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Suppose $f : B \to C$ and $g : A \to B$.

- Prove or disprove: If $f \circ g$ is onto it's target then f is onto.
  Proof: We must show that f is onto. Then by the definition of onto we must show for all $y \in C$ there exist $b \in B$ such that $f(b) = y$. Suppose $f \circ g$ is onto. Then by definition for all $y \in C$ then there exist $x \in A$ such that $f \circ g(x) = y$. By definition of function composition $f \circ g(x) = f(g(x))$. Since the x in A exist, we can define $b = g(x)$. Therefore f satisfies the definition of being an onto function.

- Prove or disprove: If $f \circ g$ is onto it's target then g is onto.
  Disproof: Let $g : \mathbb{R}_{\geq 0} \to \mathbb{R}$ be given as $g(x) = x^2$ and $f \circ g : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be given by $f \circ g(x) = x^4$. Since there exist no value in $R_{\geq 0}$ in which $x^2 < 0$ then clearly g is not onto. At the same time for all $y \in \mathbb{R}_{\geq 0}$ we can get $y^{\frac{1}{4}} = x$, which allows us to write $f \circ g(x) = x^4 = (y^{\frac{1}{4}})^4 = |y| = y$, which demonstrates that $f \circ g$ is onto despite g not being so.