ChEn 3603 Homework 3

Problem 1 (10 pts)

Consider flow in a pipe as depicted in Figure 1 where the axial velocity profile varies radially as

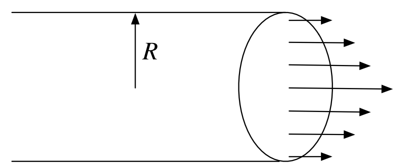

Figure 1: Pipe flow

$$v_z = \frac{\Delta p R^2}{4\mu L}\left[1-\left(\frac{r}{R}\right)^2\right] = v_z^{\max}\left[1-\left(\frac{r}{R}\right)^2\right]. \tag{1}$$

Assuming that the molar concentration of species A varies as

$$c_A = c_{A_0}\frac{r}{R}, \tag{2}$$

with c_{A_0} 25% of the total concentration, c, answer the questions below.

1. (3 pts) Plot the mole fraction of species A and the normalized molar flux of species A, $\tilde{N}_A = \frac{N_A}{c_{A_0} v_z^{\max}}$ as a function of the normalized radial position, $\tilde{r} = {}^{r}/{}_{R}$. You also need to show how you obtain these quantities.

2. (2 pts) Determine the radial location of the *maximum* molar flux of species A, $\tilde{N}_A$. Express your answer in terms of the normalized radius, $\tilde{r} = {}^{r}/{}_{R}$.

3. (5 pts) Determine the total molar flow rate of species A (N_A) *and* the total volumetric flowrate of the system $(\dot{V})$. Express your answer in terms of $v_z^{\max}$, c_{A_0} and R.

Problem 2 (20 pts)

Consider a gravity-drained tank of height H, length L as depicted in Figure 2 with a parabolic profile such that the tank's width is related to its height as $h = aw^2 + bw + c$. The width of the tank at its top is W, and that the width of the tank at $\frac{H}{4}$ is $\frac{W}{2}$.

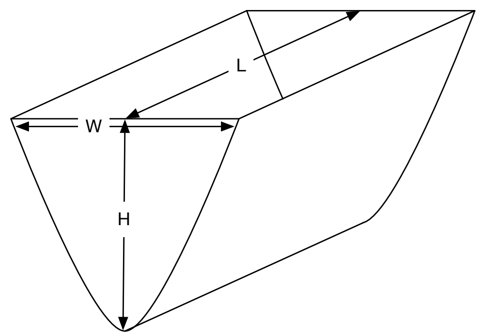

Figure 2: A parabolic tank.

1. (5 pts) Derive an expression for the rate of change of liquid height, $\frac{dh}{dt}$, in terms of h, H, L, W as well as the inlet flow rate $\dot{V}_{\text{in}}$ and the radius of the hole in the bottom of the tank, r.
2. (6 pts) Determine the radius of the outlet hole such that the tank will remain half-full by *volume* (not height) at steady-state. Express your answer in terms of H, W, L $\dot{V}_{\text{in}}$ and the tank total volume V.
3. (9 pts) For $H = W = 0.1$ m, $L = 1$ m, $r = 5$ mm and $\dot{V}_{\text{in}} = 2 \times 10^{-3}\ \frac{\text{m}^3}{\text{min}}$, and assuming that the tank starts full,
   (a) What is the steady-state liquid level? Determine this analytically.
   (b) Plot $h(t)$ for $t = [0, 3]$ minutes
   (c) Report the liquid level in the tank after 3 minutes.

You may solve the ODE numerically.

Problem 3 (20 pts)

Given the cylindrical tank in Figure 3 where a chemical reaction $A \xrightarrow{k} B$ is occurring with reaction rate given by $S_A = -kc_A^2$. Assume that you are given c_A^{in} (the inlet concentration of species A), $\dot{V}_{in}$, r (the radius of the hole in the bottom of the tank) and $u = \sqrt{2gh}$ for the outlet velocity. You may assume that the concentration is uniform throughout the tank (perfect mixing). Also assume that the reaction does not result in a change in the molar volume of the mixture.

