LESSON 11

WALLIS’ FORMULA

Objectives

- ✓ To integrate powers and product of sine and cosine using Wallis' Formula.
- ✓ To recall and apply the different trigonometric identities in transforming powers of sine and cosine; and use Wallis' Formula to shorten the solution in finding the anti - derivative of powers of sine and cosine.

Wallis' Formula

- The integral

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx$$

in which **m** and **n** are **integers ≥0**

Wallis' Formula

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx$$

$$= \frac{\left[(m-1)(m-3)\ldots or \begin{matrix} 2 \\ 1 \end{matrix}\right]\left[(n-1)(n-3)\ldots or \begin{matrix} 2 \\ 1 \end{matrix}\right]}{\left[(m+n)(m+n-2)\ldots or \begin{matrix} 2 \\ 1 \end{matrix}\right]}\alpha$$

$$\alpha = \frac{\pi}{2} \text{ if } m \text{ and } n \text{ are both even}, \alpha = 1, \text{ if otherwise.}$$

RULE:

If the first factor in any of the products to be formed in applying Wallis' Formula, for m, n≥0, is less than one, replace that product by unity.

Examples: Evaluate the following

1. $\int_0^{\pi/2} sin^8 x cos^4 x \, dx$

2. $\int_0^{\pi/2} sin^5 x cos^6 x \, dx$

3. $\int_0^{\pi/2} sin^3 x cos^5 x \, dx$

4. $\int_0^{\pi/2} sinx \, cos^7 x \, dx$

5. $\int_0^{\pi/2} sin^4 x \, dx$

6. $\int_0^{\pi/6} cos^8 3x \, dx$

7. $\int_0^{\pi/4} sin^2 4y cos^2 2y \, dy$

8. $\int_0^{3} x^2 (9 - x^2)^{\frac{7}{2}} dx$