ZAMS Project Report

Vincenzo Donofrio

Introduction

The goal of this project is to calculate the radius and luminosity of low-mass stars and see how their structure and luminosity depend on p+p reaction rate. By finding the radius at which the surface luminosity is equal to the nuclear luminosity, the zero-aged main-sequence can be determined for low mass stars. We can do so by modifying the routines we developed in the white dwarf project.

Step 1

To start, we needed to revisit the last project's goal and make sure that we have found the listed scalings in the instructions. After reworking the code from project 2 into a correct fashion, we have found the the Pc should scale with a factor of 0.77 and pc should scale with a factor of 5.99. Our values of delta, xi, and eta are also sufficiently small after a test to make sure they agree to two significant digits across a range of masses. The parameter values that work inlcude: eta = 1.0e-10, delta = 1.0e-8*Msun, and xi = 0.05. These values will be further investigated in step 8 of the instructions. Also, by rerunning the equation of state with K multiplied by 0.5 and 2, we found that the equations for Pc and pc remained satisfied. We did not write a script specific for this project to test these scenarios, as this code was crafted in project 2, but corrected that project 2 code and quickly performed these tests from that repository.

Steps 2 – 5; 8

For these steps, we needed to verify that the routines we finalized to be used for the latter parts of the project worked as intended. For the second and third steps we needed to compute the mean molecular weight for fully ionized plasma and write a routine that computes the density and temperature along an adiabat; the tests under *test_eos.py* for these two steps worked as intended. Since we are computing ZAMS that are at the start of their hydrogen burning phase we can approximate their composition to contain hydrogen, helium and nitrogen with the corresponding mass fractions of X(H)=0.706, X(He)=0.275 and H(N)=0.019; we used these exact values (they add to 1) to calculate the mean molecular weight. For step three, we needed to create a routine that would compute the heating rate of a low mass star which included in this calculation is a pp factor that gets multiplied by the pp rate; under *test_reactions.py,* the test for part four was successful. For step five, we wrote a routine that computes the central pressure, density, and temperature given a stellar mass and radius; this test under *test_reactions.py* was successful. We wrote a routine to interpolate the effective temperatures from Table 1 in the instructions and for the surface luminosity; these tests were successful under *test_zams.py.*

These tests all can be ran by deleting the necessary line in their respective test routine and then running *testing.py*. For notice, the routines in *structure.py* are fitted so they can produce correct results for the testing, while the updated routines for central_thermal and Teff are in *structure_for_main.py* and are used with unit inputs that fit better with the main files.

Step 6

For this part, we needed to copy four routines from the white dwarf project and modify them for use in this one. Besides updating the routines so they now fit this project, the main difference was requiring mass and radius as inputs instead of Pc for central_values and integrate; while perhaps these can be computed with Pc, this seemed like a necessary change considering that the new routine of central_thermal took in mass and radius as inputs. For example, in project two, central_values includes get_scaled_rho which input P, so central_values needed to input Pc, as well; however, since central_thermal inputs m and r and is necessary likewise for central_values in this project, central_values should take in mand r instead of Pc. Additionally, instruction specific modifications included adding a luminosity array and differential, including a new scale for stepsize, and a boundary condition for luminosity. As stated previously, the two routines for central thermal values and effective temperature are updated in *structure_for_main.py* for easier use with the main files.

Step 7

For this part, we needed to test the convergence of the integration by varying parameters eta, xi, and delta to ensure that our integration satisfied precision requirements. Our script under *test_convergence.py* involved two results: a return of arrays for the four integration return arrays (mass, radius, pressure, and luminosity) and a mass_keep array that returned each variation of eta, xi, and delta that satisfied the testing mass of 0.2 Msun. While this is essentially the same as the mass array, it helps to immediately where the convergence is happening, at least for the mass parameter. In the script, one line of the three at the bottom can be uncommented to test that parameter's convergence; the default that is uncommented is for delta.

From the project 2 solutions, we know that preferred values are eta = 1.0e-10, delta = 1.0e-8*Msun, and xi = 0.05. We used these values for this project, but we still needed to confirm their correctness. For delta, the output parameters began to converge starting at delta = 1.0e-7*Msun; this means it is reasonable to use delta = 1.0e-8*Msun as it is not too small, but still corresponds to after the integration converged. For eta, the output parameters began to converge starting at eta = 1.0e-6; this means it is reasonable to use eta = 1.0e-10 as it is not too small, but still corresponds to after the integration converged. For xi, the output parameters began to converge starting at xi = 0.1; this means it is reasonable to use xi = 0.05 as it is not too small, but still corresponds to after the integration converged. As stated before, only one target mass of m = 0.2*Msun was necessary and the pp_factor used was 1.0.

