

Chapter 10 | Key Management and the Public Key Revolution

Tuesday, October 31, 2023 10:50 AM

10.1 Key Distribution and Key Management

Story:

- In Chapter 1-7 we have seen
 - how private key cryptography can be used to ensure secrecy and integrity for two parties communicating over an insecure channel—assuming the two parties are in possession of a shared, secret key.
 - The question we have deferred since Ch 1, however, is
 - How can the parties share a secret key in the first place?*
 - Clearly
 - the key cannot simply be sent over the public channel because an eavesdropping adversary would then be able to observe it en route.
 - Some other mechanism must be used instead.
- In some situations
 - the parties may have access to a secure channel that they can use to reliably share a secret key.
 - e.g.
 - two parties are co-located at some time
 - Alternatively
 - the parties might use a trusted courier service (as a secure channel)
 - Stress: private key crypto is not useless—the secure channel may not be available at all times (or may be more expensive to use repeatedly)
- The above approaches have been used
 - to share keys in government, diplomatic and military contexts.
 - E.g.
 - the "red phone" connecting Moscow and Washington in the 1960s was encrypted using a one-time pad

with keys shared by couriers
who flew from one country to the other
carrying briefcases full of print-outs.

Such approaches can also be used in corporations
e.g. to setup a shared key b/w a central database
and
a new employee on his/her first day of work
(we return to this example in the next section).

- Relying on a secure channel to distribute keys
however
does not work well in many other situations
 - E.g.
 - consider a large MNC in which *every pair*
of employees
might need the ability to communicate securely,
with their communication protected from other employees as well
 - It will be inconvenient
for each pair of employees to meet so they
can securely share a key

Especially an issue if a new employee joins
again have to share keys with everyone
 - Assuming these N employees are somehow
able to securely share keys with each other
another significant drawback is that
each employee will have to manage and store $N - 1$
secret keys
(one for each other employee).
 - In fact this may significantly undercount the keys
—need keys for secure communication with
remote resources such as
databases, servers, printers etc.
 - The proliferation of so many keys is a
significant logistical problem.
 - Moreover,
these keys must be stored securely
(harder when there are so many keys)
- Storing keys is anyway a concern
 - Smart cards, e.g., can be used
but their memories are limited
on how many keys they can store.

- Concerns above

- can be addressed in "closed" organisations
- but
 - "open interactions"
 - (e.g. sending an email to a new person
 - or
 - buying something from a merchant
 - for the first time)
- In the latter, private key crypto does not provide a solution.

Summary:

Private key cryptography has three problems

- Key distribution
- Key management (many keys arise)
- Inapplicability to open systems

10.2 A partial solution: Key-distribution Centres

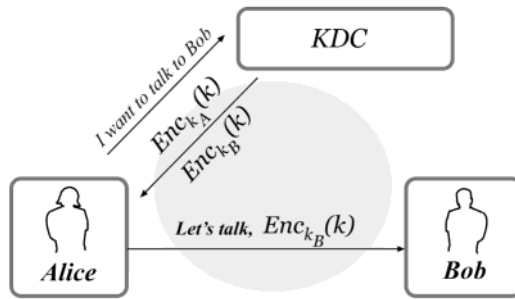
- One way to address the concerns listed previously is to use a Key Distribution Centre—to share keys.
- Idea
 - KDC is a trusted entity in an organisation
 - When the i th employee joins
 - KDC creates a key b/w itself and this new employee
 - also creates $k_1 \dots k_{i-1}$ keys
 - these are for letting the i th (new) employee communicate with all other employees
 - keys to the $1 \dots (i - 1)$ employees is
 - sent by the KDC to the employees
 - by encrypting using the key the KDC already shares with the existing employees
 - Now, everyone can communicate with each other.
- Better Idea
 - KDC sets up a key with each new employee
 - Whenever user A wants to talk to user B
 - they talk to the KDC and it issues a "session key"
 - When the users are done talking
 - they end the session
 - Advantages
 - simplifies key distribution
 - reduces key storage complexity
 - Issues:
 - KDC is a high value target (for attacks)
 - KDC is a single point of failure

- Could consider replicating the KDC
but then this means more points of failure
and more keys/updates etc.

