- Proof of 10.16
  We must show that the function $f : (0, \infty) \to \mathbb{R}$ given by $f(x) = 1 + \frac{1}{x}$ is increasing on the interval $[1, \infty)$.. By definition of increasing, we must show for all $x, y \in [1, \infty)$ if $x < y$ then $f(x) < f(y)$. Suppose $x, y \in [1, \infty)$ and $x < y$. By definition of f we must show $x + \frac{1}{x} < y + \frac{1}{y}$. Since $x \in [1, \infty)$, then x has a minimum of 1. Therefore $1 \leq x$. Inverting both sides yields $\frac{1}{x} \leq 1$. Since $1 \leq x, x < y$, then by transitivity $\frac{1}{x} < y$. Multiplying both sides by x yields $1 < xy$. Subtracting 0 yields $0 < xy - 1$. Therefore multiplying $x < y$ by $xy - 1$ will maintain the inequality. Therefore by algebraic manipulation:

$$\begin{gathered}
x(xy-1) < y(xy-1) \\
x^2y - x < xy^2 - y \\
x^2y + y < xy^2 + x \\
y(x^2+1) < x(y^2+1) \\
\frac{x^2+1}{x} < \frac{y^2+1}{y} \\
x + \frac{1}{x} < y + \frac{1}{y}.
\end{gathered}$$

- Proof of 10.17
  We must show that the function $g : (0, \infty) \to \mathbb{R}$ given by $g(x) = \sqrt{x}$ is increasing. By definition of increasing we must show for all $x, y \in (0, \infty)$ if $x < y$ then $\sqrt{x} < \sqrt{y}$. We are going to prove this by contraposition. We must show for all $x, y \in (0, \infty)$ if $\sqrt{x} \geq \sqrt{y}$ then $x \geq y$. Suppose $x, y \in (0, \infty)$, $\sqrt{x} \geq \sqrt{y}$. We must show $x \geq y$. Since for all a, b, c, d if $a \leq b, c \leq d$ then $ac \leq bd$, and we have $\sqrt{y} \leq \sqrt{x}$, then we have $\sqrt{y}\sqrt{y} \leq \sqrt{x}\sqrt{x} \Rightarrow |y| \leq |x|$. Since x, y are greater than 0, then $|y| \geq |x| \Rightarrow y \geq x$.