Let $f(x) = cos^2(\frac{\pi}{2}x) + x$ and $g(x) = sin^2(\frac{\pi}{2}x) + x$. These two function go between 0 and 1 based on the parity of x, with f for the evens and g for the odds. It is continuous on the range required and with the x terms in both function it will diverge to infinity.