Allg: Δ-Ungl: $|x+y| \le |x| + |y|$

Cauchy-Schw: $|\langle x, y\rangle| \le \|x\| \cdot \|y\|$

Bernoulli: $(1+a)^n \ge 1 + na$

Binomialk: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$\mathbb{C}$: $z = a + bi$ | $\bar{z} = a - bi$

$|z| = \sqrt{z\bar{z}}$; $\overline{e^{ix}} = e^{-ix}$

$r = |z|$; $\varphi = \begin{cases} \arccos\left(\frac{a}{r}\right), & b \ge 0 \\ -\arccos\left(\frac{a}{r}\right), & b < 0 \end{cases} \rightarrow z = r(\cos\varphi + i\sin\varphi) = e^{i\varphi} \cdot r$

$z_1 \cdot z_2 = r_1 \cdot r_2 (\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2))$

$z_1 : z_2 = r_1 : r_2 (\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2))$

$z^n = r^n(\cos(n\varphi) + i\sin(n\varphi))$

injektiv: $f(x_1) = f(x_2)$; $x_1 = x_2$

bijektiv:

surjektiv: $\forall y \in W \; \exists x \rightarrow f(x) = y$

gerade: $f(-x) = f(x)$

ungerade: $f(-x) = -f(x)$

Stetigkeit: $\lim_{x \to x_0} f(x) = f(x_0)$

Diffbar: $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$

$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$

$\cosh x = \frac{1}{2}(e^{-x} + e^{x})$

$\sinh x = \frac{1}{2}(-e^{-x} + e^{x})$

Zwischenwerts: $\forall y \in [f(a), f(b)] \; \exists x \in [a, b]: f(x) = y$

Mittelwerts: wenn f diffbar, dann $\exists x_0: f'(x_0) = \frac{f(b) - f(a)}{b - a}$

L'Hospital: $\lim_{x \to a} \frac{f'(x)}{g'(x)}$

Trigono:

$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\sin^2 x + \cos^2 x = 1$

$\tan = \frac{\sin}{\cos}$ $\cot = \frac{\cos}{\sin}$

Additio: $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin(2x) = 2\sin x \cos x$

$\cos\left(x - \frac{\pi}{2}\right) = \sin x \leftrightarrow \sin\left(x + \frac{\pi}{2}\right) = \cos x$

x	0	30	45	60	90	120	135	150	180	270	360
x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\frac{2}{3\pi}$	$\frac{3}{4\pi}$	$\frac{5}{6\pi}$	π	$\frac{3}{2\pi}$	2π
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	–	0

$\sin^2 x = \frac{1 - \cos(2x)}{2}$; $\cos^2 x = \frac{1 + \cos(2x)}{2}$

$\sin x = \sum_k (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

$\cos x = \sum_k (-1)^k \frac{x^{2k}}{(2k)!}$

Folgen:
- a_n konvergent → beschränkt
- a_n beschränkt & monoton → konvergenz
- a_n unbeschränkt → divergenz

$q^n \begin{cases} 0 & |q| < 1 \\ 1 & q = 1 \end{cases}$

$\left(1 + \frac{c}{n}\right)^n \rightarrow e^c$

$\sqrt[n]{n} \rightarrow 1$

$\frac{1}{n} \rightarrow 0$

Cauchy: $\forall \varepsilon \; \exists N \in \mathbb{N}_0: |a_n - a_m| < \varepsilon \; \forall n, m \ge N$

$(a_n) \xrightarrow{n \to \infty} a, (b_n) \xrightarrow{n \to \infty} b$:

$(a_n + b_n) \rightarrow a + b$; $(a_n b_n) \rightarrow ab$

- Konv: $\forall \varepsilon > 0 \; \exists N \in \mathbb{N}_0$: $|a_n - a| < \varepsilon \; \forall n \ge N$

Reihen: $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$; $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$; $|q| < 1$; $\sum_{n=1}^{\infty} \frac{1}{n^\alpha} = \begin{cases} \text{konv.} & \alpha > 1 \\ \text{div.} & \alpha \le 1 \end{cases}$

Kriterien: – Nullfolge: $\sum a_n$ diverg, falls $a_n \not\to 0$
- Minorante $\sum a_n$: $a_n > b_n$, $b_n \to \infty$
- Majorante $\sum a_n$: $a_n < b_n$; $b_n \to b$
- Quotient: $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$ – Wurzel: $\lim_{n \to \infty} \sqrt[n]{a_n}$ $\begin{cases} < 1 & \text{konvergiert absolut} \\ > 1 & \text{divergiert} \\ = 1 & \text{keine Aussage} \end{cases}$

Potenzreihen $f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n$

$R = \lim_{n \to \infty} \left|\frac{a_n}{a_{n+1}}\right| / \lim_{n \to \infty} \frac{1}{\sqrt[n]{a_n}} / \frac{1}{\lim\sup \sqrt[n]{a_n}}$ $\begin{cases} |x - c| < R & \text{konv abs.} \\ |x - c| > R & \text{divergiert} \\ |x - c| = R & \text{keine Auss.} \end{cases}$

