ME 493 Drone Control Homework 1

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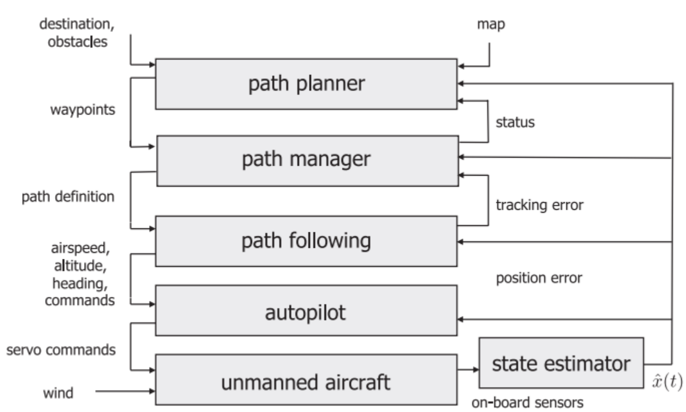
1. Search the web and find a small unmanned aircraft (fixed-wing aircraft with a wingspan of less than 5 feet). Figure out how the aircraft is steered around. What is a coordinated turn? How do you perform one?

One example of a small, fixed-wing UAV is the Lockheed Martin "Desert Hawk III"[1]. This UAV, as can be seen in the YouTube video "FPASS Desert Hawk", uses a Bank-to-Turn steering system, where an aircraft banks to direct a component of lift horizontally to provide a centripetal force which turns the aircraft. According to Beard and McLain, a coordinated turn is a turn in which "there is no lateral acceleration in the body frame of the aircraft"[2]. A turn is initiated by banking the UAV using the ailerons (or other primary roll control surfaces) an amount as determined by the required centripetal acceleration (designated by the desired turn radius and current airspeed) . To "coordinate" the turn, the aircraft is yawed using the rudder (or other primary yaw control surfaces) to counter any induced sideslip . Assuming a "steady" coordinated turn (a coordinated turn where altitude remains constant), the aircraft is also pitched slightly using the elevator (or other primary pitch control surface) to gain angle of attack and increase lift so that the vertical component of lift is equal to the weight.

2. Sketch a block diagram of the system architecture of an unmanned air vehicle (UAV). What is this useful for?

The following UAV system architecture block diagram is pulled from the textbook[3]:



The block diagram is useful because it defines clear interfaces for each portion of the system architecture. With clearly defined and consistent interfaces, any individual block/module in the system architecture can be replaced without having to rebuild the entire system. For instance, if I wanted to test a completely new autopilot system, as long as the autopilot input and output interfaces are the same, I should be able to drop

[1] https://www.airforce-technology.com/projects/deserthawkuav/

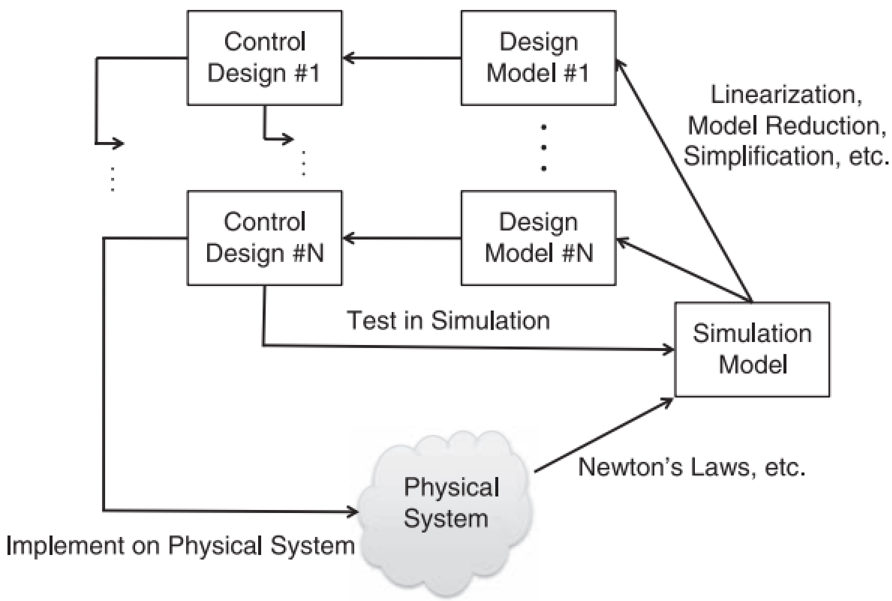
[2] Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", Chapter 5.2

[3] Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.2

the new autopilot in without issue. If instead I wanted to take my entire system and apply it to a new aircraft for some reason, I should have no problem as long as the actuation system is set up to take the same set of commands.

3. Why is it useful to develop a simulator for a UAV as we will do in this class?

Developing a simulator for is UAV is useful because it allows for a design iteration cycle.[4]



The simulator is used to develop simplified design models through linearization, model reduction, etc. which can then be used for control system design. These controllers can then be tested in the full-fidelity simulation to ensure adequate performance across the flight envelope. If performance still needs to be improved, the design process continues, either with a new controller design based on the same models or a higher fidelity model for control system design. Once satisfactory performance is achieved in simulation, the controller can be implemented on the physical system to both verify performance on actual hardware and collect data to inform higher fidelity models for future controller design iterations. When set up correctly, simulators can also be used in Software-in-the-Loop (SIL) and Hardware-in-the-Loop (HIL) testing to catch potentially catastrophic issues with implementation of embedded software or hardware interfaces before the physical system is actually flown.

