Give an example of an infinite sequence $(A_j : j \in \mathbb{Z}_{>0})$ *that satisfies the three requirements: (1) Each* A_j *is a nonempty subset of the interval set* $[-100, 100]$.
(2) For each $j \geq 1, A_{j+1} \subseteq A_j$.
(3) There is no number that belongs to all of the sets.
Let $A_j = (0, \frac{1}{2^j})$. Clearly the sequence satisfies (1), $\frac{1}{2^j}$ is comfortably under 100 for $j \geq 1$. For (2), since $\frac{1}{2^j}$ is monotonically decreasing and the left side bound of 0 doesn't change, then $(0, \frac{1}{2^{j+1}}) \subseteq (0, \frac{1}{2^j})$. For property (3), one can show that for picking any value $\epsilon \in (0, 1)$ that one can choose a value for j such that $\frac{1}{2^j} < \epsilon$, as j can be constructed to be $j > \frac{ln(\epsilon)}{ln(2)}$ therefore for all the $j-1$ intervals that contain the element ϵ, $\epsilon \notin A_j$. Therefore there does not exist a number common to all of the intervals.