CMPSC 465	Data Structures & Algorithms	
Spring 2024	Mehrdad Mahdavi and David Koslicki	HW 1

Due January 18, 10:00 pm

Instructions: You are encouraged to solve the problem sets on your own, or in groups of up to five people, but you must write your solutions strictly by yourself. You must explicitly acknowledge in your write-up all your collaborators, as well as any books, papers, web pages, etc. you got ideas from.

Formatting: Each problem should begin on a new page. Each page should be clearly labeled with the problem number. The pages of your homework submissions must be in order. You risk receiving no credit for it if you do not adhere to these guidelines.

Late homework will not be accepted. Please, do not ask for extensions since we will provide solutions shortly after the due date. Remember that we will drop your lowest two scores.

This homework is due Thursday, January 18, at 10:00 pm electronically. You need to submit it via Gradescope (Class code XX7RVV). Please ask on Campuswire about any details concerning Gradescope and formatting.

1. (5 pts.) **Getting started.** Please read the course policies on the syllabus, especially the course policies on collaboration. If you have any questions, contact the instructors. Once you have done this, please write "I understand the course policies." on your homework to get credit for this problem.

2. (36 pts.) **Comparing growth rates.** In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Give a one sentence justification for each of your answers.

	$f(n)$	$g(n)$
a)	$6n \cdot 2^n + n^{100}$	3^n
b)	$\log(2n)$	$\log(3n)$
c)	$\sqrt{n}$	$\sqrt[3]{n}$
d)	$\frac{n^2}{\log n}$	$n(\log n)^4$
e)	$n \log n + n^2$	$10n^2 + (\log n)^5$
f)	$(\log_2 n)^{\log_2 n}$	$2^{(\log_2 n)^2}$
g)	$n \log(n^{20})$	$\log(3n!)$
h)	$\log(n^9 + \log n)$	$\log(2n)$
i)	$8^n \cdot n^2$	$(\lfloor \sqrt{n} \rfloor)!$

3. (15 pts.) **Geometric progressions growth.** Prove the following:

$$\sum_{i=0}^{k} c^i = \begin{cases} \Theta(c^k) & \text{if } c > 1, \\ \Theta(k) & \text{if } c = 1, \\ \Theta(1) & \text{if } 0 < c < 1. \end{cases}$$

Hint - when $c \neq 1$, the partial sum of a geometric series $S(k) = \sum\limits_{i=0}^{k} c^i = \frac{1-c^{k+1}}{1-c}$.

4. (20 pts.) **Useful identities.** Show the following statements hold. For (b), (c), and (d), either show upper **and** lower bounds or expand the summation into a polynomial of the proper degree.

(a) Let $f(n) = \sum_{i=0}^{d} a_i \cdot n^i$ with $a_d \neq 0$. Show that: $f(n) = \begin{cases} O(n^k) & \text{if } k \geq d, \\ \Omega(n^k) & \text{if } k \leq d, \\ \Theta(n^k) & \text{if } k = d. \end{cases}$

(b) $\sum_{k=1}^{n} k^2 = \Theta(n^3)$.

(c) $\sum_{k=1}^{n} k^j = \Theta(n^{j+1})$ for any constant $j > 0$. Note that $k \leq n$ for all terms and $k \geq \frac{n}{2}$ for many terms.

(d) $\sum\limits_{i=1}^{n} \sum\limits_{j=1, j\neq i}^{n} ij = \Theta(n^4)$.

5. (24 pts.) **Practice with the definitions.**

(a) Prove that $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.

(b) Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

(c) Using the basic definition of Θ, prove that $\log_a n = \Theta(\log_b n)$ for all $a, b > 1$.