Kepler Project Report

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Background of Parts 1-3

The goal of this project was to implement three methods of integration to solve Kepler's problem. The three methods of integration are Forward Euler, Second-Order Runge-Kutta (RK2) and Fourth-Order Runge-Kutta (RK4). The first task was writing each ODE as its own function that advanced the solution by one step size. These functions exactly computed the output present in the sample file. Then we had to complete the functions in the kepler.py file that computes the potential, kinetic and total energies. The kepler.py file also contained two routines. The first, *integrate_orbit*, allowed us to integrate the equations of motion of a particle from t=0 to t=tend. The second, *set_initial_conditions*, allowed us to compute the orbital period, energy, position and velocity for a set of initial values of the semi-major axis, eccentricity and mass. Combined these functions allowed us to complete parts 4 and 5 of the project which are discussed below.

Part 4 Qs: Plot the error. Does it scale as expected? Is it better to use linear or logarithmic axes in plotting the error?

Yes, the relative error in the energy as a function of step size scales as expected. We can see in **Figure 1** that the RK4 integration method begins at an initial error that is larger than the RK2 and Euler methods, but as step size heads toward zero, the RK4 method approaches zero relative error the quickest. In terms of scaling of the error, it is much better in this scenario to use logarithmic axes (in this case, for the y-axis). Looking at **Figure 1** and **Figure 2** below, we can see that the initial relative error value for the RK4 integration method is nearly a factor of 5 times larger than the second largest initial error value (Euler). Using a linear scale, the axis would span 800 units in this scenario which would largely mask the trend observable for Euler and RK2; this is observable in **Figure 2.** Also, logarithmic scaling basically allows one method to compare its progressive trend to itself by representing the percentage change of its data, which ultimately gives a much more meaningful result as in **Figure 1.**

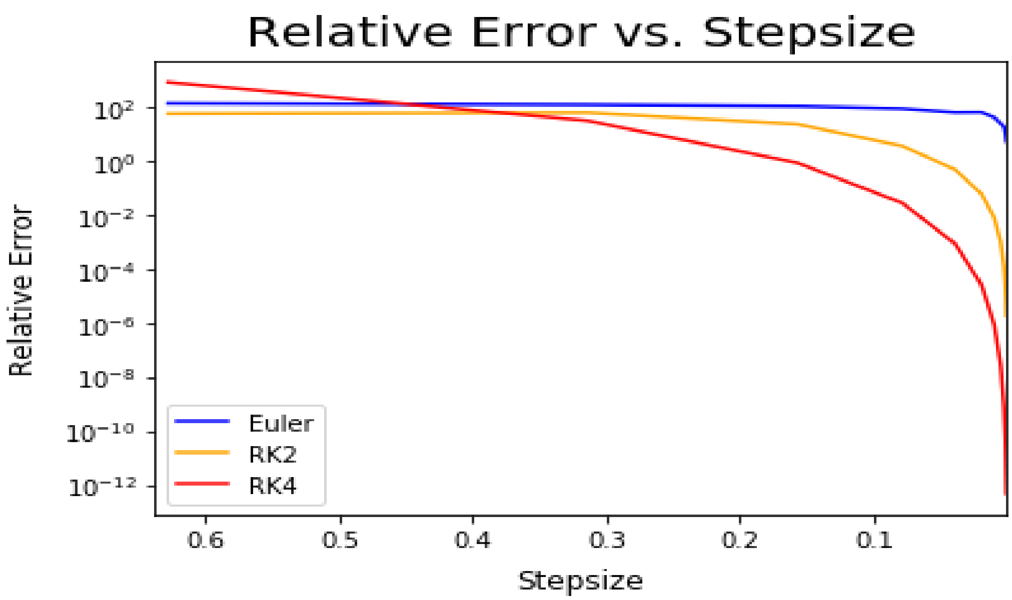

Figure 1: This figure represents the comparative distributions of the relative error in the energy as a function of step size for three integration methods: Euler, RK2, RK4. The use of logarithmic scaling allows for each method's error to be easily observable.

