For a relation R on a set X and $x, y \in X$ we have the following definitions:

- *An R-walk from x to y is a list of elements $(a_0, a_1, \cdots, a_k)$ of elements in X with $x = a_0$ and $y = a_k$ so that for $i \in \{1, \cdots, k\}$, $a_{i-1} R a_i$ (k is a allowed to be 0, so a list consisting could be $(a0)$.)*
- *An R-walk $a_0, \cdots, a_k$ is an R-path if all of the elements appearing are distinct.*
- *We say y is R-reachable from x, denoted $x \longrightarrow_R y$ if there is an R-walk from x to y.*
- *The R-reachability relation, $R^{\longrightarrow}$ is the relation on X whose pair set is all (x, y) such that $x \longrightarrow_R y$.*

1. For all relations R on X and $x, y \in X$, if $x \longrightarrow_R y$ then there is an R-path to from x to y.
   Suppose for arbitrary $x, y \in X$ that $x \longrightarrow_R y$. We want to show that there exist an R-path from x to y. By definition of $\longrightarrow_R$ there exist an R-walk from x to y. Therefore by the definition of R-path, we must show that there exist an R-walk from x to y with a list of unique elements in X. Assume the R-walk is not a list of distinct elements. Therefore there exist at least one element a_d that appears in the list $(a_0, a_1, \cdots, a_k)$ more then once. Suppose that a_d has occurrences at indices i and j. Therefore the R-walk can be written as $(a_0, \cdots, a_{i-1}, a_d, a_{i+1}, \cdots, a_{j-1}, a_d, a_{j+1}, \cdots, a_k)$. By definition of R-walk, $a_d R a_{i+1}$ and $a_{j-1} R a_d$. Therefore if you removed all of the elements from index $i+1$ to index j from the list, it would still satisfy the definition of being an R-walk as the chain of relations from x to y is unbroken. Since the list $(a_0, \cdots, a_{i-1}, a_d, a_{j+1} \cdots, a_k)$ consist of entirely unique elements, and has already been established as an R-walk, then by definition it is an R-path.

2. For all relations R, $R^{\longrightarrow}$ is transitive and reflexive.

   - Suppose that the relation R has a subset that is an R-reachability relation. We want to show that $R^{\longrightarrow}$ is transitive. By definition of R-reachability we have $x, y, z \in R^{\longrightarrow}$, where $x \longrightarrow_R y$ and $y \longrightarrow_R z$. By definition of transitivity we must show for all $x, y, z \in R^{\longrightarrow}$ that if $x \longrightarrow_R y$ and $y \longrightarrow_R z$ then $x \longrightarrow_R z$. By definition of R-walk there exist lists $(a_0, \cdots, a_k)$ and $(b_0, \cdots b_l)$ where $a_0 = x, a_k = b_0 = y, b_l = z$ in which each element is related to the next. Therefore $a_{k-1} R y$ and $y R b_0$. Thus the list taken by removing b_0 from the second list and concatenating with the first list $(a_0, \cdots, a_{k-1}, a_k, b_1, \cdots, b_l)$ satisfies the definition of R-walk, as every single pair of successive elements is related. Therefore $x \longrightarrow_R z$.
   - Suppose that the relation R has a subset that is an R-reachability relation. We want to show that $R^{\longrightarrow}$ is transitive. By definition we want to show that for all $x \in X$, $(x, x) \in R^{\longrightarrow}$. Therefore by definition of $R^{\longrightarrow}$ we must show that for all $x \in X$, $x \longrightarrow_R x$. Suppose x is an element of X. By definition of the R-reachability relation, it contains all pairs from $(x, y) \in X^2$ for which $x \longrightarrow_R y$. By definition of R-walk, an R-walk consisting of the list (x) constitutes a valid R-walk. Since the first and last element are both the same, then by definition xRx. Therefore R-reachability is reflexive.