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Assignment 3 problem 7
300H

Below are some false universal assertions. Find counterexamples to each one. For each one, formulate a modified universal assertion that is true.

- *For all choices of four real numbers a, b, c, d, if $a > b$ and $c > d$ then $ab > cd$*
  Let $a = 10, b = 1, c = 1000, d = 100$. While $a > b, c > d$, the assertion that $10 > 100000$ is clearly false. A correct assertion would be if $a > b$ and $c > d$ then $ac > bd$.

- *For any set $\mathcal{C}$ of intervals of $\mathbb{R}$, if no interval in $\mathcal{C}$ is disjoint from all of the other intervals of $\mathcal{C}$ then the union of the intervals in $\mathcal{C}$ is an interval*
  Let $\mathcal{C} = \{A, B, C, D\}, A \cap B \neq \emptyset, C \cap D \neq \emptyset, (A \cup B) \cap (C \cup D) = \emptyset$. By this construction every set in $\mathcal{C}$ isn't disjoint to every other set , however their union does not form as a set as the two pairs of sets above form "islands", which is non-continuous if their union is taken. To correct this, have the requirement that every interval in $\mathcal{C}$ not be disjoint to every other interval in the set.
  (note: this one stumped me just by misreading the quantifiers, I thought you were specifying the correction I made to the rule, and with this frustration I went to the group chat, where the hints I received were "this doesn't involve the empty set" and that "the minimum number of sets required for the counter-example was four", all else is my work.)

- *For all lists $(a_1, \cdots, a_k)$ of real numbers the average of the squares of the numbers is greater than the square of the average.*
  Let $a = (1, 1, 1)$. Therefore,

$$\frac{1^2 + 1^2 + 1^2}{3} > (\frac{1 + 1 + 1}{3})^2$$
$$1 > 1.$$

  This is a contradiction, as 1 is not greater than 1. This statement would be true if you changed the greater than sign to a greater than or equal to sign.