4.8 (a) Solving for $v(x,y) = (0,0)$ yields two solutions of the form $(\frac{-1}{2}, \frac{1}{2})$ and $(\frac{3}{2}, \frac{1}{2})$. Putting these values into the jacobian of v corresponding to $\begin{bmatrix} 2x-1 & -2y-1 \\ 2x-1 & 3-2y \end{bmatrix}$ yields the matrices $\begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ respectively. These have eigenvalues of $\pm\sqrt{2}$ and $2 \pm 2i$ respectively, therefore since both have eigenvalues with a real part greater than 0 then they are both unstable.

(b) Solving for $v(x,y) = (0,0)$ yields two solutions of the form $(\frac{-1}{2}, \frac{1}{2})$ and $(\frac{3}{2}, \frac{1}{2})$. Putting these values into the jacobian of v corresponding to $\begin{bmatrix} 2x-1 & 3-2y \\ 2x-1 & -2y-1 \end{bmatrix}$ yields the matrices $\begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$. These have eigenvalues of $\pm 2\sqrt{2}$ and $-2 \pm 2i$. Therefore $(-\frac{1}{2}, \frac{1}{2})$ is stable and $(\frac{3}{2}, \frac{1}{2})$ is unstable.

4.10 (a) Solving for $v(x,y) = (0,0)$ yields three solutions of the form $(0,0), (1,-1), (1,-2)$. Putting these values into the jacobian of v corresponding to $\begin{bmatrix} -2(y+2) & -2(y+2)-2(x+y) \\ y & x-1 \end{bmatrix}$ yields $\begin{bmatrix} -4 & -4 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -2 & -2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ respectively. These correspond with the eigenvalues $\{-4,-1\}$, $-1 \pm \sqrt{3}$, and $\pm 2i$. Therefore $(0,0)$ is stable, $(1,-1)$ is unstable. However $(1,-2)$ has unknown behavior.

(b) Solving for $v(x,y) = (0,0)$ yields three solutions of the form $(0,0), (1,-1), (1,-2)$. Putting these values into the jacobian of v corresponding to $\begin{bmatrix} 2(y+2) & 2(y+2)+2(x+y) \\ y & x-1 \end{bmatrix}$ yields $\begin{bmatrix} 4 & 4 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$ respectively. These correspond with the eigenvalues $\{4,-1\}$, $1 \pm i$, and ± 2. Therefore all of the points are unstable.

5.1
1 $\mathbf{X}_1 = \Psi(\mathbf{X}_0) = x_0 + \int_0^t v(x_0, s)ds = \int_0^t 2sds = t^2$
2 $\mathbf{X}_2 = \Psi(\mathbf{X}_1) = x_0 + \int_0^t v(x_1, s)ds = \int_0^t 2s(1+s^2)ds = t^2 + \frac{1}{2}t^4$
3 $\mathbf{X}_3 = \Psi(\mathbf{X}_2) = x_0 + \int_0^t v(x_2, s)ds = \int_0^t 2s(1+s^2+\frac{1}{2}s^4)ds = t^2 + \frac{1}{2}t^4 + \frac{1}{6}t^6$
4 $\mathbf{X}_4 = \Psi(\mathbf{X}_3) = x_0 + \int_0^t v(x_3, s)ds = \int_0^t 2s(1+s^2+\frac{1}{2}s^4+\frac{1}{6}s^6)ds = t^2 + \frac{1}{2}t^4 + \frac{1}{6}t^6 + \frac{1}{24}t^8$

These terms correspond exactly with the solution of $e^{2t} - 1$ as

$$-1 + e^{2t} = -1 + \sum_{k=0}^{\infty} \frac{t^{2k}}{k!} = \sum_{k=1}^{\infty} \frac{t^{2k}}{k!} = t^2 + \frac{1}{2}t^4 + \frac{1}{6}t^6 + \frac{1}{24}t^8 + \cdots$$

which shows the first four terms of the series we've found manually via Picard iteration.

5.2