BDA hw2

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(2) Probability of a girl birth given placenta previa

We can set up the variable in the following way.

$$\begin{aligned} y &: \text{event of female birth; 437} \\ n &: \text{total number of births; 980} \\ \theta &: \text{probability of placenta previa} \end{aligned}$$

Then we can construct a Binomial model to see whether the claim that the proportion of female births in the population of placenta previa is less than 0.485.

Prior : Uniform(0,1)

First, we can use a Uniform(0,1) non-informative prior for θ. Since, Uniform(0,1) distribution is a special case of the Beta distribution($Beta(\alpha, \beta)$), we can directly derive the posterior distribution by conjugacy.

$$\begin{aligned} \theta|y &\sim Beta(437+1, 980-437+1) \\ &\sim Beta(438, 544) \end{aligned}$$

The mean and variance of the posterior distribution can be derived mathematically.

$$E[\theta|y] = \frac{438}{438+544} \approx 0.4460$$

$$Var(\theta|y) = \frac{438 * 544}{(438+544)^2(438+544+1)} \approx 0.0002513602 \approx 0.0159^2$$

The 95% credible interval can be computed in the following way.

```
(lower_bound <- qbeta(0.025, 438, 544))
```

```
## [1] 0.4150655
```

```
(upper_bound <- qbeta(1-0.025, 438, 544))
```

```
## [1] 0.4771998
```

```
domain <- seq(0,1,0.001)
image <- dbeta(domain, 438, 544)
plot(domain, image,
     main='Posterior distribution\nBeta(438,544)',
     frame.plot = FALSE,
     type = 'l',
     )
polygon(c(domain[domain > lower_bound & domain < upper_bound], upper_bound, lower_bound),
```

```r
        c(image[domain > lower_bound & domain < upper_bound], 0, 0),
        col = 'grey')
```