PHYS 488
Exercises Week 5 (27/02 and 28/02/2017)

Please complete the following exercises and submit a report as PDF document (e.g. exported from MS Word) to VITAL as soon as the work is finished. The deadline is the **end of Monday, 06/03/2017**.

Please attend the practical sessions on Tuesday and Wednesday morning in CTL6-PCTC-Orange. The group report on this weeks work should show the code that has been produced and briefly discuss the role that each group member played in performing these tasks.

Task 1 **[2 marks]**

As discussed in the lecture, the average energy loss per distance of an electrically charged particle passing through matter can be computed with the Bethe-Bloch formula

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = K z^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2 m_e \beta^2 \gamma^2 W_{\mathrm{max}}}{I^2} \right) - \beta^2 \right]$$

Here, the variables have the following meaning:

$\beta, \gamma -$ usual relativistic variables of incident particle (see e.g. week 1 tasks)

$K = 0.307075\,\mathrm{MeV\ cm^2}$

$z, M -$ incident particle charge and mass (e.g. 1 and 106 MeV for a muon)

$\rho -$ absorber material: density ($\mathrm{g/cm^{-3}}$), e.g. for Iron ρ = 7.87 $\mathrm{g/cm^{-3}}$

$Z, A -$ absorber material: atomic number and atomic mass (e.g. for Iron Z=26, A=55.845)

$m_e = 0.511\,\mathrm{MeV} -$ electron mass

$W_{\mathrm{max}} = \dfrac{2 m_e \beta^2 \gamma^2}{1 + 2\gamma m_e / M + (m_e / M)^2} -$ maximum energy transfer in single collision

$I \approx 0.0000135\,\mathrm{MeV}\,Z -$ mean excitation energy

The parameter of the Gaussian distribution to describe the multiple coulomb scattering (MCS) is given by:

$$\theta_0 = \frac{13.6\,\mathrm{MeV}}{\beta p} z \sqrt{x/X_0} \left(1 + 0.038 \ln(x/X_0)\right)$$

($\theta_t = \sqrt{2}\theta_0$ is the appropriate parameter in a 2D model and θ_0 the one for a 3D model)

In addition to parameters given already above, here the symbols have the following meaning:

$p -$ incident particle momentum

$x/X_0 -$ material thinkness measured in radiation lenthgs

$X_0 \approx \dfrac{716.4\,A}{\rho Z (Z+1) \ln(287/\sqrt{Z})}$, e.g. $X_0 = 1.797\,\mathrm{cm}$ for Iron

The task today is to produce Java code for these formulas.

Write two classes, **EnergyLoss** and **MCS,** where separate instances represent separate materials. The instantiation of these classes should look something like this:

```
EnergyLoss ironEloss = new EnergyLoss(parameters for iron);
EnergyLoss copperEloss = new EnergyLoss(parameters for copper);
MCS ironMS = new MCS(parameters for iron);
MCS copperMS = new MCS(parameters for copper);
```

Please turn over

Once you have written the classes **EnergyLoss** and **MCS**, write a small program that uses these classes to compute <dE/dx> and θ_t for a **muon** traveling through **iron**:

```
double dEdx_Iron = ironEloss.getEnergyLoss(momentum);
double thetaT_Iron = ironMS.get ThetaT(momentum, thickness);
```

Show in your report, that you can reproduce the following table of values:
dE/dx = 81.51 MeV/cm for p=30.0MeV
dE/dx = 11.58 MeV/cm for p=300.0MeV
dE/dx = 15.25 MeV/cm for p=3000.0MeV
dE/dx = 17.71 MeV/cm for p=10000.0MeV
dE/dx = 19.81 MeV/cm for p=30000.0MeV
dE/dx = 21.96 MeV/cm for p=100000.0MeV

Iron X0 1.797cm
For a material thickness of 1cm:
thetaT = 0.0287 for p=500.0MeV
thetaT = 0.0141 for p=1000.0MeV
thetaT = 0.00468 for p=3000.0MeV

Task 2

On Vital you will find the Java class **TrackMuon.java**, that simulates muons travelling through an iron sheet. When it exits the back of the iron, it travels to two counters situated 10 and 20 cm beyond the back. The muon generates 'hits' in the detectors that are smeared by an assumed detector resolution in the y-direction of 0.1cm. In a real experiment, these hits would be the experimental data.

The muon is assumed to start inside the iron at (x, y) = (0, 0) and has a fixed energy at the start, startEnergy, which the user selects. The particle is then taken through the iron in small steps (configurable) and at each stage the energy loss and angular smearing by MCS are smulated. As the particle is followed, the (x, y) co-ordinates of each point reached by the muon are stored in a 2D array called 'trackOfMuon'. The muon is followed until:
- it leaves the back of the iron sheet,
- or, the muon has lost so much energy it stops in the iron,
- or, too many steps have been taken (safety measure to avoid infinite loops).

When a muon comes out of the back of the sheet, some information is passed to the method **lookAtThisMuon**, e.g. to writte the track to disk for plotting/analysis analysis.
The program is incomplete in 3 respects:
- the correct code to find the energy loss is not yet included
- the correct code for calculating the MCS angle θ_0 is not coded
- the code for writing data to disk is not there.

Task 2.1 **[1 mark]**
Read the program and try to understand how it works. Ask for help over any points you are not sure about. Complete the code so it makes use of the classes you wrote in part 1, for calculating the energy loss and MCS angle.

Write 10 sample muon tracks (i.e. the (x,y) pairs) into a file on disk and draw them using an x-y scatter plot chart in EXCEL. Discuss what you observe.

Please turn over

Task 2.2 **[1 mark]**

Make histograms (based on a sufficiently large number of muons) of the energy and y-position of the muons as they exit from the back of the iron. Try a range of muon energies (low, medium and high). Discuss what you observe. Why is the energy distribution very narrow (discuss possible limitations in the way it is simulated).

Task 2.3 **[1 mark]**

Add in an extra detector 30 cm beyond the iron and generate hits in that. Make a histogram of the hit positions in y in detectors 1, 2 and 3. Discuss what you observe.

Task 2.4 **[1 mark]**

If the iron were 1 m thick, estimate (using the program) the minimum muon energy for which at least 50% of the muons pass through the iron without being stopped. Discuss the method you could use to estimate this value. Include an estimate/discussion on the uncertainty(ies) on your result.