Spatio-temporal Data Analysis HW1

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Q1

(a)

Prove $E(Y) = E(\mu + LZ)$ and $V(Y) = V(\mu + LZ)$

Proof.

$$
\begin{aligned}
E(Y) &= E(\mu + LZ) \\
&= \mu + E(LZ) \\
&= \mu + LE(Z) && \text{(linearity property)} \\
&= \mu \\
V(Y) &= E([(Y - E(Y))(Y - E(Y))']) \\
&= E([(Y - \mu)(Y - \mu)']) \\
&= E([(LZ)(LZ)']) \\
&= E([LZZ'L']) \\
&= LE(ZZ')L' \\
&= LV(Z)L' \\
&= LIL' \\
&= LL' \\
&= \Sigma
\end{aligned}
$$

□

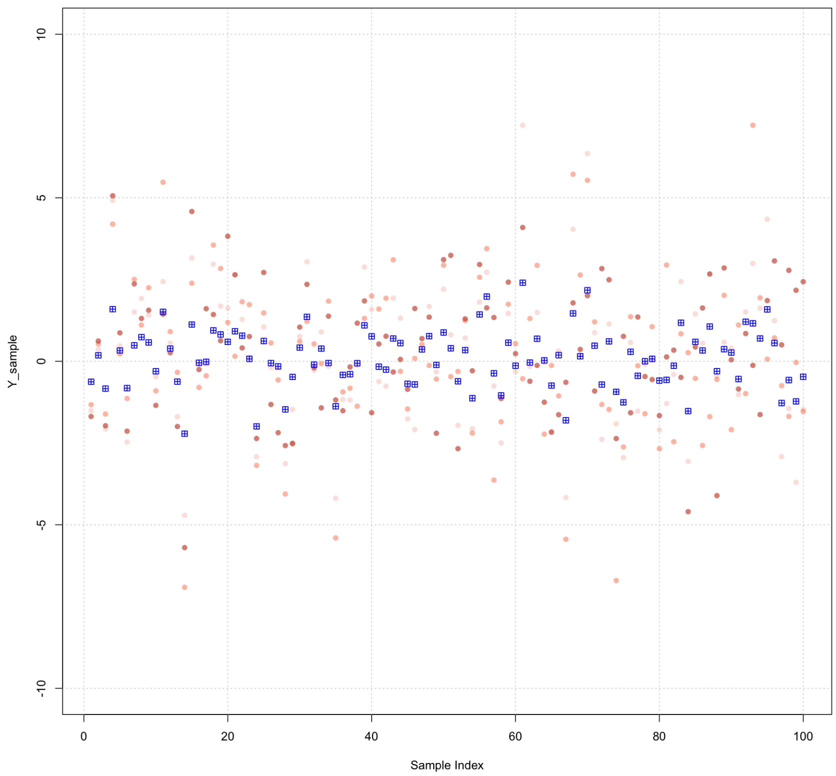
(b)

```r
create_multivariate_normal <- function (mu, Sigma) {
  lower_triangle_matrix <- t(chol(Sigma))
  dimension <- nrow(Sigma)
  Z <- rmvnorm(n = 1, mean = rep(0, dimension), sigma = diag(dimension))
  Y = mu + lower_triangle_matrix %*% Z
  return(Y)
}
```

(c)

```r
library(mvtnorm)
library(clusterGeneration)
library(yarrr)
generate_z ← function(dimension, seed= 1) {
  set.seed(seed)
  Z ← rmvnorm(n = 1, mean = rep(0, dimension), sigma = diag(dimension))
  return(Z)
}

generate_multivariate_normal_with_Z ← function (mu, Sigma, Z, seed=1) {
  set.seed(seed)
  lower_triangle_matrix ← t(chol(Sigma))
  Y = mu + (lower_triangle_matrix %*% t(Z))
  return(Y)
}


dimension ← 100
Z_sample ← generate_z(dimension)

plot(x=1,
     type="n",
     xlim = c(1,100),
     ylim = c(-10,10),
     pch = 16,
     xlab="Sample Index",
     ylab="Y_sample",
     )
grid()

set.seed(1)
Sigma ← genPositiveDefMat(dimension, covMethod="eigen")$Sigma
Y_sample ← generate_multivariate_normal_with_Z(mu = rep(0,100), Sigma = Sigma, Z = Z_sample)
points(1:100, Y_sample,
       pch = 16,
       col = transparent("coral2", trans.val = .8))

set.seed(1)
Sigma ← genPositiveDefMat(dimension, covMethod="onion")$Sigma
Y_sample ← generate_multivariate_normal_with_Z(mu = rep(0,100), Sigma = Sigma, Z = Z_sample)
points(1:100, Y_sample,
       pch = 16,
       col = transparent("coral", trans.val = .5))

set.seed(1)
Sigma ← genPositiveDefMat(dimension, covMethod="unifcorrmat")$Sigma
Y_sample ← generate_multivariate_normal_with_Z(mu = rep(0,100), Sigma = Sigma, Z = Z_sample)
points(1:100, Y_sample,
       pch = 16,
       col = transparent("coral3", trans.val = .3))