[4]Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.4

4. Consider a drone flying in still air that is about to go into a descending banked turn. The center of mass of the drone is located 10 m above a reference point on earth, its velocity components in the body frame are (15,1,0.5)m/s, its orientation is given by the Euler angles: yaw 2, pitch 10, and roll 20.

Let:

- $\vec{p}^F_{a/b}$ be the position of a with respect to b, in the F frame
- $\vec{v}[i]$ be the i$^{\text{th}}$ component of the vector

(a) A battery on the drone is sitting 0.2 m away from the center of mass (COM) in the nose direction(measured in body frame). What is its location (position vector) with respect to the earth-fixed frame?

Given:

- $\vec{p}^B_{battery/B} = [0.2, 0, 0]$ m
- $\vec{p}^I_{V/I} = [0, 0, -10]$ m
- $R^{BV} = R^b_v(\phi, \theta, \psi)$ as defined in "Beard"

Solution:

By vector addition, we know:

$$\vec{p}^I_{battery/I} = \vec{p}^I_{battery/B} + \vec{p}^I_{B/V} + \vec{p}^I_{V/I}$$

By definition of the body and vehicle frames, we have:

$$\vec{p}^I_{B/V} = 0$$

$$R^{BI} = R^{BV}$$

Through frame rotation we have:

$$\vec{p}^I_{battery/B} = \vec{p}^B_{battery/B} R^{BI}$$

Thus:

$$\vec{p}^I_{battery/B} = \vec{p}^I_{V/I} + \vec{p}^B_{battery/I} R^{BV}$$

Substituting in the given values yields:

$$\vec{p}^I_{battery/B} = [0.1968, 0.0069, -10.0347]m$$

$$\vec{p}^I_{battery/B} \approx [0.20, 0.01, -10.03]m$$

(b) What is the velocity in the earth-fixed frame?

Given:

- $\vec{v}^B_{UAV} = [15, 1, 0.5]$ m/s
- $R^{BV} = R^b_v(\phi, \theta, \psi)$ as defined in "Beard"

Solution:

By frame rotation, we have:

$$\vec{v}^I_{UAV} = \vec{v}^B_{UAV} R^{BI}$$

By definition of the body and vehicle frames, we have:

$$R^{BI} = R^{BV}$$

Thus:

$$\vec{v}^I_{UAV} = \vec{v}^B_{UAV} R^{BV}$$

Substituting in the given values yields:

$$\vec{p}^I_{battery/B} = [14.8772, 1.2887, -1.8052] m/s$$

$$\vec{p}^I_{battery/B} \approx [14.88, 1.29, -1.81] m/s$$

(c) What is the flight-path angle (in degrees)?

Given:

- $\vec{v}^I_{UAV}$ from part (b)
- $\dot{h} = V_g * sin(\gamma)$ [5]
  - $V_g = \|\vec{V_g}^I\|$

Solution:

$\dot{h}$ is the change in altitude of the UAV, where $\dot{h} > 0$ represents increasing altitude. Thus:

$$\dot{h} = -\vec{v}^I_{UAV}[3]$$

Substituting this into the given equation and solving for gamma yields:

$$-\vec{v}^I_{UAV}[3] = V_g * sin(\gamma)$$

$$\gamma = sin^{-1}(\frac{-\vec{v}^I_{UAV}[3]}{V_g})$$

Solving With the values from part (b) gives:

$$\gamma = 6.8929°$$

$$\gamma \approx 6.90°$$

(d) What is the angle of attack?

Given:

- $\vec{v}^B_{UAV} = [15, 1, 0.5]$ m/s

[5] Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.21

- $\vec{V_a^b} = \begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \begin{bmatrix} u - u_w \\ v - v_w \\ w - w_w \end{bmatrix}$ [6]
- $\alpha = tan^{-1}(\frac{w_r}{u_r})$ [7]

Solution:

Given we are in still air, we have:

$$\vec{V_a^b} = \begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \begin{bmatrix} u - u_w \\ v - v_w \\ w - w_w \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Plugging in to our formula for angle of attack gives:

$$\alpha = tan^{-1}(\frac{w}{u})$$

$$u = \vec{v}^B_{UAV}[1]$$
$$w = \vec{v}^B_{UAV}[3]$$
$$\alpha = tan^{-1}(\frac{\vec{v}^B_{UAV}[3]}{\vec{v}^B_{UAV}[1]})$$

Substituting in the given values gives:

$$\alpha = 1.9092°$$

$$\alpha \approx 1.91°$$

(e) What are the heading and course angles? Explain the difference.

Given:

- $\vec{v}^I_{UAV}$ from part (b)
- $\vec{V_g}^I = V_g \begin{bmatrix} cos(\chi)cos(\gamma) \\ sin(\chi)cos(\gamma) \\ -sin(\gamma) \end{bmatrix}$ [8]
  - $V_g = \|\vec{V_g}^I\|$

Solution:

Heading angle (also known as yaw angle) is defined as the angle of rotation about the 3-axis between the aircraft body frame and the inertial frame. Essentially, it is the angle the aircraft nose forms with north. The course angle is defined as the angle between the projection of the ground velocity vector onto the horizontal plane and true north. These will differ if there is any sideslip or horizontal component of wind relative to the aircraft body frame.

