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Let S denote the set $\{2, 10, 11, 18, 19, 27\}$. For each of the following statements, determine whether the statement is true or false.

1. *For all elements x belonging to S there is a y belonging to S such that $x + y$ is a multiple of* 5

   - Reducing all the elements in the set $\mod 5$ we observe that $\{2, 10, 11, 18, 19, 27\} \equiv \{0, 1, 2, 3, 4\} \mod 5$. Therefore since S includes $5\mathbb{Z}$ and all of the cosets of $\mathbb{Z}/5\mathbb{Z}$, whose addition operation is a group. Therefore by definition every element has an additive inverse. Therefore the statement is true.

2. *There exists an element x belonging to S such that for all elements y belonging to S, $x + y$ is a multiple of* 5.

   - The above statement is false because elements in a group, which are what S forms under division by 5, by definition have unique inverses.

3. *For all x belonging to S there is a y belonging to S such that $x + y$ is a multiple of* 7.

   - Reducing $S \mod 7$ we observe that $\{2, 10, 11, 18, 19, 27\} \equiv \{2, 3, 4, 5, 6\}$. Since $1 \mod 7 \notin S$, then there exist no element which can be added to 27 that would make it divisible by 7. Therefore the above statement is false.