3.2.10 (a) Prove for a collection of sets $\{E_\lambda : \lambda \in \Lambda\}$ that

$$(\cup_{\lambda\in\Lambda} E_\lambda)^c = \cap_{\lambda\in\Lambda} E_\lambda^c \text{ and } (\cap_{\lambda\in\Lambda} E_\lambda)^c = \cup_{\lambda\in\Lambda} E_\lambda^c$$

Proof: Suppose $x \in (\cup_{\lambda\in\Lambda} E_\lambda)^c$. We must show that $x \in \cap_{\lambda\in\Lambda} E_\lambda^c$. By definition of set compliment, $x \notin \cup_{\lambda\in\Lambda} E_\lambda$. Since x does not belong to the union of E_λ for every $\lambda \in \Lambda$, then $x \notin E_\lambda$ for each $\lambda \in \Lambda$. Therefore by definition of set compliment, $x \in E_\lambda^c$ for every $\lambda \in \Lambda$. Therefore by definition of set intersection, $x \in \cap_{\lambda\in\Lambda} E_\lambda^c$.
Next, suppose $x \in \cap_{\lambda\in\Lambda} E_\lambda^c$. We must show that $x \in (\cup_{\lambda\in\Lambda} E_\lambda)^c$. By definition of set intersection, $x \in E_\lambda^c$ for every $\lambda \in \Lambda$. Therefore by definition of set compliment, $x \notin E_\lambda$, for every λ. Since x does not belong to any of E_λ's individually, then x does not belong to the union. Therefore $x \notin \cup_{\lambda\in\Lambda} E_\lambda$. Therefore by the definition of set compliment, $x \in (\cup_{\lambda\in\Lambda} E_\lambda)^c$.
Since we have established that $(\cup_{\lambda\in\Lambda} E_\lambda)^c = \cap_{\lambda\in\Lambda} E_\lambda^c$, by defining $G_\lambda = E_\lambda^c$ for every $\lambda \in \Lambda$, then we have that $(\cup_{\lambda\in\Lambda} G_\lambda)^c = \cap_{\lambda\in\Lambda} G_\lambda^c$. If we take the compliment of the left hand side then we have that

$$\cup_{\lambda\in\Lambda} E_\lambda^c = ((\cup_{\lambda\in\Lambda} G_\lambda)^c)^c = (\cap_{\lambda\in\Lambda} G_\lambda^c)^c = (\cap_{\lambda\in\Lambda} (E_\lambda^c)^c)^c = (\cap_{\lambda\in\Lambda} E_\lambda)^c.$$

Therefore $(\cap_{\lambda\in\Lambda} E_\lambda)^c = \cup_{\lambda\in\Lambda} E_\lambda^c$

(b) i. The union of a finite number of closed sets is closed
Let $\{E_1, \cdots, E_n\}$ be a finite set of closed sets. If we consider that E_i^c is open for all $i \in [n]$, and take their intersection then we know by theorem 3.2.3 that $\cap_{i=1}^n E_i^c$ is open. Therefore if we take the compliment and and apply DeMorgan's law then we have that $(\cap_{i=1}^n E_i^c)^c = ((\cup_{i=1}^n E_i)^c)^c = \cup_{i=1}^n E_i$ is closed by theorem 3.2.13.

ii. The intersection of an arbitrary number of closed sets is closed. Let $\{E_\lambda : \lambda \in \Lambda\}$ be a collection of closed sets. Noting that E_λ^c is open for all $\lambda \in \Lambda$ then we know by theorem 3.2.3 that $\cup_{\lambda\in\Lambda} E_\lambda^c$ is open. Therefore by taking the compliment and applying DeMorgan's law we have that $(\cup_{\lambda\in\Lambda} E_\lambda^c)^c = ((\cap_{\lambda\in\Lambda} E_\lambda)^c)^c = \cap_{\lambda\in\Lambda} E_\lambda$ is closed by theorem 3.2.13.