1.2.1 (a) We must show that the square root of 3 is irrational. Suppose for contradiction that there exists $a, b \in \mathbb{Z}$ such that $\frac{a}{b} = \sqrt{3}$ and a, b are coprime. Therefore $a^2 = 3b^2$, 3 is a divisor of a^2. By Euclid's lemma 3 is a divisor of a. Therefore there exists $c \in \mathbb{Z}$ such that $a = 3c$. Thus $3c^2 = b^2$. By similar reasoning above, 3 is a divisor of b. This is a contradiction, as a, b are coprime. Therefore $\sqrt{3}$ is irrational. A similar argument works for $\sqrt{6}$, one must simply choose a single prime from the prime factorization of 6 and then get the contradiction from a, b not being coprime.

(b) The argument fails due to 4 having no prime factor of an odd power, no way to apply Euclid's lemma.

1.2.2 (a) If $A_1 \supseteq A_2 \supseteq \cdots$ are all sets containing an infinite number of elements, then the intersection of all of the sets must be infinite as well. This is false, if we have the sets $A_n = n, n+1, n+1, \cdots$ then the intersection $\bigcap_{n=1}^{\infty} A_n = \emptyset$

(b) If $A_1 \supseteq A_2 \supseteq \cdots$ are all sets containing a number of reals, then the intersection of all of the sets must be finite and non-empty. This is true

(c) $A \cap (B \cup C) = (A \cap B) \cup C$. This is false. If $A = \{1\}, B = \{2\}, C = \{2, 3\}$. Then $\emptyset = \{1\} \cap (\{2\} \cup \{2,3\}) = A \cap (B \cup C) \neq (A \cap B) \cup C = \emptyset \cup \{2,3\} = \{2,3\}$

(d) true

(e) true

1.2.10 Let $y_1 = 1$ and for each $n \in \mathbb{N}$ define $y_{n+1} = \frac{3y_n+4}{4}$.

(a) Use induction to prove that the sequence satisfies $y_n < 4$ for all $n \in \mathbb{N}$.
Proof: For the base case, $y_1 = 1 < 4$. By the principle of mathematical induction for all $k \in \mathbb{N}$ if $k < n$ then $y_k < 4$. We must show that $y_n < 4$. Since $n - 1 < n$, then by the induction hypothesis $y_{n-1} < 4$. Therefore,

$$\begin{aligned} y_{n-1} &< 4 \\ 3y_{n-1} &< 12 \\ 3y_{n-1} + 4 &< 16 \\ (3y_{n-1} + 4)/4 &< 4 \\ y_n &< 4. \end{aligned}$$

(b) We must show that $(y_1, y_2, \cdots)$ is increasing. For the base case, $y_1 = 1, y_2 = \frac{3+4}{4} = \frac{7}{4}, 1 < \frac{7}{4}$. By PMI for all $k \in \mathbb{N}$ if $k < n$ then $y_k < y_{k+1}$. Since $n - 1 < n$, by the induction hypothesis $y_{n-1} < y_n$. Therefore,

$$\begin{aligned} y_{n-1} &< y_n \\ 3y_{n-1} + 4 &< 3y_n + 4 \\ \frac{3y_{n-1} + 4}{4} &< \frac{3y_n + 4}{4} \\ y_n &< y_{n+1} \end{aligned}$$