

2.6.4 Assume  $(a_n), (b_n)$  are Cauchy. We must show that the series formed by  $c_n = |a_n - b_n|$  is Cauchy. Let  $\epsilon > 0$ . Since  $(a_n), (b_n)$  are Cauchy then there exists  $N_1, N_2 \in \mathbb{N}$  such that for all  $n_1, m_1 \geq N_1, |a_{n_1} - a_{m_1}| < \frac{\epsilon}{2}$  and for all  $n_2, m_2 \geq N_2, |b_{n_2} - b_{m_2}| < \frac{\epsilon}{2}$ . Let  $N = \max(N_1, N_2)$ . Therefore, for all  $m, n \geq N$ ,

$$\begin{aligned}
 |c_n - c_m| &= ||a_n - b_n| - |a_m - b_m|| \text{ definition of } c_n \\
 &\leq |a_n - b_n - a_m + b_m| \text{ reverse triangle inequality} \\
 &= |a_n - a_m + b_m - b_n| \\
 &\leq |a_n - a_m| + |b_n - b_m| \text{ triangle inequality} \\
 &< \frac{\epsilon}{2} + \frac{\epsilon}{2} \\
 &= \epsilon.
 \end{aligned}$$