Let x and y be real variables, and let $C(x,y)$ be a predicate involving x and y. Consider the two scenarios:
Scenerio 1*: Input variable x. Assumption: For every $y \in \mathbb{R}, C(x,y)$ is true.*
Scenerio 2*: Input variable y. Assumption: For every $x \in \mathbb{R}, C(x,y)$ is false.*
Observe that in the first scenario y is a bound variable, while in the second x is a bound variable. Consider the set S_1 of feasible instances to scenario 1 and S_2 of feasible instances to scenario 2. Can S_1 and S_2 both be nonempty? Explain.
If one negates S_2, then you get the statement $\neg S_2(y) := \exists x \in \mathbb{R}, C(x,y)$ is true. The funny thing is this statement is roughly equivalent to S_1 if you let S_2 vary with regards to y. If you do, that means that S_1 is non-empty, there has to exist an element which satisfies. It gets even more interesting if you consider the double negation of S_2, then there doesn't exist an element for which y makes $C(x,y)$ true. This directly contradicts the first scenerio, in which for certain x ALL y make $C(x,y)$ true. Therefore if one takes both to be true, then one must be empty.