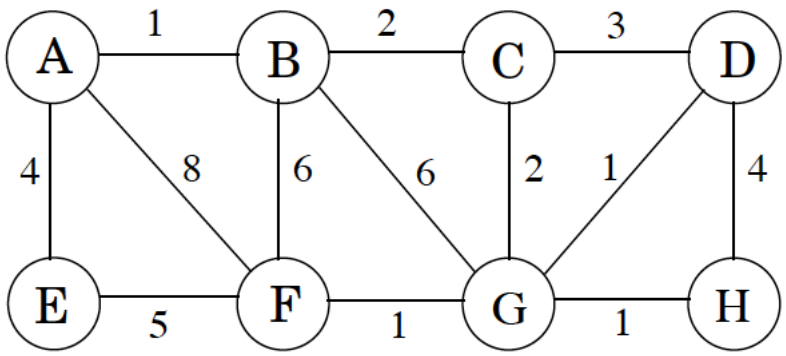
CMPSC 465 Spring 2024 — Data Structures & Algorithms — Mehrdad Mahdavi and David Koslicki

Worksheet 9

Wednesday, Mar 20, 2024

1. **Minimum Spanning Trees.** Run Prim's Algorithm to find a minimum spanning tree for the following graph. Whenever there is a choice of nodes, always use alphabetic ordering (e.g. start from node A). Show the order edges are added and the weight of the partial MST at each step.



2. **Edge Weight Incrementing.** Consider an undirected graph $G = (V, E)$ with nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s \in V$.
Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$.

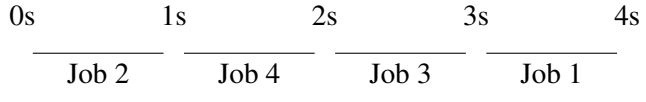
   (a) Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

   (b) Do the shortest paths change? Give an example where they change or prove they cannot change.

3. **Minimum Spanning Trees and Subgraphs.** Let $T \subseteq E$ be an MST of graph $G = (V, E)$. Given a connected subgraph $H \subseteq E$ of G, show that $T \cap H$ is contained in some MST of H.

4. **Job Scheduling.** You are given a set of n jobs to run on a computer in the next T seconds. Each job takes one second to run, and the computer runs one job at a time. Job i has an integer deadline $0 \leq d_i \leq T$ and a penalty $p_i \geq 0$. If job i isn't finished by d_i, you'll have to pay a penalty of p_i dollars. Jobs may be scheduled to start at any non-negative integer time. The goal is to schedule all jobs so as to minimize the total penalty incurred.

Penalty p_i	1	2	5	4
Deadline d_i	1	1	3	3
i	1	2	3	4



For example, suppose we have four jobs, with penalties and deadlines as shown on the left side of the figure above. If we schedule them as shown on the right, then we incur a penalty of 1 because Job 1 did not finish before its deadline $d_1 = 1$.

For each of the following greedy algorithms, either prove that it is correct, or give a simple counterexample (with at most three jobs) to show that it fails.

(a) Among unscheduled jobs that can be scheduled on time, consider the one whose deadline is the earliest (breaking ties with the highest penalty), and schedule it at the earliest available time. Repeat.

(b) Among unscheduled jobs that can be scheduled on time, consider the one whose penalty is the highest (breaking ties with the earliest deadline), and schedule it at the earliest available time. Repeat.

(c) Among unscheduled jobs that can be scheduled on time, consider the one whose penalty is the highest (breaking ties arbitrarily), and schedule it at the latest available time before its deadline. Repeat.