Given an example of a positive integer k that satisfies $2^k > k^{1000} + 1000000$.
$k = 13747$. I found this answer by getting the inequality in the form

$$\begin{aligned} 2^k &> k^{1000} + 10^6 \\ k^{1000} + 10^6 &> k^{1000} \\ 2^k &> k^{1000} \\ ln(2^k) &> ln(k^1 000) \\ k \cdot ln(2) &> 1000 \cdot ln(k) \\ \frac{k}{ln(k)} &> \frac{1000}{ln(2)} \end{aligned}$$

The bounding of the inequality by a smaller function can be done since the point at which 2^k exceeds k^{1000} will be much greater than 10^6. This fact can be justified by looking at when $k = 10$ and keeping in mind that 2^k is a monotonic funciton:

$$\begin{aligned} &2^{10} \approx 10^3 \\ &10^3 > 10^{1000} + 10^6 \end{aligned}$$

The above inequality doesn't make sense, and given that at such small values k^{1000} is exceeding the size of 10^6 makes it a negligible term. Now with the y value of $\frac{1000}{ln(2)}$ found we can now evaluate the function $\frac{x}{ln(x)}$ in a calculator. I understand that such a thing as the lambert W function exist, but I don't really know how to use that and this was easy enough. The function reached the x value of $x = 13746.809$. Now to get the nearest interger we take the ceiling and get a $k = 13747$. We can verify that this is the first interger to satisfy the inequality by looking wolfram alpha at the difference of $2^k - k^{1000}$ at $k = 13746$ and $k = 13747$. As predicted the values before and after as so massive as to make the 10^6 term insignificant.