5.5.13 Since the Jacobian of $f(x,y,z) = x^2+y^2-z^2, g(x,y,z) = x-y$ is $\begin{bmatrix} 2x & 2y & -2z \\ 1 & -1 & 0 \end{bmatrix}$ has a submatrix corresponding to $\begin{bmatrix} 2x & 2y \\ 1 & -1 \end{bmatrix}$ with determinant $-2y-2x$ then we can imply the inverse function theorem whenever $-x \neq y$. Furthermore when we can take partial derivatives, we have that for $(y+c)^2+y^2-1 = z^2$ $2(y+c)\frac{\partial y}{\partial z}+2y\frac{\partial y}{\partial z} = 2z$, thus $\frac{\partial y}{\partial z} = \frac{z}{c+2y}$. For x we have the equation $x^2+(x-c)^2-1 = z^2$, therefore $2x\frac{\partial x}{\partial z}+2(x-c)\frac{\partial x}{\partial z} = 2z$, therefore $\frac{\partial x}{\partial z} = \frac{z}{2x-c}$

5.5.14
- The lemniscate intersects the x-axis at $x = 1 \pm a$. Note that if $a \neq \mp 1$ then where the lemniscate crosses the origin is at a cusp, thus it is not differentiable there, thus it fails to be invertible. Otherwise the implicit function theorem applies.
- Note that if we convert the function to polar we find that $r = a+\cos(\theta)$, if we have $|a| > 1$ then it's impossible for $r = 0$, thus the solution around the origin will be isolated. Otherwise if $|a| \leq 1$ then we have a continuous path in and out of the origin by the continuity of cos.

5.5.15 Observe that the partial derivative of $\|\vec{x}\|$ with respect to x_l is

$$\frac{\partial\|\vec{x}\|}{\partial x_l} = p\frac{|x_l|^{p-1}}{\left(\sum_{i=1}^n |x_i|^p\right)^{1-\frac{1}{p}}}$$

The conditions under which we can use the implicit function theorem are as follows

- Consider $p \leq 1$. In this scenario, suppose that we have $\|\vec{x}\| = \|\vec{x}_0\|$ with $x_j = 0$ in $\vec{x}$, then $(\sum_{i=1}^n |x_i|^p)^{\frac{1}{p}-1} = 0$ since $x_l^p = \infty$, then $\infty^{\frac{1}{p}-1} = \infty$, thus the derivative is not invertible. Also if we have $p = 1$ then the denominator will still evaluate to
- Consider $p > 1$. Note for any coordinates which are not 0, so long as they all aren't zero, we have that $0 < (\sum_{i=1}^n |x_i|^p)^{\frac{1}{p}-1} < \infty$, as $0^p = 0$. This also gives us the condition under which we cannot apply the implicit function theorem, because if $\vec{x} = \vec{0}$ then the denominator is 0, which gives us that the derivatives are undefined.

Furthermore outside of these circumstances, the derivative as defined above is continuous, thus the function is representable as a differentiable graph on $n-1$ variables.