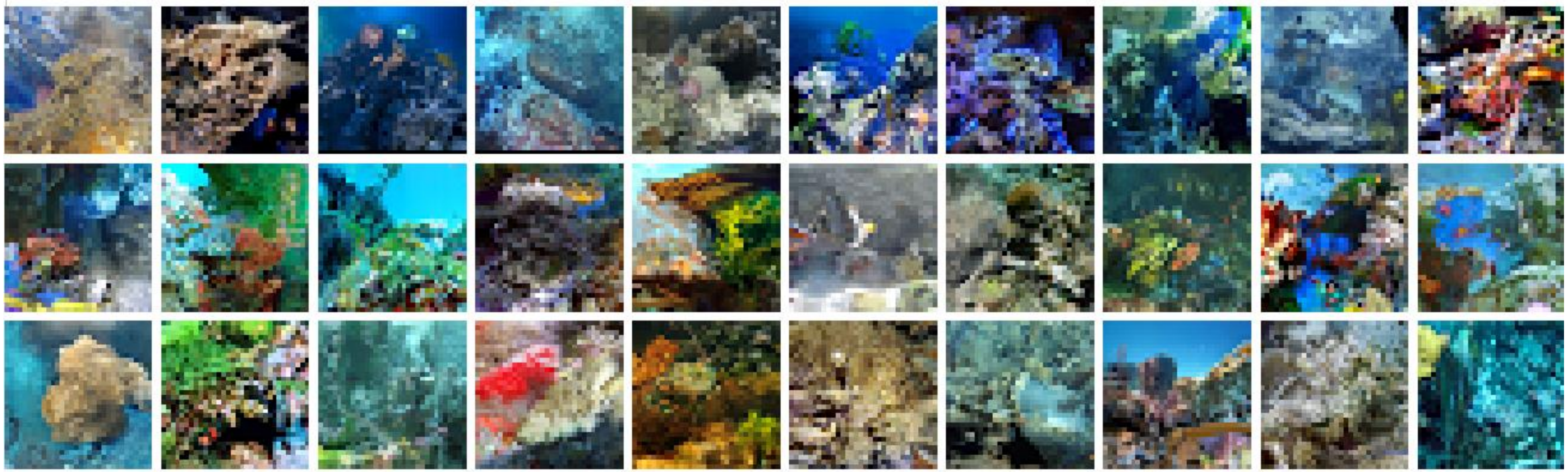
- Use two layers of convolutions stacks
  - Horizontal stack: conditions on current row and takes as input the previous layer output and the vertical stack
  - Vertical stack: conditions on all rows above current pixels
- Also replace ReLU with a $\tanh(W * x) \cdot \sigma(U * x)$





PixelCNN - Generations

- Coral reef



PixelCNN - Generation

- Sorrel horse

PixelCNN - Generation

- Sandbar

PixelCNN - Generation

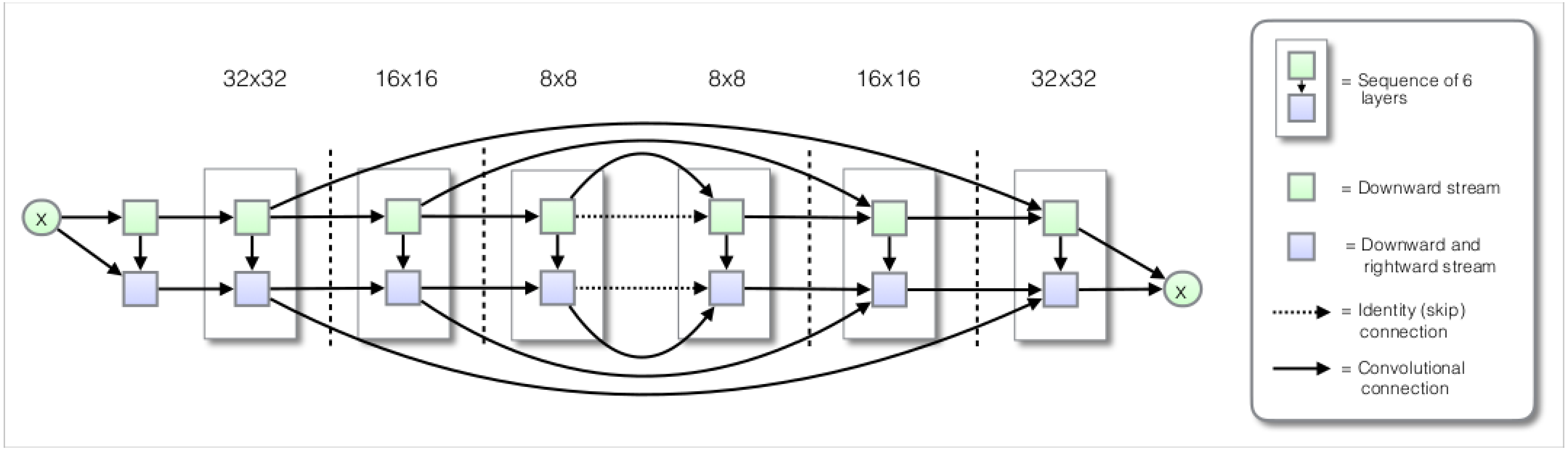
- Lhasa Apso

PixelCNN++

- Improving the PixelCNN model
- Replace the softmax output with a discretized logistic mixture lihelihood
  - Softmax is too memory consuming and gives sparse gradients
  - Instead, assume logistic distribution of intensity and round off to 8-bits
- Condition on whole pixels, not pixel colors
- Downsample with stride-2 convs to compute long-range dependencies
- Use shortcut connections
- Dropout
  - PixelCNN is too powerful a framework → can onverfit easily

