MPA Notes and Equations

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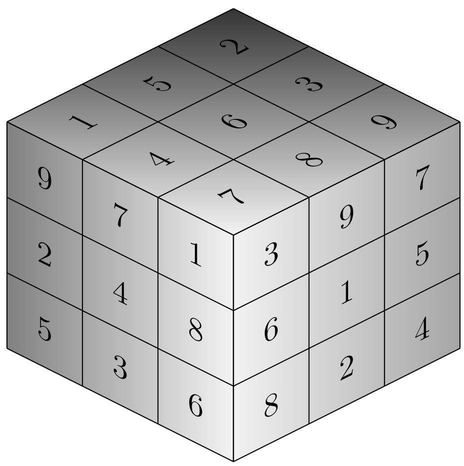


*Module Leader Mechanical Principles A

Basic Concepts and Equations for Mechanical Principles A written with LaTeX are Given in This Document!

1 Scalars and Vectors

A **scalar** has only **magnitude**, that is, it can be fully described by a single number.
Examples: Temperature, Distance, Speed, Mass.

A **vector** has both **magnitude and direction**, that is, as well as a number to quantify magnitude, we also need a way to indicate direction to describe a vector.
Examples: Displacement, Velocity, Force, Momentum. We can describe the

direction of a vector in several ways.
If the motion is only in one direction, for example, only along the x-axis and we take the usual convention of positive being to the right then we can say that a vector with a positive value (x +ve) has a direction to the right and a vector with a negative value (x -ve) has a direction to the left.
If the motion is in two directions we can; give the direction as an angle from the horizontal or indicate the direction by giving the vector's x and y components or by using unit vectors i and j.

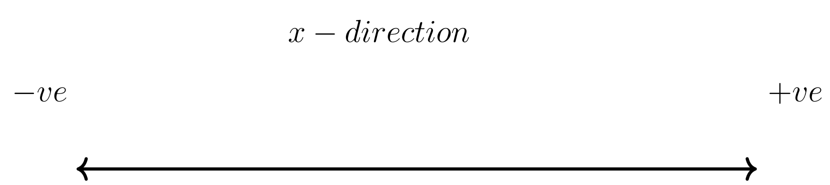


Figure 1 Indicating direction in one dimension, in this case the 'x-direction'.

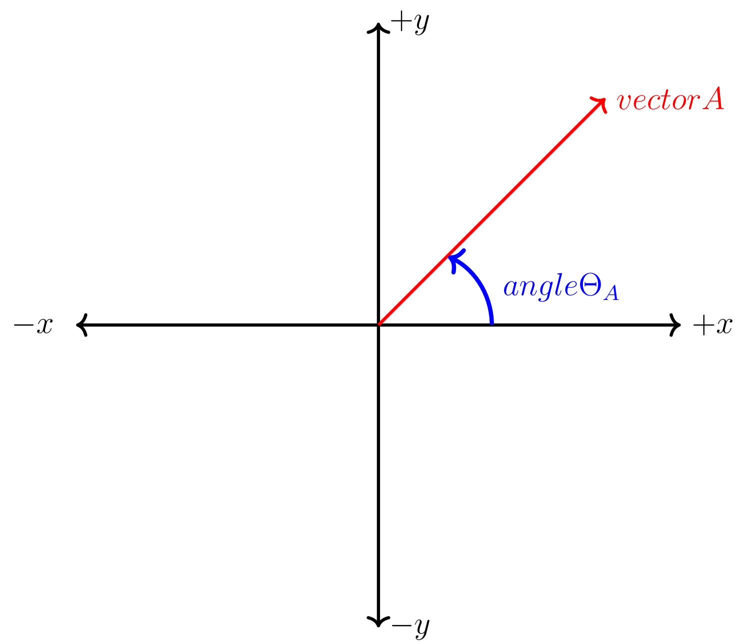


Figure 2 Indicating direction in two dimensions, the magnitude of the vector is shown by the length of the line in red and the direction of the vector is indicated by the angle in blue.