The heading angle is given in the problem statement:

$$\psi = 2°$$

[6]Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.19
[7]Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.20
[8]Beard and McLain, "Small Unmanned Aircraft: Theory and Practice", pg.22

For course angle, from Beard we have:

$$\vec{V_g}^I[1] = V_g cos(\chi) cos(\gamma)$$

$$\vec{V_g}^I[2] = V_g sin(\chi) cos(\gamma)$$

Dividing the two equations gives:

$$tan(\chi) = \frac{\vec{V_g}^I[2]}{\vec{V_g}^I[1]}$$

$$\chi = tan^{-1}(\frac{\vec{V_g}^I[2]}{\vec{V_g}^I[1]})$$

Substituting in the values from part (b) gives:

$$\chi = 4.9506^\circ$$

$$\chi \approx 4.95^\circ$$

5. Download Python files for numerical integration from MS Teams (week 2>Files). Implement a mass-spring system with parameters: m= 1, b= 0.25, and k= 1 (in appropriate units).

(b) Integrate the system first with the Euler method, and second with the Heun method. Experiment with suitable step sizes dt. Compare the numerical solution with the analytical solution (see e.g. Systems Engineering notes). Attach your plots and describe your findings.

Analytical Solution:

For an unforced mass-spring-damper system:

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\omega_n = \sqrt{k/m}$$

$$\zeta = \frac{b}{2w_n m}$$

For the given values of m, b, and k:

$$\omega_n = 1$$

$$\zeta = 0.125$$

Therefore, system is underdamped. For an underdamped mass-spring-damper system, the analytical solution is given by[9]:

$$x = 2e^{-\sigma t}[\alpha cos(\omega_d t) - \beta sin(\omega_d t)]$$

$$\dot{x} = 2e^{-\sigma t}[(-\sigma\alpha - \beta\omega_d)cos(\omega_d t) + (\sigma\beta - \alpha\omega_d) * sin(\omega_d t)]$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

