Suppose that $(b_1, b_2, \ldots, b_k)$ is an arbitrary list of numbers. Prove that $\prod_{i=1}^{k}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k\}} \prod_{j\in S} b_j$.

We must show for all lists $(b_1, \ldots, b_k)$ of real numbers that $\prod_{i=1}^{k}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k\}} \prod_{j\in S} b_j$. Suppose $(b_1, \ldots, b_k)$ is a list of real numbers. We must show that $\prod_{i=1}^{k}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k\}} \prod_{j\in S}$. By principal of mathematical induction for lists of real numbers $(e_1, \ldots, e_m)$ if $m < k$ then $\prod_{i=1}^{m}(1+e_i) = \sum_{S\subseteq\{1,\ldots,m\}} \prod_{j\in S} e_j$. We now have two cases:

- Assume $k = 1$. Then we have the list (b_1). Therefore $\sum_{S\subseteq\{1\}} \prod_{j\in S} b_j = \sum_{\emptyset,\{1\}} \prod_{j\in S} b_j = \prod_{j\in\emptyset} b_j + \prod_{\in\{1\}} b_j = 1 + b_1 = \prod_{i=1}^{1}(1+b_i)$.

- Assume $k > 1$. Since $k-1 < k$ by the induction hypothesis we have for $(b_1, \ldots, b_{k-1})$ that $\prod_{i=1}^{k-1}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k-1\}} \prod_{j\in S} b_j$. Since every subset of $\{1, \ldots, n\}$ either includes or excludes k, and removing k from S makes S a subset of $\{1, \ldots, k-1\}$, then we must show $\prod_{i=1}^{k}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k-1\}} \prod_{j\in S} b_j + \sum_{\substack{(S\setminus k)\subseteq\{1,\ldots,k-1\} \\ k\in S}} \prod_{j\in S} b_j$. Since for the second sum k is always in S, then the b_k term will always be present in every term of the sum, therefore we must show $\prod_{i=1}^{k}(1+b_i) = \sum_{S\subseteq\{1,\ldots,k-1\}} \prod_{j\in S} b_j + b_k \sum_{S\subseteq\{1,\ldots,k-1\}} \prod_{j\in S} b_j$. Therefore we must show $\prod_{i=1}^{k}(1+b_i) = (1+b_k) \sum_{S\subseteq\{1,\ldots,k-1\}} \prod_{j\in S} b_j$. Since $k-1 < k$, by the induction hypothesis we must show $\prod_{i=1}^{k}(1+b_i) = (1+b_k) \prod_{i=1}^{k-1}(1+b_i)$. Therefore by reindixing the product $(1+b_k)\prod_{i=1}^{k-1}(1+b_i)$ we have $(1+b_k)\prod_{i=1}^{k-1}(1+b_i) = \prod_{i=1}^{k}(1+b_i)$.