

Due April 25, 10:00 pm

Instructions: You are encouraged to solve the problem sets on your own, or in groups of up to five people, but you must write your solutions strictly by yourself. You must explicitly acknowledge in your write-up all your collaborators, as well as any books, papers, web pages, etc. you got ideas from.

Formatting: Each problem should begin on a new page. Each page should be clearly labeled with the problem number. The pages of your homework submissions must be in order. You risk receiving no credit for it if you do not adhere to these guidelines.

Late homework will not be accepted. Please, do not ask for extensions since we will provide solutions shortly after the due date. Remember that we will drop your lowest two scores.

You need to submit it via Gradescope (Class code XX7RVV). Please ask on Campuswire about any details concerning Gradescope and formatting.

For each algorithm question, explain your algorithm and analyze its correctness and running time. Pseudocode is not required, but you may include it if you feel it makes your written explanation more clear.

1. (25 pts.) **Solving a Linear Program.** Consider the following linear program.

$$\begin{array}{ll}\text{maximize} & 5x + 3y \\ \text{subject to} & 5x - 2y \geq 0 \\ & x + y \leq 7 \\ & x \leq 5 \\ & x \geq 0 \\ & y \geq 0\end{array}$$

- (a) Plot the feasible region.
- (b) Find the maximum objective function value and optimal variable assignments by running the simplex algorithm by hand.
- (c) Construct the dual of the original linear program and show that its optimal objective function value is equal to that of the primal linear program (strong duality holds). Additionally, give the optimal values of the dual variables. You don't need to show your work to find these values.
2. (25 pts.) **Spaceship.** A spaceship is being designed to take astronauts to Mars and back. This ship will have three compartments, each with its own independent life support system. The key element in each of these life support systems is a small *oxidizer* unit that triggers a chemical process for producing oxygen. However, these units cannot be tested in advance, and only some succeed in triggering this chemical process. Therefore it is important to have several backup units for each system. Because of differing requirements for the three compartments, the units needed for each have somewhat different characteristics. A decision must now be made on just *how many* units to provide for each compartment, taking into account design limitations on the *total* amount of *space*, *weight*, and *cost* that can be allocated to these units for the entire

ship. The following table summarizes these limitations as well as the characteristics of the individual units for each compartment:

Compartment	Space (cu in.)	Weight (lb)	Cost (\$)	Probability of failure
1	40	20	30,000	0.35
2	50	15	35,000	0.45
3	30	10	25,000	0.3
Limitation	400	100	300,000	

The objective is to *minimize the probability* of all units failing in all three compartments, subject to the above limitations and the further restriction that each compartment has a probability of no more than 0.05 that all its units fail.

Formulate a linear programming model for this problem. (*Hint:* Use logarithms.)

3. (25 pts.) **Vertex Cover.** Given an undirected graph $G = (V, E)$, a Vertex Cover is a set of vertices $V' \subseteq V$ such that every edge $e \in E$ is incident on (touches) some vertex in V' .
 - (a) Write the vertex cover problem as a linear program. The variable assignments need not be integral.
 - (b) Design an instance of the vertex cover problem where your LP outputs a non-integral solution. Your instance must use 3 nodes or less.
4. (25 pts.) **NP and EXP.** Prove that $NP \subseteq EXP$.
5. (0 pts.) **Acknowledgments.** The assignment will receive a 0 if this question is not answered.
 - (a) If you worked in a group, list the members of the group. Otherwise, write “I did not work in a group.”
 - (b) If you received significant ideas about the HW solutions from anyone not in your group, list their names here. Otherwise, write “I did not consult with anyone other than my group members.”
 - (c) List any resources besides the course material that you consulted in order to solve the material. If you did not consult anything, write “I did not consult any non-class materials.”