4.3.6 (a) Suppose $c \in \mathbb{Q}$. Thus $h(c) = 1$. Consider $\pi^{-n} + c = y_n$. Since π is transcendental then every y_n is irrational. Therefore $h(y_n) \to 0$.

Suppose $c \notin \mathbb{Q}$. Therefore $h(c) = 0$. Consider $x_n = \frac{p_n}{q_n}$, which is the nth decimal expansion of c. Since each $x_n \in \mathbb{Q}$, then $h(x_n) \to 1$.

Since every possible $\mathbb{R}$ value is disconntinous, then h is a nowhere continuous function.

(b) Suppose $c \in \mathbb{Q}$. Consider $x_n = c - \pi^{-n}$. As shown above, each x_n is irrational, thus $t(x_n) \to 0$, not to the non-zero value of $t(c)$.

(c) Let $\epsilon > 0, i \in \mathbb{R}\backslash\mathbb{Q}$, and consider the set $T = \{x \in \mathbb{R} : t(x) \geq \epsilon\}$. Since all elements in T are rational, then we know something about their denominators, in particular that they are bounded below by ϵ. Looking at $[i - \frac{1}{2}, i + \frac{1}{2}] \cap T$, we have a finite number of elements, therefore looking to $\min t([i - \frac{1}{2}, i + \frac{1}{2}] \cup T) = \frac{1}{m}$, then we have for each $x \in [i - \frac{1}{2}, i + \frac{1}{2}] \cap T$, $V_{\frac{1}{2m}}(x) \cap T = \{x\}$, otherwise multiple points would imply denominators smaller than $\frac{1}{m}$. Therefore if we choose $\delta < \frac{1}{2m}$, then for all $x \in [i - \frac{1}{2}, i + \frac{1}{2}] \cap T$, $x \notin V_\delta(c)$. Therefore if $y \in V_\delta(c), y \notin T$, thus $t(y) < \epsilon, y \in V_\epsilon(0)$. Thus we have convergence for $t(i)$.