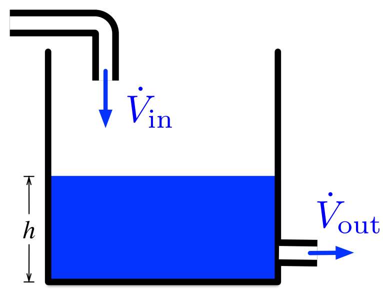

Figure 3: Cross-section of a tank of length L.

1. (6 pts) Show that
$$\frac{\mathrm{d}c_A}{\mathrm{d}t} = \frac{\dot{V}_{in}}{A_c h}\left(c_A^{in} - c_A\right) - kc_A^2,$$
where A_c is the cross-sectional area of the tank.

   Hints:

   - Note that both c_A and V are changing in time.
   - You will need to derive an equation for $\frac{\mathrm{d}V}{\mathrm{d}t}$ which you can substitute.

   Note that you need to show work that demonstrates that you can derive this (working backward from the answer won't get you too far). Otherwise you won't receive credit.

2. (8 pts) Determine $c_A(t)$ and $h(t)$ given: a tank diameter of 0.5 m and height of $H = 1$ m, $c_A(0) = 0\,\mathrm{mol/m^3}$, $c_A^{in} = 1\,\mathrm{mol/m^3}$, $k = 10^{-2}\,\mathrm{s}^{-1}$, a hole in the bottom with radius $r = 1$ cm, an inlet flow rate tank such that at steady state the tank is half full, and an initial liquid level of 5% and 95% the tank height. Plot the concentration of A, the moles of A and the height of the liquid in the tank over a 10-minute time period. Put the results for the two initial liquid levels on the same plot, but provide separate plots for $c_A(t)$, $N_A(t)$ and $h(t)$.

3. (2 pts) For the numbers in part 2, determine the steady-state concentration of A. Hint: at steady state, $\frac{\mathrm{d}c_i}{\mathrm{d}t} = 0$.

Problem 4 (16 pts)

Consider a situation where we have evaporation from a beaker tube as shown schematically in Figure 4. We will study this problem in more detail soon. For now, I will give you the solution to the problem (species profiles and molar fluxes) and here you will analyze the results. The link on the class web page provides data for the mole fraction profiles, $x_i(z)$ with z in meters, as well as the molar fluxes of each species, N_i $\left(\mathrm{mol/m^2{\cdot}s}\right)$, which are constant. You may assume that the gas phase is at 298 K and 1 atm. The species are:

1. Acetone

2. Methanol

3. Air (Here we treat air as a single species. To determine its molecular weight, assume it is comprised of 21% O_2 and 79% N_2).

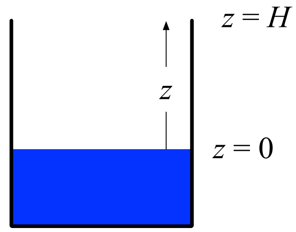

Figure 4: Schematic of the beaker with evaporating liquid.

Using this data,

1. (4 pt) Plot $x_i(z)$ on a single plot. Plot $\omega_i(z)$ on a separate plot. Be sure to label axes and provide a legend.

2. (4 pts) Show the species velocities, $v_i(z)$. Be sure to also include the equation you use to determine these. Explain the behavior you observe. Hint: you may want to omit the last point from your plot - you will see why when you try to plot it.

3. (4 pts) Plot the mass-averaged velocity, v, and molar averaged velocity, v_M as functions of z on the same plot. Include the equations you use to obtain these quantities. Discuss and explain your observations.

4. (4 pts) Plot the species molar diffusive fluxes relative to a molar averaged velocity, J_i. Also show the convective flux of each species.

   (a) What do you observe about the sum of convective and diffusive fluxes for each species?

   (b) Specifically for air, what do you observe about the convective versus diffusive fluxes?