Protocols for key distribution using a KDC.

Story:

- Many protocols exist for secure key distribution using a KDC
 - E.g. Needham-Schroeder protocol
which forms the core of Kerberos
an important and widely used service
for performing authentication and supporting secure communication
(Kerberos is used in many universities and corporations
and is the default mechanism for supporting secure networked authentication and communication in Windows and many UNIX systems).
 - We only highlight one feature of this protocol.
 - When Alice contacts the KDC
and asks to communicate with Bob
the KDC does not send the encrypted session key to both Alice and Bob (like we described earlier).
 - Instead
the KDC sends to Alice the session key encrypted under Alice's key
in addition to the session key encrypted under Bob's key
 - Alice then forwards the second ciphertext to Bob as in the figure below
 - The second ciphertext is sometimes called a ticket
and can be viewed as a credential that allows Alice to talk to Bob
(and allows Bob to be assured that he is talking to Alice)



- Indeed
 - although we have not stressed this point
 - a KDC-based approach can provide a useful means of performing authentication as well.
- Note also that Alice and Bob need not both be users
 - Alice might be a sure
 - and
 - Bob a resource
 - such as a server or a remote disk etc.
- The protocol was designed in this way to reduce the load on the KDC.
 - In the protocol as described
 - the KDC does not need to initiate a second connection to Bob
 - and need not worry whether Bob is online when Alice initiates the protocol
 - Moreover
 - if Alice retains the ticket (and her copy of the session key)
 - then she can re-initiate secure communication with Bob
 - by simply re-sending the ticket to Bob
 - without the involvement of the KDC at all
 - (In practice
 - tickets expire and eventually need to be renewed.
 - But a session could be re-established within some acceptable time period).
- We conclude by noting that
 - in practice the key that Alice shares with the KDC
 - might be a short, easy-to-memorise password.
 - In this case
 - many additional security problems arise that must be dealt with.
 - We have also been implicitly assuming an attacker
 - who only passively eavesdrops
 - rather than one who might actively try to interfere with the protocol

- We refer the interested reader to the references (at the end) for more info on how to address these.

10.3 Key Exchange and the Diffie-Hellman Protocol

Story:

- KDCs and protocols like Kerberos are commonly used in practice
 - But these approaches to the key-distribution problem still require (at some point) a private and authenticated channel that can be used to share keys.

(In particular we assumed the existence of such a channel b/w KDC and the employees on their first day).
 - Thus they still cannot solve the problem of key distribution in open systems, like the Internet where there may be no private channel available b/w two users who wish to communicate.
- To achieve private communication without ever communicating over a private channel a radically different approach is needed.
 - In 1976, Whitfield Diffie and Martin Hellman published a paper with the innocent-sounding title "New Directions in Cryptography".
 - In that work they observed that there is often *asymmetry* in the world. In particular there are certain actions that can be easily performed but not easily reversed.

E.g. padlocking without a key (i.e. easily) but then cannot be reopened (easily).

more strikingly it is easy to shatter a glass vase but extremely difficult to put it back together again

Algorithmically (and more germane for our purposes)
it is easy to multiply
two large primes
but
difficult to recover those primes
from their product.

This is exactly the factoring problem
discussed in previous chapters.

- Diffie and Hellman
realised that such phenomena could be used
to derive interactive protocols
for *secure key exchange*
that allow two parties to
share a secret key
via communication over a public channel
by having the parties perform operations that
they can reserve but that
an eavesdropper cannot.
- The existence of
secure key-exchange protocols is quite amazing
 - It means that you
and a friend could agree on a secret
by simply shouting across a room
(and performing some local computation);

the secret would be unknown to anyone else
even if they had listened to everything that was said.
 - Indeed,
until 1976, it was generally believed that
secure communication could not be achieved
without first sharing some secret information
using a private communication channel.
- The influence of Diffie and Hellman's paper was enormous
 - In addition to introducing a fundamentally new way of looking at cryptography
it was one of the first steps
towards moving cryptography out of the private domain
and into the public one.

