6 Let $\epsilon > 0$ be given. We will show that $\sum_{n=1}^{\infty}(-1)^n\frac{x^2+n}{n^2}$ converges uniformly on the interval (a,b). Let $B = \max\{|a|,|b|\}$ Note that the series $\sum_{n=1}^{\infty}\frac{B^2}{n^2} = \frac{\pi^2}{6}$ and $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n} = -\log(2)$. Since both of these separate series converges then they both satisfy the Cauchy criterion for series. Therefore there exists $N_1 \in \mathbb{N}$ such that for all $k \geq m \geq N_1, \sum_{n=m}^{k}\frac{B^2}{n^2} < \frac{\epsilon}{2}$ and there exists $N_2 \in \mathbb{N}$ such that $q \geq p \geq N_2, |\sum_{n=p}^{q}(-1)^n\frac{1}{n}| < \frac{\epsilon}{2}$. Therefore if we take $N = \max\{N_1, N_2\}$ then for all $r \geq s \geq N$,

$$|\sum_{n=s}^{r}(-1)^n\frac{x^2+n}{n^2}| \leq \sum_{n=s}^{r}\frac{B^2}{n^2} + |\sum_{n=s}^{r}(-1)^n\frac{1}{n}| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Since the series satisfies the Cauchy criterion, then it is uniformly convergent.
The function does not converge absolutely since

$$\sum_{n=1}^{\infty}\frac{x^2+n}{n^2} = \sum_{n=1}^{\infty}\frac{x^2}{n^2} + \frac{1}{n} > \sum_{n=1}^{\infty}\frac{1}{n} = \infty.$$

8 Note that for each function in the sum we have that $|c_nI(x-x_n)| \leq |c_n|$. Therefore by the Weierstrass M-test the series $\sum_{n=1}^{\infty}c_nI(x-x_n)$ converges uniformly. To show the continuity of the series when $x \neq x_n$ we must consider two cases. If x is not a limit point of the sequence $\{x_n\}$ then there must exists $\delta > 0$ such that $V(x,\delta) \cap \{x_n\} = \emptyset$. Therefore if we consider the subsequence $\{x_{n_k}\}$ that is to the left of x then the value function within $V(x,\delta)$ is simply the constant function with value $\sum_{n_k}c_{n_k}$, thus making it continuous. If x is a limit point of $\{x_n\}$ then for all of the partial sums $\sum_{n=1}^{m}I(x-x_n)c_n$ is constant for some $\delta > 0$ around x. Therefore since it is true for the partial sums and since the series converges uniformly then we can apply the limit interchange theorem and get that

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