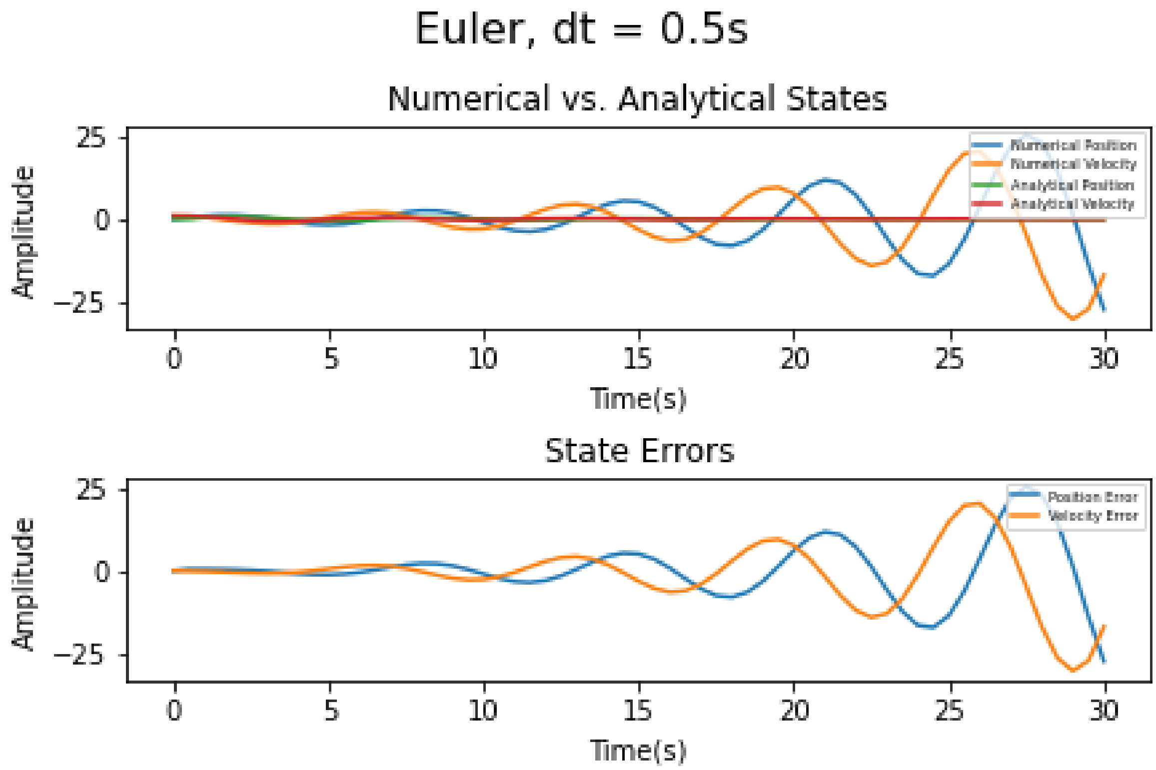
$$\alpha = x_0/2$$

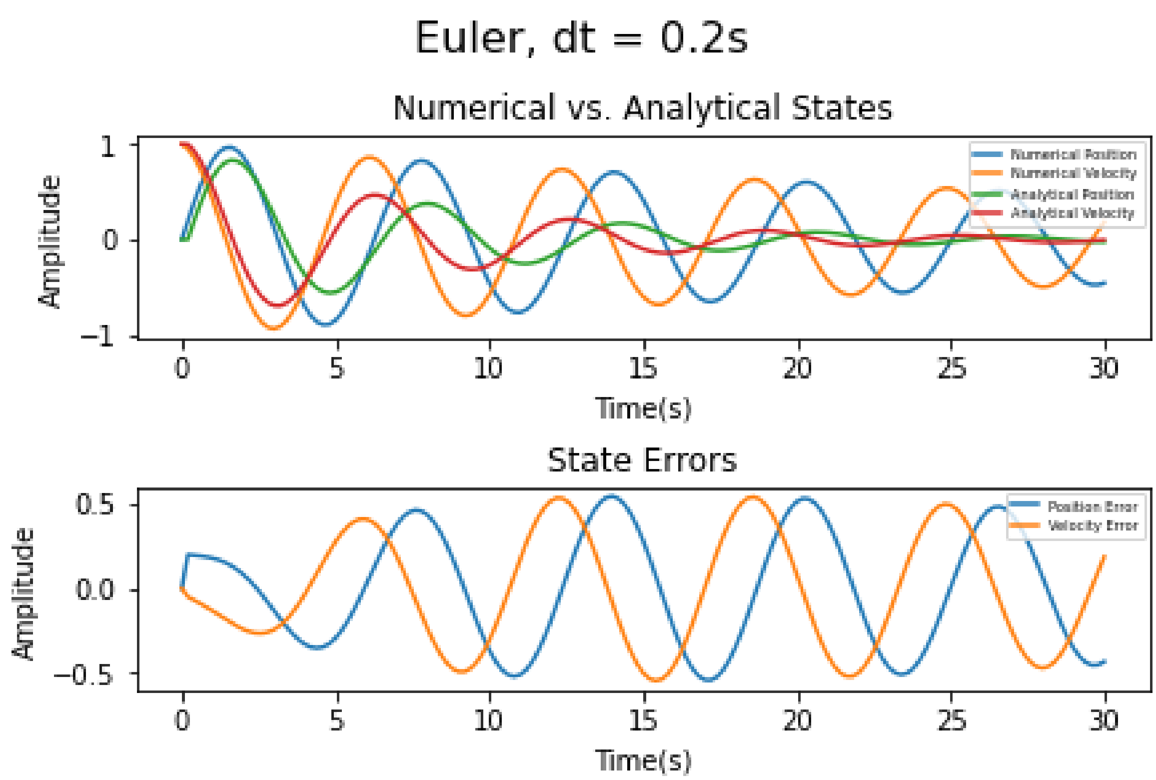
$$\beta = -\frac{(\dot{x}_0 + \sigma x_0)}{2\omega_d}$$

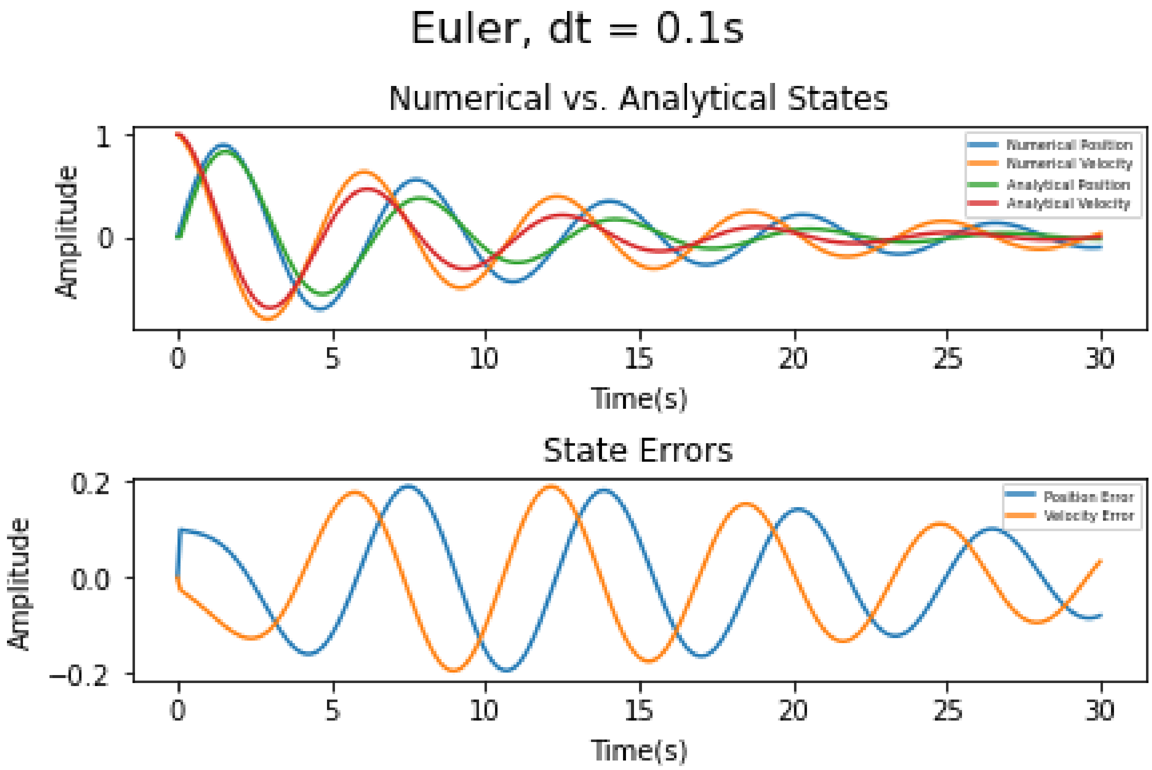
For my step-size experimentation, I will use $\vec{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