Quote (first two paragraphs of their paper):

We stand today on the brink of a revolution in cryptography. The development of cheap digital hardware has freed it from the design limitations of mechanical computing and brought the cost of high grade cryptographic devices down to where they can be used in such commercial applications as remote cash dispensers and computer terminals.

In turn, such applications create a need for new types of cryptographic systems which minimize the necessity of secure key distribution channels. ... At the same time, theoretical developments in information theory and computer science show promise of providing provably secure cryptosystems, changing this ancient art into a science.

- Diffie and Hellman were not exaggerating and the revolution they spoke of was due in great part to their work.
- In this section we present the Diffie-Hellman key-exchange protocol.

We prove its security against eavesdropping adversaries (or equivalently) under the assumption that the parties communicate over a public but *authenticated* channel (so an attacker cannot interfere with their communication).

◦ Security against an eavesdropping adversary is a relatively weak guarantee and in practice key-exchange protocols must satisfy stronger notions of security that are beyond our present scope

(moreover, we are interested here in the setting where the communicating parties have *no* prior shared information in which case there is nothing that can be done to prevent an adversary from impersonating one of the parties—we return to this point later).

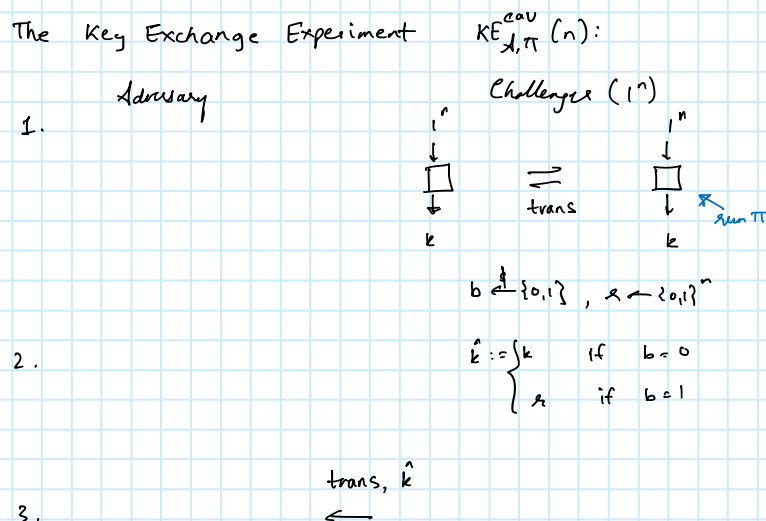
The setting and definition of security.

Story:

- We consider a setting with two parties—Alice and Bob—who run a probabilistic protocol Π in order to generate a shared secret key
- Π can be viewed as the set of instructions for Alice and Bob in the protocol.

- Alice and Bob begin by holding the security parameter 1^n
 - They then run Π using independent random bits.
 - At the end of the protocol
 - Alice and Bob output keys $k_A, k_B \in \{0,1\}^n$
 - The basic correctness requirement is that $k_A = k_B$
 - Since we only deal with protocols that satisfy this requirement we speak simply of *the* $k = k_A = k_B$ generated by an honest execution of Π .
- We now turn to defining security.
 - Intuitively, a key-exchange protocol is secure if the key output by Alice and Bob is completely unguessable by an eavesdropping adversary.
 - This is formally defined by requiring that an adversary who has eavesdropped on an execution of the protocol should be unable to distinguish the key k generated by that execution from a *uniform* key of length n .
 - This is much stronger than simply requiring that the adversary be unable to compute k *exactly*, and this stronger notion is necessary if the parties will subsequently use k for some cryptographic application (e.g. as a key for a private-key encryption scheme).
- Formalising the above
 - let Π be a key-exchange protocol
 - \mathcal{A} be an adversary
 - and n the security parameter.

Consider the following experiment.



