Time-Varying Identification of Structural Vector Autoregressions

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contributions

- Time-Varying Identification
  - a new Structural VAR
  - Markov-switching structural matrix
  - selection of exclusion restriction in regimes
  - verified identification through heteroskedasticity within regimes

contributions

- time-invariant identification
  Sims (1980)
- TVP but not TVI
  Primiceri (2005)
  Sims, Zha (2006)
- not verified TVI with fixed regimes
  Kimura and Nakajima (2016)
  Bacchiocchi, Castelnuovo, Fanelli (2017)
  Arias, Rubio-Ramirez, Shin, Waggoner (2024)
  Pagliari (2024)

TVI-SVAR

TVI-SVAR

structural VAR.

reduced form: $\mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

structural form: $\mathbf{B}(s_t, \boldsymbol{\kappa}(s_t))\boldsymbol{\varepsilon}_t = \mathbf{u}_t$

structural shocks: $\mathbf{u}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \text{diag}\left(\boldsymbol{\sigma}_t^2\right)\right)$

variances: $\sigma_{n.t}^2 = \exp\left\{\omega_n(s_t)h_{n.t}\right\}$

Markov process: $s_t \sim Markov_M\left(\mathbf{P}, \boldsymbol{\pi}_0\right)$

TVI-SVAR

stochastic volatility.

structural shocks:	$\mathbf{u}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \text{diag}\left(\boldsymbol{\sigma}_t^2\right)\right)$
variances:	$\sigma_{n.t}^2 = \exp\left\{\omega_n(s_t)h_{n.t}\right\}$
log-volatilities:	$h_{n.t} = \rho_n h_{n.t-1} + v_{n.t}$
shocks:	$v_{n.t} \sim \mathcal{N}\left(0, 1\right)$

homoskedasticity condition.

$$\omega_n(s_t = m) = 0$$

TVI

TVI: time-varying identification

structural equation.

structural form: $\mathbf{B}(s_t, \boldsymbol{\kappa}(s_t))\boldsymbol{\varepsilon}_t = \mathbf{u}_t$

TVI indicator: $\boldsymbol{\kappa}(s_t) = (\kappa_1(s_t), \dots, \kappa_N(s_t))$

TVI in n^{th} equation: $\kappa_n(s_t) = k_n \in \{1, \dots, K_n\}$

TVI: time-varying identification

exclusion restrictions.

n^{th} equation: $[\mathbf{B}(m, k_n)]_{n\cdot} = \mathbf{B}_{n.m.k_n}$

restrictions: $\mathbf{B}_{n.m.k_n} = \mathbf{b}_{n.m.k_n}\mathbf{V}_{n.m.k_n}$

$$\begin{bmatrix} b_{n.1} & b_{n.2} & 0 \end{bmatrix} = \begin{bmatrix} b_{n.1} & b_{n.2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

TVI: time-varying identification

hierarchical prior distribution.

structural: $\mathbf{b}'_{n.m.k_n} \mid \gamma_B, k_n \sim \mathcal{N}_{r_{n.m.k_n}}\left(\mathbf{0}_{r_{n.m.k_n}}, \gamma_{B.n}\mathbf{I}_{r_{n.m.k_n}}\right)$

TVI indicator: $\kappa_n(m) \sim \mathcal{M}ultinomial\left(K_n^{-1}\boldsymbol{\imath}_{K_n}\right)$

shrinkage: $\gamma_{B.n} \mid \underline{s}_{B.n} \sim \mathcal{IG}2\left(\underline{s}_{B.n}, \underline{\nu}_{B.n}\right)$

TVI: time-varying identification

inference on TVI components.

Given S posterior draws $\left\{\kappa_n(m)^{(s)}\right\}_{s=1}^{S}$ compute the posterior probability of regime-specific TVI component by:

$$\widehat{\Pr}\left[\kappa_n(m) = k_n \mid \mathbf{Y}_T\right] = S^{-1}\sum_{s=1}^{S}\mathcal{I}(\kappa_n(m)^{(s)} = k_n) \tag{1}$$

time-varying identification of US monetary policy shocks

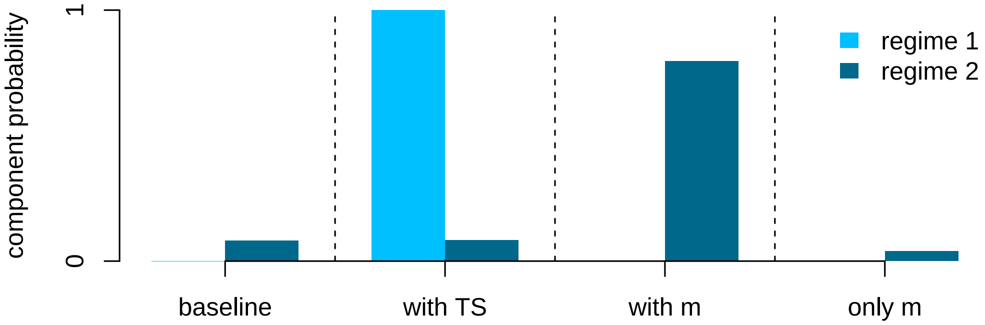
TVI for monetary policy shock

monetary policy reaction function.

