Infinite Impulse Response (IIR) Filter Design

Difference Equation and Transfer Function of IIR Filter

$$a_0 y[n] + a_1 y[n-1] + ... + a_N y(n-N) = b_0 x[n] + b_1 x[n-1] + ... + b_M x[n-M]$$

Assume

$$a_0 = 1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

Analog Filters Design using Lowpass Filter Prototype $H_P(s)$

This method converts the analog lowpass filter with a cutoff frequency of 1 radian per second, called Lowpass prototype, into practical lowpass, highpass, bandpass, and bandstop filters with their frequency specifications.

$$H_P(s) = \frac{1}{s+1}$$

Note: **s** is the Laplace Transform (complex) variable

TABLE 8.1 **Analog lowpass prototype transformations.**

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

$\omega_0 - center\ frequency$

$W\ - Bandwidth$

Table Number is in accordance with the Reference Book

Example1. Prototype Conversion to Practical Analog Filter

Given a lowpass prototype

$$H_P(s) = \frac{1}{s+1},$$

Determine each of the following analog filters

a) The highpass filter with a cutoff frequency of 40 radians per second.

b) The bandpass filter with a center frequency of 100 radians per second and bandwidth of 20 radians per second.

Analog lowpass prototype transformations.

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Solution to Example1:

Analog lowpass prototype transformations.

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$

$H_p(s) \rightarrow H_{HP}(s)$

From the table, the transformation of HPF is $\frac{\omega_c}{s}$. $\omega_c = 40 \frac{rad}{sec}$

$H_{HP}(s) = \frac{1}{\frac{\omega_c}{s}+1} = \frac{1}{\frac{40}{s}+1}$

$\boxed{H_{HP}(s) = \frac{s}{s+40}}$ Transfer function of Analog HPF in the s-domain

$H_p(s) \rightarrow H_{BPF}(s)$ From the table, the transformation of BPF is $\frac{s^2+\omega_0^2}{sW}$

$\omega_0 =$ 100 rad/sec

$BW =$ 20 rad/sec

Continuation
$H_{BPF}(s) =$

$\boxed{H_{BPF}(s) = \frac{20s}{s^2+20s+10000}}$

a) $H_p(s) \rightarrow H_{HP}(s)$

From the table, the transformation of HPF is $\frac{\omega_c}{s}$. $\omega_c = 40 \frac{rad}{sec}$

$$H_{HP}(s) = \frac{1}{\frac{\omega_c}{s}+1} = \frac{1}{\frac{40}{s}+1}$$

$$\boxed{H_{HP}(s) = \frac{s}{s+40}}$$

Transfer function of Analog HPF in the s-domain

b) $H_p(s) \rightarrow H_{BPF}(s)$

From the table, the transformation of BPF is $\frac{s^2+\omega_0^2}{sW}$

$\omega_0 =$ 100 rad/sec

$BW =$ 20 rad/sec

Continuation of b)

$$H_{BPF}(s) = \frac{1}{\frac{s^2+\omega_0^2}{sW}+1}$$

$$= \frac{1}{\frac{s^2+10000}{20s}+1}$$

$$\boxed{H_{BPF}(s) = \frac{20s}{s^2+20s+10000}}$$

The general mapping properties are summarized as following:

1. The left-half s-plane is mapped onto the inside of the unit circle of the z-plane.
2. The right-half s-plane is mapped onto the outside of the unit circle of the z-plane.
3. The positive $j\omega$ axis portion in the s-plane is mapped onto the positive half circle (the dashed-line arrow in Figure 8.8) on the unit circle, while the negative $j\omega$ axis is mapped onto the negative half circle (the dotted-line arrow in Figure 8.8) on the unit circle.

