Time-Varying Identification of Structural Vector Autoregressions

Annika Camehl Erasmus University Rotterdam
Tomasz Woźniak University of Melbourne

contributions

- time-varying identification via a data-driven search
- identification via heteroskedasticity within regimes
- data support TVI of US monetary policy shock

context of the SVAR literature

- time-invariant identification
  Sims (1980), ...
- time-vatying parameters (but not TVI)
  Primiceri (2005), Sims, Zha (2006), ...
- identification through hetroskedasticity
  Rigobon (2003), Lewis (2021), Bertshe, Braun (2022), Lütkepohl, Shang, Uzeda, Woźniak (2024), ...
- exclusion restrictions are supported by data
  Lütkepohl, Lanne, Maciejowska (2010), Lütkepohl, Woźniak (2020), ...
- not verified TVI with fixed regimes
  Kimura and Nakajima (2016), Bacchiocchi, Castelnuovo, Fanelli (2017), Arias, Rubio-Ramirez, Shin, Waggoner (2024), Pagliari (2024)

tvi: time-varying identification

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structural VAR

reduced form: $$\mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \boldsymbol{\varepsilon}_t$$

structural form: $$\mathbf{B}(s_t, \boldsymbol{\kappa}(s_t))\boldsymbol{\varepsilon}_t = \mathbf{u}_t$$

structural shocks: $$\mathbf{u}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \text{diag}\left(\boldsymbol{\sigma}_t^2\right)\right)$$

Markov process: $$s_t \sim Markov_M\left(\mathbf{P}, \boldsymbol{\pi}_0\right)$$

time-varying identification

$$\Pr\left[\boldsymbol{\kappa}(s_t) \mid \text{data}\right]$$

tvi: time-varying identification

structural equation

structural form: $\mathbf{B}(s_t, \boldsymbol{\kappa}(s_t))\boldsymbol{\varepsilon}_t = \mathbf{u}_t$

TVI indicator: $\boldsymbol{\kappa}(s_t) = (\kappa_1(s_t), \dots, \kappa_N(s_t))$

TVI in n^{th} equation: $\kappa_n(s_t) = k_n \in \{1, \dots, K_n\}$

exclusion restrictions

n^{th} equation: $[\mathbf{B}(m, k_n)]_{n\cdot} = \mathbf{b}_{n.m.k_n}\mathbf{V}_{n.m.k_n}$

$$\begin{bmatrix} b_{n.1} & b_{n.2} & 0 \end{bmatrix} = \begin{bmatrix} b_{n.1} & b_{n.2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

tvi: time-varying identification

hierarchical prior distribution

structural: $\mathbf{b}'_{n.m.k_n} \mid \gamma_B, k_n \sim \mathcal{N}_{r_{n.m.k_n}}\left(\mathbf{0}_{r_{n.m.k_n}}, \gamma_{B.n}\mathbf{I}_{r_{n.m.k_n}}\right)$

TVI indicator: $\kappa_n(m) \sim \mathcal{M}ultinomial\left(K_n^{-1}\boldsymbol{\imath}_{K_n}\right)$

time-varying identification

$$\Pr\left[\boldsymbol{\kappa}(s_t) \mid \text{data}\right]$$

inference on TVI components

Given S posterior draws $\left\{\kappa_n(m)^{(s)}\right\}_{s=1}^{S}$ compute the posterior probability of regime-specific TVI component by:

$$\widehat{\Pr}\left[\kappa_n(m) = k_n \mid \mathbf{Y}_T\right] = S^{-1} \sum_{s=1}^{S} \mathcal{I}(\kappa_n(m)^{(s)} = k_n) \tag{1}$$

identification through heteroskedasticity

stochastic volatility

structural shocks: $\mathbf{u}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \text{diag}\left(\boldsymbol{\sigma}_t^2\right)\right)$

variances: $\sigma_{n.t}^2 = \exp\left\{\omega_n(s_t)h_{n.t}\right\}$

log-volatilities: $h_{n.t} = \rho_n h_{n.t-1} + v_{n.t}$

shocks: $v_{n.t} \sim \mathcal{N}\left(0, 1\right)$

homoskedasticity condition

Lütkepohl, Shang, Uzeda, Woźniak (2024)