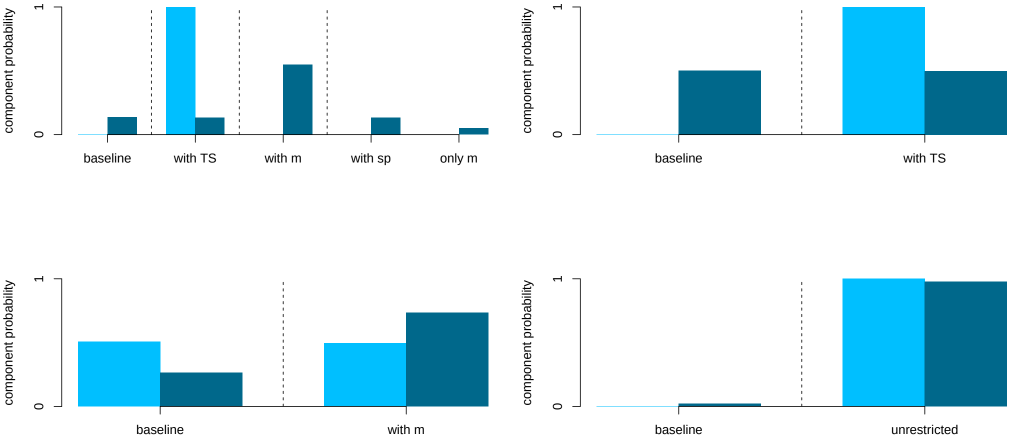
	y_t	π_t	R_t	TS_t	m_t	sp_t
benchmark	$*$	$*$	$*$	0	0	0
with TS	$*$	$*$	$*$	$*$	0	0
with m	$*$	$*$	$*$	0	$*$	0
only m	0	0	$*$	0	$*$	0

