4.2.8 Assume $f(x) \geq g(x)$ for all $x \in A \subseteq \mathbb{R}$ on which f, g are defined. Show that for any limit point $c \in A$ we must have

$$\lim_{x\to c} f(x) \geq \lim_{x\to c} g(x).$$

Let $\lim_{x\to c} f(x) = L, \lim_{x\to c} g(x) = M$. Suppose $(x_n) \subseteq A, (x_n) \to c$. Therefore by theorem 4.2.3, $\lim f(x_n) \to L, \lim g(x) \to M$. Since $(x_n) \subseteq A$ then $f(x_n) \geq g(x_n)$ for all $n \in \mathbb{N}$. Therefore by the order limit theorem, since $f(x_n) \geq g(x_n)$ then $L \geq M$. Therefore $\lim_{x\to c} f(x) \geq \lim_{x\to c} g(x)$