

NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Combination with Repetition - Problems

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Example 1: In how many ways can we write 100 as sum of 4 non-negative numbers?

Solution: That is, $A+B+C+D = 100$.

We can have: $30+30+20+20 = 100$

This is one possibility.

Please note: $30+20+30+20 = 100$

This is another possibility. You see change of ordering leads to another possibility. That is how we are counting here. So you probably are wondering why should we consider these two as different possibilities? That is a rule led by us. I mean that's how we are going to count. So in how many ways can we write 100 as $A+B+C+D$? Please note we say non-negative integer, which means 0 is also allowed.

So, $100 + 0 + 0 + 0 = 100$

$$0 + 100 + 0 + 0 = 100$$
$$0 + 0 + 100 + 0 = 100$$
$$0 + 0 + 0 + 100 = 100$$

So these are 4 different possibilities.

Let us now answer this question. Now that you are very familiar with the formula, you already see that this is nothing but 100 slots plus 3 separators. So it's going to be 100+3 slots and 3 placeholders.

So it will be $\binom{100+4-1}{4-1} = \binom{103}{3}$

Example 2: Assume I have a jar which can contain 100 candies and these candies come in 7 different colors.



And let these colors be VIBGYOR which is Violet, Indigo, Blue, Green, Yellow, Orange, Red. The question now is in how many ways I can fill this jar which can hold 100 candies with candies which come in 7 colors. Now this question is the same as in how many ways can you write 100 as

$$100 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$$

So what's the answer?

$$n = 100, r = 7$$

So the answer is $\binom{100+7-1}{7-1} = \binom{106}{6}$

I have a advice for you all and the advice goes like this, do not see the formula always. Try to solve it without the formula and try to get used to the formula. 106 slots and 6 placeholders. 6 vertical sticks in how many ways can you put six vertical sticks on 106 slots. That would be $n = 100$ and $r = 7$.

So, $\binom{n+r-1}{r-1} = \binom{100+7-1}{7-1} = \binom{106}{6}$. That's the answer.

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