CMPSC 465 Data Structures & Algorithms
Spring 2024 Mehrdad Mahdavi and David Koslicki

HW 10

Due April 18, 10:00 pm

Instructions: You are encouraged to solve the problem sets on your own, or in groups of up to five people, but you must write your solutions strictly by yourself. You must explicitly acknowledge in your write-up all your collaborators, as well as any books, papers, web pages, etc. you got ideas from.

Formatting: Each problem should begin on a new page. Each page should be clearly labeled with the problem number. The pages of your homework submissions must be in order. You risk receiving no credit for it if you do not adhere to these guidelines.

Late homework will not be accepted. Please, do not ask for extensions since we will provide solutions shortly after the due date. Remember that we will drop your lowest two scores.

You need to submit it via Gradescope (Class code XX7RVV). Please ask on Campuswire about any details concerning Gradescope and formatting.

For each algorithm question, explain your algorithm and analyze its correctness and running time. Pseudocode is not required, but you may include it if you feel it makes your written explanation more clear.

1. (25 pts.) **Paths Dynamic Program.** Given a directed acyclic graph $G = (V, E)$, vertex $s \in V$, design a dynamic programming algorithm to compute the number of distinct paths from s to v for any $v \in V$. Make sure to define subproblems and base case, write the recursion, analyze the running time, and explain why it works

2. (25 pts.) **Weighted Set Cover.** It has been proven in class that, given a Set Cover instance with n elements, if the optimal solution uses k sets, the greedy algorithm will find a solution that uses at most $k \ln(n)$ sets.

Here is a generalization of the set cover problem that additionally assigns some weight to each set.

- *Input:* A set of elements B of size n; sets $S_1, \dots, S_m \subseteq B$; positive weights $w_i, \dots, w_m$.
- *Output:* A selection of the sets S_i whose union is B.
- *Cost:* The sum of the weights w_i for the sets that were picked.

Design an efficient algorithm to find the set cover with approximately the smallest cost. Prove that if there is a solution with cost k, then your algorithm will find a solution with cost $O(k \log n)$.

3. (25 pts.) **Horn Formula.**

Find the variable assignment that solves the following horn formulas. Make sure to go through each step of the greedy algorithm (i.e., initialization, each iteration of the loop, and the conclusion) to solve the formulas.

a. $(w \wedge y \wedge z) \Rightarrow x, (x \wedge z) \Rightarrow w, x \Rightarrow y, \Rightarrow x, (x \wedge y) \Rightarrow w, (\bar{w} \vee \bar{x} \vee \bar{y}), (\bar{z})$

b. $(x \wedge z) \Rightarrow y, z \Rightarrow w, (y \wedge z) \Rightarrow x, \Rightarrow z, (\bar{z} \vee \bar{x}), (\bar{w} \vee \bar{y} \vee \bar{z})$

4. (25 pts.) **Longest Common Substring.** Given two strings $x = x_1x_2\dots x_n$ and $y = y_1y_2\dots y_m$, we wish to find the length of their longest common substring, that is, the largest k for which there are indices i and j with $x_ix_{i+1}\dots x_{i+k-1} = y_jy_{j+1}\dots y_{j+k-1}$. Give an $O(mn)$ time algorithm to solve this problem.

5. (0 pts.) **Acknowledgments.** The assignment will receive a 0 if this question is not answered.
(a) If you worked in a group, list the members of the group. Otherwise, write "I did not work in a group."
(b) If you received significant ideas about the HW solutions from anyone not in your group, list their names here. Otherwise, write "I did not consult with anyone other than my group members."
(c) List any resources besides the course material that you consulted in order to solve the material. If you did not consult anything, write "I did not consult any non-class materials."