$$\omega_n(s_t = m) = 0$$

time-varying identification of US monetary policy shocks

tvi of monetary policy shock

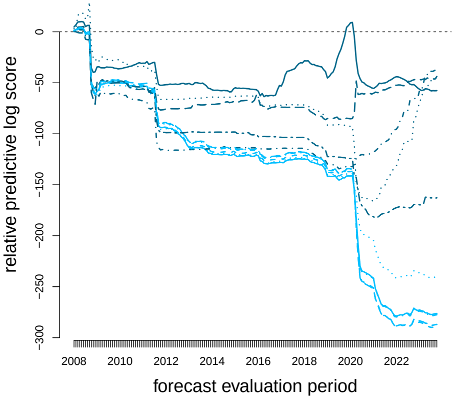
monetary policy reaction function

	y_t	π_t	R_t	TS_t	m_t	sp_t
benchmark	$*$	$*$	$*$	0	0	0
with TS	$*$	$*$	$*$	$*$	0	0
with m	$*$	$*$	$*$	0	$*$	0
only m	0	0	$*$	0	$*$	0

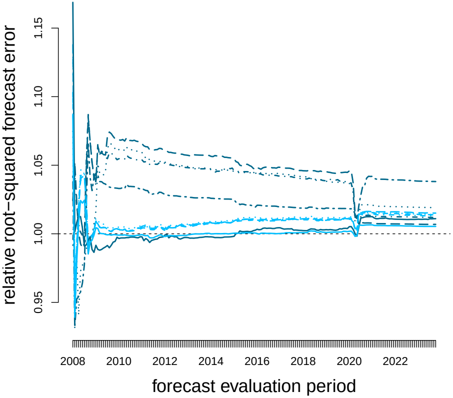
- data sample from January 1959 to June 2023 empirical evidence

forecasting performance

density forecasting



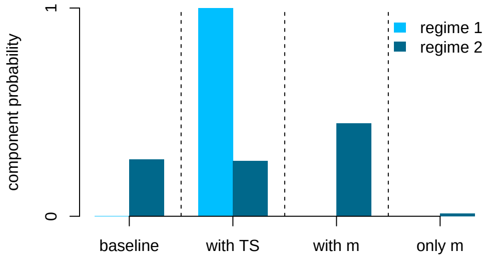
point forecasting



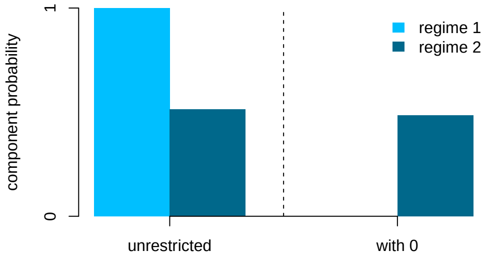
- The benchmark model with two regimes and TVI is optimal for point and density forecasting

TVI posterior probabilities

monetary policy shock



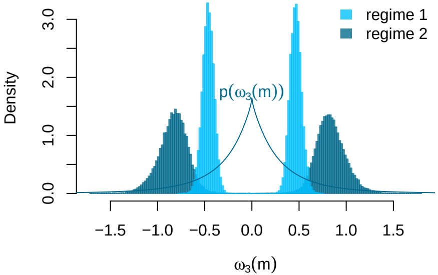
term-spread shock



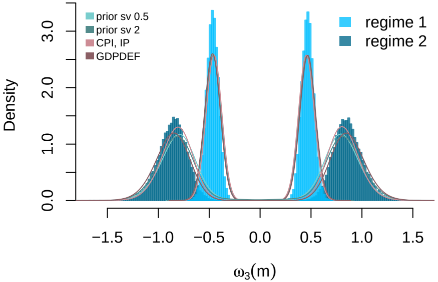
- Posterior probability of TVI: 0.87
- robustness w.r.t. TVI setup, prior distributions, alternative measurements

