Separable Differential Equations and Homogeneous First-Order DE's

Donner Hanson Jack McGrath Sarah Townsend

Anna Greene Caitlin Felts

November 27, 2020

Instructor:	Dr. Katherine Evans
Class:	Linear Algebra and Differential Equations
Time:	Monday, Wednesday, Friday 12:00PM - 1:00PM

1.4 Separable Differential Equations

Definition 1.4.1: A first-order differential equation is called separable if it can be written in the form

$$p(y) \cdot \frac{dy}{dx} = q(x)$$

1.4.2 Theorem:

If p(y) and q(x) are continuous, then Equation (1.4.1) has the general solution

$$\int q(y) \cdot dy = \int p(x) \cdot dx + c$$

where c is an arbitrary constant.
[2-3 examples]

1.8 Homogeneous first-order DE's

1.8.1 Definition:

A function is homogeneous of degree zero if

$$f(tx, ty) = f(x, y)$$

for all positive values of t for which (tx, ty) is in the domain of f.

1.8.3 Theorem:

A function $f(x, y)$ is homogeneous of degree zero if and only if it depends on y or x only.

1.8.4 Definition:

If $f(x, y)$ is homogeneous of degree zero, then the differential equation

$$\frac{dy}{dx} = f(x, y)$$

is called a homogeneous first-order differential equation.

1.8.5 Theorem:

The change of variables $y = x \cdot V(x)$ reduces a homogeneous first-order differential equation $\frac{dy}{dx} = f(x, y)$ to the separable equation

$$\frac{1}{F(V) - V} \cdot dV = \frac{1}{x} \cdot dx$$

[2-3 examples]