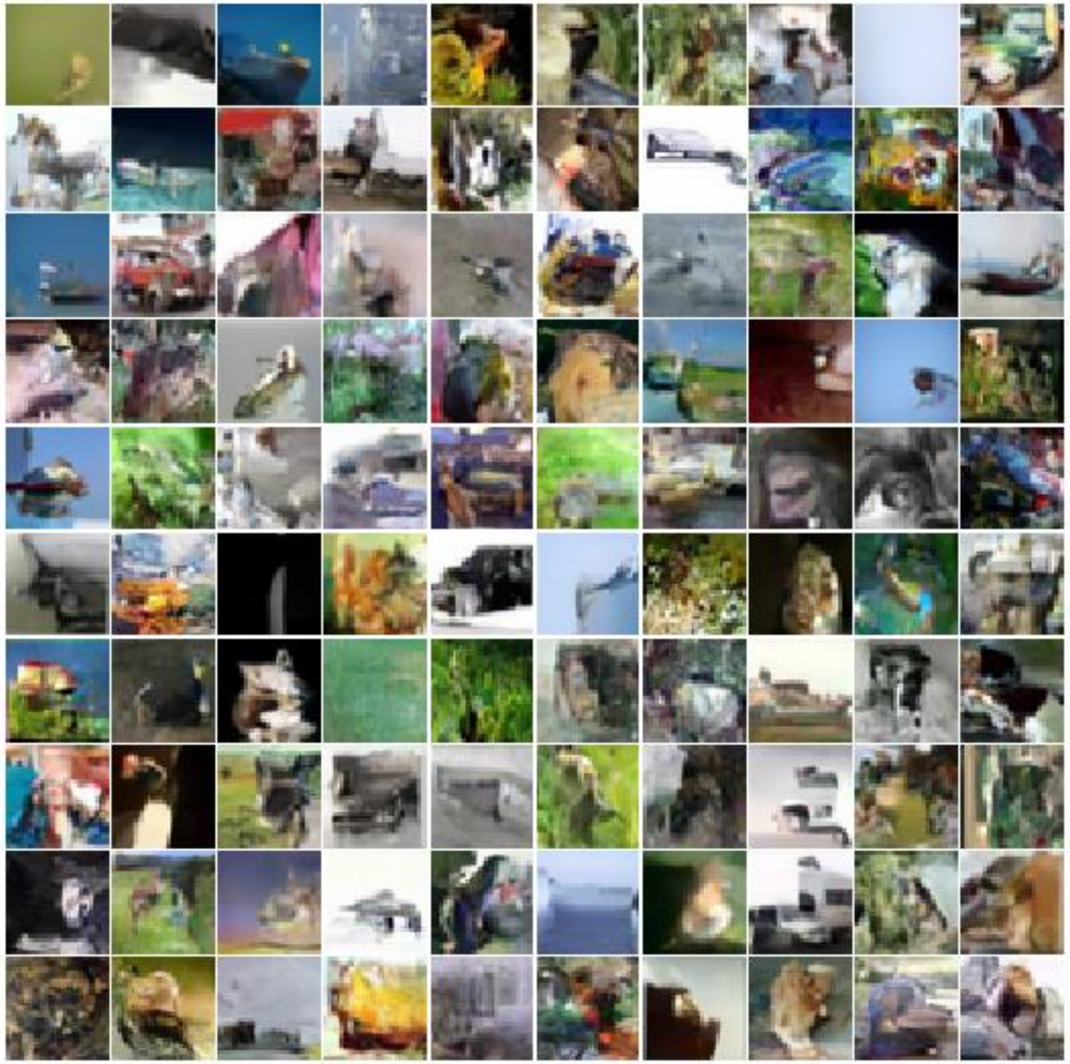
PixelCNN++: Improving the PixelCNN with Discretized Logistic, Salimans, Karpathy, Chen, Kingma, ICLR 2017

