



PREPARATORY COURSE IN MATHEMATICS, 2024

COURSE PLAN

HARPA GUÐJÓNSDÓTTIR OG STEINUNN GRÓA SIGURÐARDÓTTIR

Note: This course will be in Icelandic. However, most material (other than the Youtube lectures) is available in English, and we will also be able to help you in English during the Exercise sessions.

1. VARIOUS INFORMATION ^(5,6)

We will cover various aspects of mathematics that will come in handy in the courses *Discrete Mathematics I* and *Discrete Mathematics for Engineering Students*. Students must bring with them:

- Paper or book to calculate in
- Internet-connected computer to be able to work on interactive examples

It is good, but not necessary, to have the book *Discrete mathematics and its applications* by Kenneth H. Rosen, 7th or 8th edition, [Ros07] [Ros19], which is the same book as are used in the courses *Discrete mathematics I* and *Discrete mathematics for engineering students*. Articles specified in the reading material and problem sets refer to this book.

The main teachers of the course are Harpa Guðjónsdóttir (harpagud@ru.is) and Steinunn Gróa Sigurðardóttir (steinunngróa@ru.is). For students who are in Akureyri, the teacher's assistant will be Daníel (danielj@unak.is). Ólafur Jónsson (olafurj@unak.is) oversees students in Akureyri and the east.

At Reykjavik University, we will be in room M101, on the first floor. For students who do not have access to RU's student WiFi, you can use the „Reykjavik University“ WiFi by entering an e-mail address (does not have to be a ru e-mail address). In Akureyri, the course will be in L102.

Useful Links. Here are links that can come in handy in the course.

Dictionary of the Icelandic Mathematics Association

MyOpenMath: Interactive Examples

Lectures (pdf)

YouTube - see below for links directly to the lectures (note: Lectures are in Icelandic)

When you first enter MyOpenMath, select „Register as a new student“. Fill in the information required. At the bottom of the page you find „Course ID“ which should be „235573“, and „Enrollment Key“ is „LFG“. As a result, you should you be enrolled in the course „RU Math Prep 2024“.

2. OVERVIEW OF THE COURSE STRUCTURE

The course is three days, Friday 16. August, Saturday 17. August, and Sunday 18. August. Teaching takes place in HR, but lectures are also busy on YouTube. The schedule for each day is as follows:

Last updated: 14. ágúst 2024.

Friday

16:00–17:00	Time to watch lectures	60 min
17:00–17:50	Exercise session	50 min
17:50–18:20	Review	30 min
18:20–18:40	Break	20 min
18:40–19:30	Exercise session	50 min
19:30–20:00	Review	30 min

Saturday

11:00–12:00	Time to watch lectures	60 min
12:00–12:50	Exercise session	50 min
12:50–13:20	Review	30 min
13:20–13:40	Break	20 min
13:40–14:30	Exercise session	50 min
14:30–15:00	Review	30 min

Sunday

11:00–12:00	Time to watch lectures	60 min
12:00–12:50	Exercise session	50 min
12:50–13:20	Review	30 min
13:20–13:40	Break	20 min
13:40–14:30	Exercise session	50 min
14:30–15:00	Review	30 min

The teacher's presence in the classroom is from 17:00-20:00 on Friday and 12:00-15:00 on Saturday and Sunday. Students are expected to watch the lectures of the day before that time (on Friday, the subjects are „Algebra“ and „Propositions“, on Saturday „Set Rules“ and „Proofs and their Setup“ and on Sunday „Sequence and Series“ and „Induction“). Lectures are accessible on YouTube (see links below). Note **especially** that on each day, we cover two different subjects, so the students need to have watched both lectures of the day before the trial period. The lectures are of different lengths and some students may want to listen to some parts again, so please allocate appropriate time for that.

Note **also** that exercise session and review are not streamed and are not recorded, so they can not be watched later. However, students are free to watch the lectures and work on the examples themselves without help. The course is not graded, the course is only designed to prepare students.

DAY 1, FRIDAY 16. AUGUST

Algebra.

- Reading material: Teacher's lecture (<https://youtu.be/KgNP-dGIRp4>)
- Important: Simplifying expressions, solving equations

Exercise session 1:

Algebra 1 in MyOpenMath

Algebra 2 in MyOpenMath

Algebra 3 in MyOpenMath

Extra:

The examples at the end of this course plan

Propositions.

- Reading Material: Sections 1.1, 1.2 (https://youtu.be/xY_-7zNN3cw)
- Important: Proposition, truth table, composite propositions, equivalent propositions

Exercise session 2:

Logic in MyOpenMath

Extra:

Section 1.1: 1 a) c) d) e), 4 a) c) d), 9 a) b) d) e), 13 a) b) c) d), 16 a) b), 18 b) c), 22 a) b), 32 a) b) c) d), (40)

Section 1.2: (16), (17)

DAY 2, SATURDAY 17. AUGUST

Proofs and their Setup.

