Alex Valentino Assignment 7 problem 6
300H

Let $a_1, \cdots, a_k$ *be a list of intergers. Define the function* f *that maps a list of* k *integers to an integer by the rule* $f(x_1, \cdots, x_k) = a_1x_1 + \cdots a_kx_k$*. Let* R *be the range of the function* f*. Prove:*

- *For all* $m, n \in R$ *we have* $m + n \in R$*.* Suppose m, n are arbitrary integers in the range R. We must show that $m + n$ is in the range R. By definition, m, n have representations in the set of the list of integers of length k such that $m = f(m_1, \cdots, m_k)$ and $n = f(n_1, \cdots, n_k)$. Therefore $m + n$ can be rewritten as:
  $m + n = f(m_1, \cdots, m_k) + f(n_1, \cdots, n_k) = a_1m_1 + \cdots + a_km_k + a_1n_1 + \cdots a_kn_k = a_1(m_1 + n_1) \cdots a_k(m_k + n_k)$.
  Since the integers are closed under addition and multiplication then the list $(m_1 + n_1, \cdots, m_k + n_k)$ is a valid list of k integers, which has been shown above defines $m + n$, then $m + n \in R$.

- *For all* $n \in R$ *and* $c \in Z$ *we have* $cn \in R$*.* Suppose n, c are arbitrary integers, and $n \in R$. By definition of being in the range of R there exist a list $(n_1, \cdots, n_k)$ such that $n = f(n_1, \cdots, n_k) = a_1n_1 + \cdots a_kn_k$. Therefore multiplying n by c yields:
  $cn = c(a_1n_1 + \cdots a_kn_k) = ca_1n_1 + \cdots ca_kn_k$. Since the integers are closed under multiplication, then $(cn_1, \cdots, cn_2)$ is a valid list of k integers. Therefore f is defined over that list, and as shown above that is the representation of cn, therefore $cn \in R$.