(a) If S is a finite set, a permutation of S is a function from S to itself, whose range is all of S. Give an example of a permutation of $\{1, 2, 3, 4, 5\}$.
Example: Let $id_S : S \to S$ be given as $x \in S, id_S(x) = x$. This gives the permutation of $\{1, 2, 3, 4, 5\}$.
(b) Recall that if f and g are permutations of S then $f \circ g$ is the function from S to S given by the rule $f \circ g(s) = f(g(s))$ for all $s \in S$. The identity permutation is the permutation that maps every element to itself. Give an example of two different permutations f and g of $\{1, 2, 3, 4, 5\}$ such that neither is the identity permutation, and $f \circ g = g \circ f$.
Let $f : S \to S$ be given as $f(S) = \{2, 1, 3, 4, 5\}$ and $g : S \to S$ be given as $g(S) = \{1, 2, 3, 5, 4\}$. Then $f(g(S)) = \{2, 1, 3, 5, 4\}$, and $g(f(S)) = \{2, 1, 3, 5, 4\}$.
(c) Give an example of two different permutations f and g of $\{1, 2, 3, 4, 5\}$such that neither is the identity permutation and such that $f \circ g = g \circ f$ and $f \circ f = g \circ g$.
The previous example functions work, as $f \circ f(S) = \{1, 2, 3, 4, 5\}$ and $g \circ g(S) = \{1, 2, 3, 4, 5\}$