1.3.2 (a) A real number s is a greatest lower bound for a set $A \subseteq \mathbb{R}$ if

i. s is a lower bound for A

ii. if for every lower bound b, $b \leq s$

(b) Lemma 1.3.7 for infimums:
Suppose $s \in \mathbb{R}, A \subseteq \mathbb{R}$ where s is a lower bound for A, then $inf(A) = s$ if and only if for all $\epsilon > 0$ there exists $a \in A$ such that $a < s + \epsilon$.

i. ($\Rightarrow$) Suppose $s = inf(A), \epsilon > 0$. Then the number $s + \epsilon$ is not a lower bound as that would contradict being less than or equal to s. Since it is not a lower bound on A, then there is a smaller element a of A. Therefore $a < s + \epsilon$

ii. ($\Leftarrow$) Suppose for all $\epsilon > 0$ there exists $a \in A$ such that $a < \epsilon + s$. We must show that $s = inf(A)$. Since s is already a lower bound, we simply need to show that any lower bound is less than or equal to s. Suppose b is a lower bound on A. Since we have already shown that for any number greater than s, it can't be a lower bound, then conversely for b, since b is a lower bound then $b \leq s$.

1.3.3 Suppose $A, B \subseteq \mathbb{R}$, $sup(A) < sup(B)$. We must show that there exists $b \in B$ such that b is an upper bound of A. By theorem 1.3.7 for all $\epsilon > 0$ there exists $b \in B$ such that $sup(b) - \epsilon < b$. Since $sup(A) < sup(B)$ then $0 < sup(B) - sup(A)$. Therefore let $\epsilon = sup(B) - sup(A)$. Therefore there exists $b \in B$ such that $sup(A) < b$. By definition of supremum b is an upper bound on A.

1.3.9 (a) A finite, non-empty set always contains its supremum
True

(b) If $a < L$ for every element a in A then $sup(A) < L$.
False, if $L = sup(A)$ and A does not contain it's supremum then we satisfy the preposition, however $sup(A) < sup(A)$ is clearly false.

(c) If A and B are sets with the property that $a < b$ for every $a \in A$ and $b \in B$, then it follows that $sup(A) < inf(B)$.
False, if $A = [0, 1), B = (1, 2]$ then it is true for all $a \in A, b \in B$ that $a < b$. However $sup(A) = 1, inf(B) = 1$ as by theorem 1.3.7 subtracting any $\epsilon > 0$, there exists $a \in A$ such that $1 - \epsilon < a$ and similarly for the result proved in exercise 1.3.2. Therefore $sup(A) = inf(A)$.

(d) True

(e) True, proved in the previous exercise above.