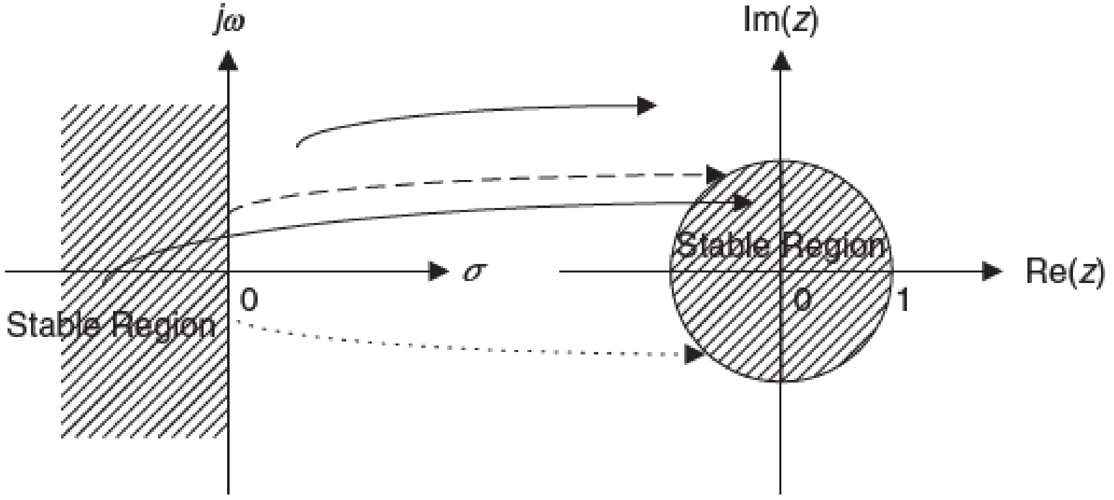


FIGURE 8.8 Mapping between the s-plane and the z-plane by the bilinear transformation.

Bilinear Transformation (BLT)

- Bilinear Transformation converts an analog filter into a digital filter.
- The bilinear transform maps the left half of the complex s-plane to the interior of the unit circle in the z-plane

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

$$z = \frac{1+sT/2}{1-sT/2}$$

T=sampling period or
sampling interval in second

$$s = \frac{2}{T}\frac{z-1}{z+1} \qquad z = \frac{1+sT/2}{1-sT/2}$$

Example2. Transforming s to z

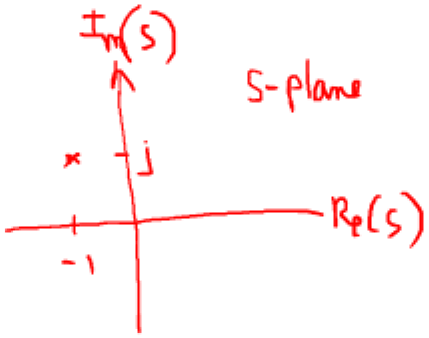
a) Given T=2sec, convert the pole s = -1+j on the s- plane to the z-plane.

Solution :

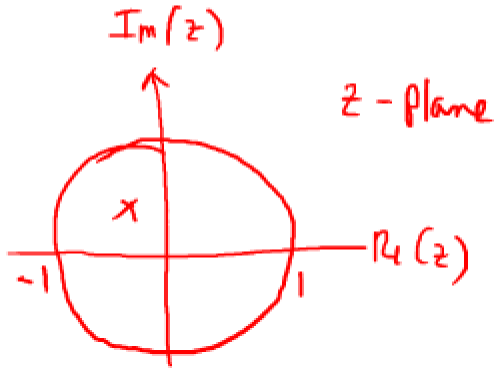
a) $$z = \frac{1 + \frac{(-1+j)(2)}{2}}{1 - \frac{(-1+j)(2)}{2}}$$

$$z = \frac{j}{2-j}$$

$$z = -0.2 + 0.4j = 0.4472\angle 2.03 \text{ or } 0.4472\angle 116.57^\circ$$



Stable because the pole is on the left half of the S-plane



Stable because the pole is Inside the unit circle

Example2b. Transforming the transfer function from s-domain to z-domain, H(s) to H(z)

$$s = \frac{2}{T}\frac{z-1}{z+1} \qquad z = \frac{1+sT/2}{1-sT/2}$$

b) Given the analog filter whose transfer function is

$$H(s) = \frac{10}{s+10}$$

Convert it to a digital filter transfer function and difference equation, when the sampling period is T=0.01 sec.

