3.2.10

i A countable set contained in $[0, 1]$ with no limit points
This can't occur, as if we enumarate a countable set, then we have a sequence, and since it's contained within $[0, 1]$ then we have a bounded sequence. Therefore by the Bolzano-Weierstrauss theorem we have a convergent subsequence, i.e. a limit point of the set. Thus this description can't occur

ii A countable set contained in $[0, 1]$ with no isolated points
This can occur! Take $A = [0, 1] \cap \mathbb{Q}$. We know by the density of $\mathbb{Q}$ in $\mathbb{R}$ that we have a sequence of rational numbers to every real number. Since a rational number is a real number, then for every element in the set A we can generate a sequence with a sufficent ϵ contained within $[0, 1]$ for which an arbitrary rational number satisfies the definition of a limit point. And the countablility of the rationals is a given.

iii The issue arises with the fact that for each $x \in A$, there is an associated $\epsilon_x > 0$ in which $V_{\epsilon_x}(x) \cap A = \{x\}$. If we have some minimum $\epsilon_x > 0$ then we have a most a collection of neighborhoods which cover the entire real line. Arguing from geometry, it's clear that only a countable number of these uncountable "tiles" need to overlay the 1d bathroom floor that is the real line. Therefore, alike in the example showing all of the points of $\{\frac{1}{n} : n \in \mathbb{N}\}$ are isolated, we must find a formula for ϵ_x based on x. The issue arises with the fact that x is an uncountable variable, where we have an uncountable set of elements from x with progressively smaller ϵ_x's. Once again, arguing from geometry, one can see how one can set up our ϵ_x's in order to turn one of our allegedly isolated points into a limit point.