Lambda Calculi With Explicit Substitutions

slides: https://github.com/donovancrichton/Talks

Donovan Crichton

Februrary 2025

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The goal of this talk

- How to read (theory) syntax.
- STLC - Simply Typed Lambda Calculus.
- Substituions in anger - De-Bruijn Indicies.
- Explicit Subsitutions.
- Explicit Substitutions in modern type theory.

The goal of this talk

To introduce and motivate underlying concepts, and applications, of explicit substitutions.
- How to read (theory) syntax.
- STLC - Simply Typed Lambda Calculus.
- Substituions in anger - De-Bruijn Indicies.
- Explicit Subsitutions.
- Explicit Substitutions in modern type theory.

Idea: Explicit Subsitutions bridge the gap between theory and implementation for substitution.

Reading Syntax: Grammar

Natural Numbers

$$\mathbb{N} ::= Z \mid S\,\mathbb{N}$$

Reading Syntax: Grammar
Natural Numbers
$\mathbb{N} ::= Z \mid S\,\mathbb{N}$

Reading Syntax: Grammar

Our <symbol> name...

Natural Numbers

$$\boxed{\mathbb{N}} ::= Z \mid S\,\mathbb{N}$$

Reading Syntax: Grammar
Our <symbol> name...
Natural Numbers
$\boxed{\mathbb{N}} ::= Z \mid S\,\mathbb{N}$

Reading Syntax: Grammar

...is defined in the following ways:

Natural Numbers

$$\mathbb{N} ::= Z \mid S\,\mathbb{N}$$

Reading Syntax: Grammar
...is defined in the following ways:
Natural Numbers
$\mathbb{N} ::= Z \mid S\,\mathbb{N}$

Reading Syntax: Grammar

The letter Z by itself.

Natural Numbers

$$\mathbb{N} ::= \boxed{Z} \mid S\,\mathbb{N}$$

Reading Syntax: Grammar
The letter Z by itself.
Natural Numbers
ℕ ::= Z | S ℕ

Reading Syntax: Grammar

...or...

Natural Numbers

$$\mathbb{N} ::= Z \mid S\,\mathbb{N}$$

Reading Syntax: Grammar
...or...
Natural Numbers
$\mathbb{N} ::= Z \mid S\,\mathbb{N}$

Reading Syntax: Grammar

The letter S followed by a space, followed by any $\mathbb{N}$.

Natural Numbers

$$\mathbb{N} ::= Z \mid \boxed{S\,\mathbb{N}}$$

Reading Syntax: Grammar
The letter S followed by a space, followed by any ℕ.
Natural Numbers
ℕ ::= Z | S ℕ

Examples of Grammar

The Natural Number 3

$$3 = S(S(SZ))$$

Examples of Grammar
The Natural Number 3
3 = S(S(SZ))

Examples of Grammar

The Natural Number 3

$$S(S(SZ))$$

Natural Numbers in Haskell

```haskell
data Nat = Z | S Nat

three :: Nat
three = S (S (S Z))
```


Examples of Grammar
The Natural Number 3
$S(S(SZ))$
Natural Numbers in Haskell
data Nat = Z | S Nat
three :: Nat
three = S (S (S Z))

Reading Syntax: Typing Rules

Reading Syntax: Typing Rules

Examples of Typing Rules

Examples of Typing Rules

STLC: The Simply Typed Lambda Caluclus

STLC: The Simply Typed Lambda Caluclus

STLC 2

Substitution and Computation

Substitution and Computation

Alpha Equivalence, Free Variables, Beta Reduction

Alpha Equivalence, Free Variables, Beta Reduction

De-Bruijn Indices

De-Bruijn Indices

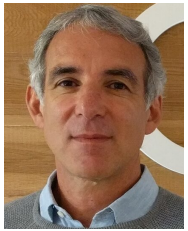
Lift-and-shift

Lift-and-shift

A lot of work at the meta-level

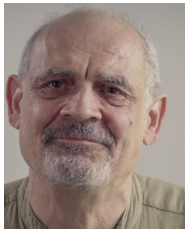
A lot of work at the meta-level

Explicit Subsitutions (Paper)



Martin Abadi

Luca Cardelli



Pierre-Louis Curien

Jean-Jacques Levy

Explicit Subsitutions (Paper)
Martin Abadi Luca Cardelli Pierre-Louis Curien Jean-Jacques Levy

Extending the STLC with Explicit Substitutions

Syntax cont.

Syntax cont.

ES rules.

ES rules.

Evaluating ES

Evaluating ES 2

Evaluating ES 2

References

References