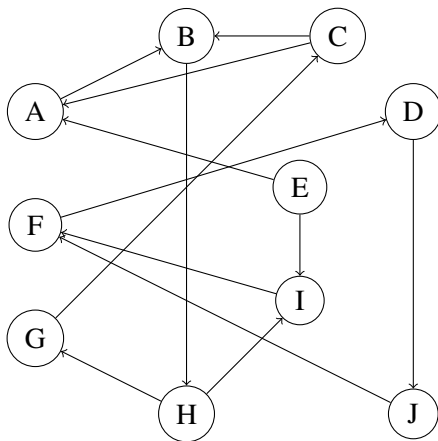


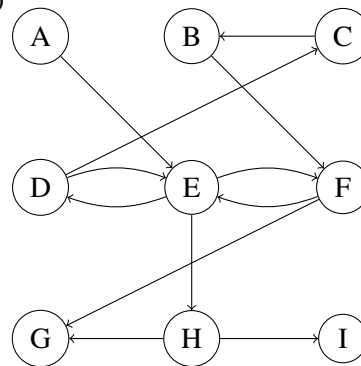
Wednesday, Feb 21, 2024

1. **Strongly Connected Components.** Run the strongly connected components algorithm on the following directed graphs and draw their SCC metagraphs. When doing DFS on the reverse graph  $G^R$ : whenever there is a choice of vertices to explore, always pick the one that is alphabetically first.

(i)



(ii)



2. **Award Ceremony.** Your job is to prepare a lineup of  $n$  awardees at an award ceremony. You are given a list of  $m$  constraints of the form: awardee  $i$  wants to receive his award before awardee  $j$ . Design an algorithm to either give such a lineup that satisfies all constraints, or return that it is not possible. Your algorithm should run in  $O(|V| + |E|)$  time.
3. **Hamiltonian Path.** Given a directed acyclic graph  $G$ , write a linear-time algorithm to determine whether there exists a path that touches every vertex exactly once (a Hamiltonian Path).
4. **Odd Cycle.** Give a linear-time algorithm to find an odd-length cycle in a *directed* graph. (*Hint:* First solve the problem under the assumption that the graph is strongly connected.)