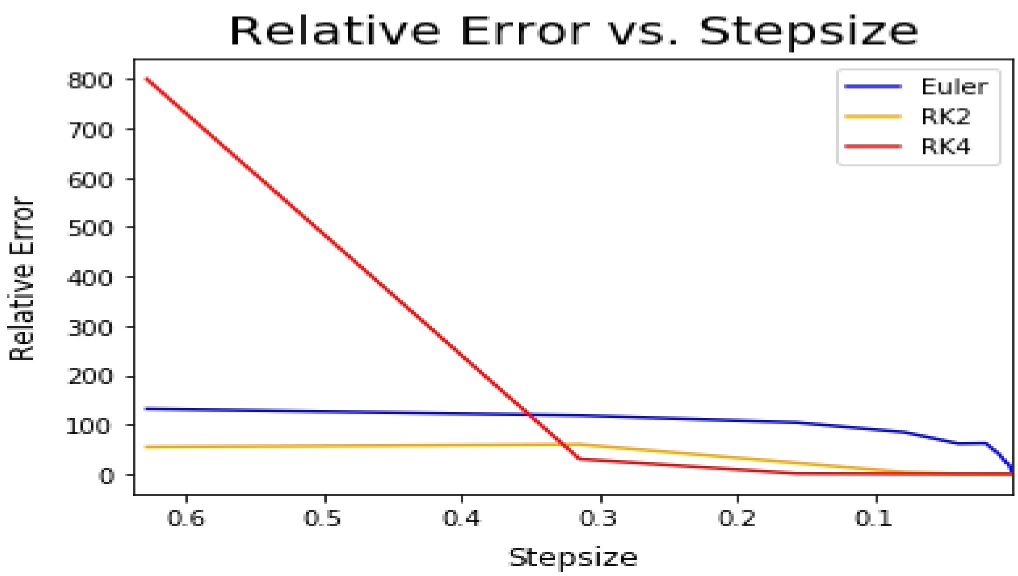

Figure 2: This figure represents the distributions of the relative error in the energy as a function of step size for three integration methods: Euler, RK2, RK4. The use of linear scaling allows for only the much larger RK4 error trend to be somewhat observable.

Part 5 Qs: For each of the three integration methods plot the particle trajectory. Does the orbit close? Is it an ellipse? Does it have the correct semi-major axis?

The particle's trajectory for the maximum value, h, using the Euler integration method is an open orbit. The trajectory for the minimum value, h/1024, using the Euler integration method results in a closed orbit and it forms an ellipse with the correct semi-major axis of 1.5 AU (when comparing to x0 = (1+e)a. For the RK2 and RK4 integration methods the particle's trajectory forms a closed elliptical orbit with a semi-major axis of 1.5 AU for the minimum step size, but output open, unstable orbits for the maximum step size.

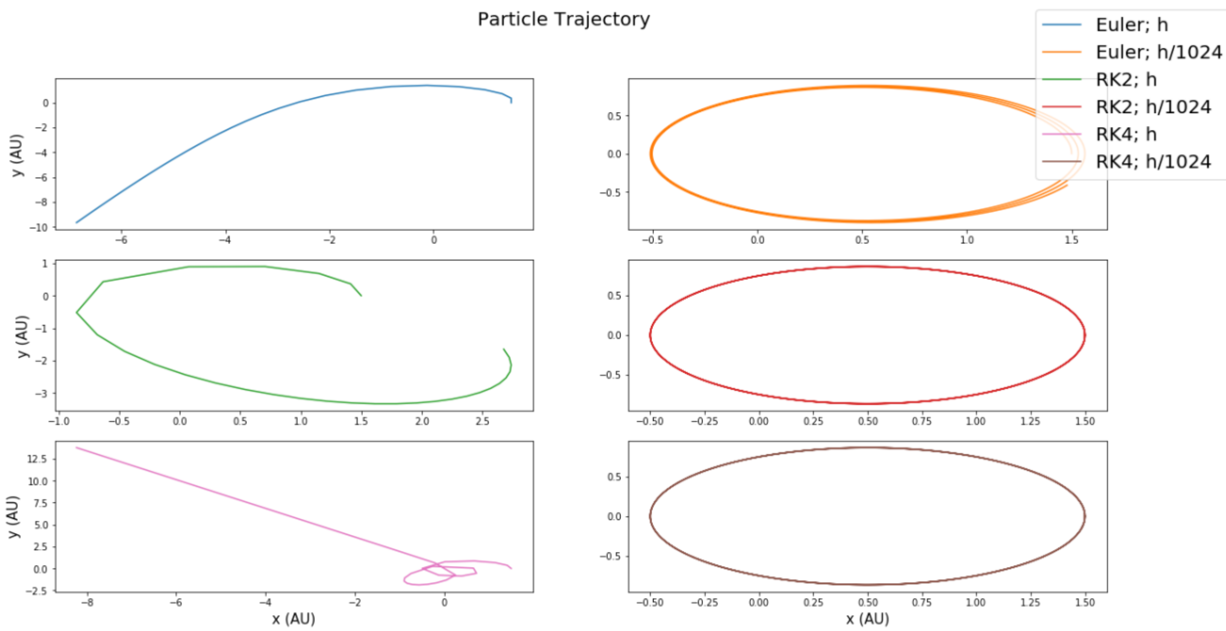

Figure 3: This subplot represents the particle trajectories of the Euler, RK2, and RK4 integration methods. The step sizes measured in this problem include the maximum and minimum values, h and h/1024, respectively.