Solution: $H(s) \Rightarrow H(z)$

$$H(z) = \frac{10}{\frac{2}{T}\left[\frac{z-1}{z+1}\right] + 10}$$

$$= \frac{10}{\frac{2}{0.01}\left[\frac{z-1}{z+1}\right] + 10}$$

$$= \frac{10}{\frac{200(z-1) + 10(z+1)}{z+1}}$$

Continuation:

$$H(z) = \frac{10(z+1)}{210z - 190}$$

$$H(z) = \frac{10(z+1)}{210z - 190} \cdot \frac{\frac{1}{210z}}{\frac{1}{210z}}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{0.04762 + 0.04762z^{-1}}{1 - 0.904762z^{-1}}}$$

Transfer function

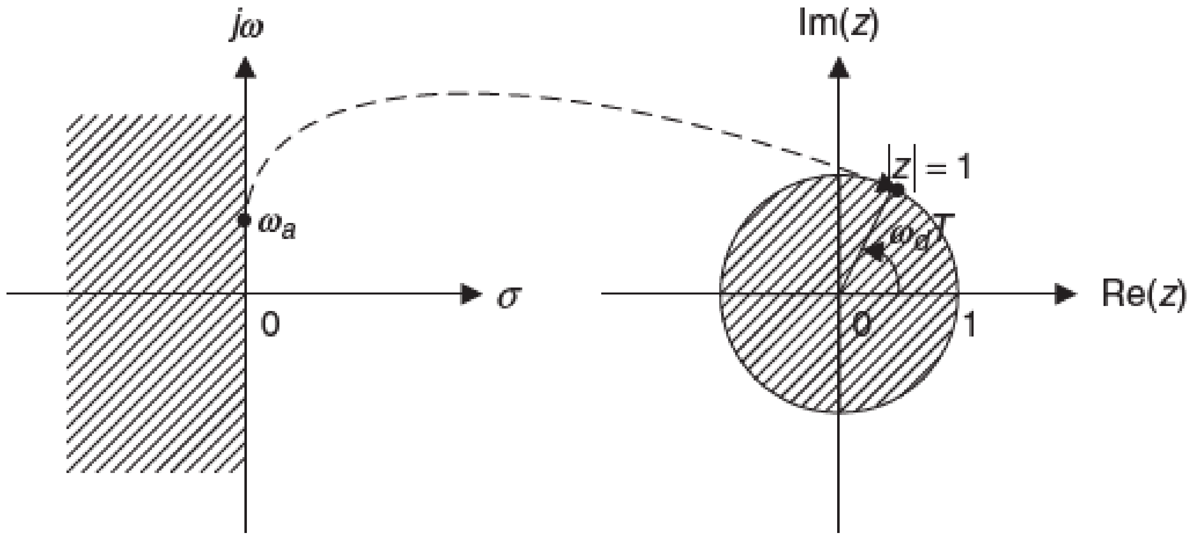
Continuation

$$\boxed{y[n] = 0.04762x[n] + 0.04762x[n-1] + 0.904762y[n-1]}$$

Frequency Mapping between s-plane and the z-plane

The bilinear transform maps the left half of the complex s-plane to the interior of the unit circle in the z-plane

We substitute $s = j\omega_a$ and $z = e^{j\omega_d T}$ into the BLT



ω_a = analog frequency

ω_d = digital freq.

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

$$j\omega_a = \frac{2}{T}\frac{e^{j\omega_d T}-1}{e^{j\omega_d T}+1}$$

$$\boxed{\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)}$$

$$j\omega_a = \frac{2}{T}\left[\frac{e^{\frac{j\omega_d T}{2}}\left(e^{j\frac{\omega_d T}{2}} - e^{-j\frac{\omega_d T}{2}}\right)}{e^{\frac{j\omega_d T}{2}}\left(e^{j\frac{\omega_d T}{2}} + e^{-j\frac{\omega_d T}{2}}\right)}\right]$$

recall: $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

so,

$$j\omega_a = \frac{2}{T}\left[\frac{2j\sin\left(\frac{\omega_d T}{2}\right)}{2\cos\left(\frac{\omega_d T}{2}\right)}\right]$$

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

IIR Filter Design Procedure using Bilinear Transformation

Step 1. Given the digital filter frequency specifications, prewarp the digital frequency specifications to the analog frequency specifications.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right).$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right), \ \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

where

$$\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, \ W = \omega_{ah} - \omega_{al}$$

ω_a= analog freq. (cut-off)
T= sampling period
ω_d= digital freq. (cut-off)
ω_o= center freq
W = bandwidth

Step 2. Perform the prototype transformation using the lowpass prototype $H_p(s)$.

Analog lowpass prototype transformations.

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Step 3. Substitute the BLT to obtain the digital filter

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

Example3. Design using BLT

$$H_P(s) = \frac{1}{s+1}.$$

Analog lowpass prototype transformations.

