CMPSC 465 Data Structures & Algorithms
Spring 2024 Mehrdad Mahdavi and David Koslicki

HW 5

Due February 22, 10:00 pm

Instructions: You are encouraged to solve the problem sets on your own, or in groups of up to five people, but you must write your solutions strictly by yourself. You must explicitly acknowledge in your write-up all your collaborators, as well as any books, papers, web pages, etc. you got ideas from.

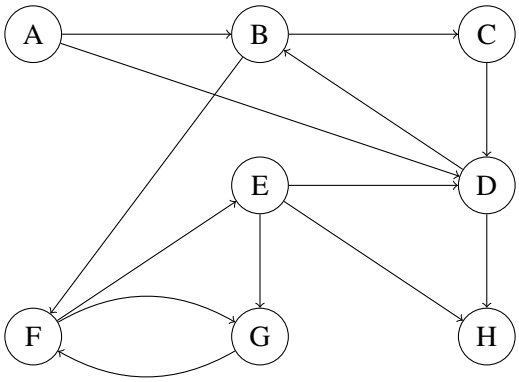
Formatting: Each problem should begin on a new page. Each page should be clearly labeled with the problem number. The pages of your homework submissions must be in order. You risk receiving no credit for it if you do not adhere to these guidelines.

Late homework will not be accepted. Please, do not ask for extensions since we will provide solutions shortly after the due date. Remember that we will drop your lowest two scores.

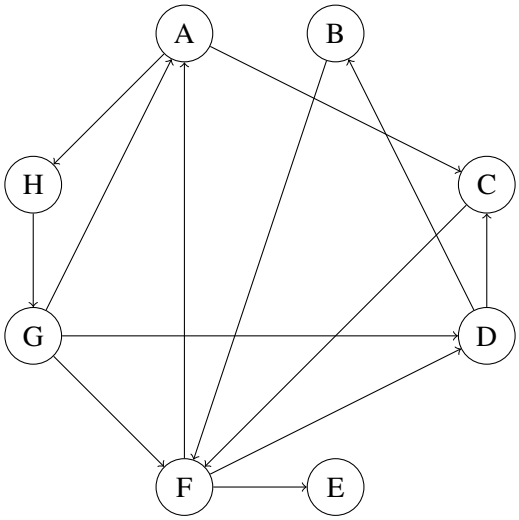
This homework is due Thursday, February 8, 10:00 pm electronically. You need to submit it via Gradescope (Class code XX7RVV). Please ask on Campuswire about any details concerning Gradescope and formatting.

1. (21 pts.) **Depth-First Search.** Perform depth-first search on each of the following graphs; whenever there's a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge, forward edge, back edge, or cross edge, and give the pre and post number of each vertex.

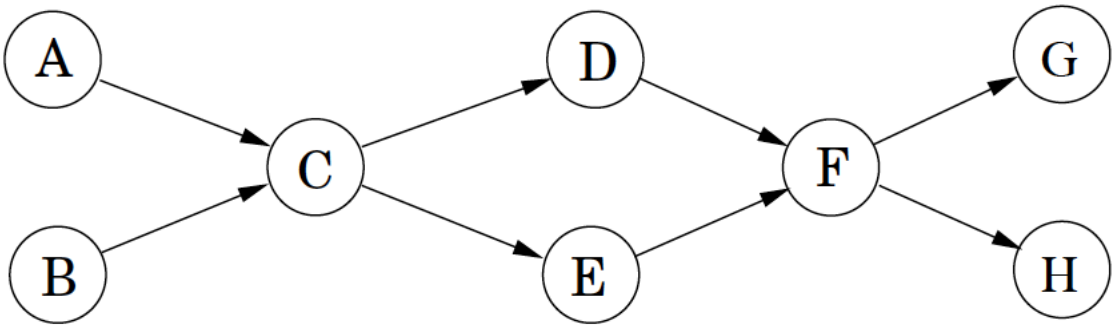
(a)



(b)



2. (20 pts.) **Topological Sort.** Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that is alphabetically first.