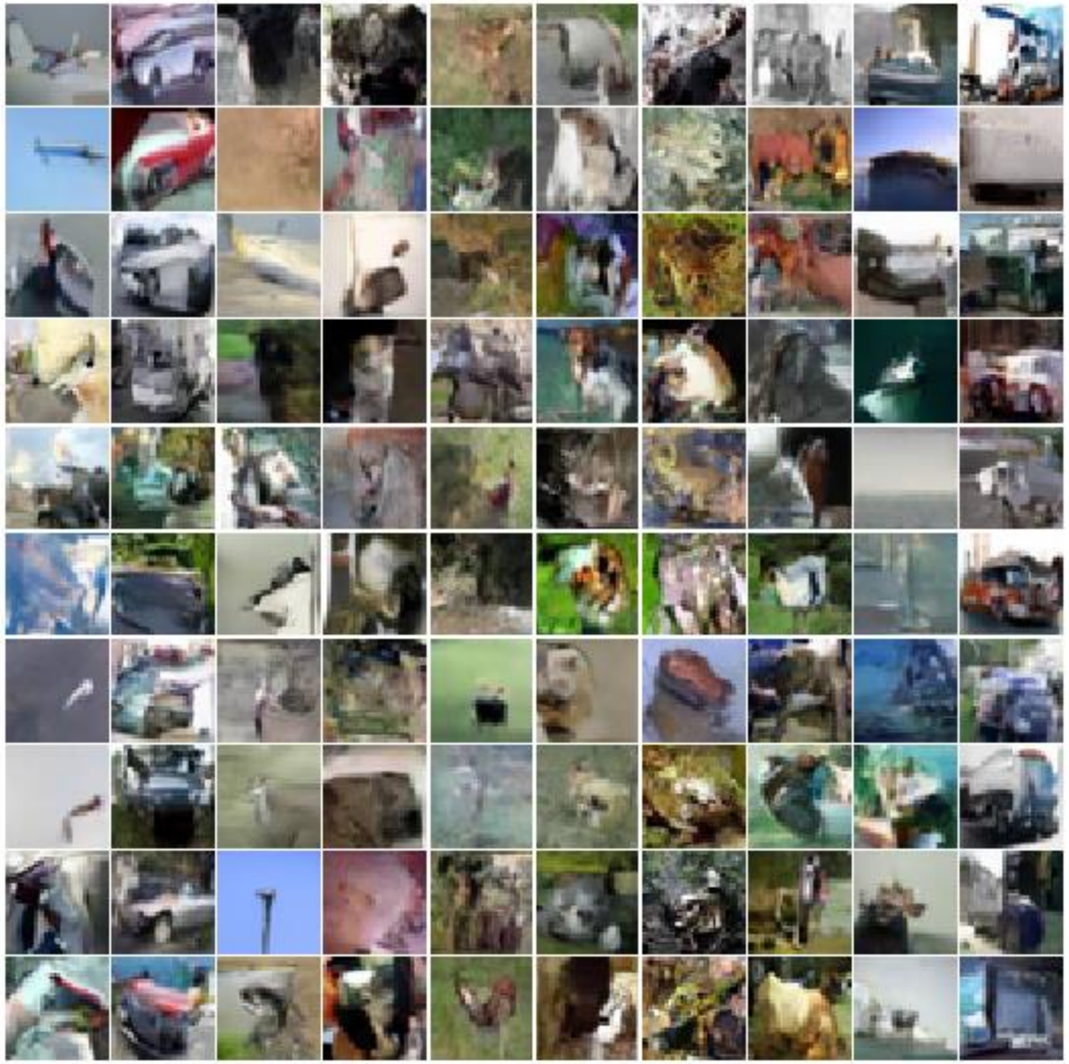
PixelCNN++



32x32
16x16
8x8
8x8
16x16
32x32
x
x
= Sequence of 6 layers
= Downward stream
= Downward and rightward stream
= Identity (skip) connection
= Convolutional connection

