ON THE BALLISTIC MODEL OF AGGREGATION

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A simple model of aggregation is discussed. It is argued on the example of this model that the aggregates may exhibit interesting scale invariant structure while having a "trivial" fractal dimension (i.e. equal to the dimensionality of space).

In recent years, there has been a renewed interest in the question of how complex spatial and temporal structures may arise dynamically as a result of simple nonlinear evolution processes. An attractive example of such a process is furnished by the simple model proposed by Witten and Sander [1]. This model describes growth by diffusion limited aggregation (DLA): the particles diffuse from far away until they encounter the cluster, whereupon they "stick". The clusters grown that way in numerical simulations (see refs. [1–3]) exhibit highly ramified, branched structure which appears to be "fractal" and self-similar. The latter is usually expressed as a statement that the cluster possesses a nontrivial Hausdorff or "fractal" dimension. This dimension may be crudely defined through the scaling of the number of particles $n(R)$ in a sphere of radius R: $D_H = \log n(R)/\log R$, and may be determined numerically in a variety of ways (see refs. [1,2]).

The purpose of this letter is to suggest that some models of aggregation exhibit interesting and non-trivial behavior (i.e. possess structure on all length scales) while having a "trivial" Hausdorff dimension, equal to the dimensionality of space. One example of such a model is provided by ballistically driven aggregation (BDA) (also referred to as the Sutherland–Vold model [3]) in which the particles move along random straight lines. We have simulated BDA on a square lattice and concluded that in contrast with previous reports [4] the Hausdorff dimension appears to be trivial. That is, the number of particles within a sphere of radius R around the origin, $n(R)$, is proportional to R^2 for $R \to \infty$ in two spatial dimensions. (This conclusion was independently reached by Meakin [5].) However the limiting form is approached extremely slowly (which may account for previous misinterpretations) and clearly indicates a non-trivial and perhaps universal behavior, possibly of the nature:

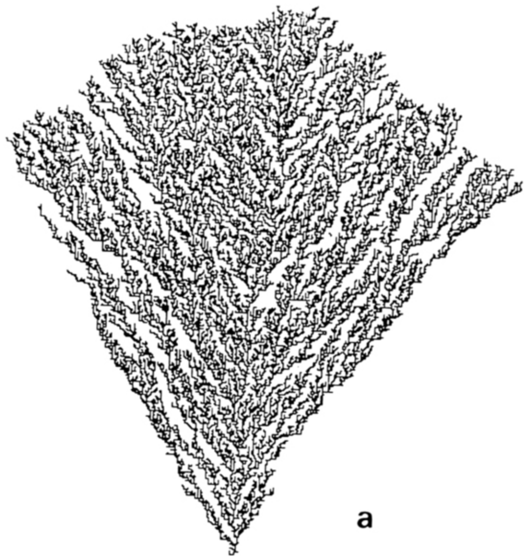
$$n(R) = \rho_0 R^2 (1 + A|\log R|^{-\alpha}) , \qquad (1)$$

or

$$n(R) = \rho_0 R^2 (1 + AR^{-\alpha}) . \qquad (2)$$

It appears that a more convenient model to study is a ballistic model modified in the following fashion: let the incoming particles all move along one direction only (say $-z$) with randomness in their "lateral" coordinate (say x) only. If one starts with a seed at point $x = z = 0$, this "rain" model generates objects like those shown in figs. 1a and 1b. The upper and the central part of these clusters seem to have uniform density and one expects therefore the fractal dimension to come out equal to 2. However, the large scale structures on the sides of the "fan" suggest that the objects are still non-trivial [‡1], with large fluctuations from realization to realization and interesting scaling corrections. This is confirmed by a numerical simulation which is particularly easy thanks to the simplicity of the mode. We obtained the function $n(R)$ with an

[‡1] One of the interesting features of the structure is the opening angle of the "fan", which empirically appears to be equal to $2 \tan^{-1} (1/2)$.



