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Assignment 5 problem 5
300H

We begin with some definitions. We say that A is a neighbor of B if $A \oplus B$ consists of exactly one element.
A list of sets is a *neighborly list* of sets if each set is a neighbor of the set following it in the list, and the last set is a neighbor of the first set.

Here is an interesting theorem: For all positive integers n, there is a list consisting of subsets of $\{1, ..., n\}$ such that (1) every subset of $\{1, ..., n\}$ appears precisely once on the list and (2) the list is neighborly.

Prove the special cases of the theorem with $n = 1, n = 2, n = 3$ and $n = 4$. (We'll prove the theorem for all n later, but if you feel ambitious you can try it now.)

1. $n = 1$:
   $\{\emptyset, \{1\}\}$

2. $n = 2$:
   $\{\emptyset, \{1\}, \{1,2\}, \{2\}\}$

3. $n = 3$:
   $\{\emptyset, \{2\}, \{1,2\}, \{1,2,3\}, \{2,3\}, \{3\}, \{1,3\}, \{1\}\}$

4. $n = 4$:
   $\{\{2\}, \{1,2\}, \{1,2,3\}, \{2,3\}, \{2,3,4\}, \{1,2,3,4\}, \{1,2,4\}, \{2,4\}, \{4\}, \{1,4\}, \{1,3,4\}, \{3,4\}, \{3\}, \{1,3\}$