PixelCNN++ - Generations





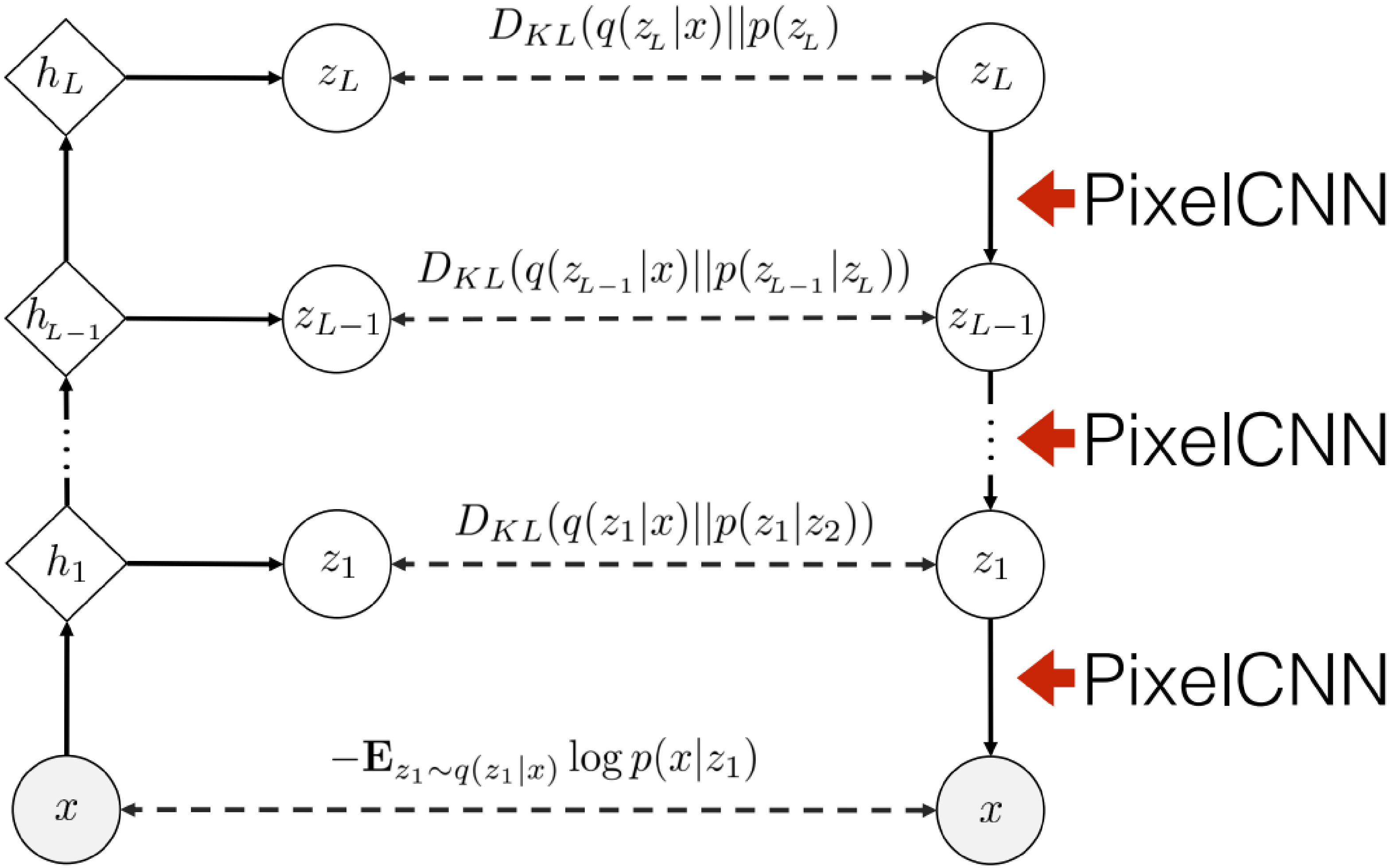
PixelVAE

- A standard VAE with a PixelCNN generator/decoder

- Be careful. Often the generator is so powerful, that the encoder/inference network is ignored ← Whatever the latent code z there will be a nice image generated

PixelVAE: A Latent Variable Model for Natural Images, Gulrajani et al., ICLR 2017

PixelVAE



$D_{KL}(q(z_L|x)||p(z_L))$
h_L
z_L
z_L
PixelCNN
$D_{KL}(q(z_{L-1}|x)||p(z_{L-1}|z_L))$
h_{L-1}
z_{L-1}
z_{L-1}
PixelCNN
$D_{KL}(q(z_1|x)||p(z_1|z_2))$
h_1
z_1
z_1
PixelCNN
$-\mathbf{E}_{z_1 \sim q(z_1|x)} \log p(x|z_1)$
x
x