3. $\xleftarrow{\text{trans, } \hat{k}}$

4. $\xrightarrow{b'}$ output 1 if $b' = b$
0 else

The key-exchange experiment $\text{KE}_{\mathcal{A}, \Pi}^{\text{eav}}(n)$:

1. Two parties holding 1^n execute protocol Π . This results in a transcript \mathbf{trans} containing all the messages sent by the parties, and a key k output by each of the parties.
2. A uniform bit $b \in \{0, 1\}$ is chosen. If $b = 0$ set $\hat{k} := k$, and if $b = 1$ then choose $\hat{k} \in \{0, 1\}^n$ uniformly at random.
3. \mathcal{A} is given \mathbf{trans} and \hat{k} , and outputs a bit b' .
4. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise. (In case $\text{KE}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1$, we say that \mathcal{A} succeeds.)

Remarks:

- \mathcal{A} is given trans to capture the fact that \mathcal{A} eavesdrops on the entire execution of the protocol and thus sees all messages exchanged by the parties.
 - In the real world
 - \mathcal{A} would not be given any key; in the experiment the adversary is given \hat{k} only as a means of defining what it means for \mathcal{A} to "break" the security of Π .
 - That is,
 - the adversary succeeding in "breaking" Π if it can correctly determine whether the key \hat{k} is the real key corresponding to the given execution of the protocol
 - OR
 - whether \hat{k} is a uniform key that is independent of the transcript.
 - As expected, we say Π is secure if the adversary succeeds with probability that is at most negligibly greater than $1/2$.
 - i.e.

Definition 10.1

A key-exchange protocol Π is *secure in the presence of an eavesdropper* if
 for all PPT adversaries \mathcal{A}
 there is a negligible function negl such that

$$\Pr[\text{KE}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1] \leq \frac{1}{2} + \text{negl}(n).$$

Story:

- The aim of a key-exchange protocol is almost always to generate a shared key k that will be used by the parties for some further cryptographic purpose e.g. to encrypt and authenticate their subsequent communication using say an authenticated encryption scheme.
- Intuitively using a shared key generated by a secure key exchange protocol should be "as good as" using a key shared over a private channel.
- It is possible to prove this formally; see Exercise 10.1

10.1 Let Π be a key-exchange protocol, and (Enc, Dec) be a private-key encryption scheme. Consider the following *interactive* protocol Π' for encrypting a message: first, the sender and receiver run Π to generate a shared key k . Next, the sender computes $c \leftarrow \text{Enc}_k(m)$ and sends c to the other party, who decrypts and recovers m using k .

- (a) Formulate a definition of indistinguishable encryptions in the presence of an eavesdropper (cf. Definition 3.8) appropriate for this interactive setting.
- (b) Prove that if Π is secure in the presence of an eavesdropper and (Enc, Dec) has indistinguishable encryptions in the presence of an eavesdropper, then Π' satisfies your definition.

The Diffie-Hellman key-exchange protocol

Story:

- We now describe the key-exchange protocol that appeared in the original paper by Diffie and Hellman (although they were less formal than we will be here).
- Let \mathcal{G} be a PPT algorithm that

on input 1^n
outputs

- a description of a cyclic group \mathbb{G}
- its order q (with $||q|| = n$) and
- a generator $g \in \mathbb{G}$.

< guessing $||q||$ refers to
the number of bits
needed to store n .

- The Diffie-Hellman key exchange is as follows

Construction 10.2

- Common input: Security parameter 1^n
- The protocol:

Alice

Bob

1. Runs $\mathcal{G}(1^n)$ to obtain
 (\mathbb{G}, q, g)

2. Chooses $x \leftarrow \mathbb{Z}_q$ uniformly
and
computes $h_A := g^x$.

3. **Sends (\mathbb{G}, g, q, h_A)**

4. Receives (\mathbb{G}, g, q, h_A)

Chooses $y \leftarrow \mathbb{Z}_q$ uniformly
computes $h_B := g^y$.

Sends h_B to Alice

Outputs the key $k_B := h_A^y$

5. Receives h_B and
outputs the key $k_A := h_B^x$

(NB for babies: both $k_A = k_B = g^{xy}$)

Story:

- In our description
we assumed that Alice generates (\mathbb{G}, q, g)
and
sends these parameters to Bob
as part of her first message.

