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Let $A \in M_{n\times n}F$ and let B be the matrix obtained from A by adding a scalar multiple of one row to the other, then $det(A) = det(B)$. Proof:

Let $A = \begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_j \\ \vdots \\ a_n \end{bmatrix}$ and $B = \begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ ca_i + a_j \\ \vdots \\ a_n \end{bmatrix}$. Therefore

$$det(B) = det(\begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ ca_i + a_j \\ \vdots \\ a_n \end{bmatrix}) = det(\begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_j \\ \vdots \\ a_n \end{bmatrix}) + c \cdot det(\begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_i \\ \vdots \\ a_n \end{bmatrix})$$

Since a determinant of a matrix with identical rows is 0, then we have that

$$det(B) = det(\begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_j \\ \vdots \\ a_n \end{bmatrix}) = det(A)$$

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