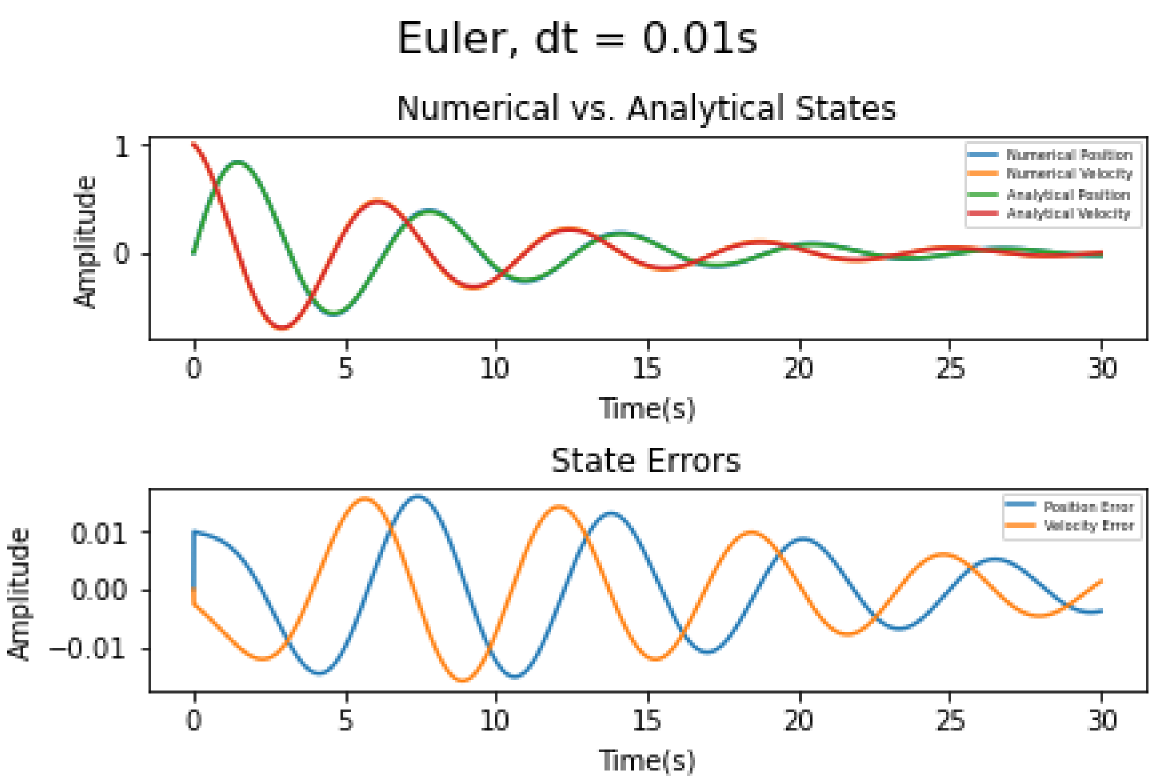
[9] https://ocw.mit.edu/courses/mechanical-engineering/2-003-modeling-dynamics-and-control-i-spring-2005/readings/notesinstalment2.pdf

