300H
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Assignment 1 problem 5

Now, suppose $g : B \to C$ is a 1-1 function. For each of the following two statements, prove the statement or give a counterexample.

1. *For any two functions $h_1 : A \to B$ and $h_2 : A \to B$ if $g \circ h_1 = g \circ h_2$ then $h_1 = h_2$.*
   - Proof: By definition $b_1, b_2 \in B, g(b_1) = g(b_2) \implies b_1 = b_2$. Therefore given that $g \circ h_1 = g \circ h_2$, then $h_1 = h_2$.
2. For any two functions $f_1 : C \to D$ and $f_2 : C \to D$ if $f_1 \circ g = f_2 \circ g$ then $f_1 = f_2$.
   - Counterexample: Let $f_1 : \mathbb{Z} \to \mathbb{Z}, f_1(x) = sin(\frac{\pi}{2}x), f_2 : \mathbb{Z} \to \mathbb{Z}, f_2(x) = 0, g : \mathbb{Z} \to \mathbb{Z}, g(x) = 2x$. While $sin(\pi x) = 0$ for all $x \in \mathbb{Z}$, here $sin(\frac{\pi}{2}) = 1 \neq 0$.