Alex Valentino Assignment 5 problem 6
300H

The following theorem appears as part 1 of Corollary 6.37 in the book. For any two functions $f : B \to C$ and $g : A \to B$ if f is one-to-one and g is one-to-one then $f \circ g$ is one-to-one. There is a proof given that uses the equivalence of one-to-one and left-invertibility. Give an alternative proof that works directly from the definition of one-to-one without using left-invertibility. (This is Exercise 6.4.8 in the book.)

Suppose $f : B \to C$ and $g : A \to B$ are arbitrary, injective, functions. We must show that $f \circ g$ is injective. By definition of injectivity we must show for arbitrary elements x_1, x_2 that if $f \circ g(x_1) = f \circ g(x_2)$ then $x_1 = x_2$. By the definition of function composition we must show for arbitrary elements x_1, x_2 that if $f(g(x_1)) = f(g(x_2))$ then $x_1 = x_2$. Suppose x_1, x_2 are arbitrary elements in A. Then by the definition of a function $g(x_1), g(x_2)$ are arbitrary members of $range(g)$, which since $range(g) \subseteq B$ would make them elements in B. Therefore by the definition of injectivity, since $f(g(x_1) = f(g(x_2))$ then $g(x_1) = g(x_2)$. Therefore by the definition of injecticity since $g(x_1) = g(x_2)$ then $x_1 = x_2$.