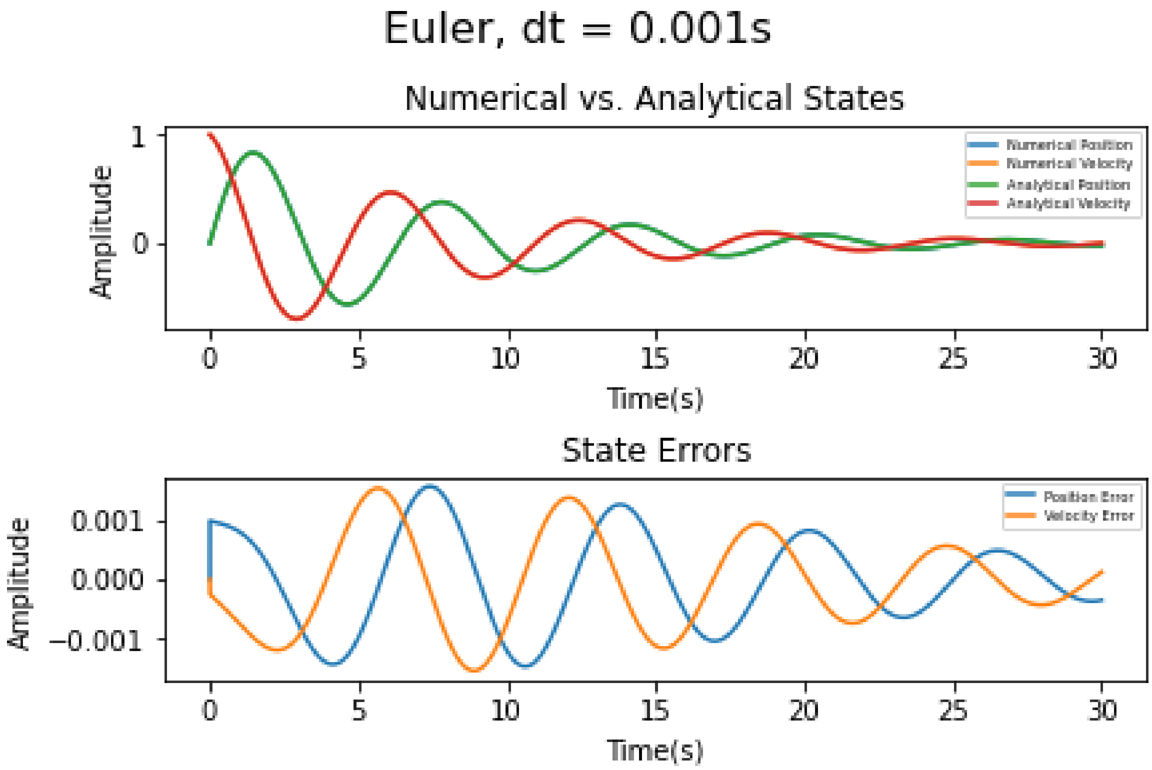
<u>Euler Plots:</u>





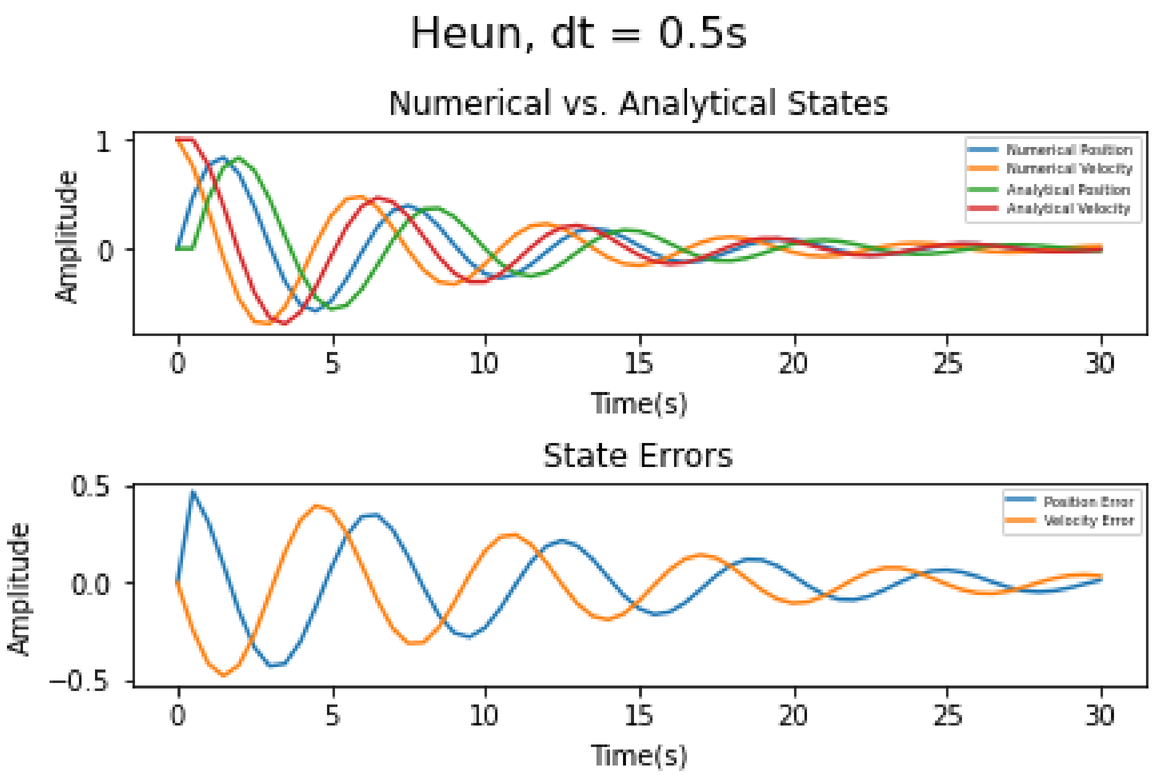

Euler, dt = 0.1s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)

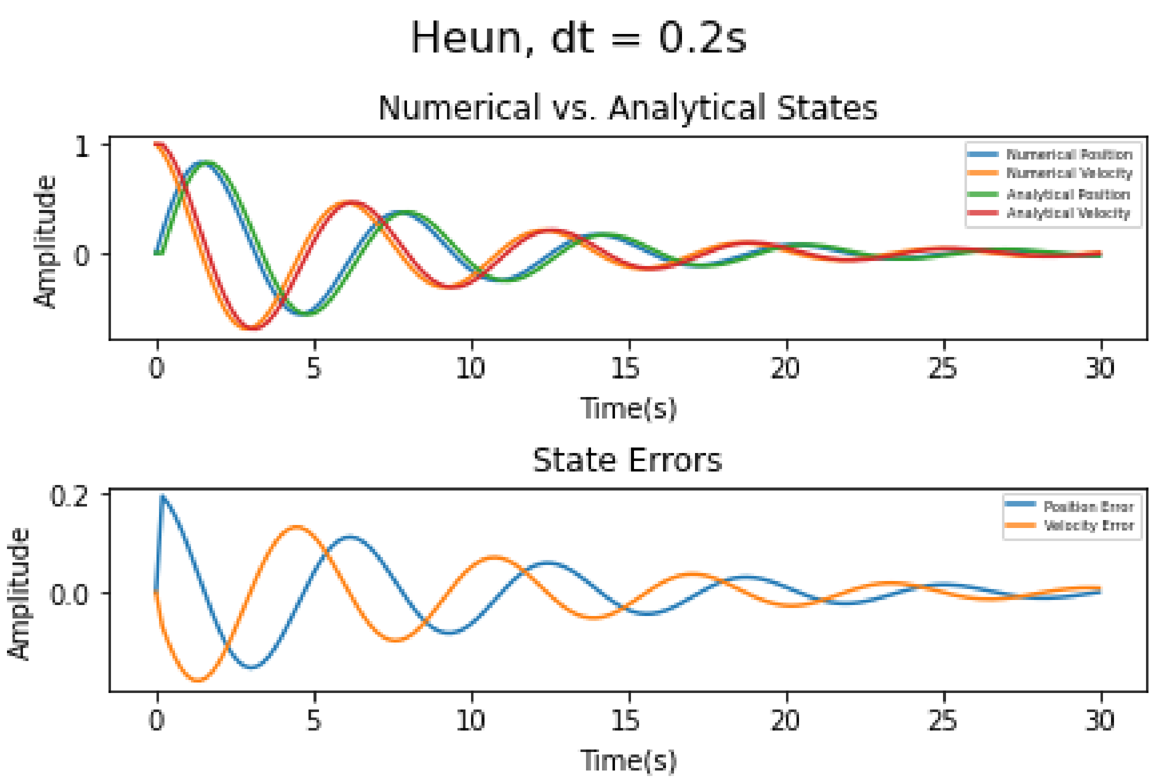

Euler, dt = 0.01s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)

