Introduction
DS351

Main principle

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- Response Y

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Assumption: There's some function f and error ϵ such that

$$Y = f(X) + \epsilon$$

Here, Y and ϵ are **random variables**.

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Performance of $\hat{f}$ is measured by

$$\begin{aligned}\mathbf{E}(Y - \widehat{Y})^2 &= \mathbf{E}[f(X) + \epsilon - \hat{f}(X)]^2 \\ &= [f(X) - \hat{f}(X)]^2 + \mathrm{Var}(\epsilon).\end{aligned}$$

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- Find the relationship between X_i and Y e.g. would increasing the value of X_i increases the value of Y?

Bias-Variance trade-off

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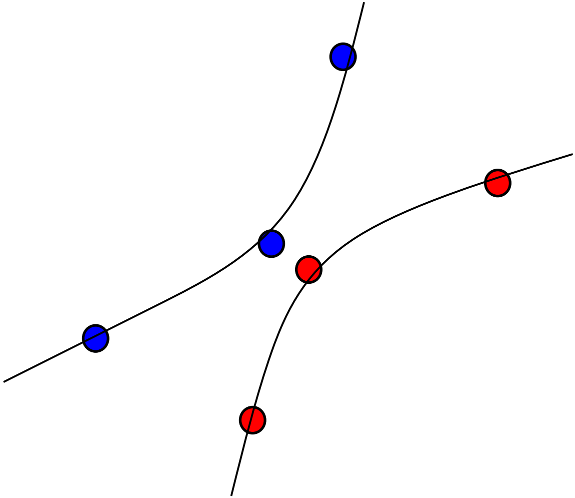
Treat $\hat{f}$ as **random** depending on the training set that we sampled from the population. It can be shown that

$$\mathbf{E}\big(y_0 - \hat{f}(x_0)\big)^2 = \mathrm{Var}(\hat{f}(x_0)) + [\mathrm{Bias}(\hat{f}(x_0))]^2 + \mathrm{Var}(\epsilon).$$

Example



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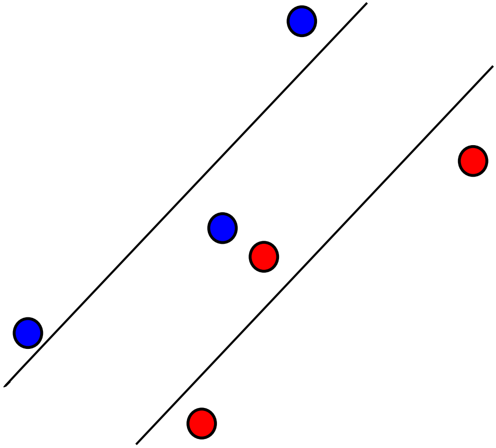

Bias-Variance tradeoff