Discrete and Continuous-Time Signals

ECSE 313 Group 16 Lab 1 Report

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Write an introduction here

1 Results

1.1 Continuous-Time Vs. Discrete-Time

Compute the integral of:

$$\int_0^{2\pi} \sin^2(5t)\,dt$$

$$\sin^2(5t) = \frac{1-\cos(10t)}{2}$$

$$\int_0^{2\pi} \frac{1-\cos(10t)}{2}\,dt = \int_0^{2\pi} \frac{1}{2}\,dt - \int_0^{2\pi} \frac{\cos(10t)}{2}\,dt$$

$$= \frac{t}{2}\Big|_0^{2\pi} - \frac{\sin(10t)}{20}\Big|_0^{2\pi}$$

$$= [\frac{2\pi}{2} - 0] - [\frac{\sin(20\pi)}{20} - \frac{\sin(0)}{20}] = \pi$$

Compute the integral of:

$$\int_0^1 e^t\,dt$$

$$= e^t\big|_0^1 = [e^1 - e^0] = e - 1$$

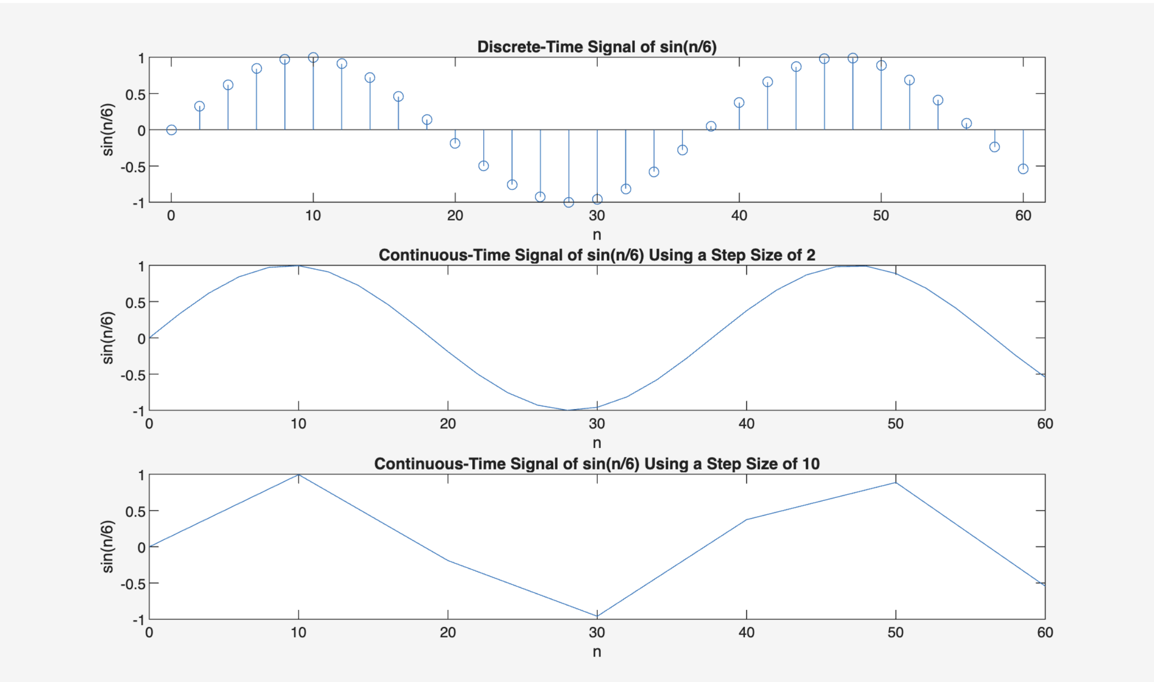

Figure 1: Discrete Vs. Continuous-Time of sin(n/6)

In Figure 1, the first continuous plot with a step size of 2 presents a smooth, continuous-looking representation of the graph sin(n/6). The second continuous plot, however, with a step size of 10 is more rigid and more closely resembles a triangular function. This observation is consistent with the mathematical analyses of sin(n/6), where the angular frequency is $w = 1/6$ and the period is $T = (2\pi)/w = (2\pi)/(1/6) = 12\pi$. Moreover, the first continuous-time signal has $(12\pi)/2 \approx 18.8$ samples per period whereas the second continuous-time signal has $(12\pi)/10 \approx 3.77$ samples per period. Therefore, since the first continuous-time signal has more samples per period, the function is more accurate to the sin(n/6) function.