→ bei $x = c - R \,|\, c + R$ konvergenz überprüfen

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$

Integration • **Partiell:** $\int uv' = uv - \int u'v$

$\int \lambda f(x) + \mu g(x) = \lambda \int f(x) + \mu \int g(x)$

$\int_1^{\infty} \frac{1}{x^\alpha} \begin{cases} \frac{1}{\alpha - 1}, & \alpha > 1 \\ \infty, & \alpha \le 1 \end{cases}$; $\int_0^1 \frac{1}{x^\alpha} \begin{cases} \frac{1}{\alpha - 1}, & \alpha < 1 \\ \infty, & \alpha \ge 1 \end{cases}$

F(x)	f(x)	f'(x)
$\frac{2\sqrt{ax^3}}{3}$	$\sqrt{ax}$	$a/2\sqrt{ax}$
$x\ln(ax) - x$	$\ln(ax)$	$\frac{1}{x}$
$a^x/\ln(a)$	a^x	$a^x \ln(a)$
$-\ln\lvert\cos x\rvert$	$\tan(x)$	$1/\cos^2 x$
$\ln\lvert\sin x\rvert$	$\cot(x)$	$-1/\sin^2 x$
$x\arcsin(x) + \sqrt{1 - x^2}$	$\arcsin(x)$	$\frac{1}{\sqrt{1 - x^2}}$
$x\arccos(x) - \sqrt{1 - x^2}$	$\arccos(x)$	$-\frac{1}{\sqrt{1 - x^2}}$
$x\arctan(x) - \frac{1}{2}\ln\lvert 1 + x^2\rvert$	$\arctan(x)$	$\frac{1}{1 + x^2}$
$x\,\text{arccot}(x) + \frac{1}{2}\ln\lvert 1 + x^2\rvert$	$\text{arccot}(x)$	$-\frac{1}{1 + x^2}$
$\sinh(x)$	$\cosh(x)$	$\sinh(x)$
$\cosh(x)$	$\sinh(x)$	$\cosh(x)$

Integration rationaler f.:

• $\int \frac{1}{x} = \ln|x|$

• $\int \frac{1}{(x - a)^n} \begin{cases} \ln|x - a|, & n = 1 \\ \frac{-1}{(n-1)(x-a)^{n-1}}, & n \ge 2 \end{cases}$

• $\int \frac{1}{x^2} = -\frac{1}{x}$

• $\int \frac{1}{(a + x)^2} = -\frac{1}{a + x}$

• $\int \frac{1}{a - x} = -\ln|a - x|$

Partialbruchzerlegung:

$\frac{A}{x - x_0} + \frac{B}{(x - x_0)^2} + \ldots + \frac{Gx + H}{(x^2 + px + q)} + \frac{Ix + J}{(x^2 + px + q)^2} \ldots$

Sonstige Sachen:

Indize Versch: $\sum_{i=1}^{n} a^{i-1} \rightarrow \sum_{j=0}^{n-1} a^j$

$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

Binomisch. Lehrsatz: $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}} \ge \frac{1}{4n}$

Max/Min: in I enthalten

Sup/Inf: kleinste/größte Grenze

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\left(e^{ix}\right)^n \rightarrow 1$

$\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} = 0$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Taylor-Entwicklung

$T_m(x_0, x) = \sum_{i=0}^{m} \frac{f^{(i)}(x_0)}{i!} (x-x^0)^i$

Restglied

$R_{m+1}(x) := f(x) - T_{m,f}(x_0; x) = \frac{f^{(m+1)}(\xi)}{(m+1)!} \cdot (x-x_0)^{m+1}$

Fehlerabschätzung: ξ, so dass R_{m+1} maximal wird.

Landau

$f(x) = o(g(x)),\ x \to a \iff \lim_{x \to a} \frac{f(x)}{g(x)} = 0$

$f(x) = O(g(x)),\ x \to a \iff \frac{|f(x)|}{|g(x)|} \leq C$

$f = Of$

$f = og \Rightarrow f = Og$

$f_1 = o(g_1) \wedge f_2 = o(g_1) \Rightarrow f_1 + f_2 = o(g_1)$

$f_1 = Og \wedge f_2 = Og \Rightarrow f_1 + f_2 = Og$

$f_1 = Og_1 \wedge f_2 = Og_2 \Rightarrow f_1 \cdot f_2 = O_{g_1 g_2}$

$f_1 = og \wedge f_2 = og \Rightarrow f_1 \cdot f_2 = o_{g_1 \cdot g_2}$

$e^x = \sum_{k=0}^{m} \frac{x^k}{k!} + O(x^{m+1})$; $\sin x = \sum_{k=0}^{m} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + O(x^{2m+3})$

$\cos x = \sum_{k=0}^{m} (-1)^k \frac{x^{2k}}{(2k)!} + O(x^{2m+2})$

Kurven

$\gamma : [a,b] \to \mathbb{R},\ t \mapsto \begin{pmatrix} \gamma_1(t) \\ \gamma_k(t) \end{pmatrix}$

- Bogenlänge: $L(\gamma) = \int_a^b \|\gamma'(t)\| \, dt$

Umparam: $s(t) = \int_a^t \|\gamma'(t)\| dt$

$\bar{\gamma} = \gamma(s^{-1}(t))$