identified via heteroskedasticity

benchmark model



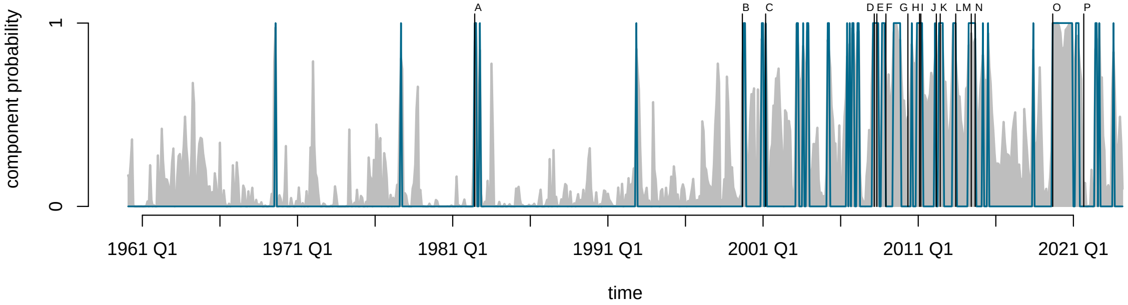
result robustness



- mp shock identified via heteroskedasticity
- similarly term spread shock

regime probabilities

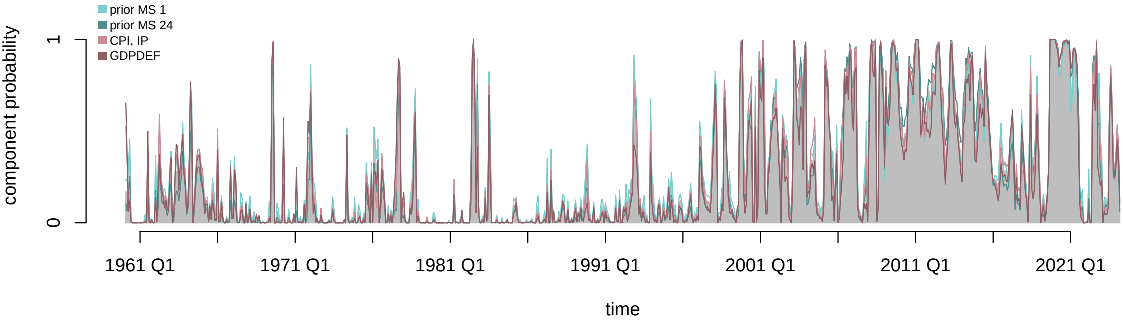
estimated probabilities of the second regime



- high-probability periods correspond to major events
- regime of higher volatility and quantitative easying

regime probabilities

robustness of the second regime probabilities



regime interpretation

regime-specific sample moments

	Regime 1			Regime 2		
	mean	sd	$\mathrm{cov}(\cdot, R_t)$	mean	sd	$\mathrm{cov}(\cdot, R_t)$
R_t	5.64	3.62		2.54	2.73	
TS_t	0.84	1.69	-3.79	1.43	1.34	-2.15
m_t	6.33	3.86	1.43	7.38	8.63	-2.03

regime interpretation

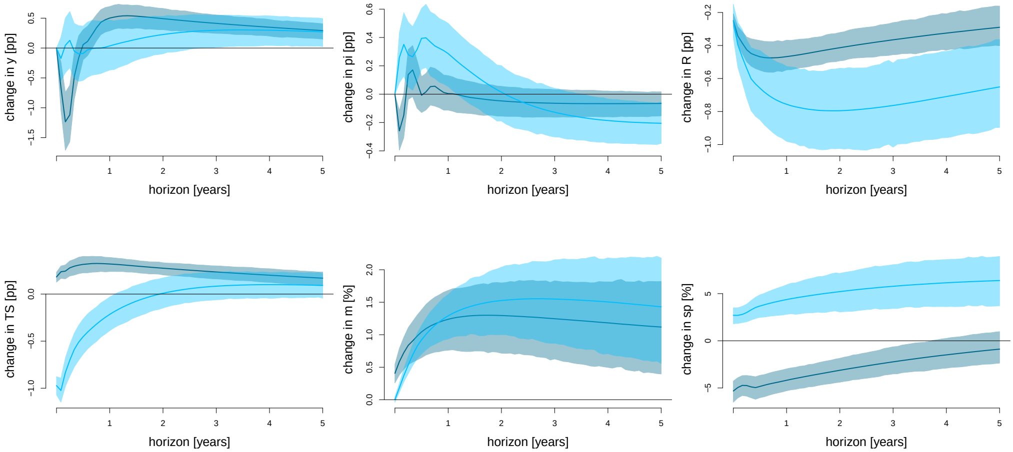
monetary policy reaction function estimates

Regime 1: $0.01y_t - \mathbf{0.07}\pi_t + \mathbf{3.64}R_t + \mathbf{4.09}\,TS_t = \cdots + \hat{u}_t^{mps}$

Regime 2: $-0.03y_t - 0.02\pi_t + \mathbf{14.00}R_t - \mathbf{0.45}m_t = \cdots + \hat{u}_t^{mps}$

regime interpretations

impulse responses to the mp shock



bsvars.org

github.com/donotdespair

@tomaszwwozniak

@tomaszwozniak.bsky.social

@tomaszwozniak@fosstodon.org