4.10 Since scoring goals is a "somewhat" rare event, we can model this process with a Poisson distribution. Let g denote the number of goals scored in a game. Since $\mathbb{P}(g \geq 1) = 0.5$, then $0.5 = 1 - \mathbb{P}(g \geq 1) = \mathbb{P}(g = 0)$. Therefore $0.5 = e^{-\lambda}, \lambda = \log(2)$.
Thus

$$\begin{aligned}
\mathbb{P}(g \geq 3) = 1 - \mathbb{P}(g < 3) &= 1 - (\mathbb{P}(g = 0) + \mathbb{P}(g = 1) + \mathbb{P}(g = 2)) \\
&= 1 - (\frac{1}{2} + \frac{1}{2}\log(2) + \frac{1}{2}\frac{\log(2)^2}{2}) \approx 0.033
\end{aligned}$$

.

4.14 Note that since the expectation is 1000, and the formula for the expectation of an exponential variable is $\frac{1}{\lambda}$ then $\lambda = \frac{1}{1000}$

(a) $\mathbb{P}(t > 2000) = e^{-\frac{2000}{1000}} = e^{-2} \approx 0.1353$

(b) $\mathbb{P}(t > 2000 | t > 500) = \mathbb{P}(t > 1500) = e^{-\frac{3}{2}} \approx 0.2231$

4.34 Since the average can be interpreted as the mean, and one can view 3 times a week as being somewhat rare, makes the Poisson distribution an ideal model. Since the mean of the poisson distribution is the rate, then assuming $\lambda = 3$ gives the probability of at most 2 accidents happening next week is $e^{-3}(1 + 3 + \frac{9}{2}) \approx 0.42319$

5.2 (a)

$$\begin{aligned}
\mathbb{E}[X] = M'(0) &= \frac{5}{6} - \frac{4}{3} = -1/2 \\
\mathbb{E}[X^2] = M''(0) &= \frac{25}{6} + \frac{16}{3} = \frac{57}{6} \\
Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 &= \frac{57}{6} - \frac{1}{4} = \frac{38-1}{4} = \frac{37}{4}
\end{aligned}$$

5.6

$$\mathbb{P}(X = 4) = \frac{1}{7}, \mathbb{P}(X = 1) = \frac{2}{7}, \mathbb{P}(X = 9) = \frac{4}{7}$$

5.12 We are assuming $t < 1$. Therefore

$$\begin{aligned}
\mathbb{E}[e^{tX}] &= \int_{\infty}^{\infty} e^{tx} f(x) dx \\
&= \int_0^{\infty} \frac{1}{2} x^2 e^{(t-1)x} dx \\
&= \frac{1}{2}\frac{1}{t-1} x^2 e^{(t-1)x}\Big|_0^{\infty} + \frac{1}{1-t}\int_0^{\infty} x e^{(t-1)x} dx \\
&= \frac{-1}{(1-t)^2} x e^{(1-t)x}\Big|_0^{\infty} + \frac{1}{(1-t)^2}\int_0^{\infty} e^{(t-1)x} dx \\
&= \frac{-1}{(1-t)^3} e^{(t-1)x}\Big|_0^{\infty} \\
&= \frac{1}{(1-t)^3}
\end{aligned}$$

5.26 Let $Y = X(X-3)$. We want to find Y's pdf. Thus we must compute $\mathbb{P}(Y \leq a)$. Note that the inequality $X(X-3) \leq a$ is equivalent to $X \in [\frac{3-\sqrt{9+4a}}{2}, \frac{3+\sqrt{9+4a}}{2}]$. Note that since X is non-zero in $[0,3]$ a must have the restriction $[\frac{-9}{4}, 0]$ Thus we can compute the cdf of Y via $\int_{\frac{3-\sqrt{9+4a}}{2}}^{\frac{3+\sqrt{9+4a}}{2}} \frac{2}{9}x dx = \frac{\sqrt{9+4a}}{3}$. Thus the pdf of Y is given by
$$f_y(a) = \begin{cases} \frac{2}{3\sqrt{9+4a}} & a \in [\frac{-9}{4}, 0] \\ 0 & \text{otherwise} \end{cases}$$