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$

Use the given $H_p(s)$ and the BLT to design a corresponding digital IIR lowpass filter with a cutoff frequency of 15 Hz and a sampling rate of 90 Hz.

Write the transfer function and difference equation of the designed IIR Filter.

Step1. Prewarp the digital freq to analog freq.

digital cutoff is 15 Hz , $F_s = 90\text{Hz}$

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

$$= \frac{2}{T}\tan\left(\frac{\frac{2\pi F}{F_s}}{2}\right)$$

$$= \frac{2}{1/90}\tan\left(\frac{15\pi}{90}\right) = 103.923 \ \frac{\text{rad}}{\text{sec}}$$

$$\omega_a = 60\sqrt{3}\ \frac{\text{rad}}{\text{sec}}$$

Step2 Perform the prototype transformation

$$H_{LP}(s) = \frac{1}{\frac{s}{\omega_c}+1} = \frac{\omega_c}{s+\omega_c}$$

where ω_c is the prewarped analog freq, ω_a.

that is, $\omega_c = \omega_a$

$$H_{LP}(s) = \frac{60\sqrt{3}}{s+60\sqrt{3}} = \frac{103.923}{s+103.923}$$

* $H_{LP}(s)$ is the transfer function of the Analog LPF

Continuation.

Step 3. Apply the BLT $\quad H_{LP}(s) = \dfrac{60\sqrt{3}}{s + 60\sqrt{3}} = \dfrac{103.923}{s + 103.923}$

$H_{LP}(s) \rightarrow H(z)$

$$H(z) = \left.\frac{103.923}{s + 103.923}\right|_{s = \frac{2}{T}\left[\frac{z-1}{z+1}\right]}$$

$$= \frac{103.923}{\frac{2}{1/90}\left[\frac{z-1}{z+1}\right] + 103.923}$$

$$= \frac{103.923(z+1)}{180(z-1) + 103.923(z+1)}$$

$$= \frac{103.923(z+1)}{283.923z - 76.077} \cdot \frac{\frac{1}{283.923z}}{\frac{1}{283.923z}}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{0.366 + 0.366z^{-1}}{1 - 0.268z^{-1}}}$$

$y[n] - 0.268\,y[n-1] = 0.366\,x[n] + 0.366\,x[n-1]$

$$\boxed{y[n] = 0.366\,x[n] + 0.366\,x[n-1] + 0.268\,y[n-1]}$$

```
%Matlab Implementation:
Fs=90;
[num,den]=lp2lp([1],[1 1],103.923)
[b,a]=bilinear(num,den,Fs)
```

$H_p(s) = \frac{1}{s+1}$

$\hookrightarrow \Omega_a$ in the analog design (from Step 1)

coefficient of y terms

Coefficient of x terms

```
>> Fs=90;
>> [num,den]=lp2lp([1],[1 1],103.923);
>> [b,a]=bilinear(num,den,Fs)

b =

    0.3660    0.3660

a =

    1.0000   -0.2679
```

Other Mathematical Functions

- Butterworth Function
- Chebyshev Function
- Bessel Function
- Elliptic Function

Mathematical Functions

- **Bessel Function** -a mathematical function used to produce the most linear phase response of all IIR filters with no consideration of the frequency magnitude response.
- **Butterworth Function** - a mathematical function used to produce maximally flat filter magnitude responses with no consideration of phase linearity or group delay variations. Filter designs based on a Butterworth function have no amplitude ripple in either the passband or the stopband. Unfortunately, for a given filter order, Butterworth designs have the widest transition region of the most popular filter design functions.
- **Chebyshev Function** - a mathematical function used to produce passband, or stopband, ripples constrained within fixed bounds. There are families of Chebyshev functions based on the amount of ripple such as 1 dB, 2 dB, and 3 dB of ripple. Chebyshev filters can be designed to have a frequency response with ripples in the passband and flat stopbandband (Chebyshev Type I), or flat passband and ripples in the stopband (Chebyshev Type II). Chebyshev filters cannot have ripples in both the passband and the stopband. Digital filters based upon Chebyshev functions have steeper transition region roll-off but more nonlinear phase response characteristics than Butterworth filters.
- **Elliptic Function** - a mathematical function used to produce the sharpest roll-off for a given number of filter taps. However, filters designed using elliptic functions, also called *Cauer filters*, have the poorest phase linearity of the most common IIR filter design functions. The ripple in the passband and stopband are equal with elliptic filters.