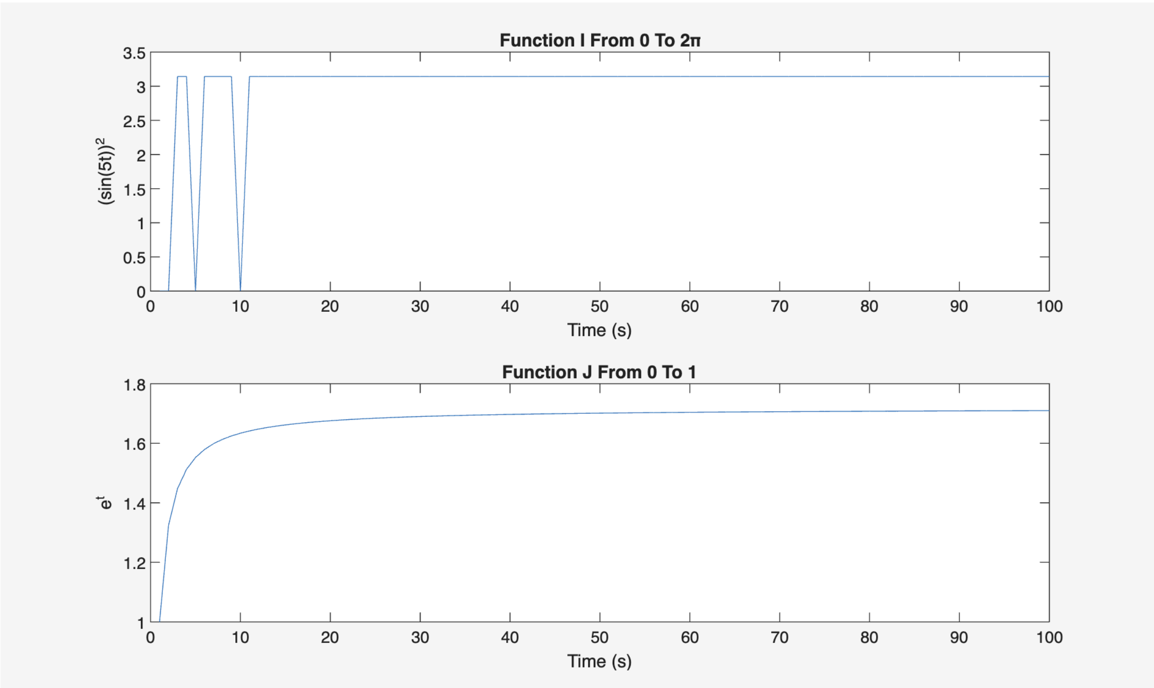

Figure 2: Plotted Graphs of $I = (sin(5t))^2$ and $J = e^t$ Functions

The graphs from Figure 2 resemble the calculations of $(sin(5t))^2$ and e^t made earlier in this section. As t increases, I levels at about $\pi \approx 3.14$ and J levels at about $e - 1 \approx 1.72$.

One point of interest for the function I is the observation that $I(5) = I(10) = 0$. The reason for this is that when $t = 5$ and $t = 10$, the sine function argument is a multiple of π, and therefore, the result is 0. Mathematicaly, when $t = 5$, Δt and the step size is $(2\pi - 0)/5 = (2\pi)/5$. Since the function starts at $t = 0$, the next t value will be $2\pi/5$, and t will continue to increment by $2\pi/5$. Furthermore, multiplying these t values with the sine function argument 5t, will have t increment by $5 * ((2\pi)/5) = 2\pi$, which is a multiple of π. Therefore, at $t = 5$, the I function is 0.

Similarly, at $t = 10$, Δt is $(2\pi - 0)/10 = \pi/5$. Multiplying this value with the function argument yields a result of $5 * (\pi)/5 = \pi$. Therefore, at $t = 10$, the J function is 0.

