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Recall the following definition: For any two sets A and B, the difference set $A \backslash B$ is the set consisting of those objects that are members of A but not members of B. Also $A \triangle B$ is equal to $A \backslash B \cup B \backslash A$.

1. Prove or disprove: For all sets A, B, C, if $A \backslash C = B \backslash C$ then $A = B$.
   Suppose A, B, C are arbitrary sets. We must show if $A \backslash C = B \backslash C$ then $A = B$. By definition of set equality we must prove if $A \backslash C = B \backslash C$ then $A \subseteq B$ and $B \subseteq A$.

   (a) Suppose A, B, C are arbitrary sets. We must show if $A \backslash C = B \backslash C$ then $A \subseteq B$. By the definition of subset we must show $\forall x \in A \Rightarrow x \in B$. Suppose x is an arbitrary element of $A \backslash C$. By definition of set equality since $A \backslash C = B \backslash C$, then $x \in B \backslash C$. By definition of set difference $x \in B$ and $x \notin C$. Therefore by the definition of subset, $A \subseteq B$.

   (b) Suppose A, B, C are arbitrary sets. We must show if $A \backslash C = B \backslash C$ then $B \subseteq A$. By definition of subset we must show $\forall x \in B \Rightarrow x \in A$. Suppose x is an arbitrary element of $B \backslash C$. By definition of set equality since $A \backslash C = B \backslash C$, then $x \in A \backslash C$. By the definition of set difference $x \in A$ and $x \notin C$. Therefore by the definition of subset, $B \subseteq A$.

   Therefore for all sets A, B, C, if $A \backslash C = B \backslash C$ then $A = B$.

2. Prove or disprove: For all sets A, B, C, if $A \oplus C = B \oplus C$ then $A = B$.
   Suppose A, B, C are arbitrary sets. We must show if $A \oplus C = B \oplus C$ then $A = B$. By definition of set equality we must prove if $A \oplus C = B \oplus C$ then $A \subseteq B$ and $B \subseteq A$.

   (a) Suppose A, B, C are arbitrary sets. We must show if $A \oplus C = B \oplus C$ then $A \subseteq B$. By the definition of subset we must show $\forall x \in A \Rightarrow x \in B$. Suppose x is an arbitrary element of $A \oplus B$. By definition of set equality since $x \in A \oplus C$ then $x \in B \oplus C$. By the definition of symmetric difference $x \in B \backslash C \cup C \backslash B$. By the definition of set union $x \in B \backslash C$ or $x \in C \backslash B$. Therefore since it's an or statement we can choose $x \in B \backslash C$. By the definition of set difference $x \in B$ and $x \notin C$. Therefore $x \in B$. This satisfies the requirement.

   (b) Suppose A, B, C are arbitrary sets. We must show if $A \oplus C = B \oplus C$ then $B \subseteq A$. By the definition of subset we must show $\forall x \in B \Rightarrow x \in A$. By definition of set equality since $x \in B \oplus C$ then $x \in A \oplus C$. By the definition of symmetric difference $x \in A \backslash C \cup C \backslash A$. By the definition of set union $x \in A \backslash C$ or $x \in C \backslash A$. Therefore since it's an or statement we can choose $x \in A \backslash C$. By the definition of set difference $x \in A$ and $x \notin C$. Therefore $x \in A$. This satisfies the requirement.

   Therefore for all sets A, B, C, if $A \oplus C = B \oplus C$ then $A = B$.