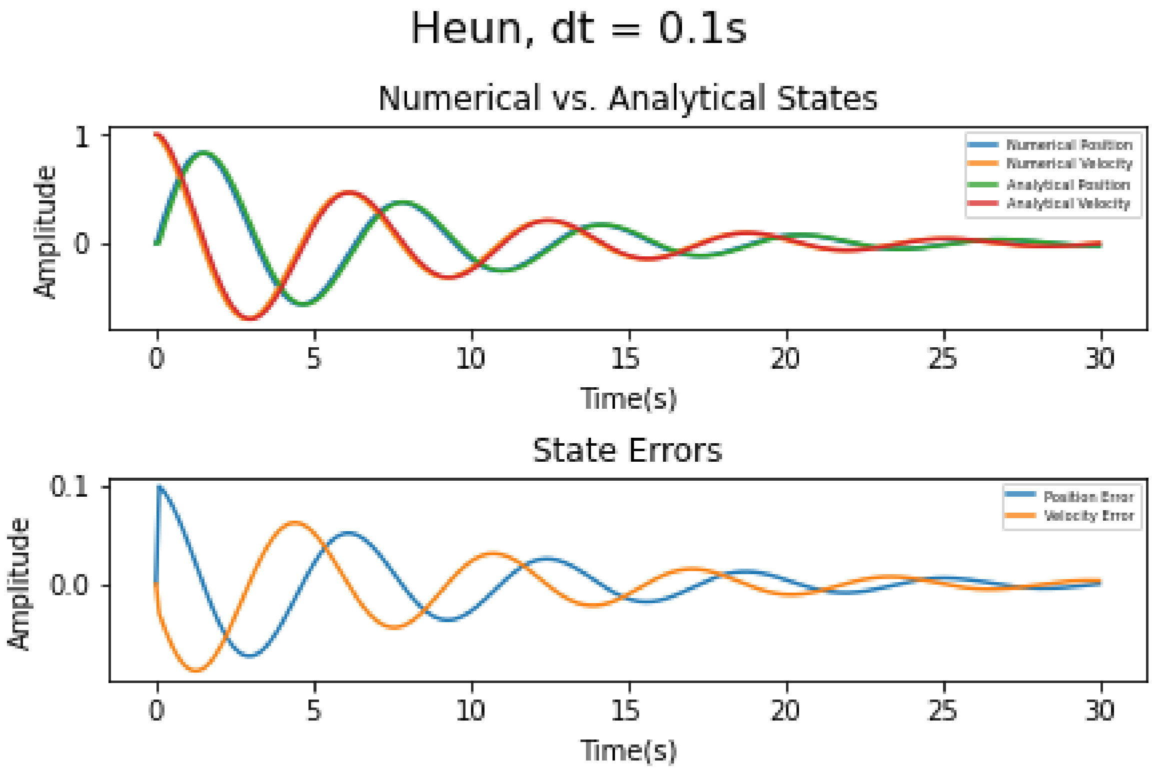


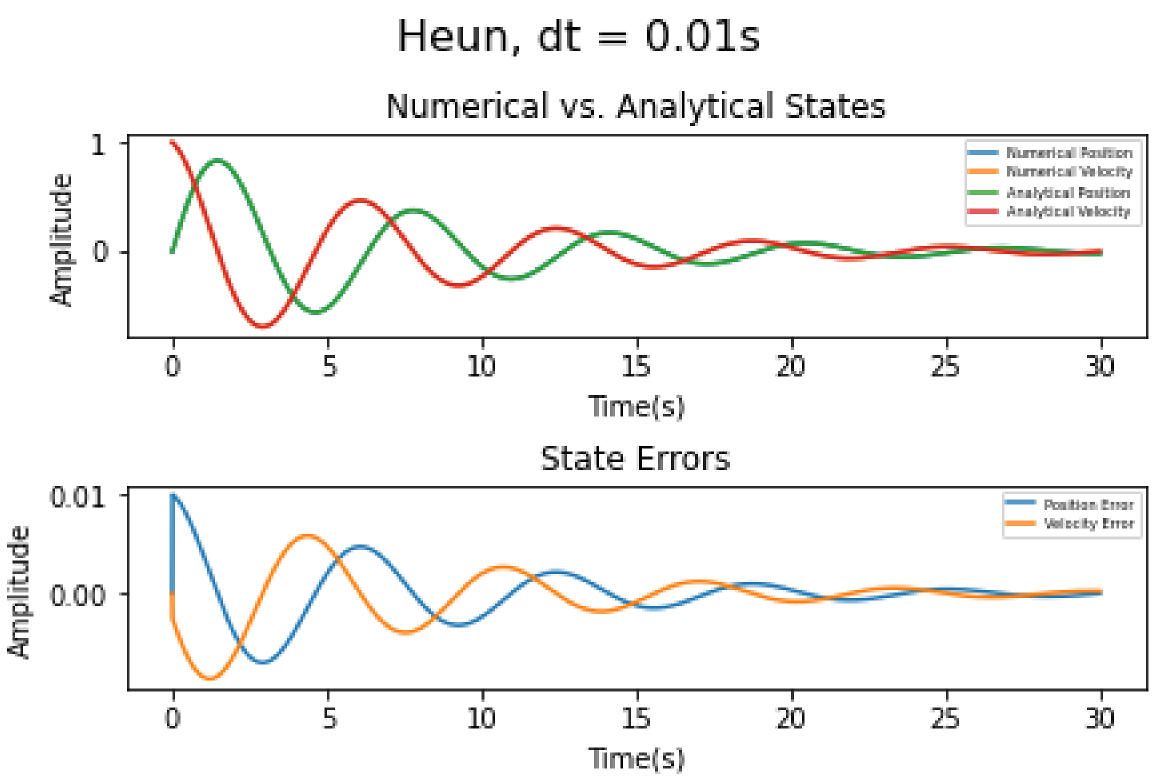
With the largest step size tested dt = 0.5s, the numerical solution quickly diverges from the analytical solution. This step size is too large to be used to solve this system with an Euler integration scheme. As the time step decreases to dt = 0.2s, the solver is seemingly stable, but errors are large and seem to grow relative to the magnitude of the analytical response as time progresses. As dt decreses from dt = 0.1 down through dt = 0.001, the state errors relative to the analytical solution get progressively smaller, and the the numerical solution diverges more and more slowly from the analytical solution (though the error magnitudes still increase relative to the magnitude of the response).

