

**Fall 2025**

## **CSCI 2011 Assignment 3: Proofs, Sets, and Functions**

**Due Tuesday 9/30/2025 at 11pm**

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Answer all 6 questions worth 2.5 points each for a total of 15 points

### **Question1:**

For this question, it can be helpful to lookup the answer to question 29 in section 1.6 from the solutions of odd numbered questions and mimic the way that the universal instantiation/ generalization and existential instantiation/ generalization are employed.

28. Use rules of inference to show that if  $\forall x (P(x) \vee Q(x))$  and  $\forall x ((\neg P(x) \wedge Q(x)) \rightarrow R(x))$  are true, then  $\forall x (\neg R(x) \rightarrow P(x))$  is also true, where the domains of all quantifiers are the same.

### **Question 2:**

Show that the product of two of the numbers  $65^{1000} - 8^{2001} + 3^{177}$ ,  $79^{1212} - 9^{2399} + 2^{2001}$ , and  $24^{4493} - 5^{8192} + 7^{1777}$  is nonnegative. Is your proof constructive or nonconstructive? [*Hint: Do not try to evaluate these numbers!*]

### **Question 3:**

Given that every even integer  $n$  can be written as  $n = 10k + r$  where  $r = 0, 2, 4, 6, 8$ , you are asked to prove that the square of every such even integer ends in 0, 4, or 6.

### **Question 4**

22. Determine whether each of these functions is a bijection from  $\mathbf{R}$  to  $\mathbf{R}$ .

- a)  $f(x) = -3x + 4$
- b)  $f(x) = -3x^2 + 7$
- c)  $f(x) = (x + 1)/(x + 2)$
- d)  $f(x) = x^5 + 1$

### **Question 5:**

22. Show that if  $A$  and  $B$  are sets with  $A \subseteq B$ , then

- a)  $A \cup B = B$ .
- b)  $A \cap B = A$ .

Hint for (b), use proof by contradiction.

**Question 6:**

2. Use set builder notation to give a description of each of these sets.

a)  $\{0, 3, 6, 9, 12\}$

b)  $\{-3, -2, -1, 0, 1, 2, 3\}$

c)  $\{m, n, o, p\}$