300H
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Assignment 1 problem 4

Prove that the $\sqrt{5}$ *is irrational.*

Assume not. Let the $\sqrt{5}$ be a rational number. Therefore $\sqrt{5} = \frac{a}{b}, a, b \in \mathbb{Z}$, where a, b are coprime. Thus,

$$(\sqrt{5})^2 = (\frac{a}{b})^2$$
$$5 = \frac{a^2}{b^2}$$
$$5b^2 = a^2.$$

Given that a and b are coprime, $5 \mid a^2$. By Euclid's lemma $5 \mid a$. Therefore $a = 5c$. Substituting this back into the previous equation yields

$$5b^2 = a^2$$
$$5b^2 = 25c^2$$
$$b^2 = 5c^2.$$

Since b and c are coprime as a result of c being a factor of a, $5 \mid b^2$. By Euclid's lemma $5 \mid b$. Therefore $b = 5d$. Substituting the two equations found for a and b back into the original expression shows that $\sqrt{5} = \frac{5c}{5d}$. This is a contradiction as a and b are supposed to be coprime. Therefore the $\sqrt{5}$ is irrational.