Prove: For any finite graph G if G is connected then $|E(G)| \geq |V(G)| - 1$.
We must show for all finite graphs G if G is connected then $|E(G)| \geq |V(G)| - 1$. Suppose G is a finite connected graph. We must show that $|E(G)| \geq |V(G)| - 1$. By the principal of mathematical induction, for all finite connected graphs H with $|V(H)| = k$, if $k < |V(G)|$, then $|E(H)| \geq |V(H)| - 1$. We now have the cases $|V(G)| \leq 1, |V(G)| > 1$.

- Assume $|V(G)| \leq 1$. Since G is anti-reflexive, and the edge set is the set of subsets of pairs of elements from $V(G)$, then $|E(G)| = 0$. Since at most $|V(G)| = 1$, then we have the inequality $0 \geq 1 - 1 = 0$, which is true.

- Assume $|V(G)| > 1$. Since G is connected and has at minimum two nodes, then every node must have at least one neighbor. Let v be an arbitrary vertex of G, and let N be the set of all of the neigbors of v. Let G' be the graph with v removed. Let the set S be given by $S = \{\{n\} \cup G'(n) : n \in N\}$ where $G'(n)$ is the set of all elements reachable by n in G'. Let W be the set of subgraphs such that $\{G'[s] : s \in S\}$. Note that each subgraph in W is connected since if they weren't then We claim that

  1. Each $s \in S$ is pairwise disjoint.
  2. Each $w \in W$ is connected.
  3. $|W| \leq |N|$.
  4. $\sum_{w \in W} |E(w)| + |N| = E(G)$.

  Proof of claim 1. We will prove this via contraposition. Suppose $s_1, s_2 \in S, s_1 \cap s_2 = A$. We must show that $s_1 = s_2$. By definition of S, there exists $n_1, n_2 \in N, n_1 \neq n_2$ such that $G'(n_1) \subset s_1, n_1 \in s_1, G'(n_2) \subset s_2, n_2 \in s_2$. Suppose $a \in A$. Since $a \in s_1, s_2$, then a is reachable by both n_1, n_2. Therefore since $a \in G'(n_1), a \in s_2$, then there exists a walk from n_1 to n_2 as there exists walks from both elements to a. Therefore by the symmetry of G', all elements reachable by n_1 include elements that are reachable by n_2 and all elements that are reachable by n_2 inlcude elements that are reachable by n_2. Therefore $s_1 = s_2$.

  Proof of claim 2. Assume for contradiction that there exists a $w \in W$ such that w is not connected. Since w is a subgraph of G, and there exists no other subgraph in W which w shares nodes with, then G must be not connected since the unique contribution of w to G is not connected. This is a contradiction.

  Proof of claim 3. Since S is pairwise disjoint, and each element of W is defined by a distinct element in S, then $|S| = |W|$. Therefore we must show $|S| \leq |N|$. Suppose for contradiction that $|S| > |N|$. Therefore there must exists a set in S that cannot be paired with an element from N. This is a contradiction as every set in S is defined to contain an element from N.

  Proof of claim 4. Since the all of the edges of G' are contained in the elements of W, then we have $\sum_{w \in W} |E(w)| = |E(G')|$. We must show that $|E(G')| + |N| = |E(G)|$. Since the only difference between G and G' is the removal of v, and v was connected to $|N|$ elements, then v had $|N|$ connections. Therefore the number of missing connections between G' and G is $|N|$. Therefore $|E(G')| + |N| = |E(G)|$.

Since we have established that each w in W is connected, and their union forms G', whose vertex set $|V(G')| = |V(G)| - 1$, which noting $|V(G')| < |V(G)|$ lets us apply the induction hypothesis for all $w \in W, |E(w)| \geq |V(w)| - 1$. Therefore adding 1 to both sides yields $|E(w)| + 1 \geq |V(w)|$. Therefore,

$$\begin{aligned}
|V(G)| - 1 = |V(G')| &= \sum_{w \in W} |V(w)| \\
&\leq \sum_{w \in W} (|E(w)| + 1) \\
&= \sum_{w \in W} |E(w)| + |W| \\
&\leq \sum_{w \in W} |E(w)| + |N| \\
&= |E(G)|.
\end{aligned}$$