```

```
Sigma <- diag(dimension)
Y_sample <- generate_multivariate_normal_with_Z(mu = rep(0,100), Sigma =
    Sigma, Z = Z_sample)
points(1:100, Y_sample,
       pch = 12,
       col = transparent("blue", trans.val = .1))
```



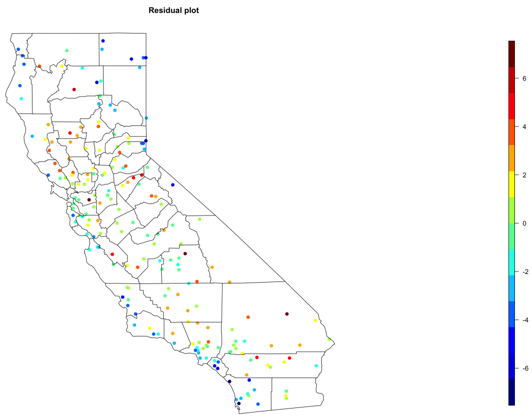
The blue squares are samples from an identity matrix sigma and others are from correlated sigmas.

Q2

(a)

```
CAtemp_with_coordinates <- cbind(CAtemp, coordinates(CAtemp))
ordinary_least_squares <- lm(avgtemp ~ lon + lat + elevation, data =
    CAtemp_with_coordinates)
predictions <- predict(ordinary_least_squares, newdata =
    CAtemp_with_coordinates)
residuals <- CAtemp_with_coordinates$avgtemp - predictions
CAtemp_with_residuals <- cbind(CAtemp_with_coordinates, residuals); names(
    CAtemp_with_residuals)[5] <- 'residuals'

range(CAtemp_with_residuals$residuals)
breaks <- -7:7
ploteqc(CAtemp_with_residuals, CAtemp_with_residuals$residuals, breaks,
    pch = 19)
```

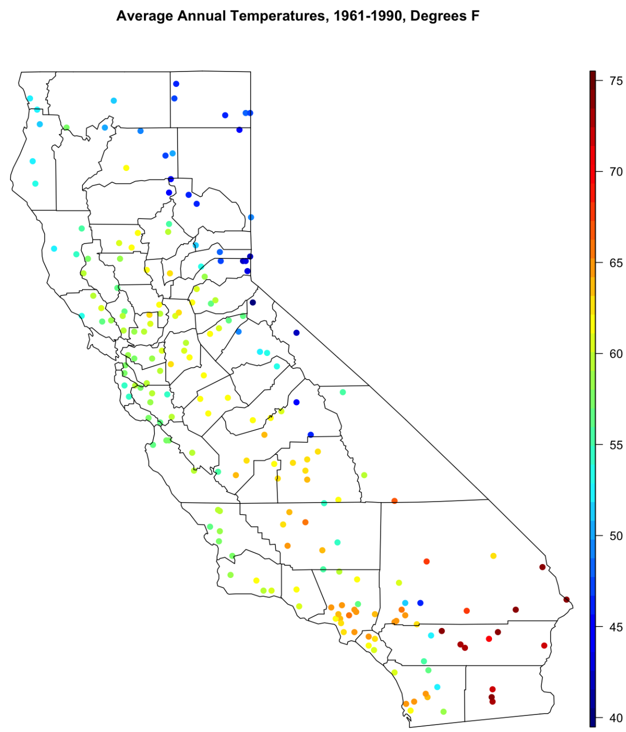
```r
map("county", region = "california", add = TRUE)
title(main = "Residual plot")
```

