Towards Verified Time Balancing of Security Protocols

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Motivation

- ASD manually verifies vendor code with containing cryptographic processes.
- Formal Methods: A mathematically based approach to the specification and verification of software.
- Can we reduce some of the resources ASD spends on manual verification by replacing with automatic verification?
- Can we also further research into secure communications protocols?
- A case study.

Formally Verifying a Time Balanced Security Protocol

- Attackers can gain information from message timing.
- Can we model a time-invariant protocol?
- A naive approach considers all operations have the same running time.
- Can we ensure that assumptions on this model hold for the implementation?

The ZRTP Protocol

- Initially started with ZRTP.
- ZRTP is complex and makes many decisions.
- Simplified version that contains just enough detail to allow us to attempt to prove some interesting things!

The Simplified Protocol

- Commit messages that contain SHA-256 hashes
- Diffie-Hellman key exchange contains modulo arithmetic.
- How can we formally guarantee the timing of operations?
- Lets cover some background before diving in!

Approach

- We can use the notion of propositions as types.
- Functional programming languages that support dependent types can act as theorem provers under a higher order constructive logic.
- We can model this protocol in such a language, Idris.
- We can express proofs about properties of the protocol.

Quick Background 1 - Currying

All functions treated as taking a single argument.

$f : \mathbb{N} \rightarrow (\mathbb{N} \rightarrow \mathbb{N})$
$f = +$

Applying an argument to a multi argument function returns the rest of the function! (Arrow associates to the right)

$f(2) : \mathbb{N} \rightarrow \mathbb{N}$
$f(2) = 2+$

Finally all arguments are applied.

$f(2, 3) : \mathbb{N}$
$f(2, 3) = 2 + 3 = 5$

Quick Background 2 - Idris Syntax and Values as Types

Building a vector type in Idris:

```
data Vec : Nat -> Type -> Type where
  Nil  : Vec 0 a
  (::) : (x : a) -> Vec n a -> Vec (n + 1) a
```

We can parameterise types over values to capture invariants in the model.

```
append : Vec n a -> Vec m a -> Vec (n + m) a
append Nil ys = ys
append (x :: xs) ys = x :: append xs ys
```

Quick Background 3 - Propositions as Types

Under the assumptions of referential transparency and totality.

Logic Term	Logic Symbol	Idris Symbol	Idris Type
Implication	$p \Rightarrow q$	`p -> q`	Function
Conjunction	$p \wedge q$	`(p, q)`	Pair / Tuple
Disjunction	$p \vee q$	`Either p q`	Tagged Union
Negation	$\neg\, p$	`p -> Void`	Void Type
IFF/Eq	$p \equiv q$, $p \iff q$	`(p -> q, q -> p)`	Pair Arrows
Universal	$\forall\, x.\ P\ x$	`p -> Type`	Π Type
Existential	$\exists\, x.\ P\ x$	`(x ** P x)`	Σ Type
		`p = q`	Type Equality

Correctness by construction

- The protocol implementations can be quite complex, possibly giving rise to equally complex proof obligations.
- We'd like to reduce this burden somehow!
- We can design a language that is just big enough to compute what we like, but restrictive enough to capture the invariants we care about.

Building a type of Prg n

Statement	Continuation	Result	Description
Halt	-	Prg 1	Terminate
AssC	Prg k	Prg (k + 1)	Asn constant.
AssV	Prg k	Prg (k + 1)	Asn variable.
UnOp	Prg k	Prg (k + 1)	Asn result of unary op.
BinOP	Prg k	Prg (k + 1)	Asn result of binary op.
Do	Prg k	Prg (m * n + k)	Run Prg m, n times.
Cond	Prg k	Prg (n + k)	Branch on Prg n.
Skip	Prg k	Prg (k + 1)	Do Nothing.

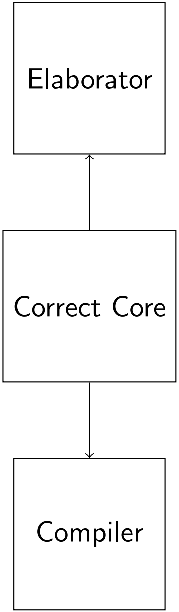
Conditionals require both branches to be Prg n. Ensuring That all branches of the program are correct by construction.

What about more expressive time parameters?

```
f : Value -> Value -> Nat
f = --some complex function to calculate the number of
    --computation steps for each input to mod.

data Prg : Nat -> Type where
  Mod : {k : Nat} -> (cont : Prg k) -> (x : Value) ->
        (y : Value) -> Prg ((f x y) + k)
```

Elaboration and Compilation of a Correct Core



- The small, correct core language can be elaborated to a more full-featured language.
- The size of the core language makes the burden of proofs much lighter.
- The compiler can map the core language expressions down to something more “real world” (e.g C, assembler).

Contributions

- Formal description of a simplified protocol.
- Prg: A small language parameterised over computational time
- Some small proofs of Prg correctness.

Further Work

- Implement the simplified protocol in Prg.
- Modulo arithmetic cases.
- Relax some of the (many) assumptions.
- Investigate elaboration and compilation with regard to invariants.