Midterm 3 - Exam A

Name:

Penn State access ID (xyz1234) in the following box:

Student ID number (9XXXXXXXX):

Lecture section time:

Instructions:

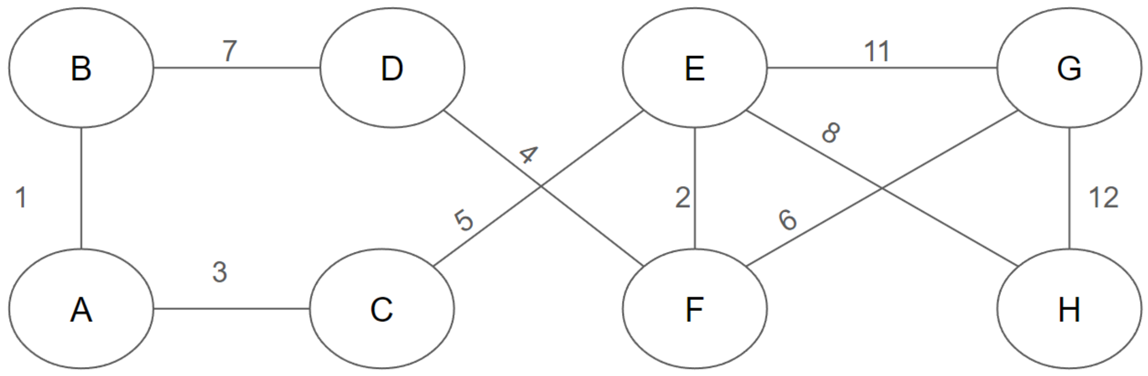
- Answer all questions. Read them carefully first. Be precise and concise. Handwriting needs to be neat. Box numerical final answers.
- Please clearly write your name and your PSU access ID (i.e., xyz1234) in the box on top of **every page**.
- Write in only the space provided. You may use the back of the pages only as scratch paper. **Do not write your solutions in the back of pages!**
- **Do not write outside of the black bounding box**: the scanner will not be able to read anything outside of this box.
- We are providing one extra page at the end if you need extra space. Make sure you mention in the space provided that you answer continues there.

Good luck!

Name:	**PSU Access ID (xyz1234):**

1 Kruskal's algorithm *(10 points)*

Use Kruskal's algorithm on the graph shown below. Ties can be broken arbitrarily.



(a) Edge (A, B) is added iteration:

[✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 ◯Never added

(b) Edge (A, C) is added iteration:

◯1 ◯2 [✓ 3] ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 ◯Never added

(c) Edge (B, D) is added iteration:

◯1 ◯2 ◯3 ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 [✓ Never added]

(d) Edge (C, E) is added iteration:

◯1 ◯2 ◯3 ◯4 [✓ 5] ◯6 ◯7 ◯8 ◯9 ◯10 ◯Never added

(e) Edge (D, F) is added iteration:

◯1 ◯2 ◯3 [✓ 4] ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 ◯Never added

(f) Edge (E, F) is added iteration:

◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 ◯Never added

(g) Edge (E, G) is added iteration:

◯1 ◯2 ◯3 ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 [✓ Never added]

(h) Edge (E, H) is added iteration:

◯1 ◯2 ◯3 ◯4 ◯5 ◯6 [✓ 7] ◯8 ◯9 ◯10 ◯Never added

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(i) Edge (F, G) is added iteration:

◯1 ◯2 ◯3 ◯4 ◯5 [✓ 6] ◯7 ◯8 ◯9 ◯10 ◯Never added

(j) Edge (G, H) is added iteration:

◯1 ◯2 ◯3 ◯4 ◯5 ◯6 ◯7 ◯8 ◯9 ◯10 [✓ Never added]

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2 Horn Formula *(8 points)*

Find the variable assignment that solves the following horn formula if it exists. If there is no valid assignment, mark all as "No valid assignment". You must use the greedy algorithm from class to find the assignment.

