5.3.8 Assume $g : (a,b) \to \mathbb{R}$ is differentiable at a point c. If $g'(c) \neq 0$ then there exists a δ-neighborhood $V_\delta(c) \subseteq (a,b)$ such that for every $x \in V_\delta(c) \backslash \{c\}$, $g(x) \neq g(c)$.
Proof: Assume $g'(c) \neq 0$. Suppose for contradiction that for every δ-neighborhood $V_\delta(c) \subseteq (a,b)$ there exists $x \in V_\delta(c) \backslash \{c\}$, $g(x) = g(c)$. If we consider $V_{\frac{1}{n}}(c) \backslash \{c\}$ then for each n we have an $x_n \in V_{\frac{1}{n}}(c)$ such that $g(x_n) = g(c)$. Therefore $\lim \frac{g(x_n)-g(c)}{x_n - c} = 0$, contradicting $g'(c) \neq 0$.