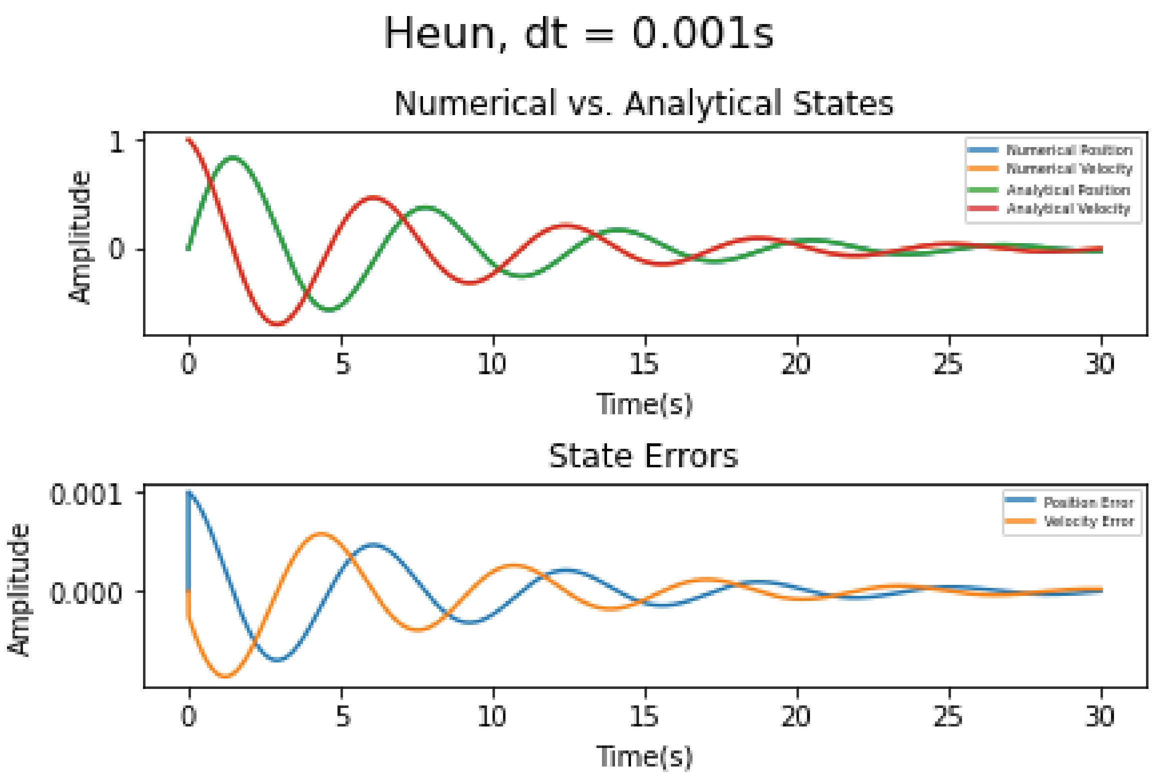
Heun Plots:


Heun, dt = 0.5s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)


Heun, dt = 0.2s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)


Heun, dt = 0.1s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)


Heun, dt = 0.01s
Numerical vs. Analytical States
Numerical Position
Numerical Velocity
Analytical Position
Analytical Velocity
Amplitude
Time(s)
State Errors
Position Error
Velocity Error
Amplitude
Time(s)



With the largest step size tested dt = 0.5s, the numerical solution seems to be the proper waveform, but shifted slightly in time. As dt decreases, the numerical solution collapses relatively quickly onto the analytical solution. The main advantage of the Heun solution seems to be that the solution does not diverge in amplitude from the analytical solution, thus errors continue to get small as the magnitude of the response decreases due to damping.

(c)(bonus)Plot the step response of the system.

The step response is the response of a system to a step input with initial conditions set to zero. In this case, to get the step response, we set $\vec{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $u = 1$ then run the program.