The component form of vector A in two dimensions can be given using unit vectors as:

$$\overrightarrow{A} = A_x\hat{i} + A_y\hat{j} \tag{1}$$

The Magnitude of vector A:

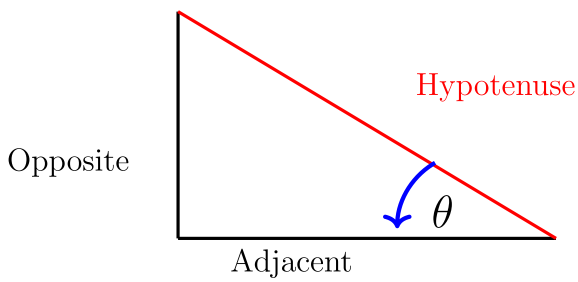
$$A = \sqrt{A_x^2 + A_y^2} \tag{2}$$

The direction angle of vector A:

$$\Theta_A = tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3}$$

It is helpful to remember the trigonometry functions; sine, cosine and tangent when dealing with vectors and one way to help remember is the mnemonic SohCahToa:

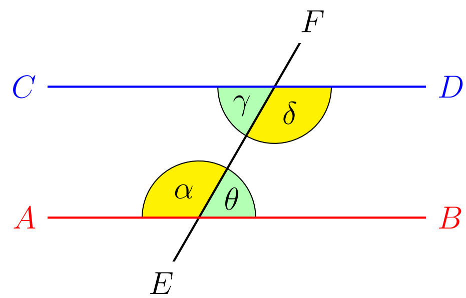
Mnemonic	Trig function	Formula	Ratio
SOH	Sine	$sin\theta = \frac{O}{H}$	Opposite over Hypotenuse
CAH	Cosine	$cos\theta = \frac{A}{H}$	Adjacent over Hypotenuse
TOA	Tangent	$tan\theta = \frac{O}{A}$	Opposite over Adjacent



In general when dealing with vectors [except when using the cross product] the problem can be tackled by:

1. break the vectors into their x and y components
2. separate and gather the x components and the y components
3. perform the operation or analysis separately for x and y components
4. write the separate results for x and y components
5. combine these x and y results together using vector addition

The following diagram might also help in selecting the correct angles.



If we assume that lines AB and CD are parallel, i. e., $AB \parallel CD$, then $\alpha = \delta$ and $\theta = \gamma$.

2 MOTION ALONG A STRAIGHT LINE

We can in many cases approximate an objects motion in space as being along a straight line. In which case we can determine aspects of its motion such as displacement, velocity and acceleration using relatively simple equations.

Displacement Δx is the change in position of an object from its initial position to its final position.

$$\Delta x = x_f - x_i \tag{4}$$

If an object moves in a number of individual 'steps' then we can calculate its *Total Displacement*

$$\Delta x_{Total} = \Sigma \Delta x_i \tag{5}$$

If an object moves from position x_1 at a time t_1 to a new position x_2 at a time t_2 we can determine its *Average Velocity*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{6}$$

Unless the motion is with constant velocity is will be different from the velocity at any particular instance in time, that is the *Instantaneous Velocity*

$$v(t) = \frac{dx(t)}{dt} \tag{7}$$

In a very similar way we can determine *Average Acceleration*

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{8}$$

and *Instantaneous Acceleration*

$$a(t) = \frac{dv(t)}{dt} \tag{9}$$

If the motion is with ***constant acceleration*** then we can use a set of equations known as the kinematic equations.

Velocity from Acceleration

$$v = v_0 + at \tag{10}$$

Position from velocity and acceleration

$$x = x_0 + v_0 + \frac{1}{2}at^2 \tag{11}$$

Velocity from distance

$$v^2 = v_0^2 + 2a(x - x_0) \tag{12}$$

When considering object falling under the effect of gravity, that is with constant acceleration $-g$ where $g = 9.8m/s^2$, then a very similar set of equations can be used replacing a with $-g$, for example;

Height of free fall from starting position y_0 and initial velocity v_0

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{13}$$

3 MOTION IN TWO AND THREE DIMENSIONS

The position of a object in 3D space can be described by its *Position Vector*

$$\overrightarrow{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{14}$$

Consider then a movement of an object from a position at t_1 to a new position at time t_2, the displacement of that object can be determined by the vector addition of the two position vectors, giving the *Displacement Vector*

$$\Delta\overrightarrow{r} = \overrightarrow{r}(t_2) - \overrightarrow{r}(t_1) \tag{15}$$

and the *Average Velocity* can be found from;

$$\overrightarrow{v_{avg}} = \frac{\overrightarrow{r}(t_2) - \overrightarrow{r}(t_1)}{t_2 - t_1} \tag{16}$$

As with motion in 1D the *Instantaneous Velocity Vector* in 3D is

$$\overrightarrow{v}(t) = \frac{d\overrightarrow{r}}{dt} \tag{17}$$

As with other vectors we can write the Velocity in terms of components;

$$\overrightarrow{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k} \tag{18}$$

This approach is very useful in problem solving where each [*orthogonal*] component x, y and z can be treated independently and only re-combined at the end to give a final result. If acceleration is constant then we can use the kinematic equations independently in each component direction.