Formula for the computation of IIR Filter Order

$$A_P\ dB = -20 \cdot \log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$A_s\ dB = -20 \cdot \log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2 v_s^{2n}}}\right)$$

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

$$n \geq \frac{\log_{10}\left(\frac{10^{0.1A_s}-1}{\varepsilon^2}\right)}{\left[2 \cdot \log_{10}(v_s)\right]}$$

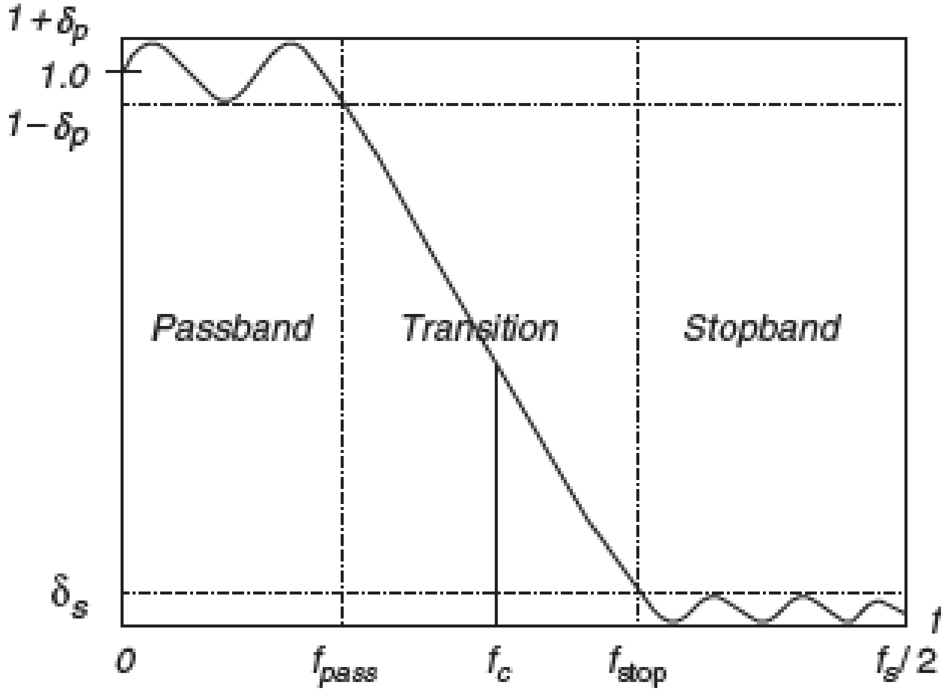
n = IIR filter order
$A_p\ dB$ – passband ripple at the normalized passband frequency edge, $v_p=1$
$A_s\ dB$ – passband ripple at the normalized stopband frequency edge , v_S
ε - absolute ripple specification
v_S – normalized stopband edge

$$\delta_p \ dB = 20 \cdot \log_{10}\left(1 + \delta_p\right)$$

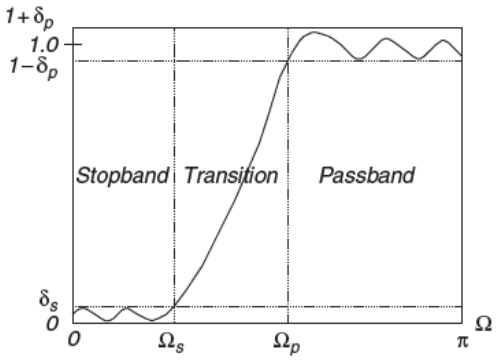
$$\delta_s \ dB = -20 \log_{10}\left(\delta_s\right)$$

δ_p _ $ripple\ (fluctuation)\ of\ the\ frequency\ magnitude\ response\ in\ the\ passband$
δ_s $-\ ripple\ of\ the\ frequency\ magnitude\ response\ in\ the\ stopband$

