Alex Valentino

Let V, W be finite dimensional vector spaces, where $\beta = (v_1, \dots, v_n)$, $\Gamma = (w_1, \dots, w_n)$ are vector spaces of V and W respectively. Let $dim(W) = dim(V) = n$. We must show that $V \cong W$. Let $L : V \to W$ be the linear transformation given by $L(v_i) = w_i$ for all $i \in [n]$. Since every basis vector in V is mapped to a non-zero vector in W, then $rank(L) = n$, therefore by the rank nullity theorem $nullity(T) = 0$. Therefore L is injective. Since L is injective and $dim(V) = dim(W)$, then L is onto. Therefore T is a bijection, thus $V \cong W$.