Step 9

For this part, we needed to create a root finder routine for stellar radius. This involved writing a function that computing the difference in nuclear and surface luminosity. We then needed to create a radius finder routine that solved for this radius in the luminosity equation. This root finder requires an interval for the radius. An effort to try and make use of the virial relation that central temperature varies very little with mass ended in either too many iterations occurring or f(a) and f(b) being of equal signs, so instead the interval of [0.001*Rsun, 4*Rsun] was utilized. Using prior knowledge about ZAMS stars we are analyzing, this interval is still sensible because we know that the minimum radius must be at least 0.001*Rsun and the maximum of 4*Rsun should cover all masses in between the intended range of [0.1*Msun, 0.3*Msun].

Step 10

a)

For this part, we needed to find the main-sequence radii, and hence L and Teff, for several masses (25) between 0.1Msun and 0.3Msun. This plot of log(L/Lsun) against log(Teff/K) required that the log(Teff/K) axis (x-axis) display as inverted to correspond to a Hertzsprung-Russell diagram.

b)

For this part, we needed to plot log(Tc/K) against log(pc/g cm^-3).

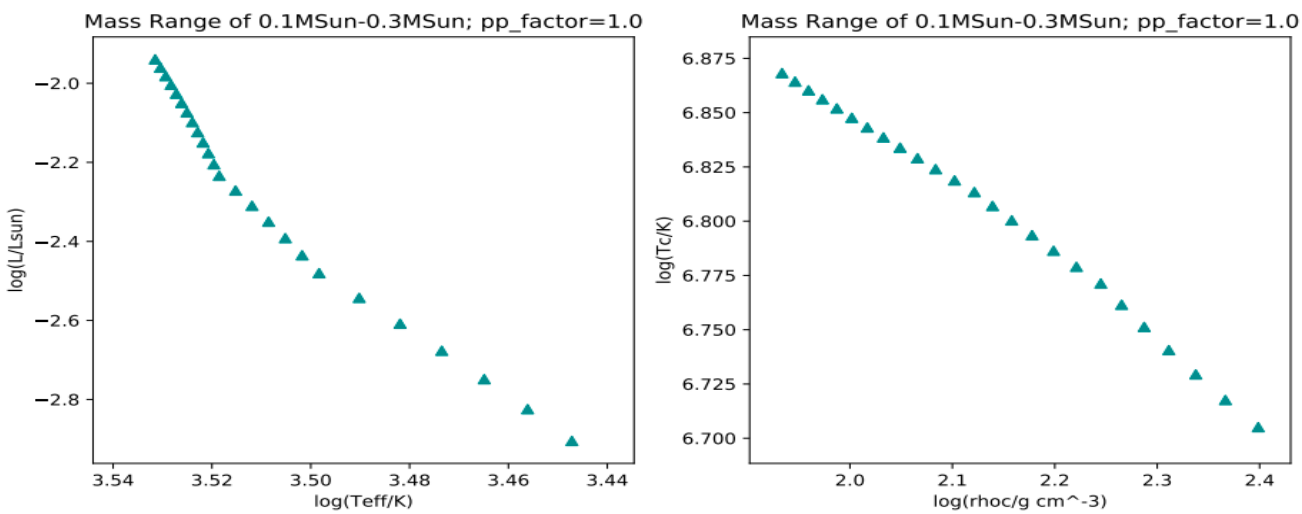


Figure 1a: Plots of log(L/Lsun) against log (Teff/K) and log(Tc/K) against log(rhoc/g cm^-3). Triangles represent 25 ZAMS stars between 0.1Msun and 0.3Msun. The pp_factor used is 1.0

The above plots are for 10a and b. We can compare to these to figures **A** and **B** in this appendix section of this report. We can see with **A** that the log(L/Lsun) against log(Teff/K) closely resembles what is expected from the ZAMS stars in the mass range 0.1Msun to 0.3Msun. At around (3.52, -2.25) on the above and **A**, we can see that gradient increases; also, the overall path of the stars follows **A**, including specific points that the stars fall on. We can see with **B** that the log(Tc/K) against log(pc/g cm^-3) closely resembles what is expected from the ZAMS stars in the mass range 0.1Msun to 0.3Msun. For the above and **B**, we can see that at around (2.2, 6.75), the gradient decreases; the overall trend is very similar as well.