- data sample from January 1959 to June 2023 empirical evidence

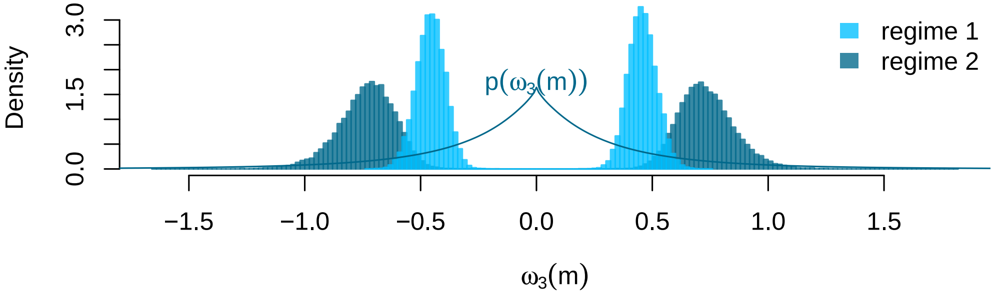
TVI posterior probabilities



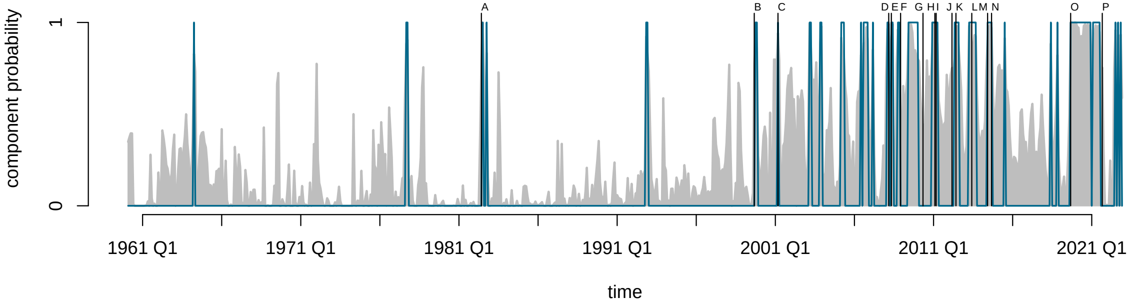
TVI posterior probabilities



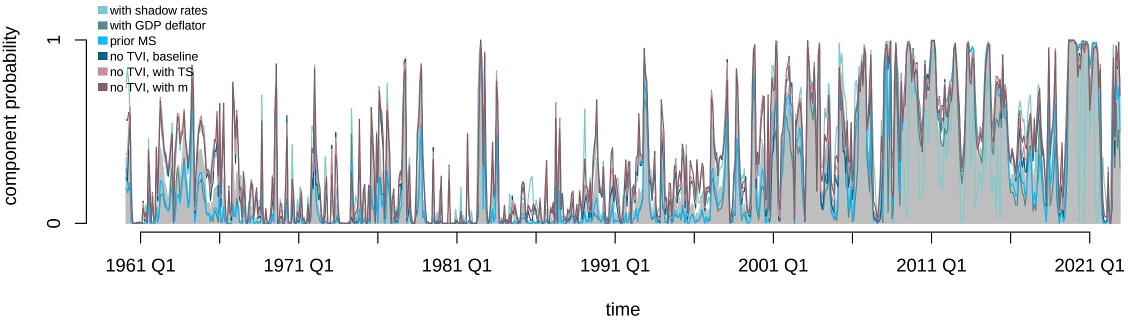
identified via heteroskedasticity



regime probabilities



regime probabilities



regime interpretations

regime-specific sample moments

	Regime 1			Regime 2		
	mean	sd	cov$(\cdot, R_t)$	mean	sd	cov$(\cdot, R_t)$
R_t	5.67	3.66		2.54	2.74	
TS_t	0.87	1.69	-3.91	1.48	1.28	-1.96
m_t	6.45	3.8	1.30	7.41	8.53	-1.55

regime interpretations

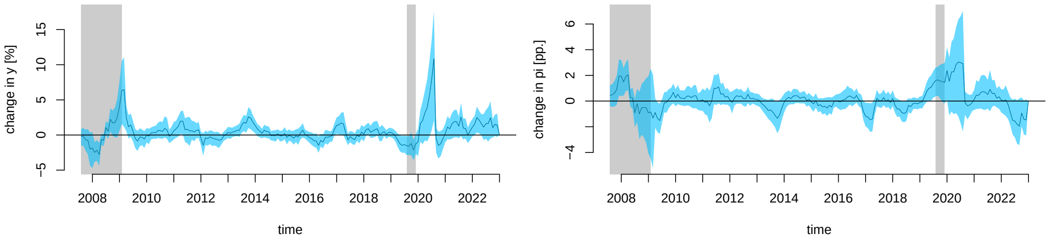
monetary policy reaction function estimates

Regime 1: $0.10 y_t - \mathbf{0.07} \pi_t + \mathbf{3.53} R_t + \mathbf{3.99} TS_t = \cdots + \hat{u}_t^{mps}$

Regime 2: $-\mathbf{0.48} y_t - 0.03 \pi_t + \mathbf{12.60} R_t - \mathbf{1.13} m_t = \cdots + \hat{u}_t^{mps}$

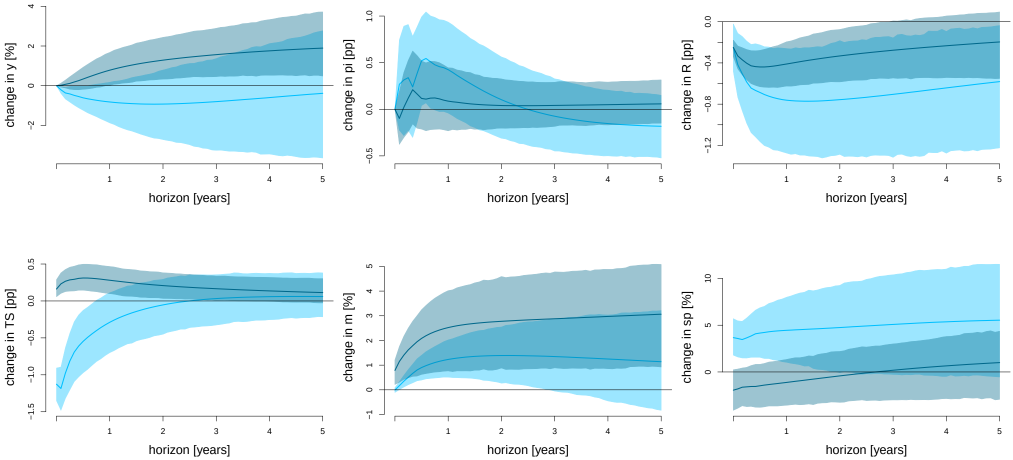
regime interpretations

the cummulative effects of last year mp shocks



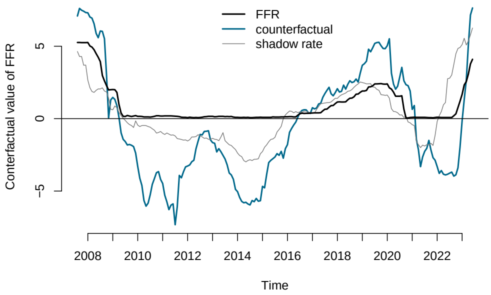
regime interpretations

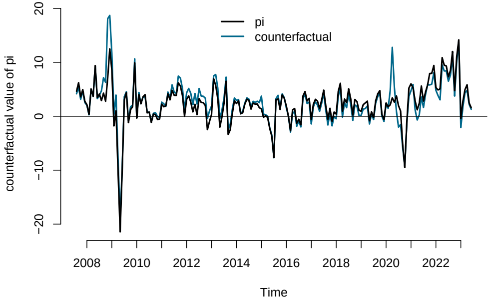
impulse responses to the mp shock



counterfactual analysis

what if the second regime never happened?





We propose a new TVI-SVAR model to show:

- time-variation in mp shock identification

  regime 1: reaction function with term spread

  regime 2: reaction function with money

- mp shock is identified via heteroskedasticity within regimes bsvars.org

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