1.2 Processing of Speech Signals

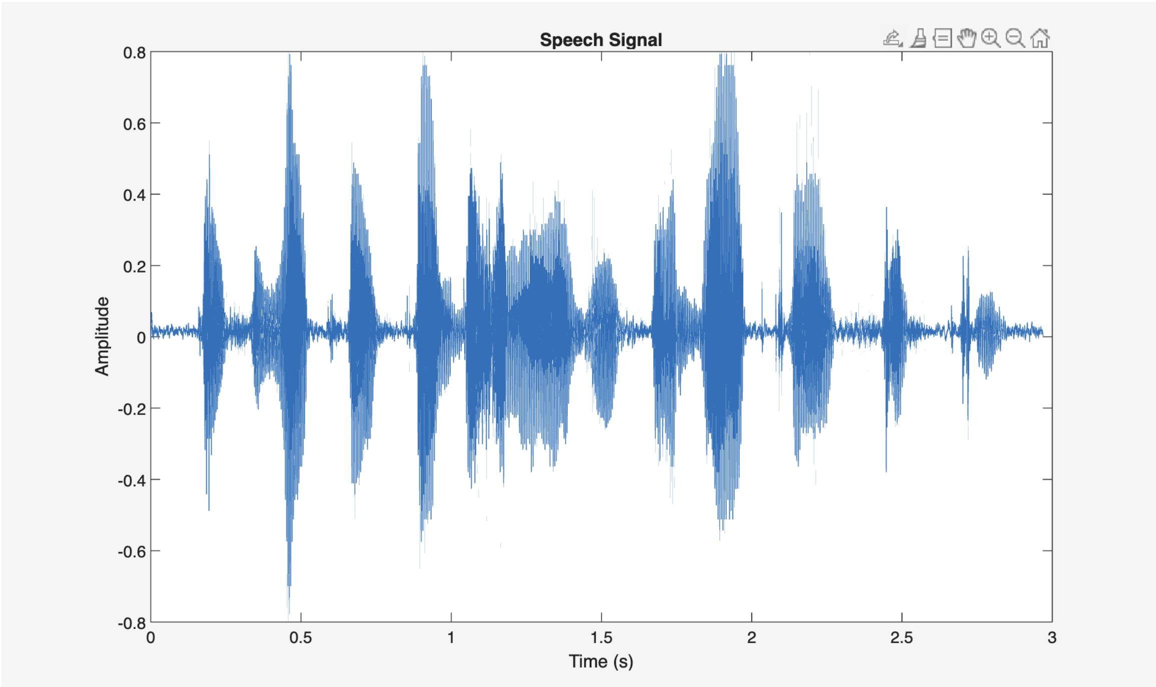

Figure 3: Speech Signal

Figure 3 represents the audio signal from the provided speech.au file. The file plays the sentence "This is a test of the emergency broadcast system."

1.3 Attributes of Continuous-Time Signals

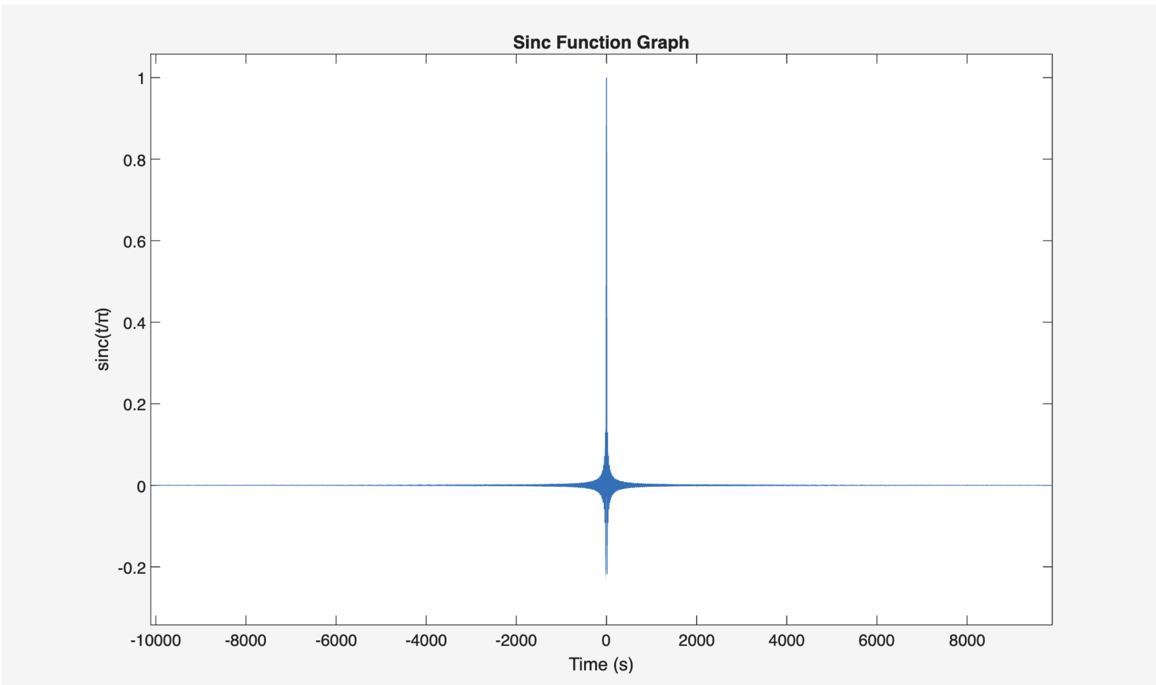

Figure 4: Sinc Function Graph

The computed values of Figure 4 are as such:

Minimum of sinc(t) signal: -0.21723
Maximum of sinc(t) signal: 0.99998
Energy of the sinc(t) signal: 3.14149

The sampling period, start time, and end time choices followed the logic where the start and end times are large values and the sample period is small. The chosen start time is -10000 and the end time is 10000. The reason for the large sample size is to accurately measure energy with more values. The chosen sample period is small at 0.01 to make the plotted function accurately mimic the sinc function, as found in Section 1.3. The small sample period logic was found in Secion 2.1 of this lab, and it made the maximum and minimum values more accurate.

find_energy.m

```
% Purpose: Find the energy of a sinc function
% Input (t): time
% Output (result): energy of a sinc function in given t range
function result = find_energy(t)
    dt = t(2) - t(1);
    func = sinc(t);
    func(isnan(func)) = 1; % Ensure no division by zero errors
    result = sum(abs(func).^2)*dt;
end
```

