In [251]:
```
import pandas as pd
import numpy as np
from matplotlib import pyplot
from gym.envs.toy_text import frozen_lake
from gym.envs.toy_text import taxi
import time
```

In [2]:

```
# https://github.com/dennybritz/reinforcement-learning/blob/master/DP/Value%20Iteration%20Solution.ipynb
def value_iteration(env, theta=0.0001, discount_factor=1.0):
    """
    Value Iteration Algorithm.

    Args:
        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all states.
        discount_factor: Gamma discount factor.

    Returns:
        A tuple (policy, V) of the optimal policy and the optimal value function.
    """

    def one_step_lookahead(state, V):
        """
        Helper function to calculate the value for all action in a given state.

        Args:
            state: The state to consider (int)
            V: The value to use as an estimator, Vector of length env.nS

        Returns:
            A vector of length env.nA containing the expected value of each action.
        """
        A = np.zeros(env.nA)
        for a in range(env.nA):
            for prob, next_state, reward, done in env.P[state][a]:
                A[a] += prob * (reward + discount_factor * V[next_state])
        return A

    V = np.zeros(env.nS)
    iterationCount = 0
    deltas = []
    while True:
        # Stopping condition
        delta = 0
        # Update each state...
        for s in range(env.nS):
            # Do a one-step lookahead to find the best action
            A = one_step_lookahead(s, V)
            best_action_value = np.max(A)
            # Calculate delta across all states seen so far
            delta = max(delta, np.abs(best_action_value - V[s]))
            # Update the value function. Ref: Sutton book eq. 4.10.
```

```
            V[s] = best_action_value
            iterationCount += 1
            deltas.append(delta)
            #print "%-4d: %f" %(iterationCount, delta)
        # Check if we can stop
        if delta < theta:
            break

    # Create a deterministic policy using the optimal value function
    policy = np.zeros([env.nS, env.nA])
    for s in range(env.nS):
        # One step lookahead to find the best action for this state
        A = one_step_lookahead(s, V)
        best_action = np.argmax(A)
        # Always take the best action
        policy[s, best_action] = 1.0

    return policy, V, iterationCount
```

In [3]:

```
# https://github.com/dennybritz/reinforcement-learning/blob/master/DP/Policy%20Iteration%20Solution.ipynb
def policy_eval(policy, env, discount_factor=1.0, theta=0.00001):
    """
    Evaluate a policy given an environment and a full description of the environment's dynamics.

    Args:
        policy: [S, A] shaped matrix representing the policy.
        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all states.
        discount_factor: Gamma discount factor.

    Returns:
        Vector of length env.nS representing the value function.
    """
    # Start with a random (all 0) value function
    V = np.zeros(env.nS)
    while True:
        delta = 0
        # For each state, perform a "full backup"
        for s in range(env.nS):
            v = 0
            # Look at the possible next actions
            for a, action_prob in enumerate(policy[s]):
                # For each action, look at the possible next states...
                for  prob, next_state, reward, done in env.P[s][a]:
                    # Calculate the expected value
                    v += action_prob * prob * (reward + discount_factor * V[next_state])
            # How much our value function changed (across any states)
            delta = max(delta, np.abs(v - V[s]))
            V[s] = v
        # Stop evaluating once our value function change is below a threshold
        if delta < theta:
            break
    return np.array(V)

def policy_improvement(env, policy_eval_fn=policy_eval, discount_factor=1.0):
    """
    Policy Improvement Algorithm. Iteratively evaluates and improves a policy
    until an optimal policy is found.

    Args:
        env: The OpenAI envrionment.
        policy_eval_fn: Policy Evaluation function that takes 3 arguments:
```

```
            policy, env, discount_factor.
        discount_factor: gamma discount factor.

    Returns:
        A tuple (policy, V).
        policy is the optimal policy, a matrix of shape [S, A] where eac
h state s
        contains a valid probability distribution over actions.
        V is the value function for the optimal policy.

    """

    def one_step_lookahead(state, V):
        """
        Helper function to calculate the value for all action in a given
 state.

        Args:
            state: The state to consider (int)
            V: The value to use as an estimator, Vector of length env.nS

        Returns:
            A vector of length env.nA containing the expected value of e
ach action.
        """
        A = np.zeros(env.nA)
        for a in range(env.nA):
            for prob, next_state, reward, done in env.P[state][a]:
                A[a] += prob * (reward + discount_factor * V[next_state
])
        return A

    # Start with a random policy
    policy = np.ones([env.nS, env.nA]) / env.nA

    iterationCount = 0
    while True:
        iterationCount += 1
        #print iterationCount
        # Evaluate the current policy
        V = policy_eval_fn(policy, env, discount_factor)

        # Will be set to false if we make any changes to the policy
        policy_stable = True

        # For each state...
        for s in range(env.nS):
            # The best action we would take under the currect policy
            chosen_a = np.argmax(policy[s])

            # Find the best action by one-step lookahead
            # Ties are resolved arbitarily
            action_values = one_step_lookahead(s, V)
            best_a = np.argmax(action_values)

            # Greedily update the policy
            if chosen_a != best_a:
```

```
                policy_stable = False
            policy[s] = np.eye(env.nA)[best_a]

        # If the policy is stable we've found an optimal policy. Return
 it
        if policy_stable:
            return policy, V, iterationCount
```