Where*Instantaneous Acceleration*

$$\overrightarrow{a}(t) = \frac{d\overrightarrow{v}(t)}{dt} \tag{19}$$

4 NEWTON'S LAWS OF MOTION

The study of how forces affect the motion of objects is known as *dynamics*, up to now we have restricted the study to equations the motion or *kinematic equations*. The most important relationships and equations in this section are based on **Newton's Laws of Motion** and to aid solve problems we will use *free body diagrams*. In many problems we can treat the mass of a

real object as acting at a point [centre of mass] and the forces acting on the object as force vectors acting on that point. This forms the basis of drawing free body diagrams.

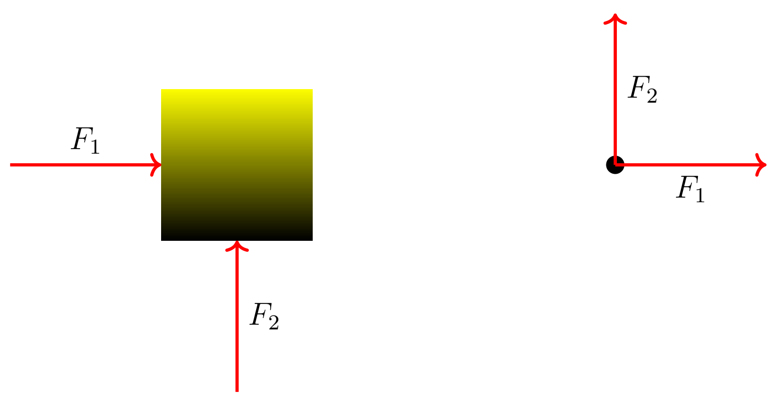


Figure 3 Two forces $\overrightarrow{F}_1$ and $\overrightarrow{F}_2$ acting on a box and the representation as a free body diagram.

This is a very simple case but we can use free body diagrams in more complex problems to help to visualise the magnitudes and directions of the forces acting on a body. Once this is clear we can calculate the *Net External Force*:

$$\overrightarrow{F}_{net} = \Sigma\overrightarrow{F} = \overrightarrow{F}_1 + \overrightarrow{F}_2 + ... \tag{20}$$

Problem-Solving Strategy: Drawing Free-Body Diagrams

1. Draw the object under consideration. If you are treating the object as a particle, represent the object as a point. Place this point at the origin of an xy-coordinate system.

2. Include all forces that act on the object, representing these forces as vectors. However, do not include the net force on the object or the forces that the object exerts on its environment.

3. Resolve all force vectors into x- and y-components.

4. Draw a separate free-body diagram for each object in the problem.

Newton's First Law of Motion

`A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force.`

We can write Newton's First Law more mathematically

$$\overrightarrow{v} = \text{constant when} \overrightarrow{F}_{net} = \overrightarrow{0}\,N \tag{21}$$

Newton's Second Law of Motion

`The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportional to its mass`

This can be written in equation form as:

$$\overrightarrow{a} = \frac{\overrightarrow{F}_{net}}{m} \tag{22}$$

It is more usual to see Newton's Second Law, written in *Vector Form* as:

$$\overrightarrow{F}_{net} = \Sigma\overrightarrow{F} = m\overrightarrow{a} \tag{23}$$

We can also write Newton's Second Law in *Component Form*, where we treat the x,y and z components separately:

$$\Sigma\overrightarrow{F}_x = m\overrightarrow{a_x},\ \ \Sigma\overrightarrow{F}_y = m\overrightarrow{a_y},\ \ \Sigma\overrightarrow{F}_z = m\overrightarrow{a_z} \tag{24}$$

By recognising that acceleration a can be written as $\frac{dv}{dt}$ and momentum $p = mv$, we can write Newton's Second Law in *Momentum Form*

$$\overrightarrow{F}_{net} = \frac{d\overrightarrow{p}}{dt} \tag{25}$$

That is, another way of expressing Newton's Second Law is that the net force acting on an object is equal to the rate of change of momentum of that object.

