6.7.2.1 Note that the surface we care about is defined via $\begin{bmatrix} 1 & 0 & -a & -c \\ 0 & 1 & -b & -d \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. It is parameterized via $\{ \begin{bmatrix} a & c \\ b & d \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} : x_3, x_4 \in \mathbb{R}\}$. Let A be the matrix of the parameterized kernel. Therefore to compute the area of the projected unit square in x_3, x_4 coordinates we need to compute $\sqrt{\det(A^T A)}$. This computation comes out to $\sqrt{\det(A^T A)} = \sqrt{a^2 + b^2 + c^2 + d^2 + (ad - bc)^2 + 1}$. Therefore we have to solve the integral $\int_0^1 \int_0^1 \sqrt{\det(A^T A)} dx_3 dx_4 = \sqrt{\det(A^T A)} = \sqrt{a^2 + b^2 + c^2 + d^2 + (ad - bc)^2 + 1}$

6.7.2.2 For the triangle defined by $\{(1, 0, 0), (\cos(\frac{\pi}{n}), \pm \sin(\frac{\pi}{n}), \frac{1}{2m})\}$, we can just recenter to the origin with the new coordinates $\{(0, 0, 0), (\cos(\frac{\pi}{n}) - 1, \pm \sin(\frac{\pi}{n}), \frac{1}{2m})\}$. Now we can compute the area of the parallelogram spanned by the two non-zero vectors, and then halve it to find the area of the triangle. If we let $a = (\cos(\frac{\pi}{n}) - 1, \sin(\frac{\pi}{n}), \frac{1}{2m}), b = (\cos(\frac{\pi}{n}) - 1, -\sin(\frac{\pi}{n}), \frac{1}{2m})$ the area computation becomes

$$\frac{1}{2}\sqrt{(a \cdot a)(b \cdot b) - (a \cdot b)^2} =$$

$$\frac{1}{2}\sqrt{(\frac{1 + 4m^2 - 8m^2 \cos(\frac{\pi}{n}) + 4m^2 \cos^2(\frac{\pi}{n})}{m^2}) \sin^2(\frac{\pi}{n})}$$

$$= \frac{1}{2} \sin(\frac{\pi}{n}) \sqrt{\frac{1}{m^2} + 2^2(1 - \cos(\frac{\pi}{n})))^2} = \frac{1}{2} \sin(\frac{\pi}{n}) \sqrt{\frac{1}{m^2} + 4^2 \sin^4(\frac{\pi}{2n})}.$$

For the triangle $\{(1, 0, 0), (\cos(\frac{\pi}{n}), \sin(\frac{\pi}{n}), \pm \frac{1}{2m})\}$ we do the same process, where instead we have $a = (\cos(\frac{\pi}{n}) - 1, \sin(\frac{\pi}{n}), \frac{1}{2m}), b = (\cos(\frac{\pi}{n}) - 1, \sin(\frac{\pi}{n}), -\frac{1}{2m})$, yielding the computation

$$\begin{aligned} \frac{1}{2}\sqrt{(a \cdot a)(b \cdot b) - (a \cdot b)^2} &= \sqrt{\frac{1}{m^2} - \frac{2\cos(\frac{\pi}{n})}{m^2} + \frac{\cos^2(\frac{\pi}{n})}{m^2} + \frac{\sin^2(\frac{\pi}{n})}{m^2}} \\ &= \sqrt{\frac{1}{m^2} \sin^2(\frac{\pi}{2n})} \\ &= \frac{\sin(\frac{\pi}{2n})}{m} \end{aligned}$$

6.7.2.3

6.7.3.3

6.7.3.4

6.7.4.3