4.3.6 (a) Suppose $c \in \mathbb{Q}$. Thus $h(c) = 1$. Consider $y_n = \frac{1}{\pi^n} + c$. By the fact that π is transcendental, each y_n is irrational, and the sequence converges to c. Therefore, since each y_n is irrational we have that $h(y_n) \to 0$.
Suppose $i \notin \mathbb{Q}$. Therefore $h(i) = 0$. If we consider the sequence (a_n) given by the truncated decimal expansion of i, clearly each a_n is rational. Therefore $h(a_n) \to 1$. Therefore $h(x)$ is a nowhere continuous function.

(b) Suppose $c \in \mathbb{Q}$. Consider the sequence once more of $y_n = \frac{1}{\pi^n} + c$. Since each y_n is irrational, then $t(y_n) \to 0$. This goes against $h(c) = \frac{1}{n}$. Therefore $t(x)$ is not continuous at every rational number.

(c) Consider $i \in \mathbb{R} \backslash \mathbb{Q}$, and let $\epsilon > 0$. If we consider the set $T = \{x \in \mathbb{R} : t(x) \geq \epsilon\}$, we note that since each $t(x)$ is positive, then T is a set of rational numbers. If we apply the archamedian principle to ϵ, we find that $m \in \mathbb{N}, \epsilon > \frac{1}{m}$. Therefore, for all $x \in T$, $V_{\frac{1}{2m}}(x) \cap T = \{x\}$, otherwise if two numbers from T were in the neighborhood then one would be guarenteed to have a larger denominator than m, which would contradict being a member of T. Therefore if we choose $\delta < \frac{1}{2m}$ then $x \in T$ implies that $x \notin V_\delta(i)$. Therefore if $x \in V_\delta(i)$, then $t(x) \in V_\epsilon(t(i))$. Therefore $t(x)$ converges for every irrational number.