Step 11

For this part, we needed to plot T(r), T(m), L(r), and L(m) for a star with M = 0.3Msun. For this part, we needed implement a different temperature equation as in part 10 as T is different than Teff and of course Tc. In fact, the instruction manual provided the equation of T = Tc(P/Pc)^(1-1/gamma), where Tc and Pc are constants that depend on the star and gamma is an absolute constant of gamma = 5/3. This temperature is computed in the routine get_rho_and_T, however, we add it as a simple equation in our main execution file to compute T when mass is at 0.3 Msun.

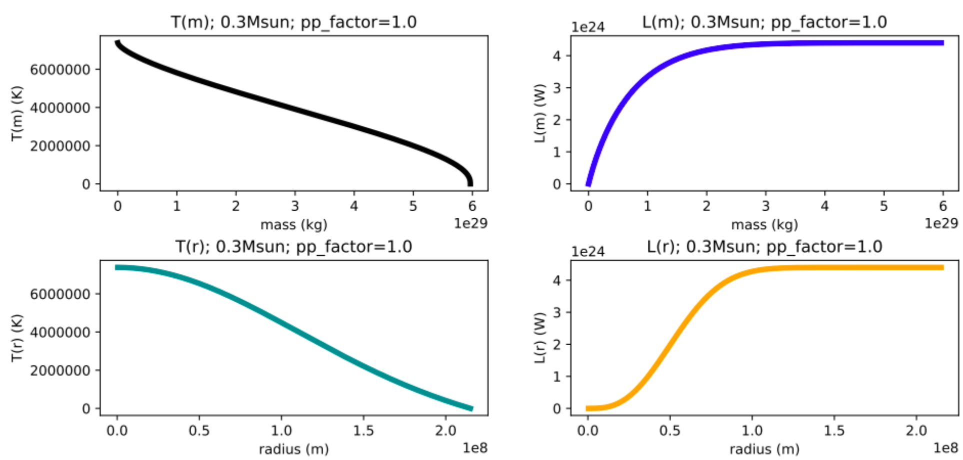


Figure 1b: Plots of T (K) against m (kg), T (K) against r (m), L (W) against m (kg), L (W) against r (m). Star used is of mass 0.3Msun. The pp_factor used is 1.0.

We can analyze each plot; while there are no comparison plots, we can reason why each behaves as they do as a good test of the content learned in the course. For T(r), we can see that

that as the radius of the star increases, the temperature decreases; this is sensible considering that the central temperature will be much hotter than the surface as radius is smaller and density is larger and the kinetic energy of particles is greater which allows temperature to increase. For T(m), we can see that as the mass of the star we integrate over increases, the temperature decreases. This is sensible considering that as we include more mass in the star, radius and mass increase. Since density is much more dependent on radius, it decreases as we collect more mass which makes temperature decrease as total mass increase. For T(r), we can see a clear decrease in gradient around 0.5e8 meters which means that location has reached large enough radius for the star to begin greatly declining in temperature. For L(r), we can see that the luminosity greatly increases with radius and then reaches a constant value. From the surface luminosity equation it is sensible that luminosity increases with radius increasing, but the constant value it approaches could correspond to eventual surface value it settles upon. For L(m), we can see that luminosity increases but with a declining gradient. The overall trend of increasing luminosity makes sense because as more mass is integrated over, so is more radius, where Luminosity greatly increases with. For L(m) and L(r), the luminosity seems to reach its constant level around the same location; this is sensible considering these plots are integrations over a star, so the luminosity should reach its constant value near the same integrated location; this is the case for the temperature as well.

We were also tasked to find what radius that L(r) reaches 90% of its final value, as well as what fraction of mass is enclosed by this radius; these values can be seen as print statements from the main files. Including them here, as well, L(r) reaches 90% of its final value at approximately a radius of 8.54e+7 meters; the fraction of mass enclosed by this radius is approximately 0.275 of the stars mass. These values seem to correspond to the plot above and the radius size makes sense as it at least should be smaller than of course its own maximum radius, but also for a sanity check, a lot smaller than the sun’s maximum radius; however, it should not be too small – say 8.54e+5 meters.

