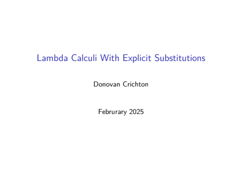
Lambda Calculi With Explicit Substitutions

Donovan Crichton

Februrary 2025

2025-01-30



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Preliminaries

- Slides and Examples available at: https://github.com/donovancrichton/Talks
- This talk: BFPG/LambdaCalculiWithExplicitSubstituions

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Welcome to the talk!

About me

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Griffith UNIVERSITY

ASD AUSTRALIAN SIGNALS DIRECTORATE

- PhD Candidate
- Computing Foundations
- School of Computing
- Visiting Scholar
- Trusted Systems Lab
- IIIS
- ASD Co-Lab Scholar

└About me

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A test definition for some concept.

This is a test example.

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The Identity Function

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f : a -> a
f x = x
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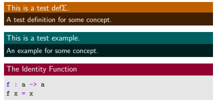

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Untyped Lambda Calculus Syntax

1. A quick refresher on the untyped lambda calculus
2. Smallest turing-complete language.
3. First we need a set of variables.
4. Grammar/Syntax has 3 terms.
5. Looks scary? You can read this already, clearly inspires data declarations in ML languages

Untyped Lambda Calculus Syntax

Our set of variables

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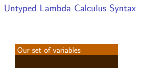

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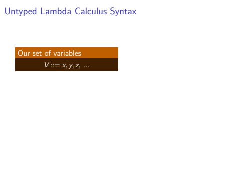
Our set of variables

$$V ::= x, y, z, \ \ldots$$



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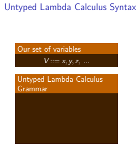
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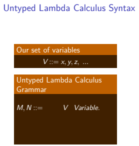
$$M, N ::= \qquad V \quad \textit{Variable.}$$



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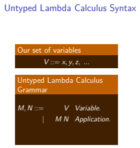
$$\begin{array}{rcll} M, N ::= & & V & \textit{Variable.} \\ & \mid & M\ N & \textit{Application.} \end{array}$$



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$M, N ::= V$ *Variable.* $\mid M\ N$ *Application.*

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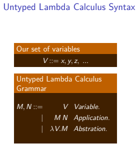

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Code Tie-In

```
-- Our 'set' of variables.
V : Type
V = String

-- Our Lambda (Λ)
-- Calculus Syntax.
data Λ = Var V
       | App Λ Λ
       | Abs V Λ
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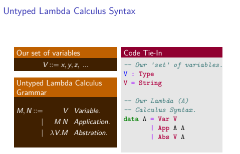

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Untyped Lambda Calculus Computation

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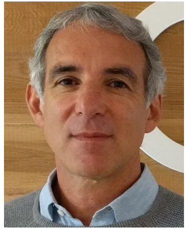
The Problem

Implementation GapFormal Reasoning

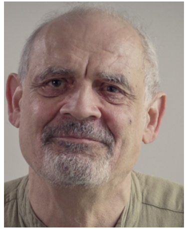
There is usually a non-trivial disconnect between how lambda calculus is presented in mathematics, to how it is implemented in a programming language. Substitution in traditional presentations of lambda calculus is a meta operation, which makes formal reasoning about it difficult.

Explicit Subsitutions (Paper)



Martin Abadi

Luca Cardelli



Pierre-Louis Curien

Jean-Jacques Levy

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└Explicit Subsitutions (Paper)

References