- In practice,
 - these parameters are *standardised*
 - and
 - are fixed
 - and
 - are known to both parties
 - before the protocol begins.
- In that case, Alice need only send h_A and Bob need not wait to receive Alice's message before computing and sending h_B .
- It is not hard to see that the protocol is correct: (noted above already).
 - The observant reader will note that the shared key is a group element not a bit-string
We return to this point later
- Diffie and Hellman did not prove security of their protocol —indeed, the appropriate notions (both the definitional framework as well as the idea of formulating precise assumptions) were not yet in place.
 - Let's see what sort of assumption will be needed in order for the protocol to be secure.
 - A first observation (due to DH) is that a *minimal* requirement for security here is that the **discrete-logarithm problem** be hard relative to \mathcal{G} (recall: \mathcal{G} was generating the tuple (\mathbb{G}, q, g)).
 - If not, then an adversary given the transcript (which in particular includes h_A) can compute the secret value of one of the parties (i.e. x) and then easily compute the shared key using that value.

Side remark

- Claim: with only classical communication and an unbounded eavesdropper no Π for exchanging keys can be secure.
- Proof idea:
The adversary can simply simulate Alice and Bob consistent with the transcript and thus learn the keys they generate.

- So,
 - hardness of the discrete-log problem
is necessary for the protocol to be secure.
- It is, not, however
 - sufficient—as it is possible that there are
other ways of computing the key
 $k_A = k_B$ without
explicitly computing x or y .
- **Computational Diffie-Hellman assumption**
 - requires that the key g^{xy} is hard to compute in its
entirety
from the transcript.
 - NB: this does not suffice either
- What is required by Definition 10.1 is that the shared
key g^{xy} should be *indistinguishable from uniform* for
any
adversary given g, g^x, g^y .
 - This is the **decisional Diffie-Hellman assumption**
introduced in Section 8.3.2
- As we will see,
 - a proof of security for the protocol follows
almost immediately from the decisional Diffie-
Hellman assumption.
- This should not be surprising
 - as the Diffie-Hellman assumptions were
introduced
well after Diffie and Hellman published their paper

as a way of abstracting the properties underlying
the
(conjectured) security of the Diffie-Hellman
protocol.
- Given this
 - it is fair to ask whether anything is gained by
defining and proving security here.
- By this point in the book
 - hopefully you're convinced the answer is yes.
 - (and I was still not! □)

- Precisely defining secure key exchange
 - forces us to think about
 - exactly what security properties we require
 - specifying a precise assumption
 - (namely the decisional Diffie-Hellman assumption)
 - means
 - we can study this assumption
 - independently of any particular application
 - and—once we are convinced of its plausibility—
 - construct other protocols based on it.
- finally proving security shows that
 - the assumption does, indeed, suffice
 - for the protocol to meet our
 - desired notion of security.
- In our proof of security
 - we used a modified version of Definition 10.1
 - in which it is required that the shared key
 - be indistinguishable from a
 - uniform element of \mathbb{G}*
 - rather than a
 - uniform n -bit string*.
 - This discrepancy will need to be addressed
 - before the protocol can be used in practice
 - after all,
 - group elements are not typically useful as
 - cryptographic keys
 - and the representation of a uniform group
 - element
 - will not (in general)
 - be a uniform bit-string—
 - and we briefly discuss one standard way to do so
 - following the proof.
 - For now
 - we let $\widehat{\text{KE}}_{\mathcal{A}, \Pi}^{\text{Eav}}(n)$ denote a
 - modified experiment
 - where if $b = 1$ the
 - adversary is given \hat{k} chosen
 - uniformly from \mathbb{G}
 - instead of a uniform n -bit string.

Theorem 10.3

If the decisional Diffie-Hellman problem is hard
relative to \mathcal{G}
then the Diffie-Hellman key-exchange protocol Π
is secure in the presence of
an eavesdropper

(with respect to the modified experiment $\widehat{KE}_{\mathcal{A}, \Pi}^{eav}$)

Proof:

Let \mathcal{A} be a PPT adversary.