PixelVAE - Generations

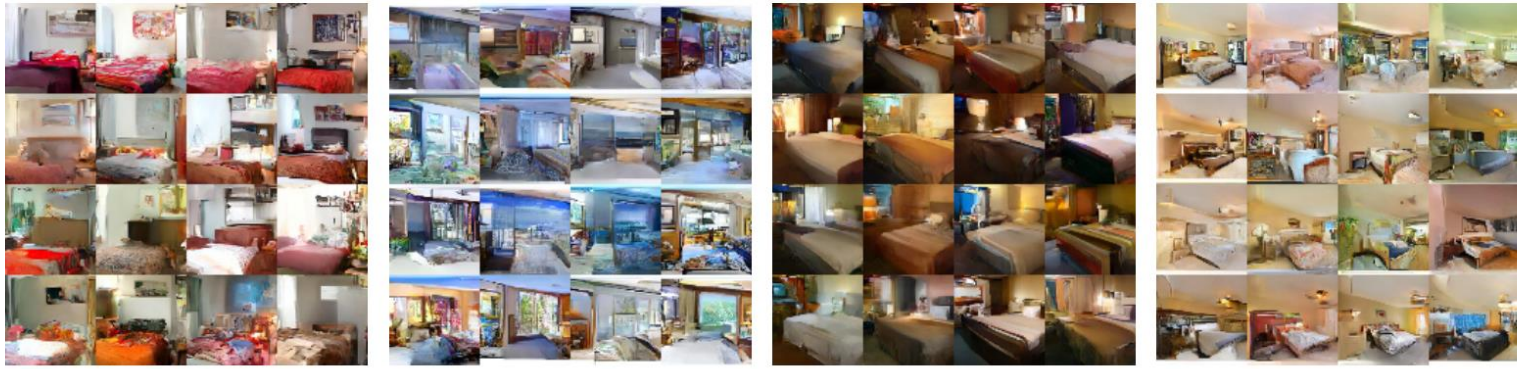
64x64 LSUN Bedrooms

64x64 ImageNet

PixelVAE - Generations

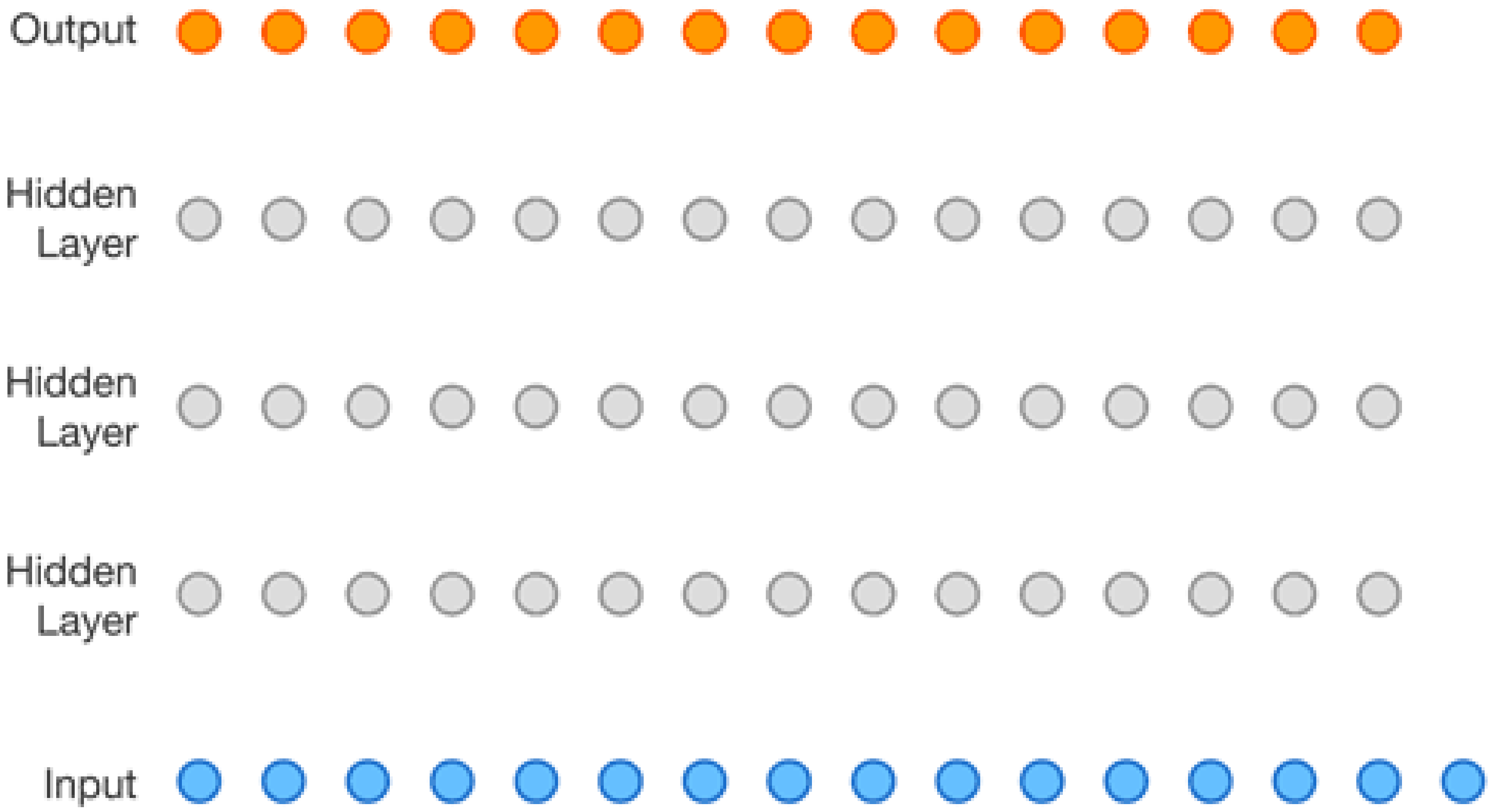
Varying top latents

Varying bottom latents



Varying pixel-level noise

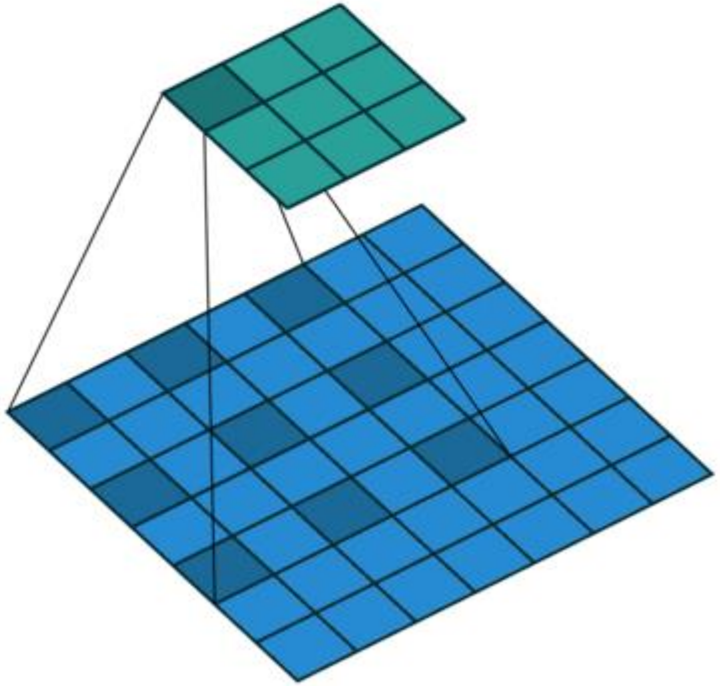
