

# Russel vs Tarski Universes

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# Preliminaries

- ▶ Slides and Examples available at:  
<https://github.com/donovancrichton/ANU-TT>
- ▶ This talk: /RusselVsTarskiUniverses

# Origin - Per Martin-Löf

[[Martin-Löf and Sambin, 1984](#), p. 87-91]



# Transfinite Types

- ▶ MLTT has  $\text{Type} : \text{Type}$ .
- ▶ Martin-Löf introduces Universes to allow transfinite type formers.
- ▶ Without Universes we have Girard's Paradox [Girard \[1972\]](#).

# Unsoundness in Idris from `Type : Type`

- ▶ Hurken provides us with a simpler proof based around Burali-Forti's Paradox [Hurkens \[1995\]](#).
- ▶ Apparently this encoding represents well-orderings/well-foundedness... I don't see it!
- ▶ Let's see the [code](#).

# References

- J.-Y. Girard. *Interprétation fonctionnelle et élimination des coupures de l'arithmétique d'ordre supérieur*. PhD thesis, Éditeur inconnu, 1972.
- A. J. Hurkens. A simplification of girard's paradox. In *International Conference on Typed Lambda Calculi and Applications*, pages 266–278. Springer, 1995.
- P. Martin-Löf and G. Sambin. *Intuitionistic type theory*, volume 9. Bibliopolis Naples, 1984.