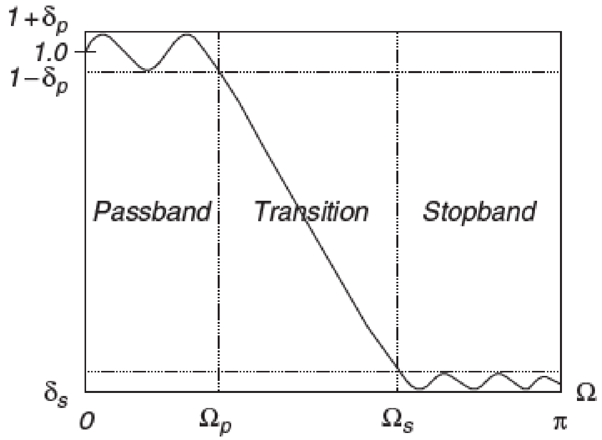
f_{pass} – passband frequency
f_{stop} – stopband frequency
f_C – cutoff frequency



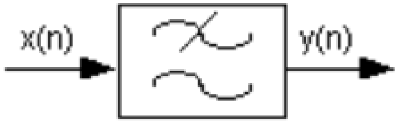
Frequency Magnitude Response and Filter Symbol

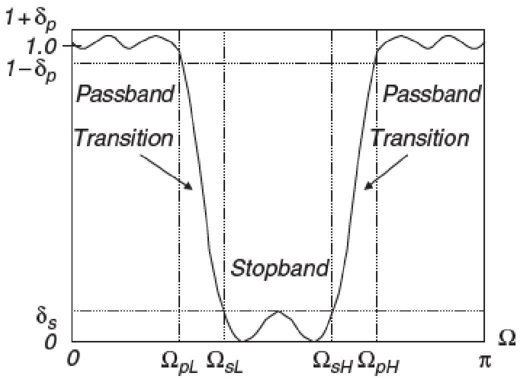


Magnitude response of the normalized highpass filter.

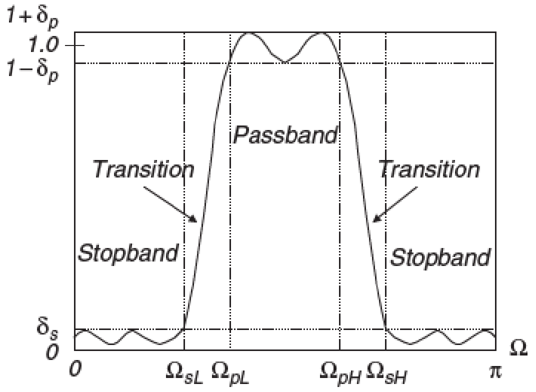


Magnitude response of the normalized lowpass filter.





Magnitude of the normalized bandstop filter.



Magnitude response of the normalized bandpass filter.

TABLE 8.1 **Analog lowpass prototype transformations.**

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Table Number is in accordance with the Reference Book

TABLE 8.3 **3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)**

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$
4	$\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\frac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\frac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table Number is in accordance with the Reference Book

TABLE 8.4 **Chebyshev lowpass prototype transfer functions with 0.5 dB ripple ($\varepsilon = 0.3493$)**

n	$H_P(s)$
1	$\dfrac{2.8628}{s+2.8628}$
2	$\dfrac{1.4314}{s^2+1.4256s+1.5162}$
3	$\dfrac{0.7157}{s^3+1.2529s^2+1.5349s+0.7157}$
4	$\dfrac{0.3579}{s^4+1.1974s^3+1.7169s^2+1.0255s+0.3791}$
5	$\dfrac{0.1789}{s^5+1.1725s^4+1.9374s^3+1.3096s^2+0.7525s+0.1789}$
6	$\dfrac{0.0895}{s^6+1.1592s^5+2.1718s^4+1.5898s^3+1.1719s^2+0.4324s+0.0948}$

Table Number is in accordance with the Reference Book

TABLE 8.5 **Chebyshev lowpass prototype transfer functions with 1 dB ripple ($\varepsilon = 0.5088$)**

n	$H_P(s)$
1	$\dfrac{1.9652}{s+1.9652}$
2	$\dfrac{0.9826}{s^2+1.0977s+1.1025}$
3	$\dfrac{0.4913}{s^3+0.9883s^2+1.2384s+0.4913}$
4	$\dfrac{0.2456}{s^4+0.9528s^3+1.4539s^2+0.7426s+0.2756}$
5	$\dfrac{0.1228}{s^5+0.9368s^4+1.6888s^3+0.9744s^2+0.5805s+0.1228}$
6	$\dfrac{0.0614}{s^6+0.9283s^5+1.9308s^4+1.20121s^3+0.9393s^2+0.3071s+0.0689}$

Table Number is in accordance with the Reference Book

TABLE 8.6 **Conversion from analog filter specifications to lowpass prototype specifications.**

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_p = 1, v_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_p = 1, v_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

Table Number is in accordance with the Reference Book

Example 4. Design using BLT (Filter Order is not given)

Design a digital lowpass Butterworth filter with the following specifications:

1. 3 dB attenuation at the passband frequency of 1.5 kHz
2. 10 dB stopband attenuation at the frequency of 3 kHz
3. Sampling frequency of 8,000 Hz.

