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PSS5
350H

Let β, γ be ordered bases of V where V is a finite dimensional vector space over F. Let L be a linear operator on V. We want to show that $det([L]_\beta^\beta - x\mathbb{I}_V) = det([L]_\gamma^\gamma - x\mathbb{I}_V)$. Let Δ be the standard basis. Then $det([L]_\beta^\beta - x\mathbb{I}_V) = det([\mathbb{I}_V]_\Delta^\beta [L]_\Delta^\Delta [\mathbb{I}_V]_\beta^\Delta - x[\mathbb{I}_V]_\Delta^\beta [\mathbb{I}_V]_\Delta^\Delta [\mathbb{I}_V]_\beta^\Delta) = det([\mathbb{I}_V]_\Delta^\beta ([L]_\Delta^\Delta - x[\mathbb{I}_V]_\Delta^\Delta)[\mathbb{I}_V]_\beta^\Delta) = det([\mathbb{I}_V]_\Delta^\beta) det([L]_\Delta^\Delta - x[\mathbb{I}_V]_\Delta^\Delta) det([\mathbb{I}_V]_\beta^\Delta)$