Newton's Second Law is used in our definition of **weight**, where in Vector Form, the *Definition of Weight* can be given as:

$$\overrightarrow{w} = m\overrightarrow{g} \tag{26}$$

Or simply in Scalar Form:

$$w = mg \tag{27}$$

An important thing to notice here is that weight is a force and will have SI units [N].

Newton's Third Law of Motion

`Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts`

This can be written in equation form as:

$$\overrightarrow{F}_{AB} = -\overrightarrow{F}_{BA} \tag{28}$$

Newton's Third Law of Motion is used commonly in the following:

Normal force on an object resting on a horizontal surface, vector form.

$$\overrightarrow{N} = -m\overrightarrow{g} \tag{29}$$

Normal force on an object resting on a horizontal surface, scalar form.

$$N = -mg \tag{30}$$

Normal force on an object resting on an inclined surface θ from the horizontal, in scalar form.

$$N = -mgcos\theta \tag{31}$$

5 APPLICATION OF NEWTON'S LAWS

When applying Newton's Laws to solving problems it is useful to follow a consistent strategy or process.

Problem-Solving Strategy: Applying Newton's Laws of Motion

1. Identify the mechanical principles involved; list the information and values given and the unknowns to be calculated.
2. Sketch the problem, using arrows to represent all forces.
3. Determine the "system of interest". Represent this in a free-body diagram with component x, y and z forces.
4. Apply Newton's second law to solve the problem in component form. (If necessary, apply appropriate kinematic equations for motion along a straight line).
5. Treat as vectors to get a result.
6. Check that the solution is reasonable.

In these problems we may also have to deal with the forces associated with some form of resistance or friction which opposes the motion. The two most common are friction between two surfaces and the resistance due to motion through a fluid [drag].

Magnitude of static friction

$$f_s \leq \mu_s N \tag{32}$$

Magnitude of kinetic friction

$$f_k \leq \mu_k N \tag{33}$$

Drag Force

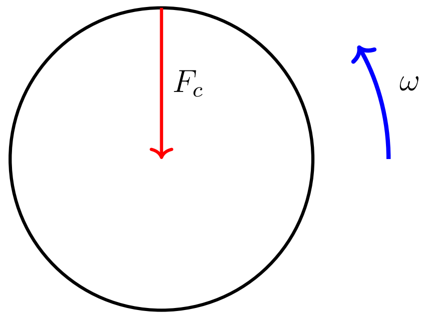
$$F_D = \frac{1}{2} C_D \rho A v^2 \tag{34}$$

Stokes' Law

$$F_s = 6\pi r \eta v \tag{35}$$

A body undergoing **Uniform Circular Motion** will experience *Centripetal Force*

$$F_c = m\frac{v^2}{r} = mr\omega^2 \tag{36}$$



where ω is the *angular velocity* and r is the *radius of motion*

$$\omega = \frac{\Delta\theta}{\delta t} \tag{37}$$

6 WORK AND KINETIC ENERGY

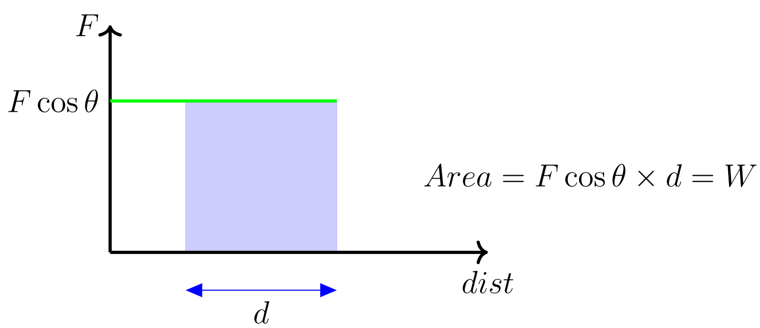
The work done on a system by a constant force is the product of the component of the force in the direction of motion times the distance through which the force acts.

Work Done by a constant force in line with displacement

$$W = F \cdot d \tag{38}$$

$$W = Fd\cos\theta \tag{39}$$

We can show this graphically.