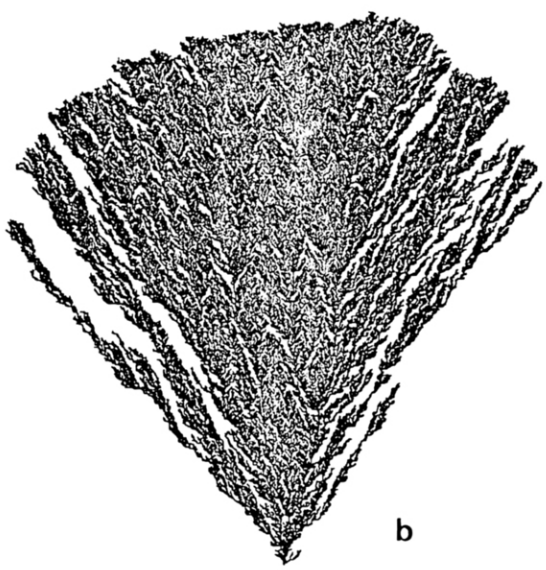


Fig. 1. (a) A cluster of about 10^5 particles generated by a "one-sided rain" BDA model. (b) A cluster of 2×10^6 particles generated by "rain" model. Notice the homogeneity of the upper and central portions of the aggregate. This uniformity seems to be responsible for the integer "fractal" dimension. However, in parallel, there are many large holes, particularly near the sides of the "fan". A comparison of figs. 1a and 1b suggests that holes of arbitrarily large size will appear in sufficiently large clusters. Hence there is nontrivial scale invariant structure superimposed upon an integer fractal dimension.

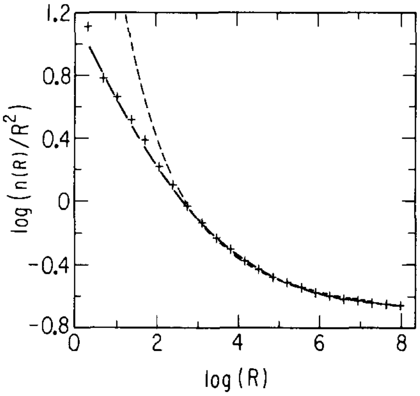


Fig. 2. Function $\log(n(R)/R^2)$ versus $\log R$; + is the data from the numerical simulation of one-sided BDA model; solid line is the fit of the form of eq. (1) with $A = 5.420$ and $\alpha = 0.6448$; dashed line – fit of the form of eq. (2) with $A = 9.640$ and $\alpha = 2.126$.

accuracy of 2% by averaging over 100 realizations of aggregates of 2×10^6 particles. We then fitted to it the functional forms suggested by eqs. (1) and (2). These fits are shown in fig. 2. Presently, in the absence of any theoretical motivation, we are unable to conclusively distinguish between the two forms of scaling corrections.

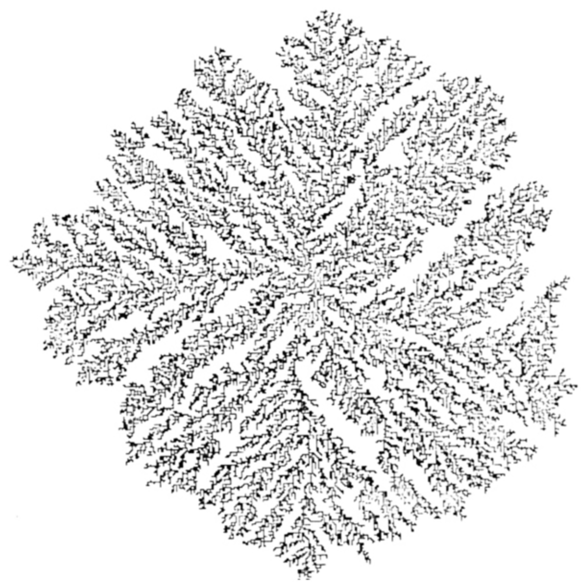


Fig. 3. A cluster of 10^5 particles generated by "four-sided rain' BDA model.

The "rain" model can be readily extended by making particles approach from many directions. For example, the cluster exhibited in fig. 3 was obtained by "shooting" particles at random from four directions. This simulation was also performed independently by Meakin [5]. Both calculations suggest that the fractal dimension of this kind of object is also trivial, while in our figure (fig. 3) one clearly observes structure on all length scales. However it is not yet understood, what an appropriate quantitative characterization of these structures should be.

The proposed models are sufficiently simple to serve as a convenient starting point for analytic study. In particular one can derive by controlled approximations of mean-field-like continuum model describing BDA [6]. This approach can then be extended in a natural way to models involving other type of particle motion, such as diffusion. In fact it is natural to consider a whole family of models interpolating from ballistic to diffusive motion, perhaps by considering "Levy flights". Of such a family we expect the BDA model to be marginally critical and as such a meaningful domain for the application of the mean field theory. These ideas will be discussed in a forthcoming publication.

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