# Linear Algebra—Winter/Spring 2026

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## Problem Set 1

**Due:** 8 PM, Tuesday January 13, 2026

### Instructions

a. *Timeline.* The first "half" of a problem set will be released by Tuesday nights (or whenever the first lecture of the week takes place). The full problem set will be released Friday nights (or whenever the second lecture of the week takes place). The full assignment will, *typically*, be due on Tuesday nights, 8 PM.

b. *Punctuality.* Aim to get the assignment done three days prior to the deadline. We will not be able to entertain any requests for extensions. If you seek clarification from us and we do not respond on time, do the following: (a) assume whatever seems most reasonable, and solve the question under that assumption and (b) ensure you have evidence for making the clarification request and report the incident to one of the PhD TAs.

c. *Resources.* Feel free to use any resource you like (including generative AI tools) to get help but please make sure you understand what you finally write and submit. The TAs may ask you to explain the reasoning behind your response if something appears suspicious.

d. *Graded vs Practice Questions.* Problems numbered using Arabic numerals (i.e. $1, 2, 3, \dots$) constitute assignment problems that will be graded. Problems numbered using Roman numerals (i.e. i,ii,iii,…) are practice problems that will not be graded.

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i. Give an example of an $m \times n$ matrix (take $n > m$) in a row-reduced echelon form, where the first $m$ rows and columns, *do not* form an identity matrix.

ii. Could we consider column-equivalent, column-reduced and column-reduced echelon matrices instead? Why or why not (there is a right answer)?

iii. Work out the solution to the problem considered in Example 5, but obtain it using the row-reduced echelon form. (Example 5 basically starts with

$$\begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 4 & 0 & -1 \\ 2 & 6 & -1 & 5 \end{bmatrix}$$

and shows you how to solve it using the "method of elimination".)

iv. Write down a general procedure for solving a system of homogeneous linear equations, given by a matrix in the row-reduced echelon form. It is preferable to use the notation we used in class. Also justify why your procedure works.

v. Can the set $\{0, 1\}$ be a subfield of the field of complex numbers? Why or why not? How about $\{-1, 0, 1\}$?

**1.** Give two non-trivial examples of subfields of the field of complex numbers, together with a proof that they are indeed subfields. You can use standard results from analysis without proof.

**2.** We considered three types of elementary row operations. Show that there is a non-trivial (i.e. non identity) elementary row operation $e'$, such that for every elementary row operation $e$ of the same type, it holds that $e(e'(A)) = e'(e(A)) = B$ but $B \neq A$ in general.

**3.** Understand and write, in your words, the proof of Theorem 4 in the textbook (i.e. Every $m \times n$ matrix over the field F is row-equivalent to a row-reduced matrix).

**4.** Consider the following two systems of linear equations.

$$\textbf{(I)} \quad \begin{cases} x_1 + 2x_2 + 5x_3 = 0, \\ x_1 + 3x_2 + 8x_3 = 0, \\ -x_1 + x_2 + 4x_3 = 0 \end{cases} \qquad \textbf{(II)} \quad \begin{cases} x_2 + 2x_3 = 0, \\ x_1 - x_3 = 0, \\ x_1 + x_3 = 0 \end{cases}$$

Are the systems **(I)** and **(II)** equivalent?

*Hint:* First solve system **(II)**. Then try to find at least one nonzero solution of system **(I)** (for example, check whether there is a solution with $x_2 = -3$).