If the force is not constant and or the path from A to B is not linear we can use the Work done by a force over an infinitesimal displacement;

$$dW = \overrightarrow{F} \cdot d\overrightarrow{r} = |\overrightarrow{F}||d\overrightarrow{r}|\cos\theta \tag{40}$$

and then calculate the Work done by a force acting along a path from A to B;

$$W_{AB} = \int_A^B \overrightarrow{F} \cdot d\overrightarrow{r} \tag{41}$$

If an object is lifted straight up from A to B against Earth's gravity near its surface at constant speed, then the force needed to lift it is equal to its weight mg . The Work done going from A to B by Earth's gravity near its surface is then;

$$W_{grav,AB} = -mg(y_B - y_A) \tag{42}$$

If we take the datum $y_A = 0$ and y_B is then some height h above the datum, we can make two related statements. One, the work done in raising the object of mass m to a height h is:

$$W = -mgh \tag{43}$$

and the gain in *Potential Energy* is:

$$PE = mgh \tag{44}$$

In a similar manner the work done going from A to B by one-dimensional spring force is:

$$W_{spring,AB} = -\frac{1}{2}k(x_B^2 - x_A^2) \tag{45}$$

and the gain in potential energy of the spring is:

$$PE_{spring,AB} = \frac{1}{2}k(x_B^2 - x_A^2) \tag{46}$$

The energy associated with a moving object is called *kinetic energy*. The translational kinetic energy (KE) of a mass m moving at a speed v . (Translational kinetic energy is distinct from rotational kinetic energy, which is considered later) can be written as:

$$KE = \frac{1}{2}mv^2 \tag{47}$$

The kinetic energy of an object can be related to its momentum

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{48}$$

The *Work-Energy Theorem* states that the net work on a system equals the change in its KE.

$$W_{net} = \frac{1}{2}mV^2 \tag{49}$$

We can also define here Power as a rate of doing work:

$$P = \frac{dW}{dt} \tag{50}$$

and Power as the dot product of force and velocity:

$$P = \overrightarrow{F} \cdot \overrightarrow{v} \tag{51}$$

7 LINEAR MOMENTUM AND COLLISIONS

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\overrightarrow{p} = m\overrightarrow{v} \tag{52}$$

Newton's second law of motion can also be written in terms of momentum, where the net external force F_{net} equals the change in momentum Δp of a system divided by the time taken for the change Δt.

$$F_{net} = \frac{\Delta p}{\Delta t} \tag{53}$$

We can rearrange this to give:

$$\Delta p = F_{net}\Delta t \tag{54}$$

The term $F_{net}\Delta t$ is know as *impulse* and is often given the symbol J.

In vector notation Impulse could be written as:

$$\overrightarrow{J} = \int_{t_i}^{t_f} \overrightarrow{F}(t)dt = \overrightarrow{J} = \overrightarrow{F}_{ave}\Delta t \tag{55}$$

and the Impulse-momentum theorem would then be written as:

$$\overrightarrow{J} = \Delta\overrightarrow{p} \tag{56}$$

we can then write the Average force from momentum as:

$$\overrightarrow{F}(t) = \frac{d\overrightarrow{p}}{dt} \tag{57}$$

An important property of momentum is that it is *conserved* in an isolated system. So we can write a Generalised conservation of momentum.

$$\sum_{j=1}^{N} \overrightarrow{p}_j = constant \tag{58}$$

An isolated system is defined to be one for which the net external force is zero ($F_{net.ext} = 0$), where:

$$\overrightarrow{F}_{net.ext} = \sum_{j=1}^{N} \frac{d\overrightarrow{p}_j}{dt} \tag{59}$$

This fact that momentum is conserved allows us to use this in the study of collisions. We will look at two basic types of collision

1. elastic collisions

2. non-elastic collisions

Beginning with elastic collisions we can define a elastic collision as one that conserves internal kinetic energy. If we consider two objects in an elastic collision with initial momentum p_1 and p_2 and final momentum p_1' and p_2' then the conservation of momentum means:

$$p_1 + p_2 = p_1' + p_2' \tag{60}$$

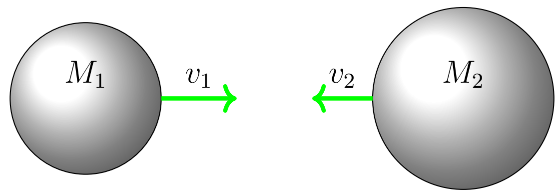
or

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{61}$$

and because in an elastic collision internal kinetic energy is conserved

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \tag{62}$$

BEFORE

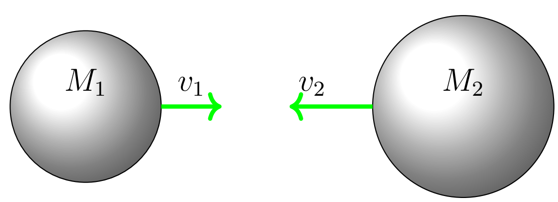


AFTER

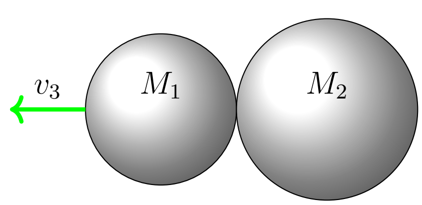
An ***inelastic*** collision is one in which the internal kinetic energy changes (it is *not* conserved). A collision in which the objects stick together is sometimes called "perfectly inelastic."