Also plot the energies—potential, kinetic, and total—as a function of time. Put the energies all

on the same plot.

For the orbits that are closed and elliptical we would expect the total energy to remain constant and the kinetic and potential energies to experience an inverse relationship as the particle moves along its trajectory. Looking at the Euler integration method for step size h, we can see that this energy relationship isn't present, which results in its corresponding unstable, open orbit; for step size of h/1024, the energy more closely follows this expectation and thus results in a closed, pretty stable orbit. For the RK2 method, the energy approximately follows this relationship for the first orbital period or so, but then becomes largely unstable which results in the more closed but still open orbit; for the step size of h/1024, the energy follows this expected relationship and outputs a closed orbit. For the RK4 method with large step size, the trend is somewhat similar to the expected, but it is clearly unstable which causes the unstable and eventually ejected orbit; with the small step size, the energy is as expected with a resulting closed orbit. From these energy plots in **Figure 4,** we can see that the most stable orbits have the step size value of h/1024 and not h. What we can see is that the Second-Order and Fourth-Order Runge-Kutta methods have the most stable orbits with the small step size.

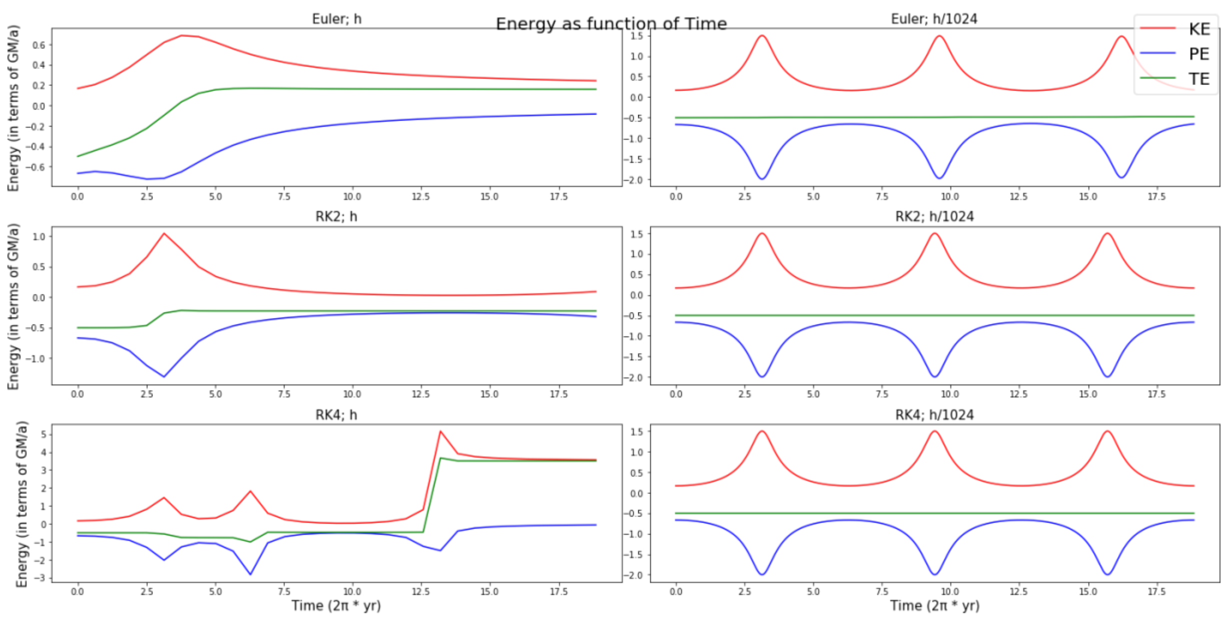

Figure 4: This subplot represents the kinetic, potential, and total energy of the Euler, RK2, and RK4 integration methods as a function of time. The step sizes measured in this problem include the maximum and minimum values, h and h/1024, respectively.