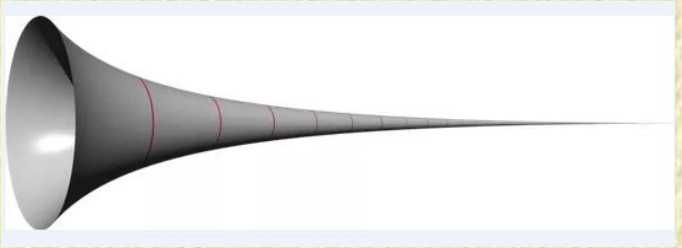
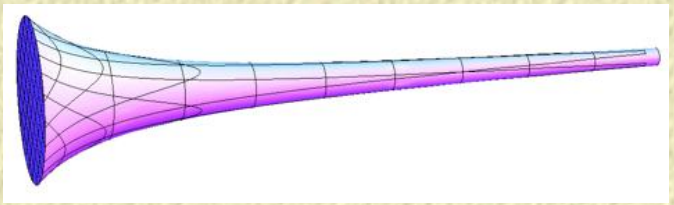
Torricelli's 小号(*Torricelli's Trumpet*)

又称*Gabriel's* 号角(*Gabriel's Horn*)问题.

Q.求由曲线$y=\frac{1}{x}$与直线$x=1, y=0$所围成的图形绕x轴一周所成旋转体的体积与其侧面积.



$$A.\ V=\pi\int_1^{+\infty}\frac{1}{x^2}dx=\pi\left(-\frac{1}{x}\right)_1^{+\infty}=\pi,$$



$$dS=2\pi f(x)\sqrt{(dx)^2+(dy)^2},$$

$$S=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}dx$$

$$=2\pi\int_1^{+\infty}\frac{1}{x}\cdot\sqrt{1+\left(-\frac{1}{x^2}\right)^2}dx>2\pi\int_1^{+\infty}\frac{1}{x}dx=+\infty,$$

$$\therefore V=\pi, S=+\infty\ .$$