In [4]:
```
frozen_lake.FrozenLakeEnv().render()
```

```
SFFF
FHFH
FFFH
HFFG
```

In [5]:
```
taxi.TaxiEnv().render()
```

```
+---------+
|R: | : :G|
| : : : : |
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| | : | : |
|Y| : |B: |
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```

In [6]:
```
print frozen_lake.FrozenLakeEnv().nS
print taxi.TaxiEnv().nS
```

```
16
500
```

In [7]:
```
%timeit print value_iteration(frozen_lake.FrozenLakeEnv())
```

```
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       0.        ]), 2976)
10 loops, best of 3: 58.2 ms per loop
```

In [8]:
```
%timeit print policy_improvement(frozen_lake.FrozenLakeEnv())
```

```
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       [0., 1., 0., 0.],
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0.82325081, 0.82337956,
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       0.76462706, 0.        , 0.        , 0.88229042, 0.94114466,
       0.        ]), 3)
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       0.        ]), 3)
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0.82325081, 0.82337956,
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10 loops, best of 3: 92.5 ms per loop
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In [10]:
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%timeit print value_iteration(taxi.TaxiEnv(), theta=0.0001, discount_fac
tor=0.99)
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1 loop, best of 3: 8.74 s per loop
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In [11]:
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       1837.76294845, 1917.25067383, 1837.76292998, 1978.99902384]), 1
0)
1 loop, best of 3: 1min per loop
```

In [339]:
```
taxi_policy_pi, taxi_values_pi, taxi_iters_pi = policy_improvement(taxi.
TaxiEnv(), discount_factor=0.99)
```

In [345]:
```
def test_policy(env, policy, episodes=1000, max_iters=100):
    rewards = []
    for _ in range(episodes):
        env.reset()
        r = 0
        done = False
        iters = 0
        while not done:
            if iters > max_iters: break
            iters += 1
            a = np.argmax(policy[env.s])
            ob, reward, done, prob = env.step(a)
            r += reward
        rewards.append(r)
    return np.mean(rewards)
```

In [326]:
```
policy = [[1., 0., 0., 0.],
       [0., 0., 0., 1.],
       [0., 0., 0., 1.],
       [0., 0., 0., 1.],
       [1., 0., 0., 0.],
       [1., 0., 0., 0.],
       [1., 0., 0., 0.],
       [1., 0., 0., 0.],
       [0., 0., 0., 1.],
       [0., 1., 0., 0.],
       [1., 0., 0., 0.],
       [1., 0., 0., 0.],
       [1., 0., 0., 0.],
       [0., 0., 1., 0.],
       [0., 1., 0., 0.],
       [1., 0., 0., 0.]]
env = frozen_lake.FrozenLakeEnv()
test_policy(env, policy)
```

Out[326]: 0.743

In [347]:
```
test_policy(taxi.TaxiEnv(), taxi_policy, episodes=1000000)
```

Out[347]: 8.456455

In [348]:
```
test_policy(taxi.TaxiEnv(), taxi_policy_pi, episodes=1000000)
```

Out[348]: 8.462363

In [302]:

```
class QAgent(object):
    def __init__(self, env, gamma=0.99, alpha=0.2, alpha_decay=1e-3, eps
ilon=1, epsilon_decay=1e-3):
        self.env = env
        self.Q = {s : {a : 0.5 for a in range(env.action_space.n)} for s
 in range(env.observation_space.n)}
        self.gamma = gamma
        self.alpha = alpha
        self.alpha_decay = alpha_decay
        self.epsilon = epsilon
        self.epsilon_decay = epsilon_decay
        self.max_change = 0

    def train(self, convergence_threshold=1e-5, max_iters=5000):
        while True:
            ob = self.env.reset()
            agent.max_change = 0
            ob_seq = [ob]
            previous_ob = None
            action = None
            reward = 0
            iters = 0
            while True:
                if iters > max_iters: break
                iters += 1
                action = agent.act(previous_ob, action, ob, reward)
                previous_ob = ob
                ob, reward, done, prob = self.env.step(action)
                ob_seq.append(ob)
                if done:
                    agent.act(previous_ob, action, ob, reward, terminal=
True) #final update
                    break
            agent.decay()
            if agent.max_change < convergence_threshold:
                break

    def act(self, previous_observation, previous_action, observation, re
ward, terminal=False):
        maxA, maxV = max([(a,v) for a,v in self.Q[observation].iteritems
()], key=lambda x: x[1])
        if terminal: maxV = 0
        if previous_observation is not None and previous_action is not N
one:
            prev = self.Q[previous_observation][previous_action]
            self.Q[previous_observation][previous_action] = \
                (1 - self.alpha) * self.Q[previous_observation][previous
_action] + \
                self.alpha * (reward + self.gamma * maxV)
            self.max_change = max(self.max_change, abs(prev - self.Q[pre
vious_observation][previous_action]))
        if np.random.random() < self.epsilon:
            chosenA = self.env.action_space.sample()
        else:
            chosenA = maxA
        return chosenA
```

```
    def decay(self, parameter='both'):
        if parameter=='alpha' or parameter=='both':
            self.alpha *= (1-self.alpha_decay)
        if parameter=='epsilon' or parameter=='both':
            self.epsilon *= (1-self.epsilon_decay)

    def run(self, n_trials=1, max_iters=100):
        rewards = []
        for _ in range(n_trials):
            self.env.reset()
            done = False
            iters = 0
            r = 0
            while not done:
                if iters > max_iters: break
                iters += 1
                a, _ = max([(a,v) for a,v in self.Q[env.s].iteritems()],
key=lambda x: x[1])
                #print self.env.s
                ob, reward, done, prob = self.env.step(a)
                r += reward
            #print self.env.s, reward
            rewards.append(r)
        return rewards
```

In [ ]:

In [260]:

```
lake_data = []
env = frozen_lake.FrozenLakeEnv()
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
#for ep in [0]:
    for al in [0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5,
 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1]:
        print "e: %f   a: %f" % (ep, al)
        agent = QAgent(env, epsilon=ep, alpha=al)
        start = time.time()
        agent.train()
        trainTime = time.time() - start
        trainScore = np.mean(agent.run(1000))
        lake_data.append({"alpha": al, "epsilon": ep, "trainTime": train
Time, "trainScore": trainScore})
env.close()
```

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```
e: 1.000000   a: 0.900000
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```
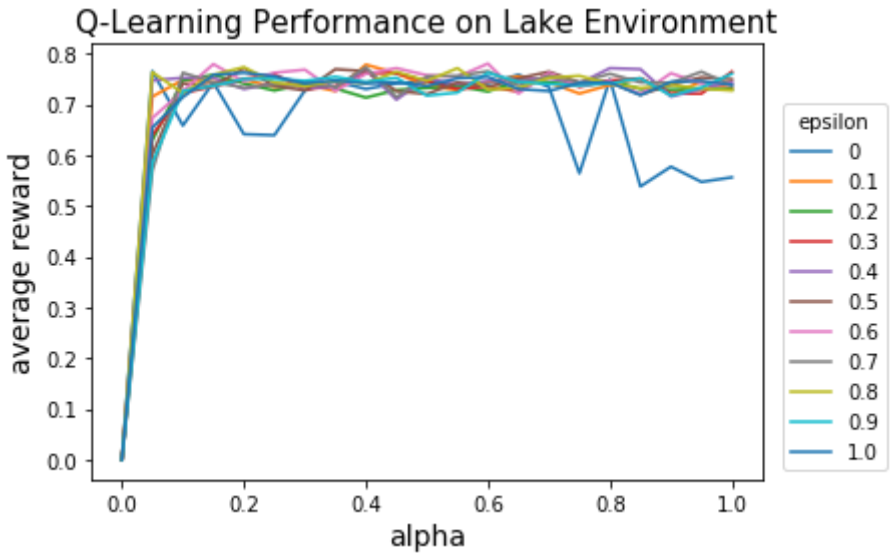
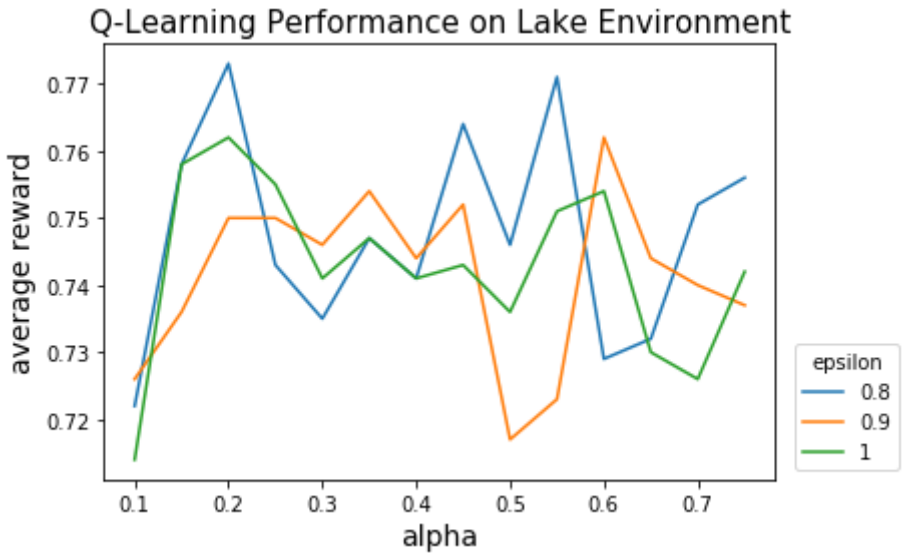

In [298]:
```
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    data = [d for d in lake_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data], [d['trainScore'] for d in da
ta])
pyplot.legend([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0], loc
=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Performance on Lake Environment", fontsize=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("average reward", fontsize=14)
#pyplot.plot([d['alpha'] for d in lake_data], [d['trainTime'] for d in l
ake_data])
```

Out[298]: Text(0,0.5,'average reward')



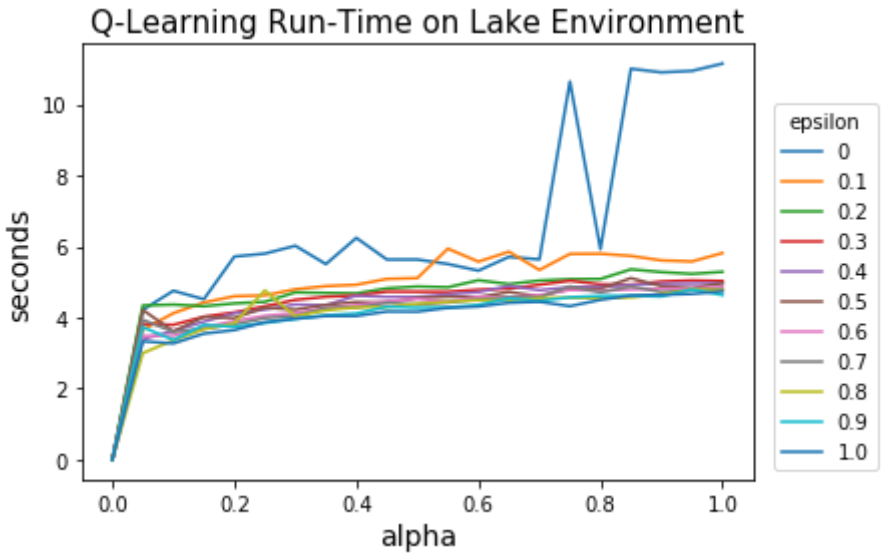
In [334]:
```
for ep in [0.8, 0.9, 1]:
    data = [d for d in lake_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data][2:16], [d['trainScore'] for d
 in data][2:16])
pyplot.legend([0.8, 0.9, 1], loc=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Performance on Lake Environment", fontsize=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("average reward", fontsize=14)
```

Out[334]:
```
Text(0,0.5,'average reward')
```



In [300]:
```
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    data = [d for d in lake_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data], [d['trainTime'] for d in dat
a])
pyplot.legend([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0], loc
=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Run-Time on Lake Environment", fontsize=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("seconds", fontsize=14)
```

Out[300]:
```
Text(0,0.5,'seconds')
```



In [269]:
```
env = frozen_lake.FrozenLakeEnv()
agent = QAgent(env, epsilon=ep, alpha=al)
start = time.time()
agent.train()
trainTime = time.time() - start
trainScore = np.mean(agent.run(1000))
print trainScore, trainTime
```