WaveNet



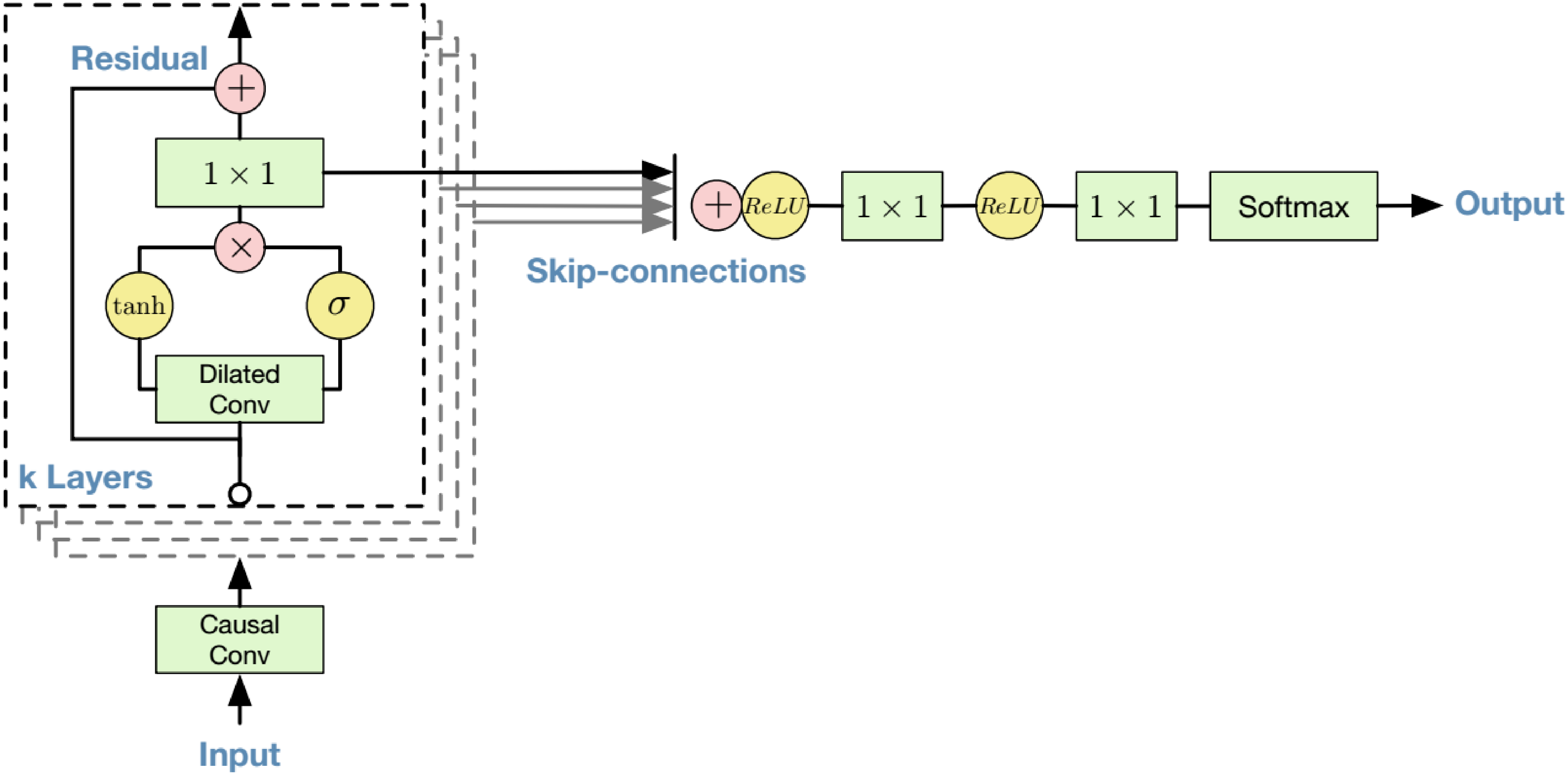
WaveNet: A Generative Model for Raw Audio, van den Oord et al., arXiv 2017

WaveNet

- Inspired by PixelRNN and PixelCNN
- Fully convolutional neural network
- Use dilated convolutions
- [Samples]

