3 In order to find the solution of the non-homogeneous equation, we must first find a solution to the homogeneous one. Guessing the form of the solution to be t^α we get that

$$\begin{aligned} \alpha(\alpha-1)t^\alpha - 2t^\alpha &= 0 \\ \alpha(\alpha-1) - 2 &= 0 \\ (\alpha-2)(\alpha+1) &= 0 \\ \alpha &\in \{-1, 2\} \end{aligned}$$

Since both t^2 and $1/t$ satisfy the differential equation, then $x_1(t) = t^2, x_2(t) = \frac{1}{t}$. Thus we have two linearly independent solutions. For the given initial condition we have the system $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, which when solved gives the solution $c_1 = 1, c_2 = 0$. Computing $\det(M(s)) = -3, \det(N(t,s))) = \frac{s^2}{t} - \frac{t^2}{s}$, we are now ready to apply the variation of constants formula:

$$\begin{aligned} x(t) &= t^2 + \int_1^t \frac{\det(N(t,s))}{\det(M(s))} r(s) ds \\ &= t^2 - \int_1^t (\frac{s^2}{t} - \frac{t^2}{s}) \frac{1}{s^3} ds \\ &= t^2 + \frac{t^3 - 3\log(t) - 1}{3t} \\ &\text{t is strictly positive, no absolute value} \\ &= \frac{4t^3 - 3\log(t) - 1}{3t}. \end{aligned}$$

4 In order to find the solution of the non-homogeneous equation, we must first find a solution to the homogeneous one. Guessing the form of the solution to be $e^{\alpha t}$ we get that

$$\begin{aligned} \alpha^2 e^{\alpha t} - \alpha \frac{t+2}{t} e^{\alpha t} + \frac{2}{t} e^{\alpha t} &= 0 \\ \alpha^2 - \alpha \frac{t+2}{t} + \frac{2}{t} &= 0 \\ \alpha^2 - 3\alpha + 2 &= 0 && \text{evaluating at } t = 1 \\ (\alpha-1)(\alpha-2) &= 0 \\ \alpha &\in \{1, 2\} \end{aligned}$$

After testing the values of α we find that only $\alpha = 1$ is a valid value. Therefore $x_1(t) = e^t$. Therefore

$$x_2(t) = x_1(t)v(t) = x_1(t) \int \frac{1}{x_1^2(t)} e^{P(t)} dt = e^t \int \frac{t^2 e^t}{e^{2t}} dt = -(t^2 + 2t + 2).$$

Thus we have two linearly independent solutions. For the given initial condition we have the system $\begin{bmatrix} e & -5 \\ e & -4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, which when solved gives the solution $c_1 = \frac{11}{e}, c_2 =$

2. Computing $\det(M(s)) = s^2e^s, \det(N(t,s))) = e^t(s^2+2s+2) - e^2(t^2+2t+2)$, we are now ready to apply the variation of constants formula:

$$\begin{aligned}
x(t) &= 11e^{t-1} - 2(t^2+2t+2) + \int_1^t \frac{\det(N(t,s))}{\det(M(s))} r(s)ds \\
&= 11e^{t-1} - 2(t^2+2t+2) + \int_1^t \frac{e^t(s^2+2s+2) - e^2(t^2+2t+2)}{s^2e^s} s^2e^s ds \\
&= 11e^{t-1} - 2(t^2+2t+2) + e^t(\frac{1}{3}t^3 - \frac{16}{3}) + e(t^2+2t+2).
\end{aligned}$$

7 In order to find the solution of the non-homogeneous equation, we must first find a solution to the homogeneous one. Guessing the form of the solution to be $e^{\alpha t}$ we get that

$$\begin{aligned}
\alpha^2e^{\alpha t} - \alpha e^{\alpha t} - 6e^{\alpha t} &= 0 \\
\alpha^2 - \alpha - 6 &= 0 \\
(\alpha+2)(\alpha-3) &= 0
\end{aligned}$$

Since both e^{-2t} and e^{3t} satisfy the differential equation, then $x_1(t) = e^{-2t}, x_2 = e^{3t}$. Computing $\det(M(s)) = 5e^s, \det(N(t,s))) = e^{3t-2s} - e^{3s-2t}$, we are now ready to apply the variation of constants formula:

$$\begin{aligned}
x(t) &= c_1e^{-2t} + c_2e^{3t} + \int_{t_0}^t \frac{\det(N(t,s))}{\det(M(s))} r(s)ds \\
&= c_1e^{-2t} + c_2e^{3t} + \frac{1}{5}\int_{t_0}^t e^{3t-2s} - e^{3s-2t} ds \\
&= c_1e^{-2t} + c_2e^{3t} + \frac{1}{5}(\frac{1}{2}e^{3t-2t_0} + \frac{1}{3}e^{3t_0-2t} - \frac{5}{6}e^t)
\end{aligned}$$

since the two terms with t_0 are multiples of x_1, x_2 our solution becomes

$$= c_1e^{-2t} + c_2e^{3t} - \frac{1}{6}e^t.$$