8.2 Let X be the value of the 4 sided die roll, let Y be the value of the 6 sided die roll, let Z be the value of the 12 sided die roll, and let $W = X+Y+Z$ be the collective value from the three die rolls. Thus $\mathbb{E}W = \mathbb{E}[X+Y+Z] = \mathbb{E}X + \mathbb{E}Y + \mathbb{E}Z = \frac{5}{2} + \frac{7}{2} + \frac{13}{2} = 12.5$

8.4 Let X be the value of the 4 sided die roll, let Y be the value of the 6 sided die roll, let Z be the value of the 12 sided die roll, and let V represent the number of fours. Since X, Y, Z are independent die rolls, then we can write V as the sum of indicators: $V = \mathbb{I}_{X=4} + \mathbb{I}_{Y=4} + \mathbb{I}_{Z=4}$. Thus $\mathbb{E}V = \mathbb{E}\mathbb{I}_{X=4} + \mathbb{E}\mathbb{I}_{Y=4} + \mathbb{I}_{Z=4} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$.

8.8 Let $X \sim Unif[1,7], Y \sim Exp[2], Z = X+Y$. We want to find $\mathbb{E}Z$ and $Var(Z)$. By the linearity of expectation, $\mathbb{E}Z = \mathbb{E}X + \mathbb{E}Y = 4 + 0.5 = 6, Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 3 + 0.25 + 0 = 3.25$

8.24 Note that $N \in \{10, \cdots, 40\}$ since there are a minimum of 10 matching pair and a max of 40 pairs where Jane follows Sam with the correct color. Note that N can be written as the sum of indicator variables $N = \sum_{n=1}^{40} \mathbb{I}_n$, where $\mathbb{I}_k$ represents the kth pairing. Note that by exchangability that all $\mathbb{I}_k$ are the same. There probabilities evaluate to $\mathbb{P}(\mathbb{I}_k) = \frac{1}{79*80}(50*49+30*29) = \frac{83}{158}$. Thus $\mathbb{E}N = \sum_{n=1}^{40} \mathbb{E}\mathbb{I}_n = 40 * \frac{83}{158} \approx 21.01$

8.42

$$\begin{aligned}
\mathbb{E}\bar{X}_n^4 &= \mathbb{E}\left(\frac{X_1 + \cdots + X_n}{n}\right)^4 \\
&= \frac{1}{n^4}\mathbb{E}[(X_1 + \cdots + X_n)^4] \\
&= \frac{1}{n^4}\mathbb{E}[\frac{4!}{1!1!1!1!}\sum_{i<j<k<l} X_iX_jX_kX_l + \frac{4!}{2!1!1!}\sum_{i<j,k\neq i,j} X_k^2X_iX_j \\
&+ \frac{4!}{2!2!}\sum_{i\neq j} X_i^2X_j^2 + \frac{4!}{3!}\sum_{i\neq j} X_i^3X_j + \frac{4!}{4!}\sum_i X_i^4] \\
&= \frac{1}{n^4}\left(6\sum_{i\neq j}\mathbb{E}[X_i^2]\mathbb{E}[X_j^2] + \sum_i \mathbb{E}[X_i^4]\right)
\end{aligned}$$

by linearity of expectation, independence of the variables, and that $\mathbb{E}X_i = 0$ for all i

$$\begin{aligned}
&= \frac{1}{n^4}(6\binom{n}{2}a^2 + nc) \\
&= \frac{1}{n^3}(3(n-1)a^2 + c)
\end{aligned}$$

8.48 Note that the joint pmf of X and Y is given by

$\frac{X}{Y}$	1	2	3
0	$\frac{9}{100}$	$\frac{81}{100}$	0
1	0	$\frac{9}{100}$	0
2	0	0	$\frac{1}{100}$

Therefore computing the expectations required to solve covariance are easy:

$$\mathbb{E}X = \frac{9}{100} + 2 \cdot \frac{90}{100} + \frac{3}{100} = \frac{192}{100} = 1.92,$$
$$\mathbb{E}Y = \frac{9}{100} + \frac{2}{100} = \frac{11}{100} = 0.11,$$
$$\mathbb{E}XY = 2 \cdot 1 \cdot \frac{9}{100} + 3 \cdot 2 \cdot \frac{1}{100} = \frac{24}{100} = 0.24.$$

Therefore $Cov(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] = 0.24 - 1.92 \cdot 0.11 = 0.0288.$