- Reading Material: Section 1.7 (https://youtu.be/HWoaN_jRToE)
- Important: Direct proofs, indirect proofs, proof by contradiction, ((if-and-only-if proofs))

Exercise session 3:

Proofs in MyOpenMath

Section 1.7: 6, 9, 18 a), 26

Extra:

Section 1.7: 1, 2, 11, 14, 18 b), 30, (38), (39), (40)

For consideration 1. Does there exist irrational numbers x and y such that x^y is a rational number? (Hint: Consider $x = \sqrt{2}$ and $y = \sqrt{2}$.)

For consideration 2. Goldbach's theorem: Show that every even integer greater than 2 can be written as the sum of two prime numbers.

Set Rules.

- Reading Material: Section 2.1, 2.2 (<https://youtu.be/DhgDP5wEu7s>)
- Important: Sets, element, \mathbb{N} , \mathbb{Z} , \mathbb{Z}^+ , \mathbb{Q} , \mathbb{R} , equal sets, Venn-diagram, subsets, cardinality

Exercise session 4:

Sets in MyOpenMath

Extra:

Section 2.1: 1 a), b) c) d), 2 a) b), 7 a) b) c) d), 11, 12, 18, 21 b), c), (22), (24)

Section 2.2: 3 a) b), 20 a) b), 26 a)

For consideration. The Frankl Theorem: Let \mathcal{M} be a finite set of sets, such that of about every two sets A, B in \mathcal{M} holds true that the combination $A \cup B$ is also in \mathcal{M} . Show that there exists an element that is in at least half of the sets in \mathcal{M} . (Here it is additionally necessary to assume that $\mathcal{M} \neq \{\emptyset\}$.)

DAGUR 3, SUNNUDAGUR 18. ÁGÚST

Sequences and Series.

- Reading Material: Section 2.4 (<https://youtu.be/dLxq1ImXh0A>)
- Important: Sequences, series, Theorem 1

Exercise session 5:

Sequences and Series in MyOpenMath

Extra:

Section 2.4: 4 a) b) c) d), 6 a)–e), 29 a) b) c), 31 a) b) c), 32 a) b), (34), (35)

Induction.

- Reading Material: Section 5.1 (<https://youtu.be/djsqsHQaySU>)
- Important: Induction

Exercise session 6:

Example: Use induction to show that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for all integers $n \geq 1$.

Section 5.1: 9, 20.

Extra:

Section 5.1: 11, 21, 31, 34.

For consideration. Show that every time a deck of cards is shuffled then you get a new order of the cards that has most likely never happened before... ever, since cards were invented. (Here it is good to keep in mind that the universe is about 14 billion years old, so if someone had picked up a deck right after a big bang and a shuffle once a second, then they're done at most 10^{15} different permutations of that deck.)

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[Ros07] Kenneth H. Rosen, *Discrete mathematics and its applications*, 7 ed., McGraw-Hill, 2007.[Ros19] ———, *Discrete mathematics and its applications*, 8 ed., McGraw-Hill, 2019.

3. EXTRA EXAMPLES FOR ALGEBRA

Problem 3.1. *Simplify the following expressions.*

- (a) $13x + 7y - 2x + 3a$.
- (b) $-2(3x - 7y) + 2y$.
- (c) $-4(3x - 7(x + y)) + 6x$.
- (d) $a(a + b) - (a - b)b$.

Problem 3.2. *Multiply together all parentheses.*

- (a) $x^2(x^3 - y)$.
- (b) $(x - 1)(x - 2)$.
- (c) $x_1(x_1 + x_2)$.
- (d) $(x - 1)^2$.
- (e) $(x - 1)^3$.
- (f) $(x - 3)^2 - (x^2 - 9)$.

Problem 3.3 (Hard). *Find a general equation for the outcome that you get when you multiply the parentheses $(x - 1)^n$.*

Problem 3.4. *Simplify the following fractions.*

- (a) $\frac{12a^2b}{(3ab^2)^2}$.
- (b) $\frac{2m^2n - 6mn}{2m}$.
- (c) $\frac{3 + \frac{1}{x}}{\frac{5}{x} + 4}$.

Problem 3.5. *Solve the equations for the unknown quantity.*

- (a) $5 - (x - 2) = 5x$.
- (b) $7 - (2 - x) = x + 2$.
- (c) *If a square has an area of 4, what is its height?*
- (d) *If a circle has an area of 1, what is its radius?*
- (e) *The distance between two train stations is 25km. At noon a train departs from the second station and at the same time stops bee off from the other, opposite the train. The train travels at a speed of 90km/h and the bee at 6km/h. When will we meet? the train and the bee?*

Problem 3.6. *Solve the equations for the unknown quantity.*

- (a) $x^2 - 7x + 10 = 0$.
- (b) $x^2 - 6x + 9 = 0$.
- (c) $x^2 + 9 = 0$.
- (d) $x^3 - 6x^2 + 9x = 0$.