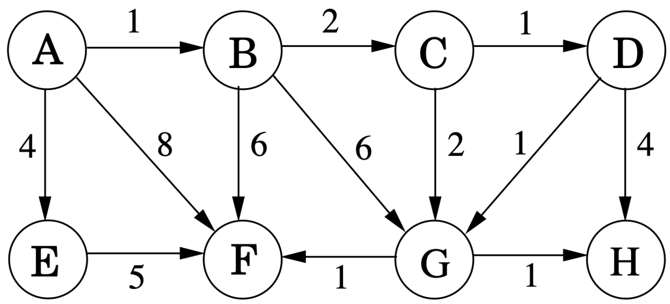
CMPSC 465	Data Structures & Algorithms	
Spring 2024	Mehrdad Mahdavi and David Koslicki	Worksheet 7

Wed, February 28, 2024

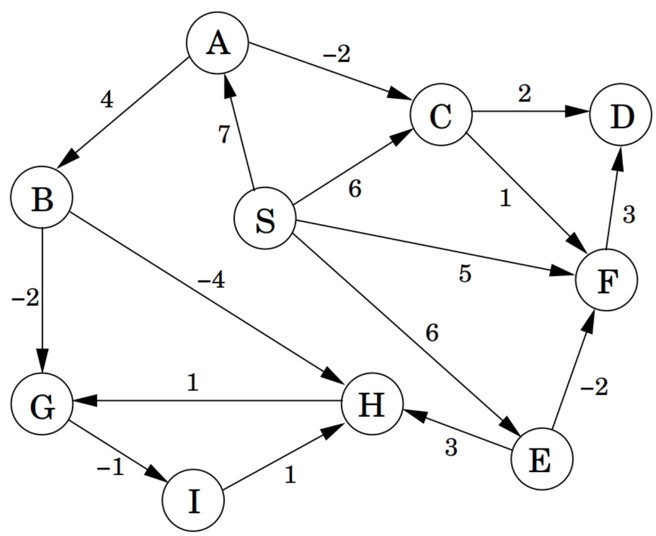
1. **Dijkstra's.** Suppose Dijkstra's Algorithm is run on the following graph, starting at node A.



   (i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

   (ii) Show the final shortest-path tree.

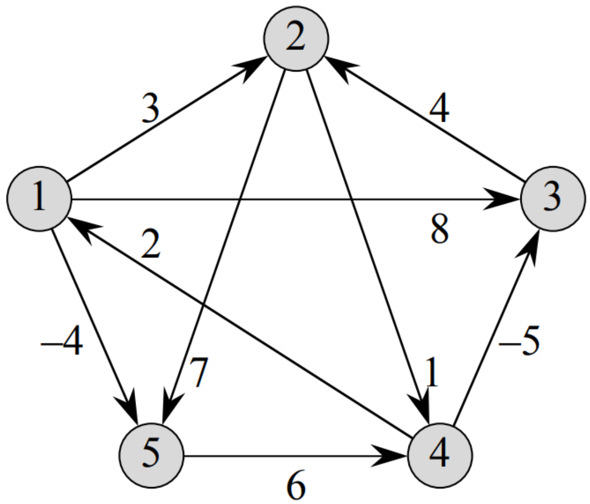
2. **Bellman-Ford.** Suppose Bellman-Ford is used to find all the shortest paths from node S.



   (i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

   (ii) Show the final shortest-path tree.

3. **Floyd-Warshall.** Run Floyd-Warshall to find all pairs of shortest paths in the following graph. Show the distance matrix for each step of the algorithm, including the initial and final matrices.



4. **Dijkstra's with Negative Edges.** Professor F. Lake suggests the following algorithm for finding the shortest path from node s to node t in a directed graph with some negative edges: add a large constant to each edge weight so that all the weights become positive, then run Dijkstra's algorithm starting at node s, and return the shortest path found to node t. Is this a valid method? Either prove that it works correctly or give a counterexample.

5. **Shortest Path via a Node.** You are given a strongly connected directed graph $G = (V, E)$ with positive edge weights along with a particular node $v_0 \in V$. Give an $O(|V|^2)$ algorithm for finding the shortest paths between all pairs of nodes, with the one restriction that these paths must all pass through v_0. Assume you may run Dijkstra's Algorithm in $O(|V|\log|V| + |E|)$ (the Fibonacci Heap implementation).

6. **Good Nodes in a Binary Tree.** Given a binary tree, a node X in the tree is named good if in the path from the root to X there are no nodes with a value greater than X. Give an $O(|V|)$ algorithm to find the number of good nodes in the binary tree.