Practical examples of writing programs and proving theorems in Idris.

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Preliminaries

Propositional Logic

- Concerned with statements of verifiable facts.
- Used daily by programmers when reasoning about Boolean values.

Symbol	Meaning	Example
T, F	True, False	Boolean values.
$p, q, r, ...$	Propositions	Let p = "It is raining."
$\neg$	Negation (Not)	$\neg p$
$\wedge$	Conjuction (And)	$p \wedge q$
$\vee$	Disjunction (Or)	$p \vee q$
$\rightarrow$	Implication (If)	$p \rightarrow q$
$\leftrightarrow$	Bi Implication (Iff)	$p \leftrightarrow q$
$\equiv$	Equivalence	$p \equiv q$
$\top$	Tautology	$p \vee \neg p \equiv \top$
$\bot$	Contradiction	$p \wedge \neg p \equiv \bot$

Definitions of Connectives

Conjunction (And)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (Or)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation (Not)

p	$\neg p$
T	F
F	T

Implication (If)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi Implication (Iff)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equivalence

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Proof Techniques
By Exhaustion

Idea: Prove by enumerating all possible cases.

Prove: $(\neg p \vee q) \leftrightarrow (p \rightarrow q)$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$	$(\neg p \vee q) \leftrightarrow (p \rightarrow q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Proof Techniques
By Appeal to Lemma

Idea: Introduce pre-proven smaller proofs (called a Lemma) to prove a larger proof.

- **Lemma 1.** $p \vee \neg p \equiv \top$.
- **Lemma 2.** $(p \equiv q) \equiv (p \leftrightarrow q)$.

Prove: $(p \leftrightarrow q) \vee \neg(p \equiv q) \leftrightarrow \top$.

$$
\begin{array}{ll}
(p \leftrightarrow q) \vee \neg(p \equiv q) \leftrightarrow \top & \text{Premise.} \\
(p \equiv q) \vee \neg(p \equiv q) \equiv \top & \text{Lemma 2.} \\
\top \equiv \top & \text{Lemma 1.} \\
\qquad\qquad\qquad\qquad\qquad \square &
\end{array}
$$