CMPSC 465 Data Structures & Algorithms
Spring 2024 Mehrdad Mahdavi and David Koslicki

Worksheet 3

Wednesday, January 31, 2023

1. **Heapsort.** Please consider the following array, which represents a min heap.

   $$A = [1,2,9,5,6,10].$$

   Suppose we remove the element at position 0 of the heap. How does the resulting heap look? Write both the array and tree representation of the heap.

2. **Heaps Operations.** Suppose we run Build-Heap on the array $A = [5,3,17,10,84,19,6,22,9]$. What is the resulting min heap ?

3. **Heaps Operations.** Show that a heap with n elements has height $\lfloor \log n \rfloor$.

4. **Creating heaps.** We can build a heap by repeatedly inserting the elements into the heap. Would this always create the same heap as Build-Heap when run on the same input array? Prove that they do, or provide a counterexample.

5. **Find k-th smallest.** Given an array of n elements and an integer $k \geq 0$ we would like to find the k-th smallest element in the array.

   1. Provide an algorithm for this problem that uses a min heap with running $O(n + k \log n)$.
   2. Provide an algorithm for this problem that uses a max heap with running $O(n \log k)$.
   3. Which algorithm has better running time?