

Use a joint truth table to compare the following logical expressions. For which pairs or expressions does one logically follow from the other. Which pairs are logically equivalent?

$$E = (A \implies B) \implies C.$$

$$F = A \implies (B \implies C).$$

$$G = (B \implies C) \implies (A \implies C).$$

$$H = (A \implies C) \implies (B \implies C).$$

$$I = (A \implies C) \iff (B \implies C).$$

		a	b	c	$((a \rightarrow b) \rightarrow c)$			
		1	1	1	1	1	1	1
		1	1	0	1	1	1	0
		1	0	1	1	0	0	1
E:		1	0	0	1	0	0	1
		0	1	1	0	1	1	1
		0	1	0	0	1	1	0
		0	0	1	0	1	0	1
		0	0	0	0	1	0	0
		a	b	c	$(a \rightarrow (b \rightarrow c))$			
		1	1	1	1	1	1	1
		1	1	0	1	0	1	0
		1	0	1	1	1	0	1
F:		1	0	0	1	1	0	1
		0	1	1	0	1	1	1
		0	1	0	0	1	1	0
		0	0	1	0	1	0	1
		0	0	0	0	1	0	1
		a	b	c	$((b \rightarrow c) \rightarrow (a \rightarrow c))$			
		1	1	1	1	1	1	1
		1	1	0	1	0	0	1
		1	0	1	0	1	1	1
G:		1	0	0	0	1	0	0
		0	1	1	1	1	1	1
		0	1	0	1	0	0	1
		0	0	1	0	1	1	1
		0	0	0	0	1	0	1
		a	b	c	$((a \rightarrow c) \rightarrow (b \rightarrow c))$			
		1	1	1	1	1	1	1
		1	1	0	1	0	0	1
		1	0	1	1	1	1	1
H :		1	0	0	1	0	0	1
		0	1	1	0	1	1	1
		0	1	0	0	1	0	0
		0	0	1	0	1	1	1
		0	0	0	0	1	0	0

	a	b	c	$((a \rightarrow c) \leftrightarrow (b \rightarrow c))$				
	1	1	1	1	1	1	1	1
	1	1	0	1	0	0	1	1
	1	0	1	1	1	1	1	0
I :	1	0	0	1	0	0	0	0
	0	1	1	0	1	1	1	1
	0	1	0	0	1	0	0	0
	0	0	1	0	1	1	1	0
	0	0	0	0	1	0	1	0

Since $I = H \wedge G$, $I \implies H$ and $I \implies G$ since as shown above none of the statements are tautologies, therefore since $I = H \wedge G$, then $H \wedge G \implies H$ and $H \wedge G \implies G$. Which happen to be tautologies, thus they logically follow. However they aren't logically equivalent as $H \implies H \wedge G$ isn't. E is a logical implication of H, and E is a logical implication of F. None of the pairs are logically equivalent.