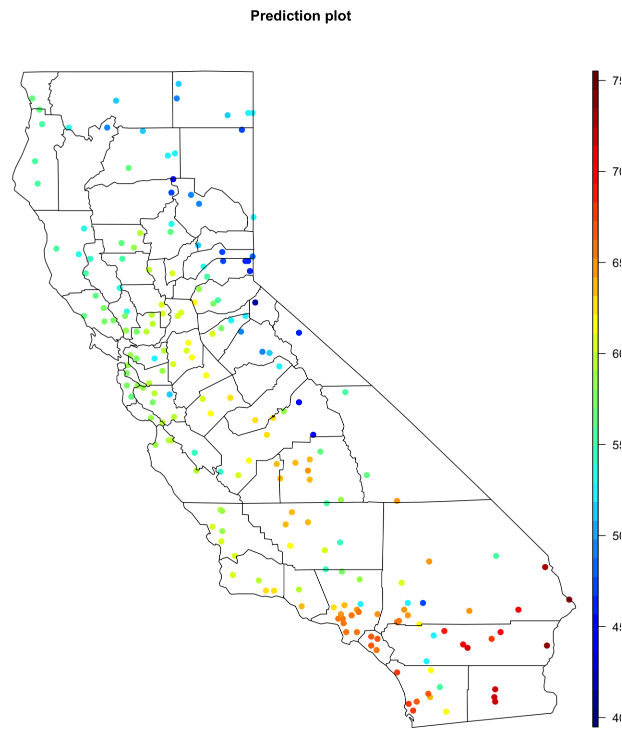


Plot of residuals as colored points onto map of CA.

Residuals seem to have a shrinking pattern as it gets closer to the border. This means that there lies unmeasured spatial dependency for the data.

```r
par(mfrow = c(1,2))
range(CAtemp$avgtemp)
breaks <- 40:75
ploteqc(CAtemp, CAtemp$avgtemp, breaks, pch = 19)
map("county", region = "california", add = TRUE)
title(main = "Average Annual Temperatures, 1961-1990, Degrees F")
ploteqc(CAtemp_with_predictions, CAtemp_with_predictions$predictions,
    breaks, pch = 19)
map("county", region = "california", add = TRUE)
title(main = "Prediction plot")
```



