CMPSC 465 Spring 2024 | Data Structures & Algorithms Mehrdad Mahdavi and David Koslicki | Worksheet 13

Worksheet 13

Wednesday, April 17, 2024

1. **Linear Program.** Consider the following linear program:

$$
\begin{aligned}
\text{maximize} \quad & x_1 + x_2 - x_3 \\
\text{subject to} \quad & x_1 - x_2 = 3 \\
& x_2 + x_3 \leq 1 \\
& x_2, x_3 \geq 0
\end{aligned}
$$

Does this linear program have an optimum? If so, what are the optimal values of x_1, x_2, x_3? (No need to show your work, but explain briefly.)

2. **Linear Programs and Simplex.** Given the following linear program:

$$
\begin{aligned}
\text{maximize} \quad & x + y \\
\text{subject to} \quad & 2x + y \leq 4 \\
& x + 3y \leq 6 \\
& x, y \geq 0
\end{aligned}
$$

(a) Plot the feasible region.

(b) Show all the vertices in the order of being visited by the simplex method with the values at those points, assuming we start at $(0,0)$ and move along x-axis first.

3. **Max Flow.** Following are two variations of the maximum flow problem. Show that they can be reformulated to linear programs.

(a) Each edge e has not only a capacity, but also a lower bound $b_e \geq 0$ on the flow it must carry.

(b) The outgoing flow from each node u is not the same as the incoming flow, but is smaller by a factor of $(1 - \varepsilon_u)$, where ε_u is a loss coefficient associated with node u.

4. **Cargo Plane.** A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below.

- Material 1 has a density of 2 tons/cubic meter, a maximum available amount of 40 cubic meters, and a revenue of $1,000 per cubic meter.

- Material 2 has a density of 1 ton/cubic meter, a maximum available amount of 30 cubic meters, and a revenue of \$1,200 per cubic meter.
- Material 3 has a density of 3 tons/cubic meter, a maximum available amount of 20 cubic meters, and a revenue of \$12,000 per cubic meter.

Write a linear program to maximize revenue while obeying these constraints.

5. **Change-Making Linear Program.** This is a continuation of worksheet 12's problem 4a. You are given an unlimited supply of coins of denominations $v_1, \ldots, v_n \in \mathbb{N}$ and a value $W \in \mathbb{N}$. Your goal is to make change for W using the minimum number of coins, that is, find the smallest set of coins whose total value is W.

   (a) Formulate a linear programming model for this problem.

   (b) Even when a problem involves integer variables such as the number of coins, the intuitive linear program may give a non-integral solution. This is a problem since it doesn't make much sense to divide a coin into fractions in the real world. Give an example input that causes your linear program to give a non-integral output.

   (c) Explain how the problem of non-integral solutions can be solved if the constraints are allowed to take on a different form.