CHEN 2450
HOMEWORK 10

BOUNDARY VALUE PROBLEMS

Submit all your homework using Jupyter notebooks. You are expected to include proper text and discussion for each and every problem including appropriate headings and formatting. You will lose points if your reports are not readable or just include code with a few print out statements. No exceptions.

Problem 1 (50 pts)

Consider the steady state heat diffusion equation in a rod of extending from $x = 0$ to $x = L$:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = -\frac{1}{\lambda} e^{-\frac{(x-a)^2}{b^2}}. \tag{1}$$

The right-hand-side of this equation is a source term that contributes to heating the rod and is a function of the x location on the rod.

1. (10 pts) Using the ***finite difference method***, discretize equation (1) at an arbitrary interior point i $(1 < i < n)$. Do this symbolically and use x_i in the source term

2. (20 pts) For $N = 5$ grid points, show the system of equations (in matrix form) that must be solved subject to the Dirichlet boundary conditions:

$$\begin{cases} T = \alpha & \text{at } x = 0 \\ T = \beta & \text{at } x = L \end{cases}. \tag{2}$$

3. (20 pts) For $N = 5$ grid points, show the system of equations (in matrix form) that must be solved subject to the Neumann boundary conditions

$$\begin{cases} \frac{\mathrm{d}T}{\mathrm{d}x} = \alpha & \text{at } x = 0 \\ \frac{\mathrm{d}T}{\mathrm{d}x} = \beta & \text{at } x = L \end{cases}. \tag{3}$$

Problem 2 (50 pts)

Consider the ODE

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}x^2} = \alpha e^{-\phi x} \tag{4}$$

with boundary conditions $\phi(0) = a$ and $\phi(L) = b$.

1. (10 pts) Linearize the source term in (4) around a guessed value of ϕ^* and show the new linearized ODE.

2. (15 pts) Using the ***finite difference*** method, show the ***linearized*** set of equations for $n = 5$ grid points (including the boundary points like we did in class). Write these in matrix form.

3. (25 pts) Write a Python code that solves the discrete linearized problem. Plot the numerical solution to this problem for $n = 100$ points, $L = 1$, $a = 0$, $b = 1$ and $\alpha = -10^4$.