Part 3)

For this part, we needed to suppose that the weak interactions were 10^5 which would increase Enuc by the same factor. We needed to recompute the main-sequence stars with this increases reaction rate; we have included a pp_factor parameter in the necessary functions to make these easier. Additionally, instead a creating an unnecessarily long function in the main loop to change the pp factor, we simply created separate main files to compute with the different pp factors.

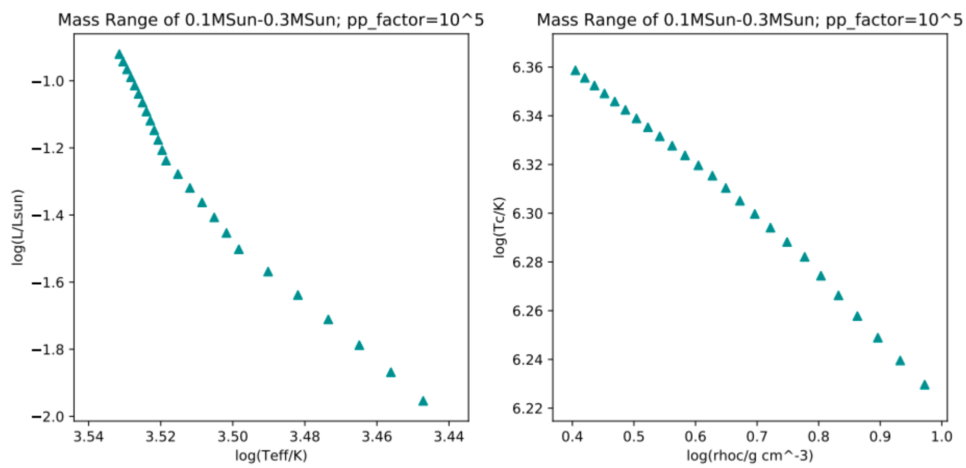


Figure 2a: Plots of log(L/Lsun) against log (Teff/K) and log(Tc/K) against log(rhoc/g cm^-3). Triangles represent 25 ZAMS stars between 0.1Msun and 0.3Msun. The pp_factor used is 10^5

We can see that the stars vary in luminosity for log(L/Lsun) against log(Teff/K). This makes sense considering that Teff does not depend on a change is pp factor. Additionally, if the weak interaction is increased and therefore, Enuc, this makes luminosity increase as the integrated luminosity depends on Enuc; if Enuc increases, then luminosity will. For log(Tc/K) against log(rhoc/g cm ^3), we can see that central temperature and density both decrease for all stars in the mass range when pp factor is increased. Obviously, the weaker interaction doesn't make the star burn as warm centrally; of the temperature is not centrally as warm, then the central density will also decrease as the reaction slows.

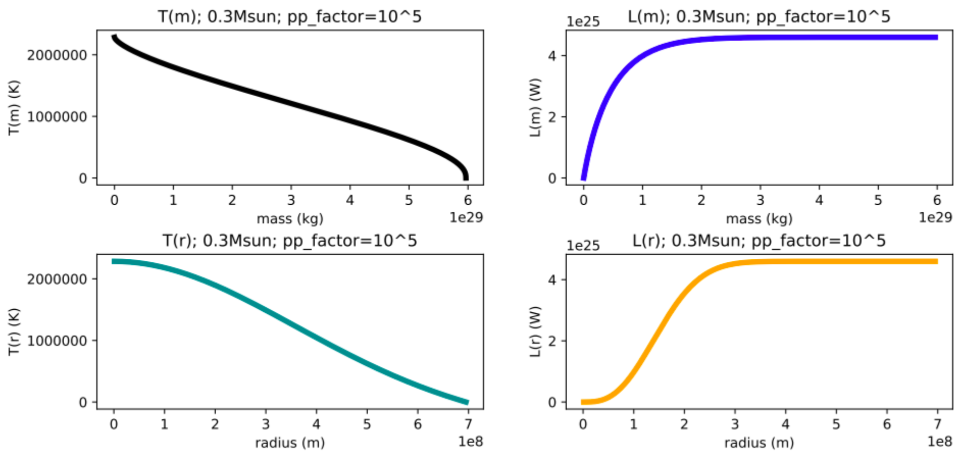


Figure 2b: Plots of T (K) against m (kg), T (K) against r (m), L (W) against m (kg), L (W) against r (m). Star used is of mass 0.3Msun. The pp_factor used is 10^5.