1.4 Special Functions

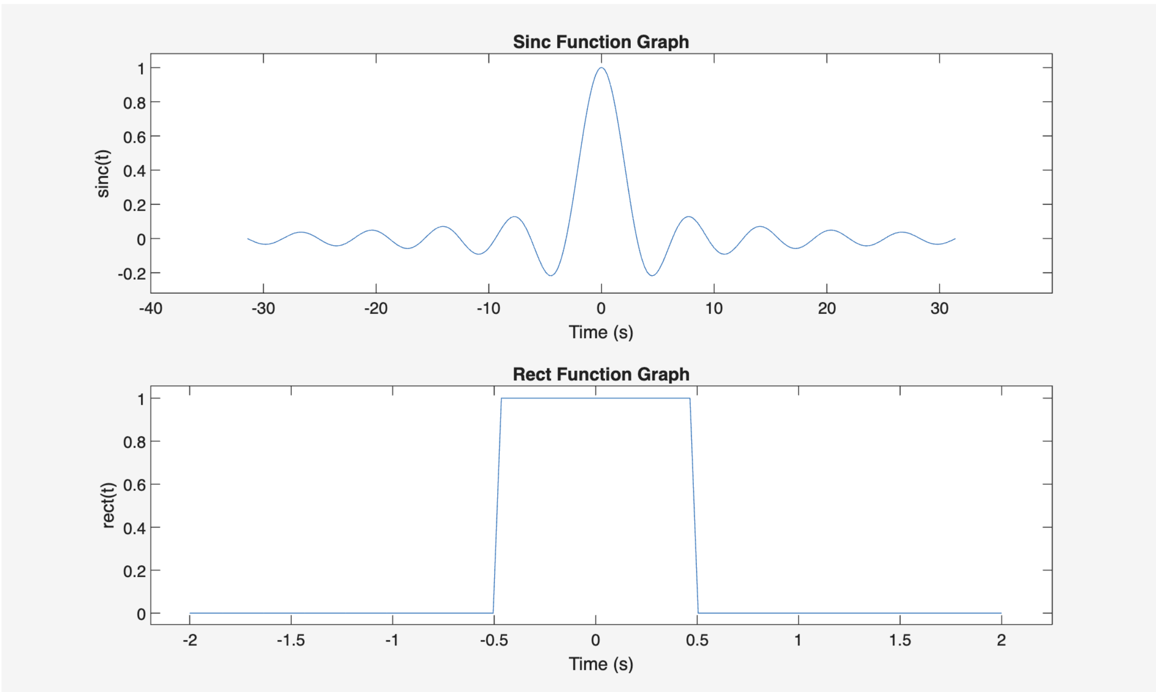

Figure 5: Plotted Graphs of the sinc(t) and rect(t) Functions

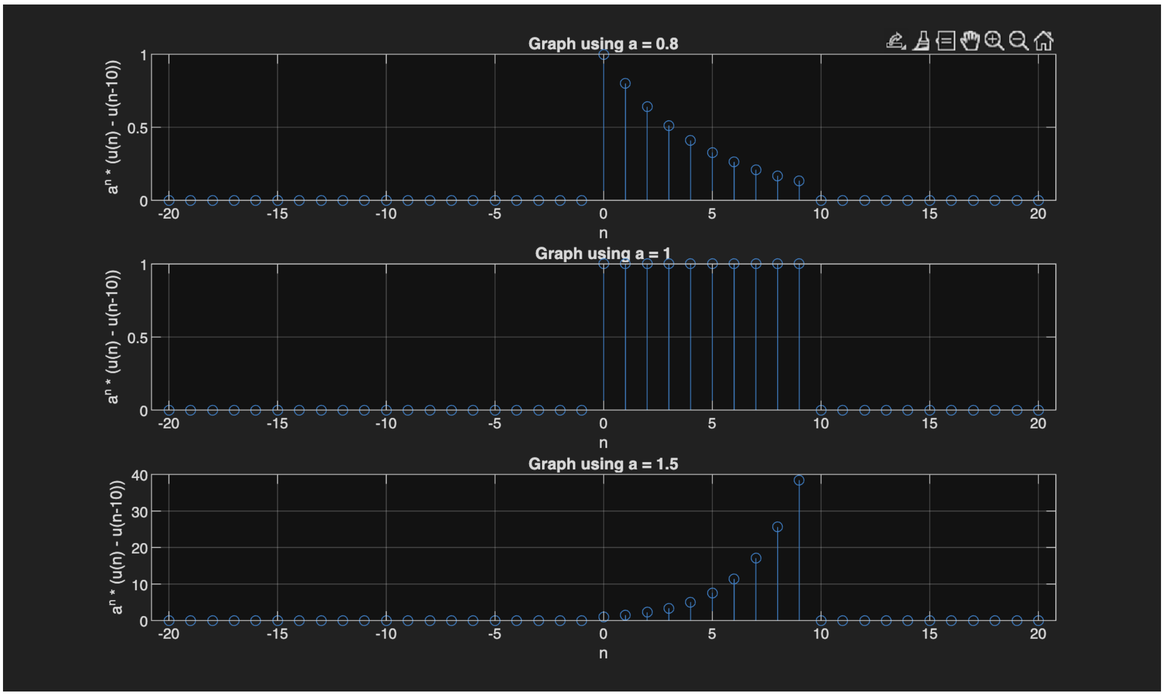

Figure 6: Stem Graphs of an Exponential Function With Different 'a' Values

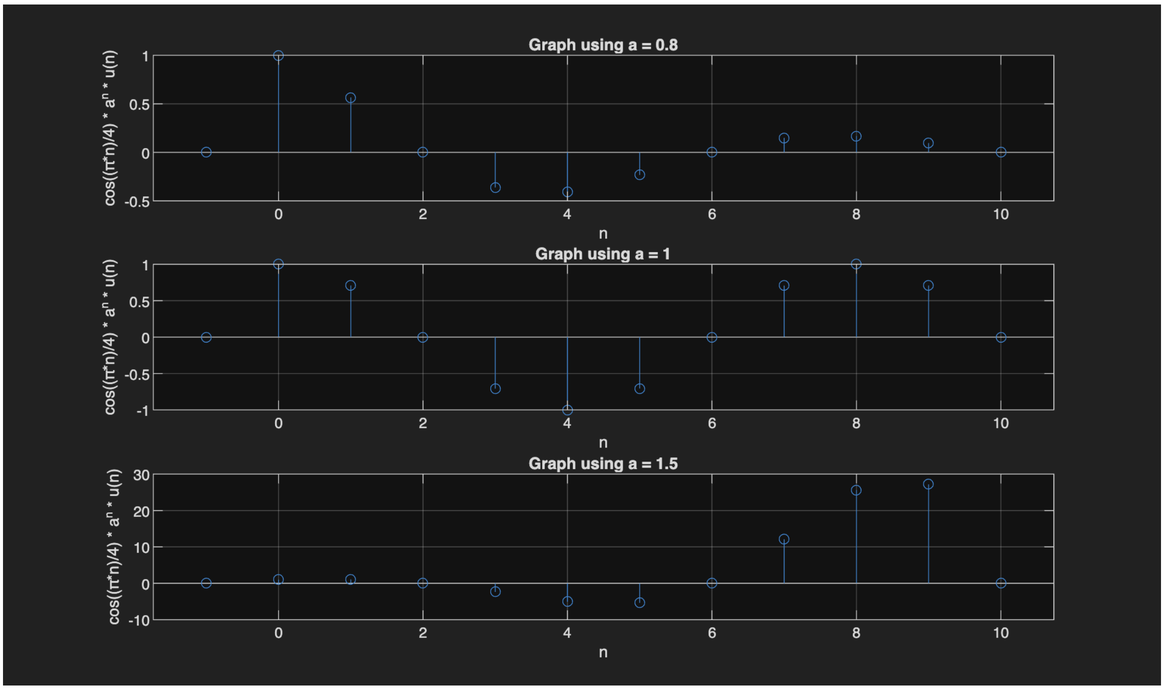

Figure 7: Stem Graphs of a Cosine Function With Different 'a' Values

main2_6.m

```
clc; clear; close all;

% First figure of sinc and rect
figure(1);

% First plot of sinc(t) from -10pi to 10pi
subplot(2, 1, 1);
t1 = linspace(-10*pi, 10*pi, 1000);
y1 = sinc(t1);
```

```
plot(t1, y1);
xlabel("Time (s)");
ylabel("sinc(t)");
title("Sinc Function Graph");

% Second plot of rect(t) from -2 to 2
subplot(2, 1, 2);
t2 = linspace(-2, 2, 100);
y2 = (abs(t2) <= 0.5);
plot(t2, y2);
xlabel("Time (s)");
ylabel("rect(t)");
title("Rect Function Graph");

% Second figure with unit step
figure(2);
orient('tall');
a_vals = [0.8, 1.0, 1.5];
n = -20:20;
u_n = (n >= 0);
u_n_delay = (n >= 10);

% a^n (u(n) - u(n-10))
for i = 1:length(a_vals)
    a = a_vals(i);
    y = (a.^n) .* (u_n - u_n_delay);
    subplot(3, 1, i);
    stem(n, y);
    grid on;

    xlabel("n");
    ylabel("a^n * (u(n) - u(n-10))");
    title(['Graph␣using␣a␣=␣', num2str(a)]);
end


% Third figure with cos
figure(3);
n = -1:10;
u_n = (n >= 0);

% cos((pi/4)n) * a^n * u(n)
for i = 1:length(a_vals)
    a = a_vals(i);
    y = cos((pi / 4) * n) .* (a.^n) .* u_n;
    subplot(3, 1, i);
    stem(n, y);
    grid on;

    xlabel("n");
    ylabel("cos((pi*n)/4) * a^n * u(n)");
    title(['Graph␣using␣a␣=␣', num2str(a)]);
end
```

Since the sinc function on matlab is within the Signal Processing Toolbox, we created our own sinc function:

sinc.m

```
% Purpose: Create the sinc function, which is sin(x)/x
% Input (x): x value of sinc function
% Output (y): y value of sinc function
function y = sinc(x)
    y = sin(x) ./ x;
end
```

