Alex Valentino Assignment 9 problem 2

300H

The Fibonacci numbers are defined by the recurrence $f_2 = f_1 = 1$ and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$. Prove that for all $n \geq 3$, $f_n = 1 + \sum_{i=1}^{n-2} f_i$.

Suppose $n \in \mathbb{N}, n \geq 3$. We must show that $f_n = 1 + \sum_{i=1}^{n-2} f_i$, where f_i is the ith fibonacci number. By the principal of mathematical induction for all $k \in \mathbb{N}$, if $3 \geq k < n$, then $f_k = 1 + \sum_{i=1}^{k-2} f_i$. The condition $n \geq 3$ provides two cases:

- Assume $n = 3$. Then $f_3 = f_2 + f_1 = 1 + 1 = 1 + f_1 = 2$.
- Assume $n > 3$. Then $f_n = f_{n-1} + f_{n-2}$. Since $n - 1 < n$, then by the induction hypothesis, $f_{n-1} = 1 + \sum_{i=1}^{n-3} f_i$. Therefore $f_n = f_{n-1} + f_{n-2} = f_{n-2} + 1 + \sum_{i=1}^{n-3} f_i = 1 + \sum_{i=1}^{n-2} f_i$.