5.5.3 Let $S(x,y) = (x^2 - y^2, 2xy)$. Note that $DS(x,y) = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$. Therefore the determinate of the jacobian is as follows: $|DS(x,y)| = 4(x^2 + y^2)$. The map can only be non-invertible when $|DS(x,y)| = 0$, however this only occurs when $x = y = 0$. Furthermore, the structure of the jacobian is all linear continuous terms, thus the jacobian is continuous everywhere. Therefore the inverse function theorem holds for S everywhere but the origin. Additionally, solving $(u,v) = S(x,y)$ for undetermined (x,y) generally, one finds that $x = \sqrt{\frac{\sqrt{u^2+v^2}+u}{2}}, y = \sqrt{\frac{\sqrt{u^2+v^2}-u}{2}}$. Note that if we consider the inverse near $(0,0)$ that the approximation still holds since $\sqrt{u^2+v^2} - u \geq u - u = 0$, thus the function is always defined since it never goes negative under the square root.

5.5.4 Let $(u,v) = f(x,y) = (e^x \cos(y), e^x \sin(y))$. Then $Df(x,y) = e^x \begin{bmatrix} \cos(y) & -\sin(y) \\ \sin(y) & \cos(y) \end{bmatrix}$. Note that the determinate $|Df(x,y)| = e^x(\cos^2(y) + \sin^2(y)) = e^x > 0$, therefore the Jacobian is always invertible. Furthermore, each term of the Jacobian is the product of continuous functions, thus making it continuous. Therefore the inverse function theorem holds everywhere. Note that generally the inverse is defined as $(\frac{1}{2}\log(u^2+v^2), \arctan(\frac{u}{v}))$. Note that issues don't arise the the inverse of the x coordinate, only the y, as $-\frac{\pi}{2} < \arctan(\frac{u}{v}) < \frac{\pi}{2}$. Therefore the inverse is well defined within $U = B((0,0), \frac{\pi}{2})$. For the point $(1,0)$ the same condition on y holds, however the disk will be centered at $(1,0)$, thus $V = B((1,0), \frac{\pi}{2})$. If we want to pick a larger open set then U or V, one could take $\mathbb{R} \times (-\frac{\pi}{2}, \frac{\pi}{2})$. Since log is onto for all real numbers uniquely, then the inverse defined for x works everywhere. Note that the given set is open since for arbitrary $(a,b) \in \mathbb{R} \times (-\frac{\pi}{2}, \frac{\pi}{2})$, for any chosen r such that $(b-r, b+r) \subseteq (-\frac{\pi}{2}, \frac{\pi}{2})$ then any points $(c,d) \in B((a,b),r)$ have that $|d-b| \leq \sqrt{(c-a)^2 + (d-b)^2} < r$, thus $d \in (b-r, b+r)$, and trivially $c \in \mathbb{R}$, thus our given set is open.

5.5.6

5.5.7