10.2

| $\frac{y}{x}$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| 1 | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

10.6 (a) $f_{X|Y}(x|y) = \frac{3}{8}\frac{x+y}{y^2}$

(b)
- $\mathbb{P}(X < \frac{1}{2}|Y = 1) = \int_0^{\frac{1}{2}} \frac{2}{3}(x+1)dx = \frac{5}{12}$
- $\mathbb{P}(X < \frac{3}{2}|Y = 1) = \mathbb{P}(X < 1|Y = 1) = \int_0^1 \frac{2}{3}(x+1)dx = 1$

(c)

$$\begin{aligned}\mathbb{E}[X^2|Y = y] &= \frac{2}{3}\int_0^y x^2\frac{x+y}{y^2}dx \\ &= \frac{7}{18}y^2\end{aligned}$$

$\int_{-\infty}^{\infty} \mathbb{E}[X^2|Y = y]f_Y(y) == \int_0^2 \frac{7}{18}\frac{3}{8}y^4dy = \frac{14}{15}$
$f_X(x) = \int_x^2 \frac{x+y}{4}dy = \frac{-1}{4}(\frac{3x^2}{2} - 2x - 2)$
$\mathbb{E}[X^2] = \int_0^2 \frac{-x^2}{4}(\frac{3x^2}{2} - 2x - 2)dx = \frac{14}{15}$

10.10 (a)
- $f_{X|N}(k|n) = \binom{n}{k}p^k(1-p)^{n-k}$
- $\mathbb{E}[X|N = n] = np$
- $\mathbb{E}[X|N] = Np$

(b) $\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|N]] = \mathbb{E}[pN] = p\lambda$

(c) Note that $\mathbb{E}[XN] = \mathbb{E}[\mathbb{E}[XN|N]] = \mathbb{E}[N\mathbb{E}[X|N]] = \mathbb{E}[pN^2] = p(Var(N) + \mathbb{E}[N]^2) = p(\lambda + \lambda^2)$, therefore
$Cov(X, N) = \mathbb{E}[XN] - \mathbb{E}[X]\mathbb{E}[N] = p(\lambda + \lambda^2) - p\lambda \cdot \lambda = p\lambda$. Thus $X, N$ are positively correlated since $\lambda, p > 0$.

10.38 (a) $f(x, y) = f_Y(y)f_{X|Y}(x|y) = e^{-y}ye^{-xy} = ye^{-(x+1)y}$ if $x, y > 0$, otherwise $f(x, y) = 0$.

(b) $f_{Y|X}(y|x) = \frac{ye^{-(x+1)y}}{\int_0^{\infty} ye^{-(x+1)ydy}} = \frac{ye^{-(x+1)y}}{\frac{1}{(x+1)^2}} = (x+1)^2ye^{-(x+1)y}$. I have never seen a distribution of this exact type. Something exponential?

10.40 (a)

$$\begin{aligned}\mathbb{P}(Y > 2|X = x) &= 1 - \mathbb{P}(Y \leq 2|X = x) \\ &= 1 - \int_0^2 xdy \text{ if } x < \frac{1}{2} \text{ otherwise it has probability } 0 \\ &= 1 - 2x\end{aligned}$$

(b) Note that

$$\begin{aligned}\mathbb{E}[Y|X=x] &= \int_0^{1/x} yxdy \\ &= x\frac{y^2}{2}]_0^{1/x} \\ &= \frac{1}{2x}\end{aligned}$$

Thus $\mathbb{E}[Y|X] = \frac{1}{2X}$. Furthermore, we know that $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]]$. Therefore $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]] = \mathbb{E}[1/2X] = \int_0^\infty \frac{1}{2x} xe^{-x}dx = \frac{1}{2}[-e^{-x}]_0^\infty = \frac{1}{2}$