Karlsruhe Institute of Technology

Institute of Stochastics

Stochastic Geometry | Summer term 2020
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Work Sheet 14

Instructions for week 14 (July 20^{th} to July 24^{th}):

- Steffen Winter will offer a discussion session via MS-Teams on July 22^{nd} at 16:00. If you want to participate in the discussion, please prepare your questions and send an e-mail to steffen.winter@kit.edu to announce your participation. Please also join the 'Team' corresponding to the exercise class via the link in our Ilias course.
- Solve Problems 1 and 2 of the exercises below. You are not supposed to submit solutions for this last exercise sheet, it shall simply serve to visualize the concept of Boolean models. Python codes and corresponding pictures will be provided at the end of the week.

Exercises for week 14:

Problem 1 (Simulating Boolean models)

Use your favorite programming language to do the following simulations (we suggest the use of R or Python). You might want to use some of the code from Problem 1 b) of Work sheet 5 as a starting point. Assume that the underlying center function is the center of the circumball of a compact set.

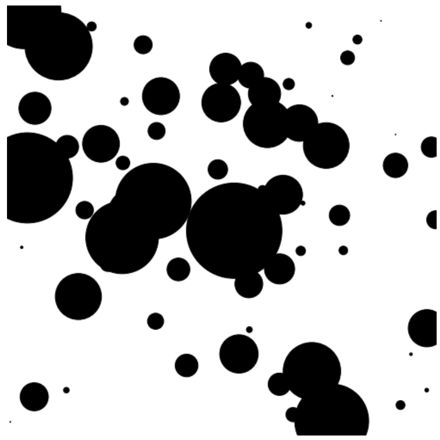
a) Simulate a stationary Boolean model in $[0,1]^2$ with intensity $\gamma = 50$ and deterministic grains that are disks with radius 0.05.

b) Vary the intensity parameter in part a). For which intensity do you get a connected component which connects the top and the bottom (or the left with the right side) of the unit square?

c) Simulate a stationary Boolean model in the square $[0,10] \times [0,5]$ with intensity $\gamma = 3$ and typical grain $Z_0 = [-S,S]^2$, where S is uniformly distributed on the interval $\left(0, \frac{1}{2}\right)$.

d) Simulate a stationary Boolean model in the cube $[0,5]^3$ with intensity $\gamma = 0.25$ and typical grain $Z_0 = B(0,R) \subset \mathbb{R}^3$, where R has an exponential distribution with parameter $\lambda = \frac{10}{3}$.

Problem 2 (Estimation in the Boolean model)



On the right hand side you find the realization of a stationary Boolean model Z observed in the unit square. You are given no information other than the fact that the picture shows the Boolean model in the unit square, $Z \cap [0, 1]^2$, and that the picture, provided under the name 'Sheet14_2' in 'png' format in Ilias, has a size of 500×500 pixels.

It is your task to find estimates for the intensity parameter γ of the Boolean model and for the expected radius of the typical grain, which apparently is a ball with some random radius.

Proceed in the following steps:

a) Visit the 'morphometer' software tool on the website Morphometry.org and make sure that the settings are put to 'Image analysis' in 'Expert' mode and that you 'analyze black phase'. The correct settings are indicated in the picture below (in red). Proceed to upload the picture of the Boolean model (the option is indicated in blue below).

b) After the analysis of the 'basic parameters' you can read off the area and perimeter of the Boolean model in the picture you have uploaded (indicted in orange in the picture below). The result is given in pixels. Use these values to calculate the intrinsic volumes of the model, $V_j\big(Z \cap [0, 1]^2\big)$, for $j = 0, 1, 2$.

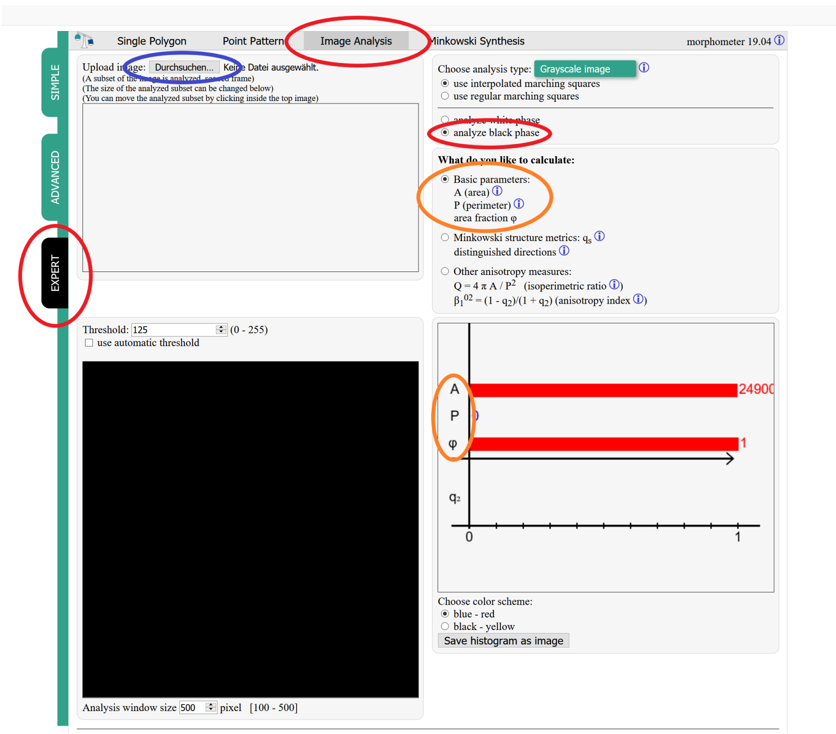
Hint: Recall that the picture has $500 \times 500 = 250000$ pixels and represents the unit square. To calculate the Euler characteristic, recall that it is just the number of connected components minus the number of holes in the set.

c) Use your results from part b) to estimate the intrinsic volume densities of the Boolean model.

Hint: The definition of $\delta_j = \delta_{V_j}$ suggests that $\widehat{\delta}_j = V_2\big([0, 1]^2\big)^{-1} \cdot V_j\big(Z \cap [0, 1]^2\big)$ is a reasonable estimator.

d) Use the relations from Remark 4.38 to estimate the intensity γ of the given Boolean model. Also use the information you have acquired so far to estimate the expected radius of the typical grain.

e) You are given the additional information that the radius of the typical grain follows an exponential distribution with some parameter $\lambda > 0$. Can you find an estimate for λ based on your previous observations?



The solutions to these problems will be uploaded on July 24th.