$(w \wedge z) \Rightarrow x, (x \wedge z) \Rightarrow w, x \Rightarrow y, \Rightarrow y, (x \wedge y) \Rightarrow w, (\bar{w} \vee \bar{x}), (\bar{z} \vee \bar{y})$

(a) w: ◯True ◯False [✓ No valid assignment]

(b) x: ◯True ◯False [✓ No valid assignment]

(c) y: ◯True ◯False [✓ No valid assignment]

(d) z: ◯True ◯False [✓ No valid assignment]

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3 Huffman Encoding *(8 points)*

Select the valid Huffman encoding for symbols a, b, c, d with the following multiplicities. You must choose that the right edges are 1 and the left edges are 0. While merging, the set with the larger number of unique symbols is the right child. If the sets are the same size, the set with the letter closest to the start of the alphabet is the right child.

$$a : 1, b : 6, c : 3, d : 2$$

(a) a: ◯0 ◯1 ◯00 ◯01 ◯10 ◯11 ◯000 ◯001 ◯010 ◯011 ◯100 ◯101 ◯110 [✓ 111]

(b) b: [✓ 0] ◯1 ◯00 ◯01 ◯10 ◯11 ◯000 ◯001 ◯010 ◯011 ◯100 ◯101 ◯110 ◯111

(c) c: ◯0 ◯1 ◯00 ◯01 [✓ 10] ◯11 ◯000 ◯001 ◯010 ◯011 ◯100 ◯101 ◯110 ◯111

(d) d: ◯0 ◯1 ◯00 ◯01 ◯10 ◯11 ◯000 ◯001 ◯010 ◯011 ◯100 ◯101 [✓ 110] ◯111

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4 Edit distance *(10 points)*

Compute the edit distance between strings S_1 and S_2 by filling out the table below using the dynamic programming procedure. The edit distance is the number of insertions, deletions, and substitutions that are needed to turn one string into another. Note that points will not be awarded for filling out the table and it is only provided to help you answer. You must fill out the associated answer bubble for each square. Squares are numbered from left to right, top to bottom.

$$S_1 = AACG, S_2 = AGC$$

	–	A	A	C	G
–	1.	2.	3.	4.	5.
A	6.	7.	8.	9.	10.
G	11.	12.	13.	14.	15.
C	16.	17.	18.	19.	20.

(a) Square 1 has which number in it?:

[✓ 0] ◯1 ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(b) Square 2 has which number in it?:

◯0 [✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

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(c) Square 3 has which number in it?:

◯0 ◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7

(d) Square 4 has which number in it?:

◯0 ◯1 ◯2 [✓ 3] ◯4 ◯5 ◯6 ◯7

(e) Square 5 has which number in it?:

◯0 ◯1 ◯2 ◯3 [✓ 4] ◯5 ◯6 ◯7

(f) Square 6 has which number in it?:

◯0 [✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(g) Square 7 has which number in it?:

[✓ 0] ◯1 ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(h) Square 8 has which number in it?:

◯0 [✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(i) Square 9 has which number in it?:

◯0 ◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7

(j) Square 10 has which number in it?:

◯0 ◯1 ◯2 [✓ 3] ◯4 ◯5 ◯6 ◯7

(k) Square 11 has which number in it?:

◯0 ◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7

(l) Square 12 has which number in it?:

◯0 [✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(m) Square 13 has which number in it?:

◯0 [✓ 1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(n) Square 14 has which number in it?:

◯0 ◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7

(o) Square 15 has which number in it?:

◯0 ◯1 [✓ 2] ◯3 ◯4 ◯5 ◯6 ◯7

(p) Square 16 has which number in it?:

◯0 ◯1 ◯2 [✓ 3] ◯4 ◯5 ◯6 ◯7

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(q) Square 17 has which number in it?:

◯0 ◯1 [✓ ●2] ◯3 ◯4 ◯5 ◯6 ◯7

(r) Square 18 has which number in it?:

◯0 ◯1 [✓ ●2] ◯3 ◯4 ◯5 ◯6 ◯7

(s) Square 19 has which number in it?:

◯0 [✓ ●1] ◯2 ◯3 ◯4 ◯5 ◯6 ◯7

(t) Square 20 has which number in it?:

◯0 ◯1 [✓ ●2] ◯3 ◯4 ◯5 ◯6 ◯7

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5 True/False *(10 points)*

True or false? Fill in the right bubble. No justification is needed.

[✓ **T**] ● **F** ○ 1. Kruskal's algorithm can always create all possible MSTs of a graph across multiple runs.

