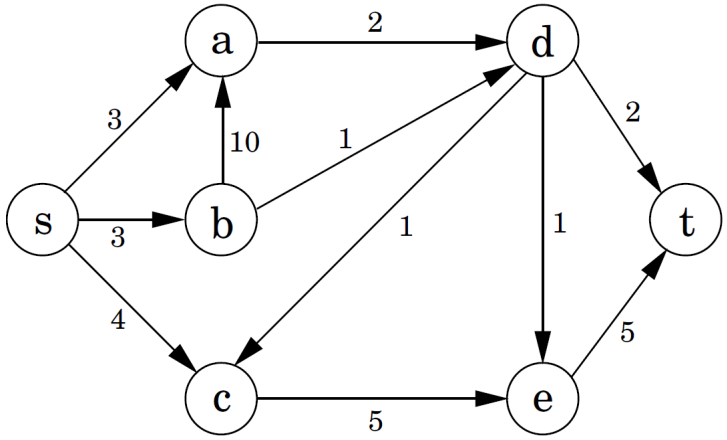
CMPSC 465 Data Structures & Algorithms
Spring 2024 Mehrdad Mahdavi and David Koslicki

Worksheet 8

Wednesday, March 13, 2024

1. **Max-Flow.** Find the maximum flow from vertex s to vertex t in the following graph using the Ford-Fulkerson Algorithm. Show the residual graph at each step of the process.



2. **Verifying a Max-Flow.** Suppose someone presents you with a solution to a max-flow problem on some network. Give a linear time algorithm to determine whether the solution does indeed give a maximum flow.

3. **Flow Variations.** Show how to reduce the following variations of the max-flow problem to the standard max-flow problem discussed in class.

   (a) There are many sources and many sinks, and we wish to maximize the total flow from all sources to all sinks.

   (b) In addition to the existing edge capacities, each *vertex* has a capacity on the maximum flow that can enter it.

4. **Flow Formulation.** Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any sidewalk that the other child has used that day. Assume every street has a sidewalk on either side and the children always walk with the street to their right. Fortunately, the children have no problem with their paths crossing at an intersection, and both the professor's house and the school are at intersections. Given a map of the town, formulate the problem of determining whether both children can go to the same school as a maximum-flow problem.