Alex Valentino Assignment 8 problem 2
300H

Let Q be the relation on the set $\mathbb{R}$ consisting of all pairs $(x, y) \in \mathbb{R}^2$ satisfying $x + 1 < y$. Let P be the relation on the set $\mathbb{R}$ consisting of of all pairs $(x, y) \in \mathbb{R}^2$ satisfying $x < y + 1$. Prove or disprove:

1. Q is antisymmetric.
   We must show that for all $x, y \in \mathbb{R}$ if $x \neq y$ and $x + 1 < y$ then $y + 1 \not< x$. Suppose $x, y \in \mathbb{R}, x \neq y, x + 1 < y$. We must show $y + 1 \not< x$. By definition of $<$, we must show that $x \leq y + 1$. Since $\mathbb{R}$ is complete under subtraction, $x < y - 1$ holds. Therefore since $y - 1 \leq y + 1$, as $-1 \leq 1$, then $x \leq y + 1$.

2. Q is transitive.
   Suppose $x = 0.9, y = 0, z = -0.5$. $0.9 < 0 + 1$, and $0 < 1 - 0.5$. However, $0.9 \not< 0.5$. Therefore Q is not transitive.

3. P is antisymmetric. Suppose $x = 1, y = 0.5$. Then $1 < 0.5 + 1$, but $0.5 < 1 + 1$. Therefore P is not antisymmetric.

4. P is transitive. Suppose $x = 1.3, y = 0.4, z = 0$. Then $1.3 < 1 + 0.4, 0.4 < 1 + 0$, however $1.3 \not< 1$.