[✓ **T**] ● **F** ○ 2. Kruskal's runs in $O(|E| \log |E|)$.

[✓ **T**] ● **F** ○ 3. Running Kruskal's algorithm on a disconnected graph gives MSTs costing as much as the MSTs produced by running Kruskal's algorithm on each connected component in the graph.

[✓ **T**] ● **F** ○ 4. Huffman encoding always gives an optimal lossless encoding.

T ○ [✓ **F**] ● 5. Huffman encodings always have at least one code of length one.

[✓ **T**] ● **F** ○ 6. Huffman encoding can be implemented with a greedy algorithm.

T ○ [✓ **F**] ● 7. The greedy set cover algorithm never outputs an optimal selection of sets.

[✓ **T**] ● **F** ○ 8. The longest increasing subsequence problem can be solved in $O(n^2)$ with dynamic programming.

T ○ [✓ **F**] ● 9. Edit distance of two strings of lengths M and N can be computed in $O(min\{M, N\})$ running time.

[✓ **T**] ● **F** ○ 10. The Floyd-Warshall algorithm can be reformulated as a dynamic programming procedure.

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6 Minimum Non-Adjacent Sum *(15 points)*

You are given an array A = $(a_1, a_2, ..., a_n)$ with n (possibly negative) integers. Design $O(n)$ running time algorithm to find a subsequence A_1 of A such that A_1 does not contain adjacent elements of A (i.e. if $a_k \in A_1$ then $a_{k-1} \notin A_1$ and $a_{k+1} \notin A_1$) and that the sum of all integers in A_1 is minimized. Explain your algorithm, the correctness, and its running time. Hint: Define dp[i] as the minimum sum that can be achieved by a non-adjacent subsequence of $(a_1, ..., a_i)$

Solution: This question is the same Worksheet 11 Q3 except finding the minimum sum rather than the maximum sum. Define dp[i] as the minimum sum that can be achieved by a subsequence of $(a_1, ..., a_i)$. For each integer a_i, we have two choices. If we choose a_i, dp[i] = dp[i-2] + a_i; if not, dp[i] = dp[i-1].

The pseudo-code, which includes three steps, is as follows.

```
Initialization:
dp[0] = 0
dp[1] = $min\{0, a_1\}$
Iteration:
for i = 2···n:
        dp[i] = $min\{dp[i-1], dp[i-2] + a_i\}$
endfor
Termination: dp[n] gives the minimum sum that can be obtained.
Running time: O(n).
```

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Spanning trees *(15 points)*

Assume you are given a graph $G = (V, E)$ with both positive and negative edge weights along with an algorithm A that can return a minimum spanning tree when given a graph with only *positive* edges. Describe an efficient way to transform G into a new graph G' containing only positive edge weights so that the minimum spanning tree of G can be easily found from the minimum spanning tree of G'. Prove that this procedure actually results in a minimum spanning tree and explain the running time.

Solution:

1. Find the smallest negative edge weight in the graph, denoted as w_l.
2. Update each edge weight W in the graph to a new weight $\hat{W}$, calculated as $\hat{W} = W - w_l + 1$.

As we subtract the smallest weight w_l from all edge weights and then add 1, the relative differences between the weights of the edges are maintained. Therefore this transformation ensures that all edge weights in G' are positive. As all the edges are positive in G', Algorithm A will return a valid MST of G'.
We can then find the MST for G by adding weight w_l to all the edges in the MST for G'. MST for G has the same edges as MST for G'.

Proof: Consider any two edges e_1 and e_2 with original weights W_1 and W_2. Without loss of generality, assume $W_1 < W_2$. After the transformation, the weights become $\hat{W}_1 = W_1 - w_l + 1$ and $\hat{W}_2 = W_2 - w_l + 1$. We can see that $\hat{W}_1 < \hat{W}_2$ still holds, which means the ordering of the edges by weight is preserved. Consequently, algorithm A applied to G' will select the set of edges that would form a valid MST for G.

Running Time: The time complexity to find the minimum edge weight w_l is $O(|E|)$ where $|E|$ is the number of edges. Updating the weight of one edge takes $O(1)$ time. Updating all the edge weights takes $O(|E|)$ time. Hence, the total running time for the transformation is $O(|E|)$.

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