(a) Indicate the *pre* and *post* numbers of the nodes. Please put your answers in the format of "$A : 1, 2$" for each vertex, with 1 as A's *pre* number you found and 2 as A's *post* number. Note this is only an example for the illustrative purpose that has nothing to do with the correct solution.

(b) What are the sources and sinks of the graphs? Please put your answers in the format of "Sources: ...; Sinks: ...".

(c) What topological ordering is found by the algorithm?

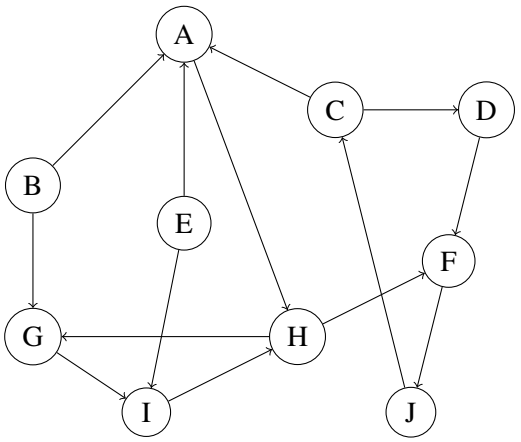
(d) How many topological orderings does this graph have? Why?

3. (20 pts.) **Finding cycles in undirected graphs.**

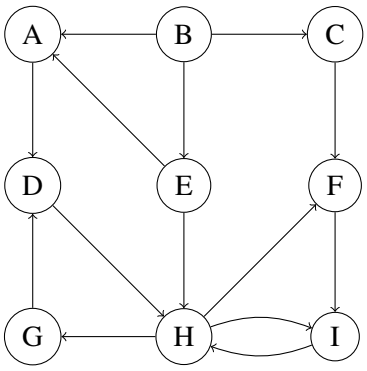
Design an algorithm to check whether an undirected graph $G = (V, E)$ has a cycle. Your algorithm should run in $O(|V|)$ time, independent of $|E|$. (Hint: Try to modify the Explore procedure and make sure you stop once you find one cycle.)

4. (20 pts.) **Strongly Connected Components (SCC).** Run the strongly connected components algorithm on the following directed graphs. When doing DFS on the reverse graph G^R: whenever there is a choice of vertices to explore, always pick the one that is alphabetically first.

(i)



(ii)



In each case answer the following questions.

(a) In what order are the strongly connected components (SCCs) found?

(b) Which are source SCCs and which are sink SCCs?

(c) Draw the "metagraph" (each "meta-node" is an SCC of G).

(d) What is the minimum number of edges you must add to this graph to make it strongly connected?

5. (19 pts.) **Funny Money.**

You are in charge of the United States Mint. The money-printing machine has developed a strange bug: it will only print a bill if you give it one first. If you give it a d-dollar bill, it is only willing to print bills of value $d^2 \pmod{600}$ or $d^2+1 \pmod{600}$. For example, if you give it a \$5 bill, it is willing to print \$25 and \$26 bills, and if you then give it a \$26-dollar bill, it is willing to print bills of value \$76 or \$77. ($76 \equiv 26^2 \pmod{600}$.)

You start out with only a \$1 bill to give the machine. Every time the machine prints a bill, you are allowed to give that bill back to the machine, and it will print new bills according to the rule described above. You want to know if there is a sequence of actions that will allow you to print a \$10 bill, starting from your \$1 bill.

(a) Model this as a graph problem: give a precise definition of the graph involved and state the specific question about this graph that needs to be answered.

(b) What algorithm should be applied to solve the problem?

6. (0 pts.) **Acknowledgments.** The assignment will receive a 0 if this question is not answered.
(a) If you worked in a group, list the members of the group. Otherwise, write "I did not work in a group."
(b) If you received significant ideas about the HW solutions from anyone not in your group, list their names here. Otherwise, write "I did not consult with anyone other than my group members."
(c) List any resources besides the course material that you consulted in order to solve the material. If you did not consult anything, write "I did not consult any non-class materials."