

### § 1.3 Matrices & Elementary Row Operations.

$\text{Mat}^n$ : Abbreviate system (1-1) as

$$\text{where } A = \begin{pmatrix} A_{11} & \dots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{m1} & \dots & A_{mn} \end{pmatrix} \quad \left. \begin{array}{l} x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \\ y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \end{array} \right\}$$

$A$  is called the matrix of coefficients of the sys.

Remarks:

(1) for now, it is just a shorthand  
(mat mult will appear later)

(2)  $A$  is strictly, is not a matrix

$A$  is a representation  
An  $m \times n$  matrix over the field  $F$   
is a function  $A$   
from  $\{(i,j)\}$  to  $F$   
 $1 \leq i \leq m$   
 $1 \leq j \leq n$   
# rows / # cols  
into the field.  
 $A(i,j)$  = "matrix entry  
at row  $i$ , col  $j$ "

Story: We consider elementary op's.

"linear combination"

In Defn. Elementary row operations on an  $m \times n$  matrix  $A$  over  $F$   
are defined as follows.

1. Multiplication of the  $r^{th}$  row of  $A$  by a non-zero scalar  $c$ .

$$A = \begin{pmatrix} & & & \\ & & & \\ & & \cancel{r^{th}} & \\ & & & \end{pmatrix}$$

2. Replacement of the  $r^{th}$  row of  $A$   
by row  $s$  plus  $c$  times row  $r$ .

3. Interchange of two rows of  $A$ .

$$\begin{array}{c} A = \\ \cancel{s^{th}} \rightarrow \\ r^{th} \rightarrow \\ \downarrow \\ s(r) + c(s) \end{array}$$