Although not intended to do so in the instructions, we have included T(r), T(m), L(r), and L(m). We can see the major changes for a 0.3Msun star includes a decrease as discussed previously in central temperature, and luminosity values increase by a factor of 10; the overall trends remain similar, as well. For this scenario, L(r) at 90% corresponds to a radius 2.39e+8 meters; this means that for a larger pp factor, the radius at which L(r) reaches 90% of its final value greatly increases. However, because the final radius also greatly increases, this means that even less of the star's mass is enclosed by this radius; 0.194 of the star's mass is enclosed by this radius.

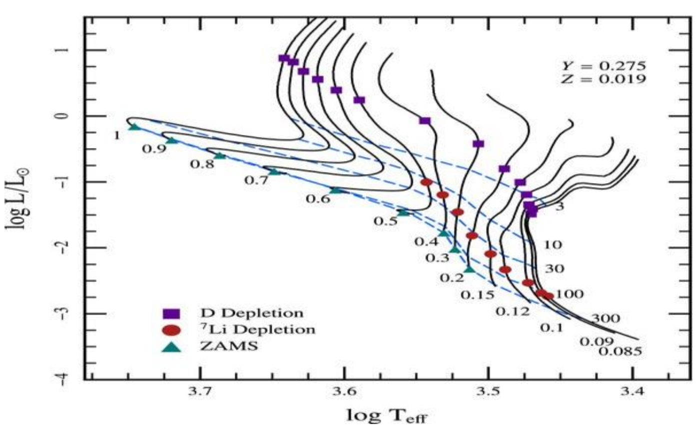
We can now speculate if the sun changed in a similar fashion to these lower-mass stars, how would surface temperatures on Earth change? We can first analyze the decrease in central temperature and temperature integrated over the star. In terms of analyzing surface temperatures on Earth, we want to consider luminosity of the sun, and although temperature varies luminosity greater than radius does, the change in temperature is not as great as the change in radius. Since the change in radius using this pp factor of the star is so greatly positive, this greatly increases the luminosity of the star. This makes sense when looking at the luminosity graphs and can see from the lot that luminosity increased by approximately a factor of ten for the main-sequence stars and the 0.3Msun star. Since this is the case, if the sun changed in a similar fashion with this increased factor, surface temperatures on Earth would greatly increase. This would likely cause life on Earth to not exist, as well as possibly further in planets be engulfed by the increased radius the sun would receive because of the increase in factor.

References

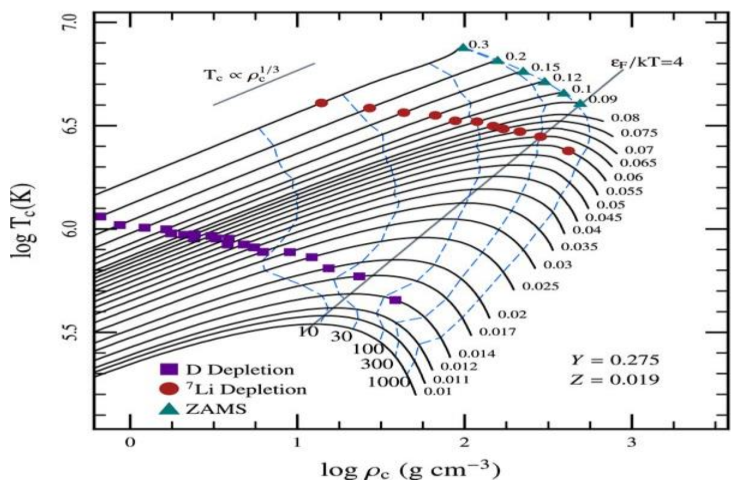
1. Bill Paxton, Lars Bildsten, Aaron Dotter, Falk Herwig, Pierre Lesaffre, and Frank Timmes. Modules for experiments in stellar astrophysics (MESA). ApJS, 192:3, January 2011.

Appendix

A[1]



B[1]



$T_c \propto \rho_c^{1/3}$
$\varepsilon_F/kT=4$
0.3
0.2
0.15
0.12
0.1
0.09
0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.017
0.014
0.012
0.011
0.01
10
30
100
300
1000
D Depletion
^{7}Li Depletion
ZAMS
$Y = 0.275$
$Z = 0.019$
log T_c(K)
log ρ_c (g cm^{-3})