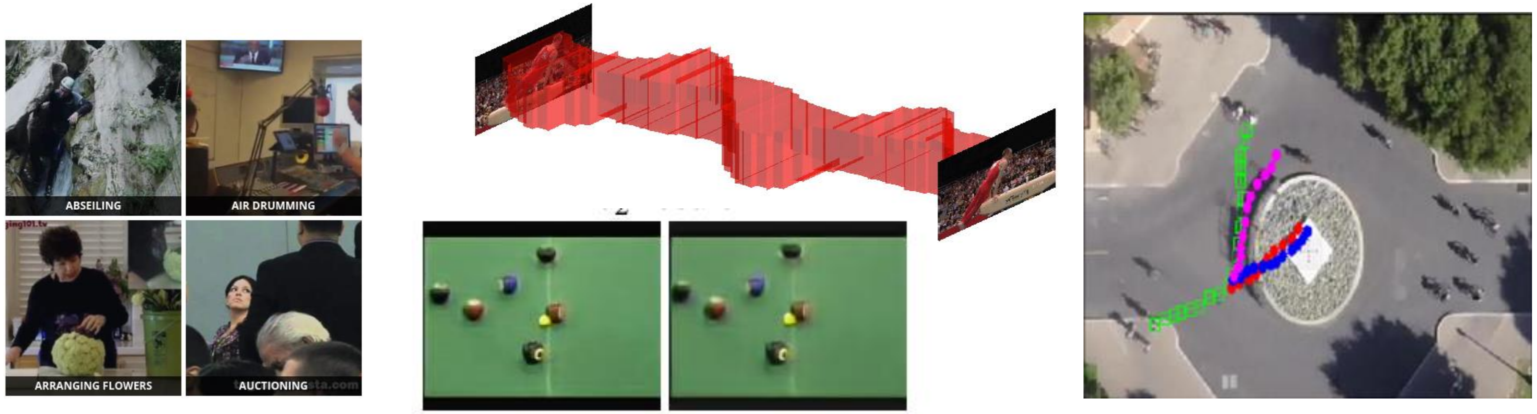


WaveNet architecture



VideoTime: Time-Aligned DenseNets

- Video Time: Properties, Encoders and Evaluation
  - A. Ghodrati, E. Gavves, C. Snoek, BMVC 2018
- How to model image sequences optimally?



Video Time: Properties, Encoders and Evaluation, Ghodrati, Gavves, Snoek, BMVC 2018

Properties of video

A standard VAE with a PixelCNN

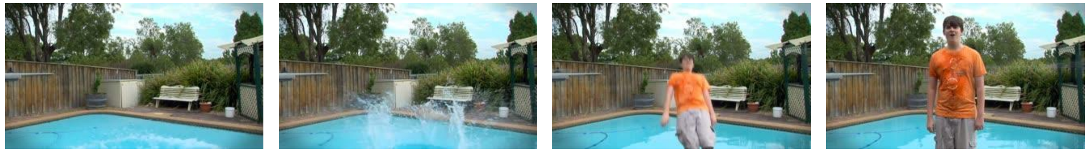
Forward

Backward

Video properties

Natural order (+)

Reverse order (-)



- Temporal asymmetry

- Temporal continuity

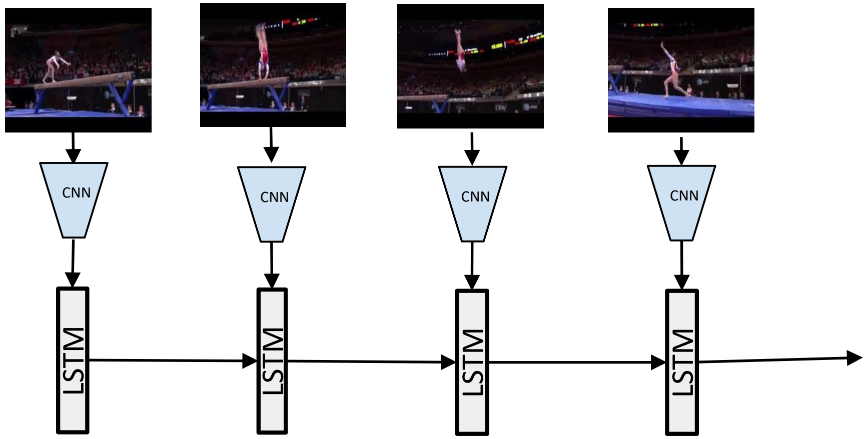
"Pretending to put something into something"

