Submit your solution on MS Teams as one (1) PDF file

Goals

- Use the equations of rigid-body motion to qualitatively explain aircraft behavior.
- Be able to implement the equations of motion in Python and simulate the motion of an aircraft subject to given forces and moments
- Familiarize yourself with some custom aircraft frame and angle conventions.
- Be able to implement your own numerical integration method in Python.

Reading

- Beard and McLain (BM), *Small Unmanned Aircraft: Theory and Practice*: Chapter 3

Excercise 1: Download Python files for numerical integration from MS Teams (see week 4 > Files > hw02). For this question you will Implement a Runge-Kutta 4 integration routine.

(a) First, run the downloaded simulation and make sure it works.

(b) Next, implement your version of the Runge Kutta 4 integrator in the file `integrators.py`.

(c) Integrate the mass-spring system with both the `Heun` and `RungeKutta4` method. Experiment with suitable step sizes `dt`. Compare the numerical solutions with the analytical solution. Attach your plots and describe your findings.

Excercise 2: Let the body-fixed frame of an aircraft be chosen such that it coincides with the principal axes, i.e. the mass moment of inertia matrix is diagonal:

$$\mathbf{J}_c^{bb} = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix}.$$

(a) Write down the three equations that follow from application of Euler's second law in the body frame, i.e.

$$\begin{aligned} m_1^b &= \dots \\ m_2^b &= \dots \\ m_3^b &= \dots, \end{aligned}$$

where m_i^b, $i = 1, 2, 3$, are the components of the total applied moment expressed in the body frame.

Using these expressions, we now study a vertical spin motion. Let the aircraft velocity $\mathbf{V}_g$ point down (the angle of attack α is very large), and also the angular velocity $\boldsymbol{\omega}$. Assume that the the pitch velocity is zero, $q = 0$.

(b) Sketch an aircraft with body frame, and indicate $\mathbf{V}_g$, $\boldsymbol{\omega}$, p and r.

(c) Assume *stationary* vertical spin motion. What is needed to maintain the spin motion? Which part of the aircraft can provide this?

(d) **Bonus** For fighter jets with low aspect ratio wings and a heavy engine in the fuselage (e.g. F-100 Super Sabre): $J_1 \ll J_2$, and $J_2 \approx J_3$. What happens when the roll velocity is large? Do you think steady vertical spin can be maintained?

Excercise 3: Loosely defined since I don't have my own Python template ready yet, and I don't like the one provided by Beard. I will share my code with you later this week.. Here is your challenge: Implement the equations of motion given in the supplementary chapter 3 notes (see MS Teams) in Python code. The inputs are the forces and moments applied to the drone in the body frame. Parameters include the mass, the moments and products of inertia, and the initial conditions for each state. Use the parameters given in Beard's book, appendix E. For possible templates / examples see the book's website Small Unmanned Aircraft: Theory and Practice.