For each of the following assertions, identify the free variables and the bound variables. Briefly explain your reasoning.

- *(Variables: integers $n, m, r,$) For every positive integer n the set* $\{m \in \mathbb{Z} : m^2 - r \mid n\}$
  n is bound and m, r are free. Clearly n is quantified with the "for every" statement and m and r are the variables for which inserting values for will turn this predicate into an assertion.

- *(Variables: real numbers x, y, ϵ, and subset $S \subseteq \mathbb{R}$) x is not a member of S and for all real numbers $\epsilon > 0$, there exist a member y of S such that $|x - y| \geq \epsilon$.* ϵ, y are bound as they come included with "for all" and "for every" respectively. x, S are the free variables as this statement is evaluating on x and that evaluation is being stored in the set S.

- *(Variables: functions $f, g$and h and real number x) There is a function g and a functon h such that for every real number x, $f(x) = g(x) + h(x)$ and $g(x) = g(-x)$ and $h(-x) = -h(x)$.*
  h, g, x are bound variables, as for the functions h, g the "there is" statement could have been rewritten as "there exist" and not change the meaning, and for the variable has a direct "for every" statement associated with it. f is the free variable, as all the other variables are constructed around this arbitrary f, thus it is the free variable on which this statement goes from a predicate to an assertion.