Runur og summur

Runa (e. sequence) er listi af tölum, mögulega óendanlega langur.

$$a_1, a_2, a_3, \dots$$

stundum táknaður $\{a_n\}$, eða $\{a_n\}_{n=1}^{+\infty}$.

Dæmi

Runan $\{a_n\}$ þar sem $a_n = \frac{1}{n}$ er

$$a_1, \quad a_2, \quad a_3, \quad a_4, \quad \dots, \quad a_n, \dots$$
$$\frac{1}{1}, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad \frac{1}{4}, \quad \dots, \quad \frac{1}{n}, \dots$$

Kvótarafir (e. geometric progression / sequence)

$$k, \quad k\cdot r, \quad k\cdot r\cdot r, \quad k\cdot r\cdot r\cdot r, \dots$$
$$k, \quad kr, \quad kr^2, \quad kr^3, \dots$$

k : upphafsgildi

r : kvóti

Almenn jafna fyrir $a_n = k\cdot r^{n-1}$

$$\left(a_1 = k\cdot r^{1-1} = k\cdot r^0 = k \right.$$
$$\left. a_2 = k\cdot r^{2-1} = k\cdot r^1 = kr\right)$$

Dæmi

$\{a_n\}$ þar sem $a_n = 2 \cdot 5^{n-1}$

$n=1 \quad a_1 = 2 \cdot 5^{1-1} = 2 \cdot 5^0 = 2$

$n=2 \quad a_2 = 2 \cdot 5^{2-1} = 2 \cdot 5^1 = 10$

$n=3 \quad a_3 = 2 \cdot 5^{3-1} = 2 \cdot 5^2 = 50$

$n=4 \quad a_4 = 2 \cdot 5^{4-1} = 2 \cdot 5^3 = 250$

$\vdots$

Dæmi

1. r

$\{a_n\}$ þar sem $a_n = (-1)^{n-1}$

$n=1 \quad a_1 = (-1)^{1-1} = (-1)^0 = 1$

$n=2 \quad a_2 = (-1)^{2-1} = (-1)^1 = -1$

$n=3 \quad a_3 = (-1)^{3-1} = (-1)^2 = 1$

$n=4 \quad a_4 = (-1)^{4-1} = (-1)^3 = -1$

$\vdots$

Mismunaruna (e. arithmetic progression)

$k, \quad k+d, \quad k+d+d, \quad k+d+d+d, \ldots$

$k, \quad k+d, \quad k+2d, \quad k+3d, \ldots$

k : upphafsgildi

d : mismunur

Almenn jafna $a_n = k + (n-1)d$

$$\left(\begin{aligned} a_1 &= k + (1-1)d \\ &k + 0 \cdot d \\ &k \\ a_2 &= k + (2-1)d \\ &= k + d \end{aligned} \right)$$

Dæmi

$\{s_n\}$ þar sem $s_n = -1 + 4n \qquad n = 1, 2, 3, \ldots$

$$s_1 = -1 + 4 \cdot 1 = 3 \qquad\qquad 3 + (n-1) \cdot 4$$
$$s_2 = -1 + 4 \cdot 2 = 7$$
$$s_3 = -1 + 4 \cdot 3 = 11$$
$$\vdots$$

Summur (e. summations / series)

Ef við erum með stök úr runu

$$a_m, a_{m+1}, \ldots, a_n$$

þarf ekki að byrja fremst

og leggjum þau saman

$$a_m + a_{m+1} + \cdots + a_n$$

þá táknum við það með

$$\sum_{j=m}^{n} a_j \qquad\qquad (j \text{ kallast vísir (e. index)})$$

Dæmi

$a_j = j^2$

$$\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$
$$= 1 + 4 + 9 + 16 + 25$$
$$= 55$$

Dæmi

$$\sum_{i=4}^{8} (-1)^i = (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$$
$$= \underbrace{1 \quad -1}_{0} \quad \underbrace{1 \quad -1}_{0} \quad 1$$
$$= 1$$

Setning (Regla)

Ef $r \neq 1$ þá er

$$\sum_{j=0}^{n} k \cdot r^j = \frac{k r^{n+1} - k}{r-1}$$

en ef $r = 1$

$$\sum_{j=0}^{n} k r^j = \sum_{j=0}^{n} k = (n+1) \cdot k$$

Dæmi

$$\sum_{j=0}^{10} \underbrace{2}_{k} \cdot \overbrace{3}^{r}{}^j = \frac{2 \cdot 3^{11} - 2}{3-1} = \frac{2(3^{11}-1)}{2} = 3^{11} - 1$$

$$\sum_{j=11}^{20} 2 \cdot 3^j = \sum_{j=0}^{20} 2 \cdot 3^j - \sum_{j=0}^{10} 2 \cdot 3^j$$

regla

$$= \frac{2 \cdot 3^{21} - 2}{3-1} - (3^{11} - 1)$$
$$= \frac{2(3^{21}-1)}{2} - 3^{11} + 1 = 3^{21} - 1 - 3^{11} + 1$$
$$= 3^{21} - 3^{11}.$$

Tvöfaldar summur

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij = \sum_{i=1}^{4}\left(\underset{j=1}{i} + \underset{j=2}{2i} + \underset{j=3}{3i}\right)$$

$$= \sum_{i=1}^{4} 6i$$

$$= 6\sum_{i=1}^{4} i = 6(1+2+3+4)$$

$$= 6 \cdot 10 = 60$$