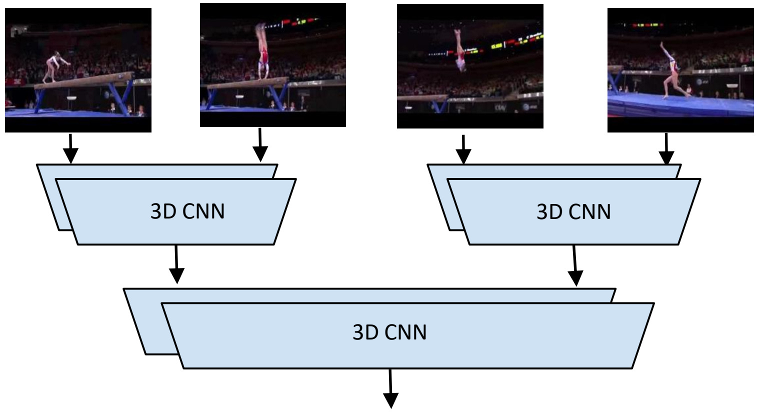
- Temporal causality

Current solutions

LSTMs learn transitions between subsequent states

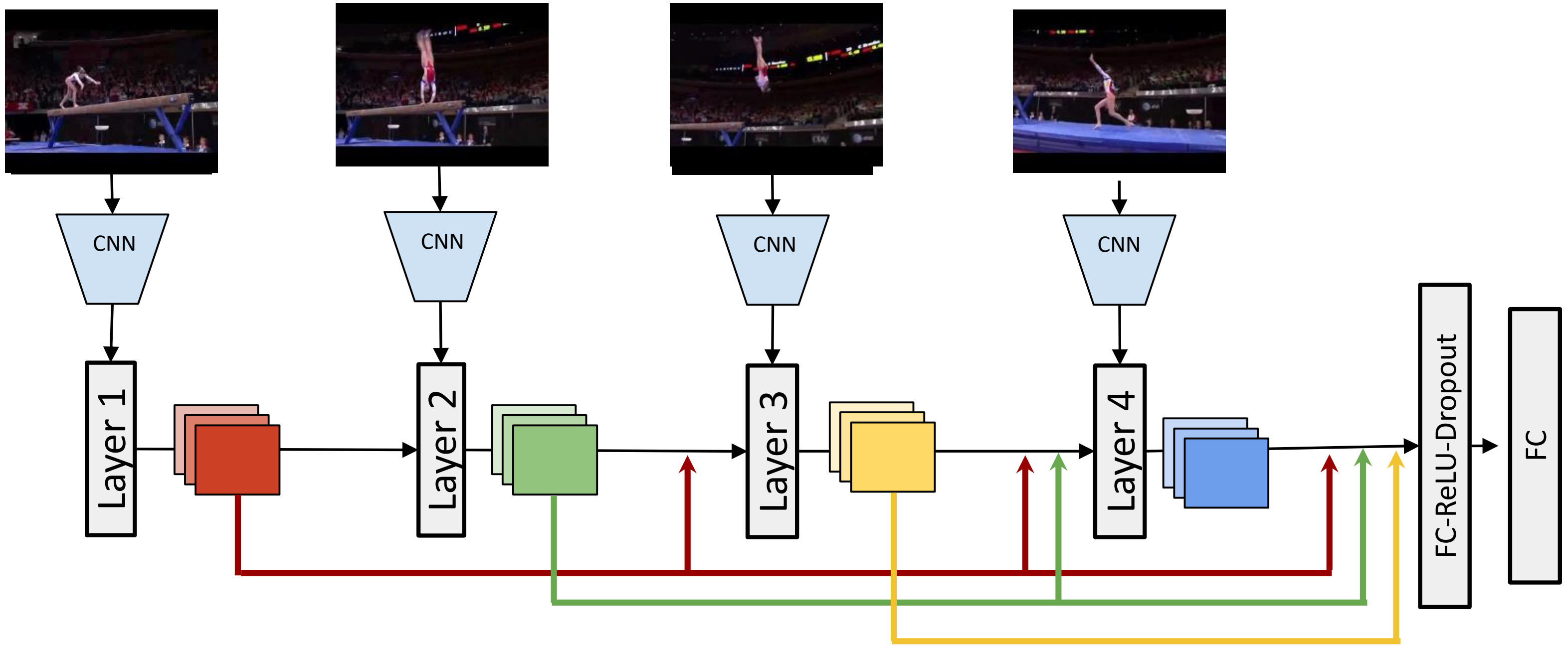
3D convolutions learn spatiotemporal patterns within a video



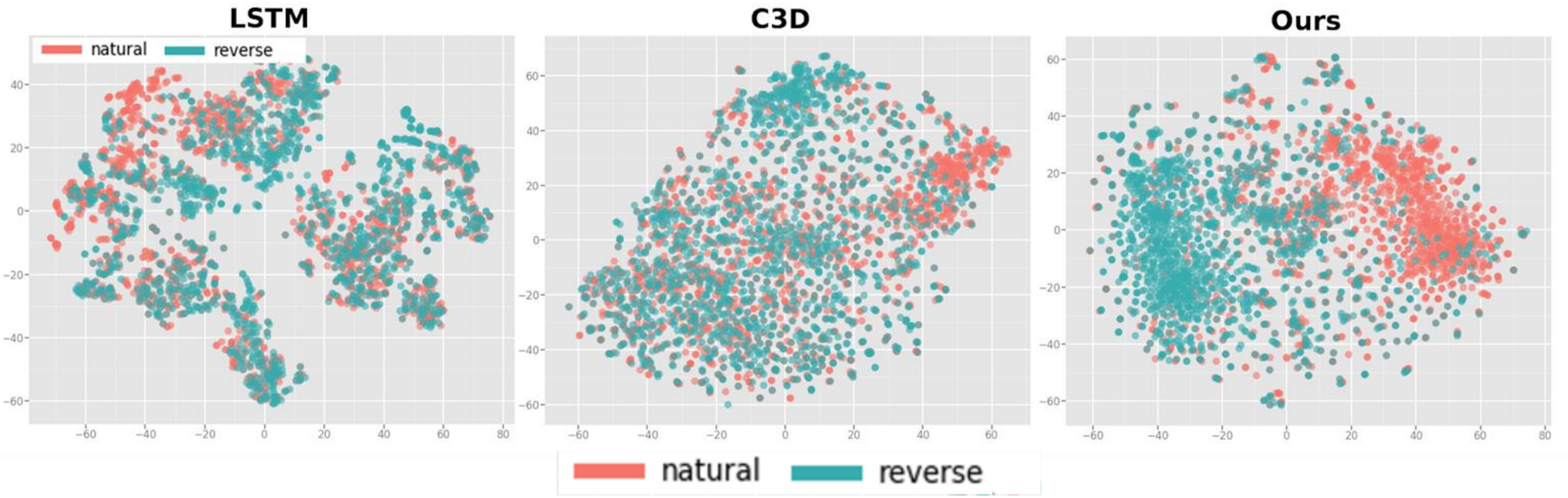


Time-Aligned DenseNets

- Autoregressive-like: each new frame directly depends on all previous frames
  - They are not generative
- No parameter sharing → like ConvNets, unlike RNNs
- Sequential → like RNNs, unlike ConvNets



Better latent space



Summary

- Autoregressive Models
- PixelCNN, PixelCNN++, PixelRNN
- WaveNet
- Time-Aligned DenseNets