CMPSC 465 Spring 2024

Data Structures & Algorithms

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Worksheet 1

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1. **Compare Growth Rates.** Order the following functions by asymptotic growth:

   (i) $f_1(n) = 3^n$

   (ii) $f_2(n) = n^{\frac{1}{3}}$

   (iii) $f_3(n) = 12$

   (iv) $f_4(n) = 2^{\log_2 n}$

   (v) $f_5(n) = \sqrt{n}$

   (vi) $f_6(n) = 2^n$

   (vii) $f_7(n) = \log_2 n$

   (viii) $f_8(n) = 2^{\sqrt{n}}$

   (ix) $f_9(n) = n^3$

2. **Prove Order of Growth.** Prove the following:

   (i) $\log(n!) = \Theta(n \log n)$

   (ii) $\sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$

3. **Analyze Running Time.** For each pseudo-code below, give the asymptotic running time in Θ notation.

   (i)

```
for i := 1 to n do
    j := i;
    while j < n do
        j := j + 5;
    end
end
```

   (ii)

```
for i := 1 to n do
    for j := 4i to n do
        s := s + 2;
    end
end
```

(iii)
```
for i := 1 to n do
    j := 2;
    while j < i do
        j := j^4;
    end
end
```

4. Polynomial and Exponential Growth. Prove the following:

$$n^c = O(a^n) \quad \forall c > 0, a > 1$$