4.4.6 (a) Consider $h(x) = \sin(\frac{1}{x})$, and $x_n = \begin{cases} \frac{1}{\frac{\pi}{2}+2\pi n} & n \text{ even} \\ \frac{1}{\frac{3\pi}{2}+2\pi n} & n \text{ odd} \end{cases}$. Clearly $x_n \to 0$, however $h(x_n)$ fluctuates between 1 and -1.

(b) This is impossible, as by theorem 4.4.2 the image of $f([0,1])$ is a compact set, and since the Cauchy sequence converges, then it must converge in it's image.

(c) Similar to (b), since the sequence is Cauchy then it is bounded, thus we are mapping a compact set to another compact set, above logic applies.

(d) Consider the function $f(x) = \frac{1}{2} - |x - \frac{1}{2}|$. At $\frac{1}{2}$ clearly f attains a maximum. f has a 0 value at $x = 0, 1$. However since those points aren't attained, then the minimum can never be attained.