Forecasting with Time-Varying Order-Invariant Structural Vector Autoregressions

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contributions

We propose **a new flexible model** that combines:

- order-invariant specification
- time-variation in parameters
- stochastic volatility and non-normal shocks
- enforcing normalisation

that entertains **excellent forecasting performance**.

context of the SVAR literature

- order-invariant SVARs
  Arias, Rubio-Ramírez, Shin (2023), Chen, Koop, Yu (2025)
- time-vatying parameters
  Primiceri (2005), Sims, Zha (2006)
- stochastic volatility of shocks
  Carriero, Clark, Marcellino (2019), Lütkepohl, Shang, Uzeda, Woźniak (2024)
- non-normal shocks
  Lanne, Meitz, Saikkonen (2017), Brunnermeier, Palia, Sastry, Sims (2021)
- time-variation + id via heteroskedasticity
  Camehl, Woźniak (2025)

a new flexible model

a new flexible model

structural VAR

reduced form: $\mathbf{y}_t = \mathbf{A}(s_t)\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

structural form: $\mathbf{B}(s_t)\boldsymbol{\varepsilon}_t = \mathbf{u}_t$

structural shocks: $\mathbf{u}_t \mid \mathbf{x}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \text{diag}\left(\boldsymbol{\lambda}_t \odot \boldsymbol{\sigma}_t^2\right)\right)$

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structural matrix

- $\mathbf{B}(s_t)$ – no restrictions imposed (order-invariant)
- $\mathbf{B}(s_t)$ – identified via heteroskedasticity and non-normality
- $\mathbf{B}(s_t)$ – time-varying and following persistent regimes s_t

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Markov process

- $s_t \sim Markov_M\left(\mathbf{P}, \boldsymbol{\pi}_0\right)$
- s_t – stationary with $M = 2$ regimes
- s_t – overfitting with $M = 20$ allowing many with 0 occurrences
- time-invariant model: $M = 1$

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structural matrix

- training sample prior for $\mathbf{B}_m = \mathbf{B}(s_t = m)$:

$$p\left(\mathbf{B}(s_t)\right) \propto \det\left(\mathbf{B}_m\right)^{T_t} \exp\left\{-\frac{1}{2}\sum_{n=1}^{N}[\mathbf{B}_m]_{n\cdot}\underline{S}_n^{-1}[\mathbf{B}_m]_{n\cdot}'\right\}$$

$$\underline{S}_n^{-1} = \sum_{t=1}^{T_t}\left(\mathbf{y}_t - \underline{\mathbf{A}}_1(s_t)\mathbf{y}_{t-1}\right)\left(\mathbf{y}_t - \underline{\mathbf{A}}_1(s_t)\mathbf{y}_{t-1}\right)'$$

a new flexible model

structural VAR

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Student-t shocks

- $\lambda_{n.t} \sim \mathcal{IG}2\left(\nu_n(s_t) - 2, \nu_n(s_t)\right)$
- $\nu_n(s_t)$ – estimated shock- & regime-specific degrees of freedom
- Prior normalisation: $\mathbb{E}[\lambda_{n.t}] = 1$
- **Problem:** normalisation $\mathbb{E}[\lambda_{n.t} \mid data] = 1$ not ensured *a posteriori*

a new flexible model

structural VAR

reduced form: $\mathbf{y}_t = \mathbf{A}(s_t)\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

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Student-t shocks

- **Problem:** normalisation $\mathbb{E}[\lambda_{n.t} \mid data] = 1$ not ensured *a posteriori*
- **Solution:** impose posterior normalisation: $\mathbb{E}[\lambda_{n.t} \mid data] = 1$
- $\lambda_{n.t} \mid data, \nu_n(s_t), s_t, \dots \sim \mathcal{IG}2\left(\nu_n(s_t) - 2, \nu_n(s_t)\right)$

a new flexible model

structural VAR

reduced form: $\mathbf{y}_t = \mathbf{A}(s_t)\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