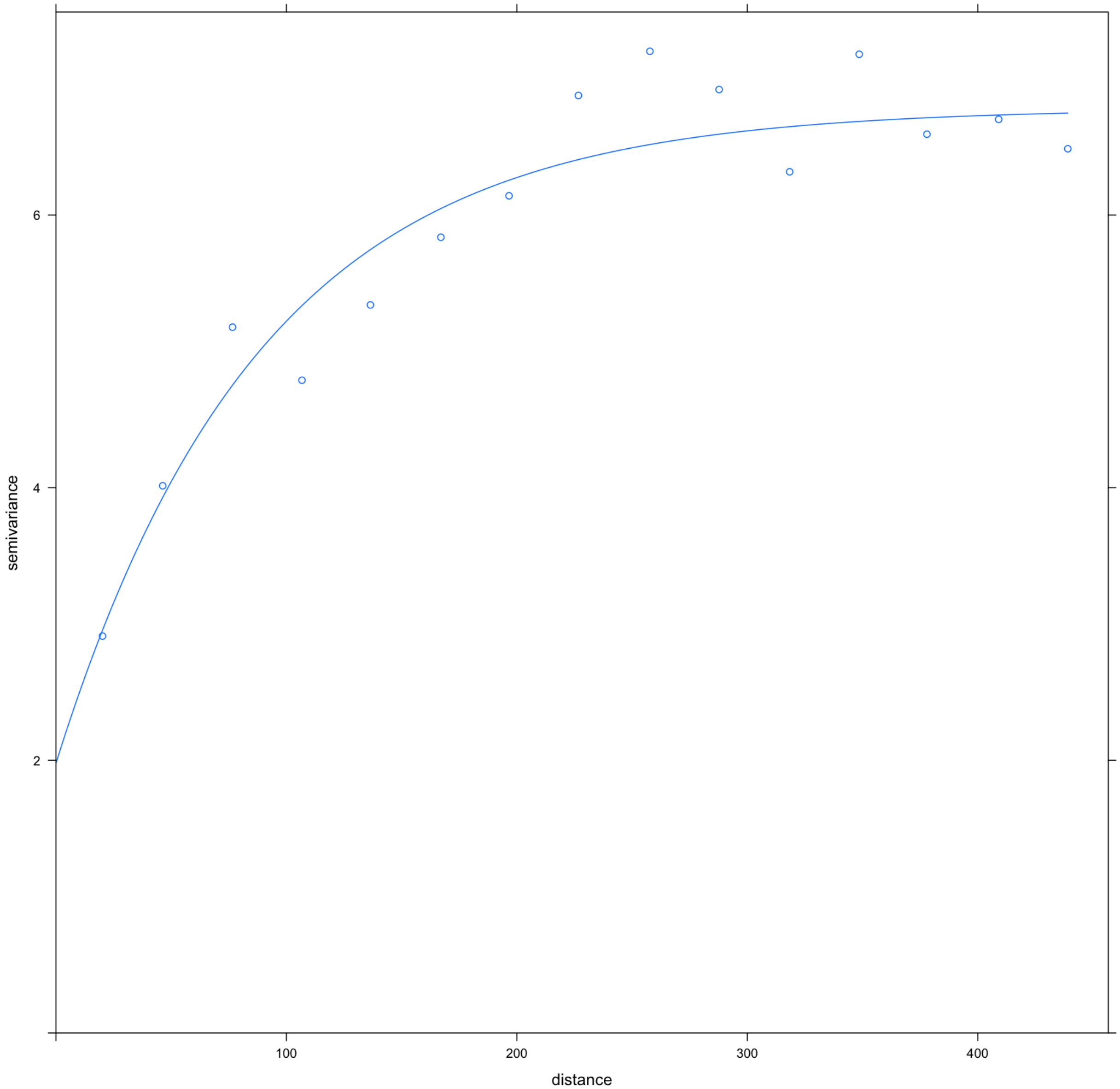


This is the prediction plot side by side with the original data.

(b)

After non-parametric variogram estimation we can fit a exponential(parametric) curve using the residuals. The parameters are as follows.
$\hat{\tau}^2 = 1.978, \hat{\sigma}^2 = 4.804, \hat{\rho}^2 = 88.937$.

```
variogram <- variogram(residuals ~ 1, data = CAtemp_with_coordinates)
variogram.fit <- fit.variogram(variogram, vgm(1, "Exp", 100, 2))
plot(variogram, variogram.fit)

sigma_sqrd_hat <- variogram.fit$psill[2]
tau_sqrd_hat <- variogram.fit$psill[1]
rho_hat <- variogram.fit$range[2]
```



(c)

$\hat{\beta}_{gls} = (X'\Sigma X)^{-1}X'\Sigma^{-1}Y$, where $\hat{\Sigma}_{i,j} = C(s_i - s_j)$
$C(s_i, s_j)$ is the exponential covariance function.

$$C(s_i, s_j) = \begin{cases} \tau^2 + \sigma^2 & \text{if } s_i - s_j = 0 \\ \sigma^2 exp(-\|s_i - s_j\|/\rho) & \text{else} \end{cases}$$

```
# Get distance matrix.
distance_matrix <- rdist(coordinates(CAtemp_with_coordinates))
dim(distance_matrix) # (200, 200)

# Create cov matrix.
exponential_covariance <- function(distance_matrix, tau_sqrd_hat,
```

```
    sigma_sqrd_hat, rho_hat){
  n = dim(distance_matrix)[1]
  matrix_with_no_nugget <- matrix(rep(0, n*n), ncol=n)
  print(dim(matrix_with_no_nugget))
  for (i in 1:n){
    for (j in 1:n){
      h = distance_matrix[i,j]
      matrix_with_no_nugget[i,j] <- sigma_sqrd_hat * exp(-h/rho_hat)
    }
  }
  matrix_with_nugget <- (sigma_sqrd_hat + tau_sqrd_hat) * diag(n)
  print(dim(matrix_with_nugget))
  return(matrix_with_no_nugget + matrix_with_nugget)
}
covariance_matrix_hat <- exponential_covariance(distance_matrix, tau_sqrd_hat, sigma_sqrd_hat, rho_hat)

# Invert covariance matrix and store.
inverse_covariance_matrix_hat <- solve(covariance_matrix_hat)

# Create X
X <- cbind(CAtemp$elevation, coordinates(CAtemp)); colnames(X) <- c('elevation', 'lon', 'lat')
y <- CAtemp$avgtemp

# Form beta_gls
(beta_gls <- solve(t(X) \%*\% inverse_covariance_matrix_hat \%*\% X) \%*\% t(X)\%*\% inverse_covariance_matrix_hat \%*\%y)
```

(d)