1. Let $T : V \to W$ be a linear transformation from a finite-dimensional vector space V to a finite-dimensional vector space W. Let β and β' be ordered bases for V, and let γ' and γ be ordered bases for W. We must show that $[T]_{\beta'}^{\gamma'} = P^{-1}[T]_\beta^\gamma Q$ where Q is the β' to β change of coordinates matrix and P is the γ' to γ change of coordinates matrix. By definition $Q = [\mathbb{I}_V]_\beta^{\beta'}$, $P = [\mathbb{I}_W]_{\gamma'}^\gamma$. Note that the statement we're trying to prove is equivalent to $P[T]_{\beta'}^{\gamma'} = [T]_\beta^\gamma Q$. Therefore,

$$P[T]_{\beta'}^{\gamma'} = [\mathbb{I}_W]_{\gamma'}^\gamma [T]_{\beta'}^{\gamma'} = [\mathbb{I}_W T]_{\beta'}^\gamma = [T]_{\beta'}^\gamma = [T\mathbb{I}_V]_{\beta'}^\gamma = [T]_\beta^\gamma [\mathbb{I}_V]_{\beta'}^\beta = [T]_\beta^\gamma Q.$$

Therefore $[T]_{\beta'}^{\gamma'} = P^{-1}[T]_\beta^\gamma Q$.

2. Suppose $A, B \in M_{m\times n}(F), P \in GL_m(F), Q \in GL_n(F), B = P^{-1}AQ$. We must show that there exists an n dimensional vector space V and an m dimensional vector space W over F, ordered bases β and β' for V and γ and γ' for W, and a linear transformation $T : V \to W$ such that
$$A = [T]_\beta^\gamma \text{ and } B = [T]_{\beta'}^{\gamma'}.$$
Let $V = F^n, W = F^m, T = L_A$, and β and γ be the standard ordered bases for F^n and F^m respectively.

3. Let V be a finite-dimensional vector space with the ordered basis β. Prove that $\psi(\beta) = \beta^{**}$. Let $\beta = \{x_1, \ldots, x_n\}$. Therefore $\beta^{**} = \{\hat{x}_1, \ldots, \hat{x}_n\}$. We must show that $\psi(\beta) = \beta^{**}$. Since