§ 1.3 Matrices & Elementary Row Operations.

Not^n: Abbreviate system (1-1) as

$AX = Y$

where $A = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ A_{m1} & \cdots & A_{mn} \end{pmatrix}$ $\quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ $\quad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$

A is called the matrix of cofficients of the sys.

Remarks:

(1) For now, it is just a shorthand (Mat mult will appear later)

(2) A is strictly, is not a matrix

it is a represent^n

An $m \times n$ matrix over the field F is a function A from (i, j) into the field.

$1 \le i \le m$
$1 \le j \le n$

rows

ys

$A(i,j)$ = "matrix entry at row i col j"

Story: We consider elementary op^{n}s. "linear combination"

Inf Defn. Elementary row operations on an $m \times n$ matrix A over F are defined as follows:

1. Multipl^n of one row of A by a non-zero scalar c.

$A = $ $c \times$

2. Replacement of the r^{th} row of A by row r plus c times row s.

$A = $ s^{th} r^{th} $R(r) + cR(s)$

3. Interchange of two rows of A.

Samyak: Audio
Soham: Board

Inf Defn: Elementary row oper^{n}s. Fix mat A.

1. Multiply one row of A by a non-zero scalar c

2. Replace r^{th} row of A by r^{th} row + c times s^{th} row.

3. Interchange two rows of A.

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story: An elementary row op is a "map" e that associates an $m \times n$ matr A w/ another $m \times n$ matrix $e(A)$.

Defn: Elementary row operations:

Let A be $m \times n$ matrices as above & let e be an arbitrary f^n of the form above.

Then e must satisfy the following (for some indices r, s & scalar $c \neq 0$):

1. $e(A)_{ij} = \begin{cases} A_{ij} & \text{for all } i \neq r \\ cA_{rj} & \text{for } i = r. \end{cases}$

2. $e(A)_{ij} = \begin{cases} A_{ij} & \forall\ i \neq r \\ A_{rj} + cA_{sj} \end{cases}$

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3. $e(A)_{ij} = A_{ij}$ $\quad i \neq r, \neq s$

$e(A)_{rj} = A_{sj}$

$e(A)_{sj} = A_{rj}$

Story: • In defining $e(A)$ we care about the # rows (and not really about # cols)

• When we speak of e we consider the class of all m-rowed matrices (over F).

• Why these operations? (after apply e on a mat A, to obtain $e(A)$, one can recover A by performing a similar op^n on $e(A)$).

Theorem 2. To each elementary row op e, there corresponds an elementary row ope.^{n} e_1, of the same type as e

s.t. $e_1(e(A)) = e(e_1(A))\ \forall A$.

In other words, the inverse operation of an elementary row op exists & it is an elementary row op^n of the same type.

proof.

We do the proof for each type separately:

(1) Suppose e is the op^n that multiplies the r^{th} row of a matrix by a non-zero scalar c.

Clearly, e_1 can be taken to be the op^n that multiplies the r^{th} row w/ $1/c$.

(want $e_1(e(A)) = A$)

(2) Suppose e is the op^n that replaces row r by row r + c. row s. $(s \neq r)$.

Clearly, one can take e_1 to be the op^n that replaces row r by row r + (-c) rows.

$e_1(e(A)) = A$