ChEn 3603 Homework 2

Problem 1 (8 pts)

For an incompressible fluid, the divergence of the velocity is zero: $\nabla \cdot \mathbf{v} = 0$.

Consider a two-dimensional velocity field with the x-velocity component (u) given as

$$u = A\beta \cos(\alpha x) \sin(\beta y)\, e^{-\left(\alpha^2+\beta^2\right)\nu t}, \tag{1}$$

where A, α and β are constants and ν is the fluid's kinematic viscosity.

1. Find the implied y-velocity component (v) for an incompressible fluid.
2. Produce contour plots of the velocity magnitude that can be *interactively controlled* via a time slider. Use unity for all constants in (1) and allow the interactive slider over the time interval $t = [0, 1]$. For any constants of integration, take their values to be 0.

Plotting Tips:

- Look at the numpy.meshgrid function
- Look at the matplotlib.pyplot.contourf function
- Look at the ipywidgets.interact function and documentation on the float slider widget. I suggest that you get your plotting working by itself first, then wrap that into a function that takes the time as an argument and generates the plot for use with the interact function.

Problem 2 (11 pts)

From heat transfer, you should recall that Fourier's law of conduction states

$$\mathbf{q} = -\lambda \nabla T$$

where λ is the thermal conductivity and $\mathbf{q}$ is the heat flux (a vector). Recall that ∇ is the gradient operator, which can be written in Cartesian coordinates as $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$.

In addition, for constant λ,

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T,$$

where $\alpha = \frac{\lambda}{\rho c_p}$ is the thermal diffusivity and ∇^2 is the Laplacian, which in Cartesian coordinates is $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Recall that ∇ represents the *slope* while ∇^2 represents *curvature*.

1. (5 pts) Complete the following table, assuming a one-dimensional temperature profile:

T profile	Heat Flux	$\frac{\partial T}{\partial t}$
Constant		
Linear, slope>0		
Linear, slope<0		
Concave up, centered at origin		
Concave down, centered at origin		

2. (6 pts) For each of the following temperature profiles, plot the corresponding temperature gradient (∇T) and scaled time-rate-of change, $\frac{1}{\alpha}\frac{\partial T}{\partial t}$. Put $T(x)$ on the left axis and ∇T (or $\frac{1}{\alpha}\frac{\partial T}{\partial t}$) on the right axis.

   (a) $T(x) = x^2$

   (b) $T(x) = \sin(2\pi x)$

   Consider the range $x = [0, 1]$. Don't worry about units on any of this. You should have two separate plots for each part (four plots total).

Problem 3 (10 pts)

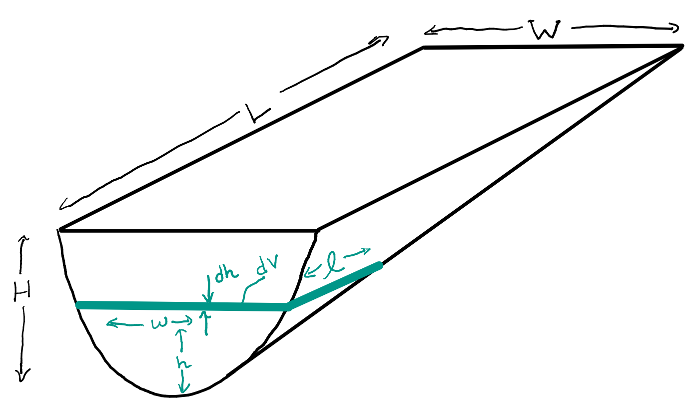
1. Find the area bounded by $y = 0$ and $y = x^2$ over the interval $x = [-a, a]$. Show all of your work for credit.
2. Find the area bounded by $y = x^2$ and $y = a^2$ on the interval $x = [-a, a]$. Show all of your work for credit.

Problem 4 (10 pts)

Consider a tank of length L with a parabolic cross-section such that the tank's width is related to its height as

$$h = aw^2 + bw + c. \tag{2}$$

The width of the tank at its top is W, and that the width of the tank at $h = \frac{H}{4}$ is $w = \frac{W}{2}$. The length of the tank varies linearly with its height. At $h = 0$ (the bottom of the tank), $\ell = 0$ whereas at the top of the tank, $\ell = L$. This id depicted schematically in the figure below.



Find the volume of the tank. Express your answer in terms of L, W and H.