3.34 *Let X be a random variable with probability mass function*

$$\mathbb{P}(X=1)=\frac{1}{2}, \mathbb{P}(X=2)=\frac{1}{3} \text{ and } \mathbb{P}(X=5)=\frac{1}{6}$$

(a) *Find a function g such that $\mathbb{E}[g(X)] = 13\ln 2 + 16\ln 5$. Your answer should give at least the values $g(k)$ for all possible values k of X , but you can also specify g on a larger set if possible.*
$$g(x) = \begin{cases} 26ln(2) & x=1 \\ 0 & x=2 \\ 72ln(5) & x=5 \end{cases}$$

(b) *Suppose $t \in \mathbb{R}$, then find a function which satisfies $\mathbb{E}[g(X)] = \frac{1}{2}e^t + \frac{2}{3}e^{2t} + \frac{5}{6}e^{5t}$*
Let $g(x) = e^{xt}$

(c) *Find a function g such that $\mathbb{E}[g(X)] = 2$.*
Let $g(x) = 2$

3.36 if $p_X(y) = \begin{cases} \frac{2}{y^2} & 1 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$ then $\mathbb{E}[X^4] = \int_{-\infty}^{\infty} y^4 p_X(y)dy = \int_1^2 2y^2 dy = \frac{2}{3}(8-1) = \frac{14}{3}$.

3.44 (a) The cdf is given by $F_S(a) = \int_0^1 \int_{-\frac{\pi}{2}}^{\arctan(a)} d\theta dr = \frac{1}{\pi}\arctan(a) + \frac{1}{2}$, which one finds by taking the area of the sector from $-\frac{\pi}{2}$ to the angle of the slope of the line ax.

(b) The pdf is given by $\frac{d}{da}(\frac{1}{\pi}\arctan(a) + \frac{1}{2}) = \frac{1}{\pi}\frac{1}{1+a^2}$

3.52

$$\begin{aligned} \sum_{k=1}^{\infty} P(X \ge k) &= \sum_{k=1}^{\infty}\sum_{i=k}^{\infty} \mathbb{P}(X=i) \\ &= \sum_{i=1}^{\infty}\sum_{k=1}^{i} \mathbb{P}(X=i) \\ &= \sum_{i=1}^{\infty} i\mathbb{P}(X=i) \\ &= \mathbb{E}X \end{aligned}$$

3.54 (a) $\mathbb{P}(X \ge k) = 1 - \mathbb{P}(X < k) = 1 - \sum_{i=1}^{k-1}(1-p)^{i-1}p = 1-(1-(1-p)^{k-1}) = (1-p)^{k-1}$

(b) $\mathbb{E}X = \sum_{k=1}^{\infty}\mathbb{P}(X \ge k) = \sum_{k=1}^{\infty}(1-p)^{k-1} = \frac{1}{1-(1-p)} = p^{-1}$

3.62 (a) $F_X(a) = \begin{cases} 0 & a < 0 \\ a & 0 \le a < 0.75 \\ 1 & a \ge 0.75 \end{cases}$

(b) $\mathbb{E}(X) = \int_0^{0.75} x dx + \int_{0.75}^1 0.75 dx = \frac{9}{32} + \frac{1}{4} = \frac{15}{32}$.

(c) $Var(X) = \mathbb{E}(X^2) - (\mathbb{E}X)^2 = \int_0^{0.75} x^2 dx + \int_{0.75}^1 \frac{9}{32} dx - \frac{225}{1024} = 0.0615234$