1.5 Sampling

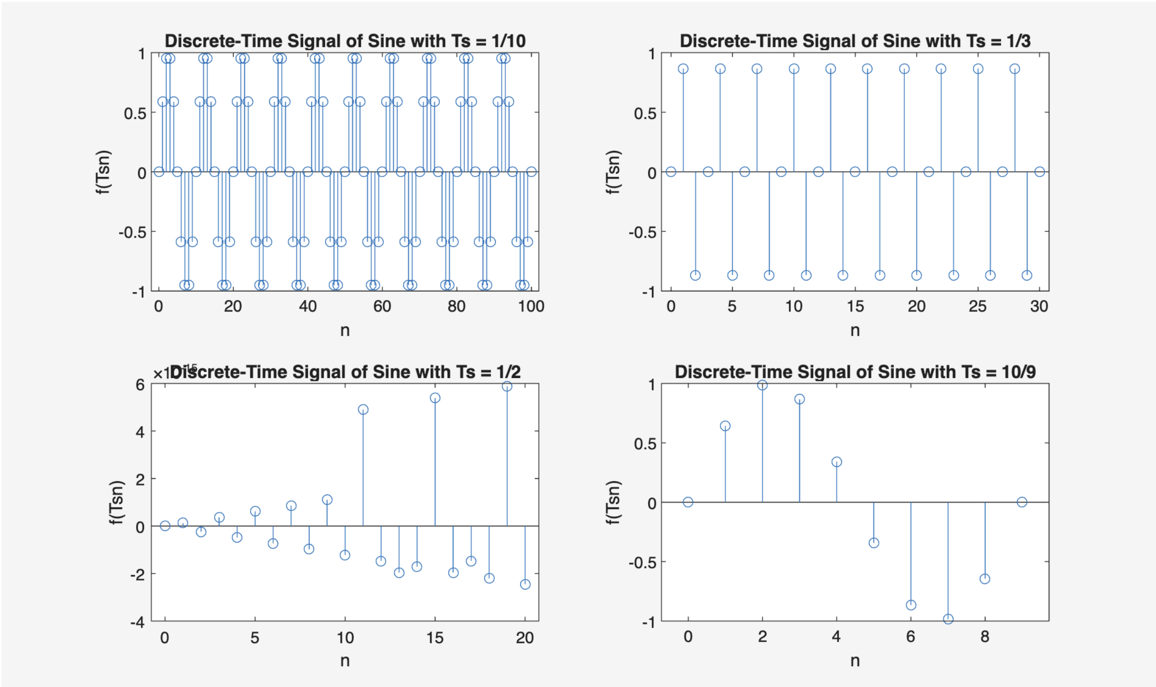

Figure 8: Discrete-Time Signal of Sine Function With Varying Periods

With the exception of the plot where $T_s = 1/2$, each plot shows characteristics of a sine wave. However, to validate the plots we will analyze how we are sampling these functions.

The Nyquist-Shannon sampling theorem states that a continuous signal can be perfectly reconstructed with the equation: $F > 2 * f_{max}$, where F is the sampling rate and f_{max} is the maximum frequency. The f_{max} of the function $sin(2\pi * T_s * n)$ is 1 Hz, so the F of each function must be greater than 2 Hz.

The frequencies for all the plots where $T_s = 1/10$, $T_s = 1/3$, $T_s = 1/2$, and $T_s = 10/9$ are $F = 10$ Hz, $F = 3$ Hz, $F = 2$ Hz, and $F = 9/10$ Hz respectively. Therfore, the bottom two plots of Figure 8 experience an aliasing effect because $1/2 < 2$ and $10/9 < 2$ and are not true sine waves.

The sampled version of the signal with $T_s = 1/10$ is oversampled compared to the other sampled versions of the signal because the plot clearly outlines the sine function

