7.5
$$x^9 - x = x(x+1)(x+2)(x^2+1)(x^2+x-1)(x^2-x-1)$$

7.7 Suppose K is a finite field, and that there exists a prime p and $r \in \mathbb{N}$ such that $|K| = p^r$. We must consider $\pi = \prod_{k\in K^*} k$. Note that for each element $k \in K^*$ which doesn't satisfy $x^2 - 1 = 0$ has a unique inverse other than itself. Therfore $\prod_{k\in K^*, k^2\neq 1} k = 1$. Note that if we multiply the previous product by $(-1)(1)$ we get π. Thus $\pi = -1$.

7.8 Let $f(x) = x^3 + x + 1, g(x) = x^3 + x^2 + 1, f(\alpha) = 0, g(\beta) = 0, \mathbb{F}_2(\alpha) = K, \mathbb{F}_2(\beta) = L$. We want to construct $\sigma : K \to L$ such that σ is an isomorphism. Note that

$$(\alpha+1)^3 + (\alpha+1)^2 + 1 = \alpha^3 + \alpha^2 + \alpha + 1 + \alpha^2 + 1 + 1 = \alpha^3 + \alpha + 1 = 0.$$

Therefore $g(\alpha+1) = 0$, thus we know by the textbook that $\sigma(\alpha+1) = \beta$ is an isomorphism since $\alpha+1, \beta$ are both roots of the irreducible polynomial of g. Furthermore since both K, L are isomorphic to $\mathbb{F}_8$ then we only need to ask the number of automorphisms of $\mathbb{F}_8$. If we consider the basis of $\mathbb{F}_8$ to be $(1, \alpha, \alpha^2)$, then we just have the isomorphisms of (1) doing nothing, (2), multiplying by α yielding $(\alpha, \alpha^2, \alpha+1)$, and (3) multiplying by α^2 yielding $(\alpha^2, \alpha+1, \alpha^2+\alpha)$. Note that we must maintain at least one root, and since there are 3 roots, and that all cases are covered by swapping the roots which in turn swaps the other implies that there are 3 automorphisms.

Bonus Let $P(x) = \prod_{i=1}^n (x - \alpha_i)$. By the defintion of the formal derivative we have by the product rule that $P'(x) = \sum_{i=1}^n \prod_{j=1, i\neq j}^n (x - \alpha_j)$. Note that $P'(\alpha_i) = \prod_{j=1,i\neq j}^n (\alpha_i - \alpha_j)$, since all of the other $n-1$ terms in the series contain $(x - \alpha_i)$, thus going to 0. Therefore $\prod_{i=1}^n P'(\alpha_i) = \prod_{i=1}^n \prod_{j=1,i\neq j}^n (\alpha_i - \alpha_j)$. Note that the normal discriminant is strictly positive, therefore for each $(\alpha_i - \alpha_j)$ there exists $(\alpha_j - \alpha_i)$ with a negative sign. Since there are $nC2 = \frac{n(n+1)}{2}$ pairings one must multiply by $(-1)^{\frac{n(n+1)}{2}}$ to counteract the signs of the negative pairings. Note that additionally once the sign issue is taken care of that we now have paired every $(\alpha_i - \alpha_j)$ with $(\alpha_j - \alpha_i)$, thus we have eliminated nearly half of the terms in the product therefore the formulation above is equivalent to $\prod_{i<j}^n (\alpha_i - \alpha_j)^2$.