Alex Valentino

3.4.7 (a) Find a disconnected set whose closure is connected:
$(1,2) \cup (2,3)$ is a disconnected set as proven by exercise 3.4.5. Clearly $[1,2] \cup [2,3]$ are not disconnected as they form the interval $[1,3]$, which by theorem 3.4.7 is in fact connected.

(b) If A is connected, is $\overline{A}$ connected? If A is perfect is $\overline{A}$ perfect too?

- As shown by theorem 3.4.7, all connected sets correspond with intervals. Therefore the only interval which will gain points would be an open interval, say $A = (a,b)$. Since $\overline{A} = [a,b]$ is still an interval, then $\overline{A}$ is connected
- If A is perfect, then by definition it is closed. Therefore $\overline{A}$ doesn't gain any new points, as all limit points of A are already self contained. Therefore $\overline{A}$ is still perfect.