1.6 Random Signals

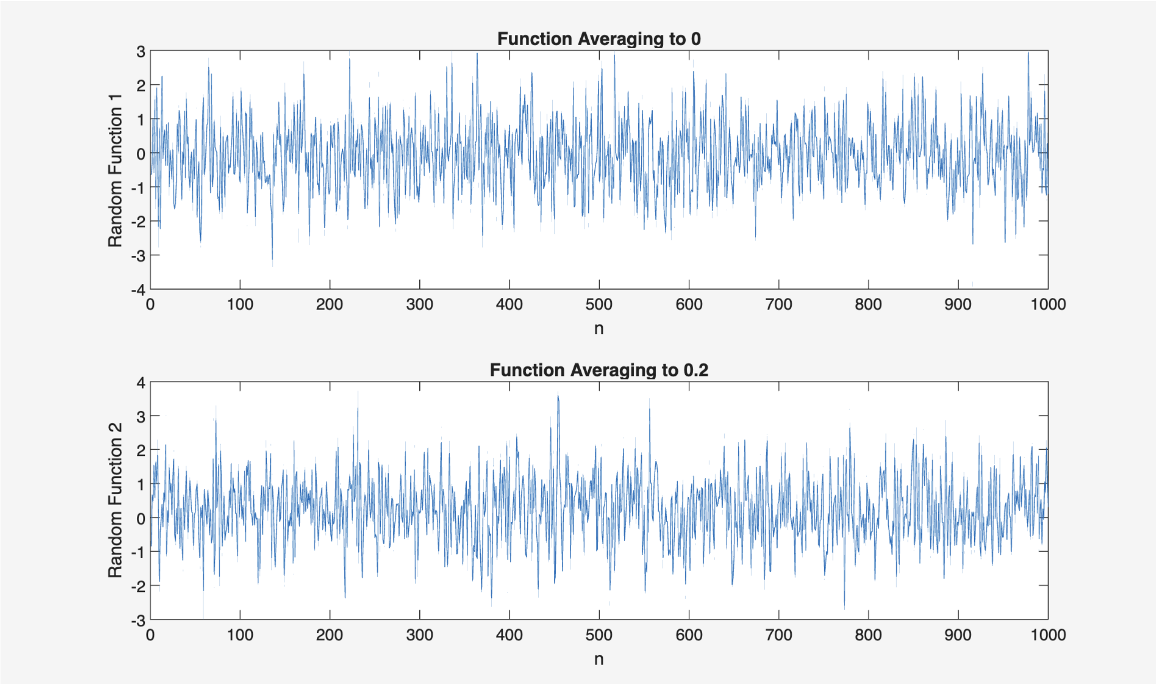

Figure 9: Randomly Generated Functions Averaging to 0 (Plot 1) or 0.2 (Plot 2)

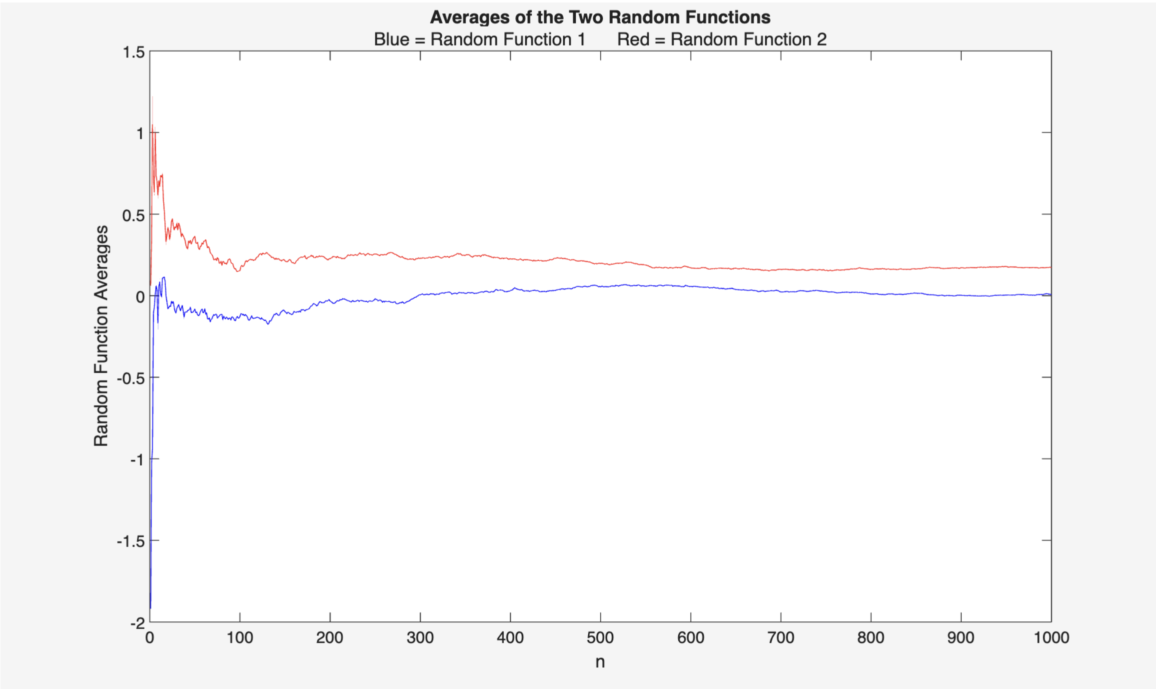

Figure 10: Averages of the Plots in Figure 9

In Figure 10, as n increases, the averages of the random functions smooth to their respective values. Moreover, the blue curve modeling the first plot in Figure 9 levels out to about 0 as n increases, and the red curve modeling the second plot in Figure 9 levels out to about 0.2 as n increases. On the other hand, at small values of n, the average of the two curves fluctuates significantly.

Although the raw images of the two plots from Figure 9 look near identical, the results from Figure 10 clearly outlines their differences in averages.