Step 1a) Calculate the Prewarped Analog freq

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right) = 2F_s \tan\left(\frac{2\pi \frac{F}{F_s}}{2}\right)$$

$$\omega_{ap} = 2(8000)\tan\left(\pi \frac{1500}{8000}\right) = 10.691\times 10^3 \text{ rad/sec}$$

$$\omega_a = 2(8000)\tan\left(\pi \frac{3000}{8000}\right) = 38.627\times 10^3 \frac{\text{rad}}{\text{sec}}$$

Step 1b) Calculate the order

$A_p = 3\,dB$

$A_s = 10\,dB$

Table 8.6

$$V_s = \frac{\omega_{as}}{\omega_{ap}} = 3.613$$

$$\varepsilon^2 = 10^{0.1A_p} - 1 = 10^{0.3(3)} - 1 = 0.995 \approx 1$$

$$n \geq \frac{\log\left(\frac{10^{0.1A_s}-1}{\varepsilon^2}\right)}{2\log V_s} = \frac{\log\left(\frac{10^{0.1(10)}-1}{1}\right)}{2\log 3.613}$$

$n \geq 0.855$ $\quad \boxed{n=1}$

Example 4. (Continuation)

step 2. Perform Prototype Transformation

$$H_p(s) = \frac{1}{s+1} \qquad H_p(s) \rightarrow H_{LPF}(s)$$

$$H_{LPF}(s) = \frac{1}{\frac{s}{\omega_{ap}} + 1} = \frac{1}{\frac{s}{10.691\times 10^3} + 1}$$

$$\checkmark \; H_{LPF}(s) = \frac{10.691\times 10^3}{s + 10.691\times^3}$$

step 3. Apply the BLT

$$s = \frac{2}{T}\left[\frac{z-1}{z+1}\right] = 2F_s\left[\frac{z-1}{z+1}\right]$$

$$H(z) = \frac{10.691\times 10^3}{2(8000)\left(\frac{z-1}{z+1}\right) + 10.691\times 10^3}$$

$$H(z) = \frac{10.691\times 10^3 (z+1)}{16000(z-1) + 10.691\times 10^3 (z+1)}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{0.40055 + 0.40055\, z^{-1}}{1 - 0.19891\, z^{-1}}}$$

$$Y(z)\left[1 - 0.19891 z^{-1}\right] = X(z)\left[0.40055 + 0.40055 z^{-1}\right]$$

diff eqtn

$$y[n] - 0.19891\, y[n-1] = 0.40055\, x[n] + 0.40055\, x[n-1]$$

coeff of x = b = [0.40055 0.40055]

coeff of y = a = [1 −0.19891]

filter(b, a, signal)

MATLAB Implementation of Example 4

```
%Design of IIR Butterworth Lowpass Filter (Example1)
%Two methods are presented
clf;
clear;
fc=1500;
Fs=8000;

%Method 1. Using command butter
order = 1;
Wc = 2*fc/Fs; %Normalizing the frequencies. Note that pi is already included in the butter command.
[b,a]=butter(order,Wc,'low') %Calculation of filter coefficients

%Method 2. Using the lp2lp and bilinear commands
[num,den]=lp2lp([1],[1 1],10.691*10^3); % Complete step 2
[B,A]=bilinear(num,den,Fs) % Complete step 3

%Plotting the filter response
freqz(b,a,512,Fs); %The 512 is the MATLAB default for number of evaluation points
title('Magnitude and Phase response of IIR Butterworth Lowpass filter');
```

Exercise.

- Design a first order digital highpass Chebyshev filter with a cutoff frequency of 3kHz and 1dB ripple on the passband using a sampling frequency of 8kHz.

Reference

- Tan, L. (2013). *Digital signal processing : fundamentals and applications* (2nd ed.). Amsterdam: Elsevier