Since $\Pr[b = 0] = \Pr[b = 1] = 1/2$

it holds that

$$\Pr[\widehat{KE}_{\mathcal{A}, \Pi}^{eav}(n)=1] = \frac{1}{2} \Pr[\widehat{KE}_{\mathcal{A}, \Pi}^{eav}(n)=1 \mid b=0] + \frac{1}{2} \Pr[\widehat{KE}_{\mathcal{A}, \Pi}^{eav}(n)=1 \mid b=1].$$

In experiment \widehat{KE}

the adversary \mathcal{A} receives

$(\mathbb{G}, q, g, h_A, h_B, \hat{k})$

where the first part (excluding \hat{k})

is the transcript

and \hat{k} is either the actual key or a uniform group element (depending on b being 0 or 1, resp.).

Distinguishing b/w these two cases is

exactly equivalent to solving the decisional Diffie-Hellman problem.

Here are the details

(basically relates distinguishing to the security game here)

$$\begin{aligned}
& \Pr \left[\widehat{\text{KE}}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1 \right] \\
&= \frac{1}{2} \cdot \Pr \left[\widehat{\text{KE}}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1 \mid b = 0 \right] + \frac{1}{2} \cdot \Pr \left[\widehat{\text{KE}}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1 \mid b = 1 \right] \\
&= \frac{1}{2} \cdot \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^{xy}) = 0] + \frac{1}{2} \cdot \Pr[\mathcal{A}(\mathbb{G}, q, g, g^x, g^y, g^z) = 1] \\
&= \frac{1}{2} \cdot \left(1 - \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^{xy}) = 1] \right) + \frac{1}{2} \cdot \Pr[\mathcal{A}(\mathbb{G}, q, g, g^x, g^y, g^z) = 1] \\
&= \frac{1}{2} + \frac{1}{2} \cdot \left(\Pr[\mathcal{A}(\mathbb{G}, q, g, g^x, g^y, g^z) = 1] - \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^{xy}) = 1] \right) \\
&\leq \frac{1}{2} + \frac{1}{2} \cdot \left| \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^z) = 1] - \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^{xy}) = 1] \right|,
\end{aligned}$$

where the probabilities are all taken over (\mathbb{G}, q, g) output by $\mathcal{G}(1^n)$, and uniform choice of $x, y, z \in \mathbb{Z}_q$. (Note that since g is a generator, g^z is a uniform element of \mathbb{G} when z is uniformly distributed in \mathbb{Z}_q .) If the decisional Diffie-Hellman assumption is hard relative to \mathcal{G} , that exactly means that there is a negligible function negl for which

$$\left| \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^z) = 1] - \Pr[\mathcal{A}(\mathbb{G}, g, q, g^x, g^y, g^{xy}) = 1] \right| \leq \text{negl}(n).$$

We conclude that

$$\Pr \left[\widehat{\text{KE}}_{\mathcal{A}, \Pi}^{\text{eav}}(n) = 1 \right] \leq \frac{1}{2} + \frac{1}{2} \cdot \text{negl}(n),$$

completing the proof. ■

□

Uniform group elements vs uniform bit-strings

(quick version)

- To go from indistinguishability from uniform group elements to that from uniform strings
the DH protocol can be modified by
having the parties apply an appropriate
key-derivation function
to their shared group element g^{xy} .

Active adversaries

- Active adversary—modify the communication b/w Alice and Bob
- Two classes of active adversaries informally considered
 - Impersonation—act as the other player
 - Man-in-the-middle—Alice and Bob communicate but the adversary intercepts and changes the messages
- We don't study these in the book
 - involved
 - cannot be achieved without *some* information shared between the parties in advance
- DH is completely insecure against man-in-the-middle—there is an attack where Alice and Bob end up with different keys k_A and k_B

both known to the adversary
but neither party detecting the adversary.

Diffie–Hellman key exchange in practice. The Diffie–Hellman protocol in its basic form is typically not used in practice due to its insecurity against man-in-the-middle attacks, as discussed above. This does not detract in any way from its importance. The Diffie–Hellman protocol served as the first demonstration that asymmetric techniques (and number-theoretic problems) could be used to alleviate the problems of key distribution in cryptography. Furthermore, the Diffie–Hellman protocol is at the core of standardized key-exchange protocols that are resilient to man-in-the-middle attacks and are in wide use today. One notable example is **TLS**; see Section 12.8.