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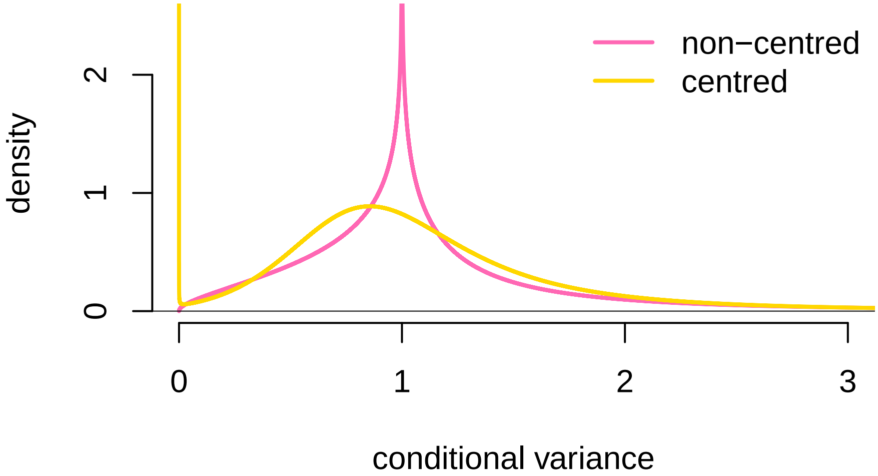
stochastic volatility

- $\sigma^2_{n.t} = \exp\left(\omega_n(s_t)h_{n.t}\right)$
- $h_{n.t} = \rho_n h_{n.t-1} + v_{n.t}$ and $v_{n.t} \sim \mathcal{N}(0,1)$
- fast-moving volatility ensures identification within regime
- verify $\omega_n(s_t) = 0$ to make certain (Camehl, Woźniak, 2025)

a new flexible model

conditional variance prior ensures normalisation



a new flexible model

alternative formulation

reduced form: $\mathbf{y}_t = \mathbf{A}(s_t)\mathbf{x}_t + \boldsymbol{\varepsilon}_t$

error terms: $\boldsymbol{\varepsilon}_t \mid \mathbf{x}_t \sim t_n\left(\mathbf{0}_N, \boldsymbol{\Sigma}_t(s_t), \boldsymbol{\nu}(s_t)\right)$

covariances: $\boldsymbol{\Sigma}_t(s_t) = \mathbf{B}(s_t)^{-1}\mathrm{diag}\left(\boldsymbol{\sigma}_t^2\right)\mathbf{B}(s_t)^{-1\prime}$

forecasting performance

forecasting performance

competing models

- **Benchmark:** TVP-SVAR-SV by Primiceri (2005)
- Our model with the followig specifications:
  - MS(1) / MS(2) / MS(20)
  - MSS / MSSA
  - order-invariant / lower-triangular structural matrix
  - SV / homoskedastic shocks
  - Student-t / normal shocks

forecasting performance

data

- **FRED-MD:** INDPRO, UNRATE, PCEPI, FEDFUNDS
- Jan 1959 – Jun 2025
- expanding window forecasting exercise from Jul 2019

forecasting performance

point forecast 1 period ahead

volatility	SV				homoskedastic			
identification	OI		LT		OI		LT	
shocks	norm	t	norm	t	norm	t	norm	t
MS(1)	2.56	1.49	1.11	1.04	2.23	0.72	1.09	165.51
MSS(2)	0.87	0.87	0.88	0.88		0.86	0.89	0.89
MSS(20)	0.87	0.85	0.89	0.86	0.87	0.85	0.89	0.86
MSSA(2)	0.92	0.85	0.92	0.85	0.92	0.84	0.86	0.85
MSSA(20)	0.84	0.84	0.85	0.85	0.84	0.84	0.84	0.85

MAFE relative to TVP-SVAR-SV

forecasting performance

density forecast 1 period ahead

volatility	SV				homoskedastic			
identification	OI		LT		OI		LT	
shocks	norm	t	norm	t	norm	t	norm	t
MS(1)	-2.21	0.13	-1.31	-0.80	-1.16	-0.38	-0.69	-92.27
MSS(2)	0.93	0.78	0.89	0.91	-Inf	0.80	0.89	0.88
MSS(20)	0.66	0.51	0.96	0.83	0.65	0.51	0.96	0.83
MSSA(2)	0.86	0.70	0.81	0.75	0.86	0.71	0.79	0.76
MSSA(20)	0.35	0.06	0.67	0.42	0.34	0.06	0.66	0.40

predictive log-score relative to TVP-SVAR-SV

forecasting performance

predictive Bayes factors

- time-variation
  - MS(2) vs. MS(1): **4.818**
  - MS(20) vs. MS(1): **4.899**
  - MS(20) vs. MS(2): **1.017**
  - MSSA vs. MSS: **0.872**

forecasting performance

predictive Bayes factors

- identification
  - OI vs. LT: **0.827**
  - t vs. norm: **0.981**
  - SV vs. homoskedastic: **1.097**

bsvars.org

github.com/donotdespair

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