Question 1 **Write up carefully argued solutions to the following problems.**

(a)

Determine if the relations are reflexive, symmetric, anti-symmetric, and/or transitive.

(a)

Prove that given R and S are transitive, $R \cap S$ is transitive.

(a) Unrolling, we get...

$(a, b) \in R \cap S \land (b, c) \in R \cap S \to (a, c) \in R \cap S$

(b)

Proof.

1. $(a, b) \in R \land (b, c) \in R \to (a, c) \in R$ (Given)
2. $(a, b) \in S \land (b, c) \in S \to (a, c) \in S$ (Given)

   3.1. $(a, b) \in R \cap S \land (b, c) \in R \cap S$ (Assumption)
   3.2. $(a, b) \in R \cap S$ (Elim $\land$: 1.1)
   3.3. $(b, c) \in R \cap S$ (Elim $\land$: 1.1)
   3.4. $(a, b) \in R \land (a, b) \in S$ (Def of Union: 1.2)
   3.5. $(a, b) \in R$ (Elim $\land$ 1.4)
   3.6. $(a, b) \in S$ (Elim $\land$ 1.4)
   3.7. $(b, c) \in R \land (b, c) \in S$ (Def of Union: 1.3)
   3.8. $(b, c) \in R$ (Elim $\land$ 1.7)
   3.9. $(b, c) \in S$ (Elim $\land$ 1.7)
   3.10. $(a, b) \in R \land (b, c) \in R$ (Intro $\land$: $1.5, 1.8$)
   3.11. $(a, b) \in S \land (b, c) \in S$ (Intro $\land$: $1.6, 1.9$)
   3.12. $(a, c) \in R$ (Modus Ponens: $1, 1.10$)
   3.13. $(a, c) \in S$ (Modus Ponens: $2, 1.11$)
   3.14. $(a, c) \in R \cap S$ (Def of Union)

3. $(a, b) \in R \cap S \land (b, c) \in R \cap S \to (a, c) \in R \cap S$ (Direct Proof)

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