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3 \tag{63}$$

BEFORE



AFTER



Collisions in 2D can be analysed using vectors and by treating the x-components and the y-components independently. For example for an ellastic collision.

$$p_{x1} + p_{x2} = p'_{x1} + p'_{x2} \tag{64}$$

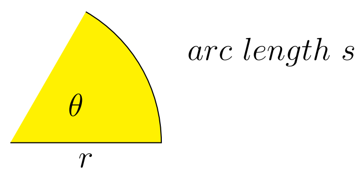
and

$$p_{y1} + p_{y2} = p'_{y1} + p'_{y2} \tag{65}$$

8 FIXED AXIS ROTATION

The *Angular Position* θ of an object rotating at a radius r which is displaced through an arc length s is given by:

$$\theta = \frac{s}{r} \tag{66}$$

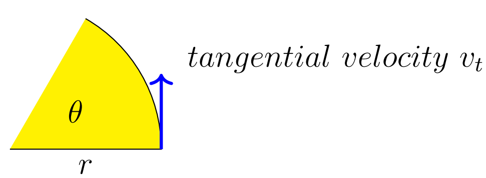


As shown earlier for uniform circular motion, [which is motion in a circle at constant speed and, hence, constant angular velocity] *angular velocity* ω was defined as the time rate of change of angle θ : Angular Velocity:

$$\omega = \frac{d\theta}{dt} \tag{67}$$

The *Tangential Velocity* v_t is then

$$v_t = r\omega \tag{68}$$



In a similar way we can write equations for *Angular Acceleration*

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}. \tag{69}$$

and *Tangential Acceleration*

$$a_t = r\alpha \tag{70}$$

The Kinematics for rotational motion is completely analogous to translational kinematics, where:

Angular Displacement

$$\theta_f = \theta_0 + \bar{\omega}t \tag{71}$$

Angular velocity with constant acceleration

$$\omega_f = \omega_0 + \alpha t \tag{72}$$

Angular displacement with constant acceleration

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{73}$$

Moment of Inertia

$$I = \sum_j m_j r_j^2$$

Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2$$

Torque Vector

$$\overrightarrow{\tau} = \overrightarrow{r} \times \overrightarrow{F}$$

Newton's 2nd Law of Rotation

$$\sum_i \tau_i = I\alpha$$

Rotational Power

$$P = \tau\omega$$

Percentage Uncertainty

$$= \frac{\delta A}{A} \times 100\%$$

VECTORS

Multiplication by a scalar

$$\overrightarrow{B} = \alpha \overrightarrow{A}$$

Cummutative Law

$$\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{B} + \overrightarrow{A}$$

Associative Law

$$(\overrightarrow{A} + \overrightarrow{B}) + \overrightarrow{C} = \overrightarrow{A} + (\overrightarrow{B} + \overrightarrow{C})$$

The component form of a vector in two dimensions

$$\overrightarrow{A} = A_x\hat{i} + A_y\hat{j}$$

The Magnitude of a vector in a plane

$$A = \sqrt{A_x^2 + A_y^2}$$

The direction angle of a vector in a plane

$$\Theta_A = tan^{-1}\left(\frac{A_y}{A_x}\right)$$

The component form of a vector in three dimensions

$$\overrightarrow{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

The Magnitude of a vector in three dimensions

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Definition of the **Scalar** product

$$\overrightarrow{A}.\overrightarrow{B} = AB\cos\theta$$

Scalar product in terms of the scalar components of vectors

$$\overrightarrow{A}.\overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$$

Dot products odf unit vectors

$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$$

Magnitude of the **vector** product

$$\mid \overrightarrow{A} \times \overrightarrow{B} \mid = AB\sin\theta$$

Cross products of unit vectors

$$\hat{i} \times \hat{j} = +\hat{k}, \hat{j} \times \hat{k} = +\hat{i}, \hat{k} \times \hat{i} = +\hat{j}$$

