Russel vs Tarski Universes

Donovan Crichton

August 2022

Preliminaries

- Slides and Examples available at:
  https://github.com/donovancrichton/ANU-TT
- This talk: /RusselVsTarskiUniverses

Origin - Per Martin-Löf

[Martin-Löf and Sambin, 1984, p. 87-91]

Transfinite Types

- MLTT has `Type : Type`.
- Martin-Löf introduces Universes to allow transfinite type formers.
- Without Universes we have Girard's Paradox Girard [1972].

Unsoundness in Idris from Type : Type

- Hurken provides us with a simpler proof based around Burali-Forti's Paradox Hurkens [1995].
- Apparently this encoding represents well-orderings/well-foundedness... I don't see it!
- Let's see the code.

References

J.-Y. Girard. *Interprétation fonctionnelle et élimination des coupures de l'arithmétique d'ordre supérieur*. PhD thesis, Éditeur inconnu, 1972.

A. J. Hurkens. A simplification of girard's paradox. In *International Conference on Typed Lambda Calculi and Applications*, pages 266–278. Springer, 1995.

P. Martin-Löf and G. Sambin. *Intuitionistic type theory*, volume 9. Bibliopolis Naples, 1984.