5.2.8 (a) If a derivative function is not constant, then the derivative must take on some irrational values.

True. Suppose $f : A \to \mathbb{R}$, $f'(x)$ exists, $f'(x) \neq c$. Since A by definition is an interval for f' to be well defined then if A is a closed interval $[a, b]$ then by the darboux theorem f' attains all values between $f'(a)$ and $f'(b)$. Since $f'(a)$ and $f'(b)$ are real numbers then by the density of the irrationals in $\mathbb{R}$ there exists i such that (WLOG) $f'(a) < i < f'(b)$. Therefore f' always attains an irrational. If the interval is open, then we can find the midpoint, $m = \frac{a+b}{2}$, take the open ball around m, $V_\epsilon(m)$ guaranteed by it's entry in an open set, then take take the set $[m - \epsilon/2, m + \epsilon/2]$, which we know contain the end points as $m - \epsilon < m - \epsilon/2$ and $m + \epsilon/2 > m + \epsilon/2$, thus constructing a closed interval on which f' is defined.

(b) If f' exists on an open interval, and there is some point c where $f'(c) > 0$, then there exists a δ-neighborhood $V_\delta(c)$ around c in which $f'(x) > 0$ for all $x \in V_\delta(c)$). False, consider the function

$$f(x) = \begin{cases} x + x^2 \sin(e^{1/|x|}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

at 0. If we evaluate the derivative manually we find $\lim_{x\to 0} 1 + x \sin(e^{1/|x|})$, which similar to evaluating $1 + x \sin(1/x)$ converges to 1 by the squeeze theorem. However with the actual evaluation of $f'(x) = 1 + 2x \sin(e^{1/|x|}) - e^{1/|x|} \cos(e^{1/|x|})$ clearly the $e^{1/|x|} \cos(e^{1/|x|})$ term will fluctuate wildly when approaching 0.

(c) True. Suppose for contradict

(d) False, take $f(x) = \frac{x^2 - x}{x}$. Clearly $\lim_{x\to 0} \frac{x^2 - x}{x} = \lim_{x\to 0} \frac{x(x-1)}{x} = -1$, however directly evaluating $f(0)$ is undefined.