The Cross product in terms of scalar components of vectors

$$\overrightarrow{A} \times \overrightarrow{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

MOTION ALONG A STRAIGHT LINE

Displacement

$$\Delta x = x_f - x_i$$

Total Displacement

$$\Delta x_{Total} = \Sigma \Delta x_i$$

Average Velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantaneous Velocity

$$v(t) = \frac{dx(t)}{dt}$$

Average Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous Acceleration

$$a(t) = \frac{dv(t)}{dt}$$

Velocity from Acceleration [constant a]

$$v = v_0 + at$$

Position from velocity and acceleration [constant a]

$$x = x_0 + v_0 + \frac{1}{2}at^2$$

Velocity from distance [constant a]

$$v^2 = v_0^2 + 2a(x - x_0)$$

Height of free fall

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

MOTION IN TWO AND THREE DIMENSIONS

Position Vector

$$\overrightarrow{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

Displacement Vector

$$\Delta\overrightarrow{r} = \overrightarrow{r}(t_2) - \overrightarrow{r}(t_1)$$

Velocity Vector

$$\overrightarrow{v}(t) = \frac{d\overrightarrow{r}}{dt}$$

Velocity in Terms of Components

$$\overrightarrow{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$$

Average Velicity

$$\overrightarrow{v}_a vg = \frac{\overrightarrow{r}(t_2) - \overrightarrow{r}(t_1)}{t_2 - t_1}$$

Instantanious Acceleration

$$\overrightarrow{a}(t) = \frac{d\overrightarrow{v}(t)}{dt}$$

Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$

NEWTON'S LAWS OF MOTION

Net External Force

$$\overrightarrow{F}_{net} = \Sigma\overrightarrow{F} = \overrightarrow{F}_1 + \overrightarrow{F}_2 + ...$$

Newton's First Law

$$\overrightarrow{v} = constant when \overrightarrow{F}_{net} = \overrightarrow{0} N$$

Newton's Second Law, Vector Form

$$\overrightarrow{F}_{net} = \Sigma\overrightarrow{F} = m\overrightarrow{a}$$

Newton's Second Law, Component Form

$$\Sigma\overrightarrow{F}_x = m\overrightarrow{a_x}, \Sigma\overrightarrow{F}_y = m\overrightarrow{a_y}, \Sigma\overrightarrow{F}_z = m\overrightarrow{a_z}$$

Newton's Second Law, Momentum Form

$$\overrightarrow{F}_{net} = \frac{d\overrightarrow{p}}{dt}$$

Defination of Weight, Vector Form

$$\overrightarrow{w} = m\overrightarrow{g}$$

Defination of Weight, Vector Form

$$w = mg$$

Newton's Thrid Law

$$\overrightarrow{F}_{AB} = -\overrightarrow{F}_{BA}$$

Normal force on an object resting on a horizontal surface, vector form.

$$\overrightarrow{N} = -m\overrightarrow{g}$$

Normal force on an object resting on a horizontal surface, scalar form.

$$N = -mg$$

Normal force on an object resting on an inclined surface, scalar form.

$$N = -mgcos\theta$$

APPLICATION OF NEWTON'S LAWS

Magnitude of static friction

$$f_s \leq \mu_s N$$

Magnitude of kinetic friction

$$f_k \leq \mu_k N$$

Centripetal Force

$$F_c = m\frac{v^2}{r} = mr\omega^2$$

Drag Force

$$F_D = \frac{1}{2}C_D\rho A v^2$$

Stokes' Law

$$F_s = 6\pi r\eta v$$

WORK AND KINETIC ENERGY

Work Done by a force in line with displacement

$$W = F \cdot d$$

Work done by a force over an infinitesimal displacement

$$dW = \overrightarrow{F} \cdot d\overrightarrow{r} = |\overrightarrow{F}||d\overrightarrow{r}|\cos\theta$$

