2.7.13 (a)

$$
\begin{aligned}
|\sum_{j=m+1}^{n} x_n y_n| &= |s_n y_{n+1} - s_m y_{m+1} + \sum_{j=m+1}^{n} s_j (y_j - y_{j+1})| && \text{Exercise 2.7.12} \\
&\leq M|y_{n+1} - y_{m+1} + \sum_{j=m+1}^{n} (y_j - y_{j+1})| && \text{Upper bound on } (s_n) \\
&\leq M|y_{n+1} + y_{m+1} + \sum_{j=m+1}^{n} (y_j - y_{j+1})| \\
&= M|y_{n+1} + y_{m+1} + y_{m+1} - y_{n+1}| && \text{Expanding the telescoping series} \\
&= 2M|y_{m+1}|
\end{aligned}
$$

(b) Dirichlet's Test proof:
We will show that the series $t_m = \sum_{j=1}^{m} x_j y_j$ converges by the Cauchy Criterion for Series. Let $\epsilon > 0$. Since (y_n) converges to 0 then there exists $N \in \mathbb{N}$ such that for all $n \geq N$ $|y_n| < \frac{\epsilon}{2M}$. Therefore for all $n > m \geq N$

$$|t_n - t_m| = |\sum_{j=m+1}^{n} x_j y_j| \leq 2M|y_{m+1}| \leq 2M|y_N| < 2M\frac{\epsilon}{2M} = \epsilon.$$

Therefore by the Cauchy Criterion for Series, (t_m) converges.

(c) The Alternating Series Test is simply the cases where $x_n = (-1)^{n+1}$, as it is a sequence bounded above and below by 1. The requirement on y_n is the exact same as in the Alternating Series Test