Alex Valentino

Assignment 5 problem 4
300H

(a) Prove: For any sets A, B, C and $D, (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
Suppose A, B, C, D are arbitrary sets. We must show $D, (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$. By the defintion of set equality, we must show $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$ and $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$

1. We must show $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$. Suppose $(x, y) \in (A \times B) \cap (C \times D)$. By the definition of set intersection $(x, y) \in (A \times B)$ and $(x, y) \in (C \times D)$. Therefore by the definition of binary relation, $x \in A$ and $y \in B$ and $x \in C$ and $y \in D$. By the definition of set intersection $x \in A \cap C$ and $y \in B \cap D$. By the definition of binary relation $(x, y) \in (A \cap C) \times (B \cap D)$.

2. We must show $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$. Suppose $(x, y) \in (A \cap C) \times (B \cap D)$. By the definition of binary relation $x \in A \cap C$ and $y \in B \cap D$. By the definition of set intersection $x \in A$ and $x \in C$, and $y \in B$ and $y \in D$. By the definition of binary relation $(x, y) \in A \times B$ and $(x, y) \in (C \times D)$. Therefore by the definition of set intersection $(A \times B) \cap (C \times D)$.

(b) Show that if we replace $\cap$ in all three places by $\cup$ in the previous assertion, then it is false.
Let $A = \{1, 2\}, B = \{3\}, C = \{1\}, D = \{2, 3\}$. $\{(1, 3), (2, 3), (1, 4)\} = (A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D) = \{(1, 3), (2, 3), (1, 4), (2, 4)\}$.

(c) Prove: For any sets A, B, C, and $D, (A \times B) \cup (C \times D) \supseteq (A \cap C) \times (B \cup D)$.
Suppose A, B, C, D are arbitrary sets. We most show for all $(x, y) \in (A \cap C) \times (B \cup D)$ implies $(x, y) \in (A \times B) \cup (C \times D)$. Suppose $(x, y) \in (A \cap C) \times (B \cup D)$. By the definition of binary relation $x \in (A \cap C)$ and $y \in (B \cup D)$. Then by the definition of set intersection $x \in A$ and $x \in C$, and by the definition of set union $y \in B$ or $y \in D$. We now have two goals: $x \notin A \times B$ and $x \notin C \times D$.

1. Assume $(x, y) \notin A \times B$. Therefore $x \notin A$ or $y \notin B$. Since we know $x \in A$, then $x \not inB$. Thus since we have $y \in B$ or $y \in D$, then y must be a member of D. Therefore by the definition of cartesean product $(x, y) \in C \times D$.

2. Assume $(x, y) \notin C \times D$. Therefore $x \notin C$ or $y \notin D$. Since we know $x \in C$, then $y \notin D$. Since we also have $y \in B$ or $y \in D$, then $y \in B$. Therefore since we already have $x \in A$, then by the definition of cartesean product we have $(x, y) \in A \times B$.