1.7 2-D Signals

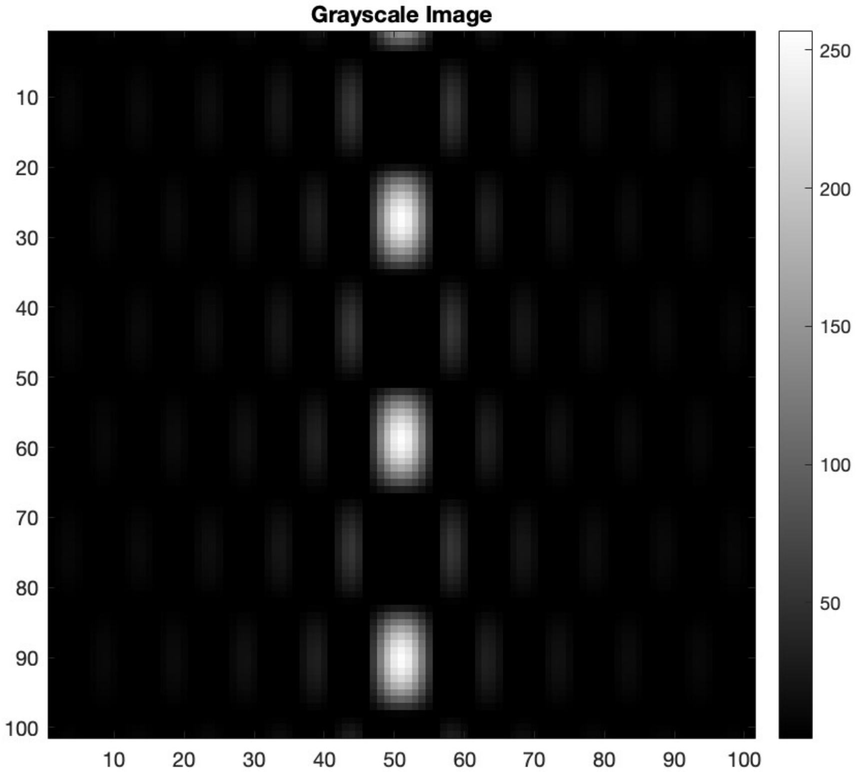

Figure 11: Grayscale Image of 2-D Image

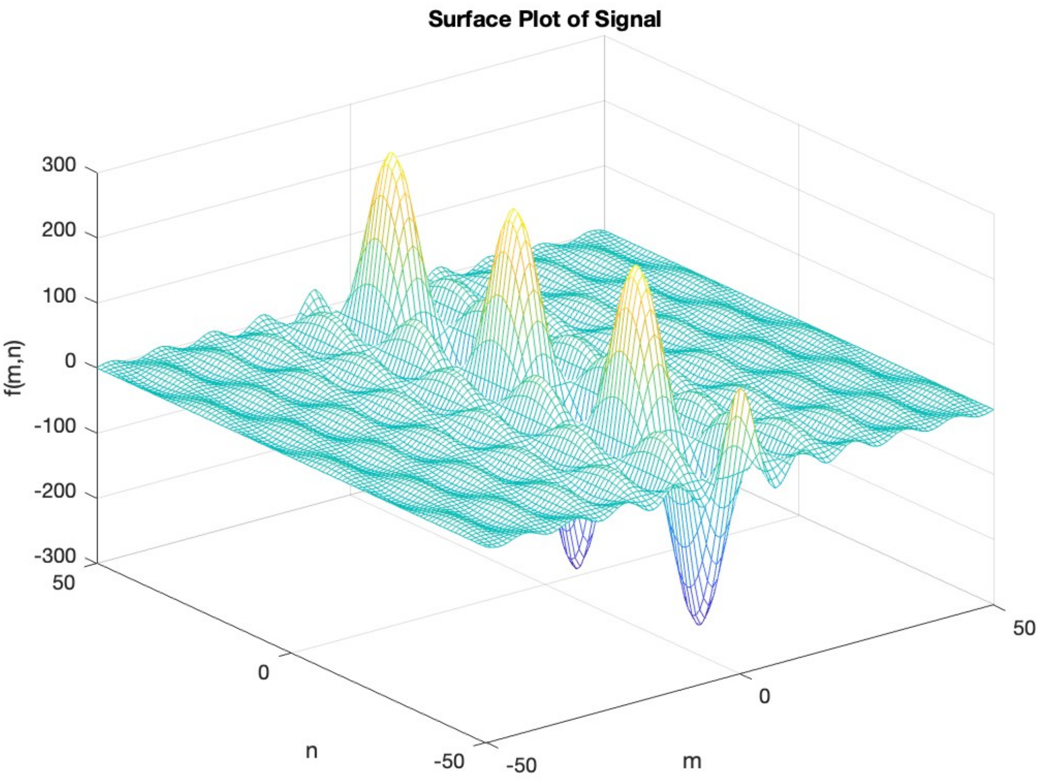

Figure 12: Surface Plot of 2-D Image

The surface plot allows for closer inspection of the 3d structure and characteristics like smoothness and bandwidth. Furthermore, it allows for easy comparison between seperate but closely related signals. On the other hand, the image plot allows for quick pattern detection and is easier to display as a 2d matrix.