NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Catalan Numbers - Part 4

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So we have seen that,
total number of paths without crossing the diagonal from (0,0) to any (n,n)
$= \binom{2n}{n} - \binom{2n}{n+1}$.

This goes by the name the n[th] Catalan number denoted as C_n .This was given by the mathematician Catalan.

$$C_n = \binom{2n}{n} - \binom{2n}{n+1}$$

After simplification we get:

$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

So let us see how the sequence C_n goes.

$C_0 = 1$

$$C_1 = \frac{1}{2}\binom{2}{1} = 1$$

$$C_2 = \frac{1}{3}\binom{4}{2} = 2$$

$$C_3 = \frac{1}{4}\binom{6}{3} = 5$$

$$C_4 = \frac{1}{5}\binom{8}{4} = 14$$

And so no. So the Catalan numbers sequence is 1, 1, 2, 5, 14, 42, 132 and so on. Let us see some interesting examples of Catalan numbers.

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