Work done by a force acting along a path from A to B

$$W_{AB} = \int_A^B \overrightarrow{F} \cdot d\overrightarrow{r}$$

Work done going from A to B by Earth's gravity near its surface

$$W_{grav,AB} = -mg(y_B - y_A)$$

Work done going from A to B byone-dimensional spring force

$$W_{spring,AB} = -\frac{1}{2}k(x_B^2 - x_A^2)$$

Kinetic Energy of a particle

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Power as a rate of doing work

$$P = \frac{dW}{dt}$$

Power as the dot product of force and velocity

$$P = \overrightarrow{F} \cdot \overrightarrow{v}$$

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Difference of potential energy

$$\Delta U_A B = U_B - U_A = -W_A B$$

Gravitational potential energy near Eath's surface

$$U(y) = mgy + const$$

Potential Energy of an ideal spring

$$U(x) = \frac{1}{2}kx^2 + const$$

Conservation of energy with no non-conservative forces

$$0 = W_{nc,AB} = \Delta(K + U)_{AB} = \Delta E_{AB}$$

LINEAR MOMENTUM AND COLLISIONS

Definition of momentum

$$\overrightarrow{P} = m\overrightarrow{v}$$

Impulse

$$\overrightarrow{J} = \int_{t_i}^{t_f} \overrightarrow{F}(t)dt = \overrightarrow{J} = \overrightarrow{F}_{ave}\Delta t$$

Impulse-momentum theorem

$$\overrightarrow{J} = \Delta\overrightarrow{p}$$

Average force from momentum

$$\overrightarrow{F}(t) = fracd\overrightarrow{p}dt$$

Conservation of momentum

$$\overrightarrow{p}_1 + \overrightarrow{p}_2 = constant$$

Generalised conservation of momentum

$$\sum_{j=1}^{N} \overrightarrow{p}_j = constant$$

External forces

$$\overrightarrow{F}_{ext} = \sum_{j=1}^{N} \frac{d\overrightarrow{p}_j}{dt}$$

FIXED AXIS ROTATION

Angular Position

$$\theta = \frac{s}{r}$$

Angular Velocity

$$\omega = \frac{d\theta}{dr}$$

Tangential Speed

$$v_t = r\omega$$

Angular Acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Tangential Acceleration

$$a_t = r\alpha$$

Angular Displacement

$$\theta_f = \theta_0 + \bar{\omega}t$$

Angular velocity with constant acceleration

$$\omega_f = \omega_0 + \alpha t$$

Angular displacement with constant acceleration

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Moment of Inertia

$$I = \sum_j m_j r_j^2$$

Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2$$

Torque Vector

$$\overrightarrow{\tau} = \overrightarrow{r} \times \overrightarrow{F}$$

Newton's 2nd Law of Rotation

$$\sum_i \tau_i = I\alpha$$

Rotational Power

$$P = \tau\omega$$

ANGULAR MOMENTUM

Displacement of centre of mass of rolling object

$$d_{CM} = R\theta$$

Velocity of centre of mass of rolling object

$$v_{CM} = R\omega$$

Acceleration of centre of mass of rolling object

$$a_{CM} = R\alpha$$

Angular Momentum of a point mass

$$\overrightarrow{l} = \overrightarrow{r} \times \overrightarrow{p}$$

Angular Momentum

$$L = I\omega$$

Conservation of angular momentum

$$\frac{d\overrightarrow{L}}{dt} = 0$$

STATIC EQUILIBRIUM AND ELASTICITY

First Equilibrium Condition

$$\sum_k \overrightarrow{F}_k = \overrightarrow{0}$$

Second Equilibrium Condition

$$\sum_k \overrightarrow{\tau}_k = \overrightarrow{0}$$

Linear relation between stress and strain

$$stress = (elasticmodulus) \times strain$$

Young's modulus

$$Y = \frac{tensilestress}{tensilestrain} = \frac{F_\perp}{A}\frac{L_0}{\Delta L}$$

Shear modulus

$$Y = \frac{shearstress}{shearstrain} = \frac{F_\parallel}{A}\frac{L_0}{\Delta x}$$

OSCILLATIONS

Relationship between frequency and period

$$f = \frac{1}{T}$$

Position in Simple Harmonic Motion with $\phi = 0$

$$x(t) = A\cos(\omega t)$$

General Position in Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

General Velocity in Simple Harmonic Motion

$$x(t) = -A\omega\sin(\omega t + \phi)$$

General Acceleration in Simple Harmonic Motion

$$x(t) = -A\omega^2\cos(\omega t + \phi)$$

Maximum displacement (amplitude) of SHM

$$x_{max} = A$$

Maximum velocity of SHM

$$|v_{max}| = A\omega$$

Maximum acceleration of SHM

$$|a_{max}| = A\omega^2$$

Angular frequency of a simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

Natural angular frequency of a simple pendulum

$$\omega_0 = \sqrt{\frac{k}{m}}$$