```
0.738 4.73475885391
```

In [303]:
```
taxi_data_bak = taxi_data[:]
```

In [304]:

```
taxi_data = []
env = taxi.TaxiEnv()
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
#for ep in [0]:
    for al in [0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5,
 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1]:
        print "e: %f   a: %f" % (ep, al)
        agent = QAgent(env, epsilon=ep, alpha=al)
        start = time.time()
        agent.train()
        trainTime = time.time() - start
        trainScore = np.mean(agent.run(1000))
        taxi_data.append({"alpha": al, "epsilon": ep, "trainTime": train
Time, "trainScore": trainScore})
env.close()
```

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```
e: 1.000000    a: 0.900000
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```
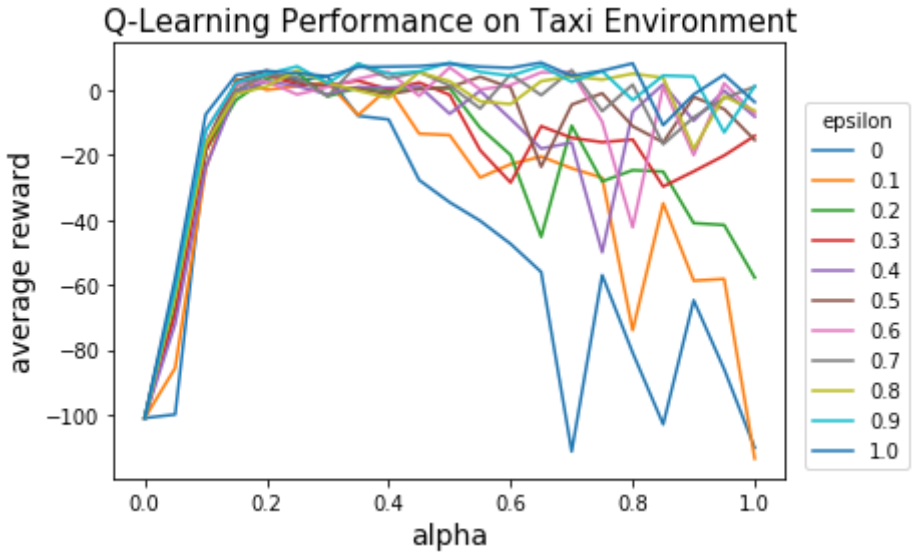

In [305]:
```
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    data = [d for d in taxi_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data], [d['trainScore'] for d in da
ta])
pyplot.legend([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0], loc
=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Performance on Taxi Environment", fontsize=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("average reward", fontsize=14)
```

Out[305]: Text(0,0.5,'average reward')



In [311]:
```
for ep in [0.8, 0.9, 1.0]:
    data = [d for d in taxi_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data][2:16], [d['trainScore'] for d
 in data][2:16])
pyplot.legend([0.8, 0.9, 1.0], loc=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Performance on Taxi Environment (Zoomed)", font
size=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("average reward", fontsize=14)
```

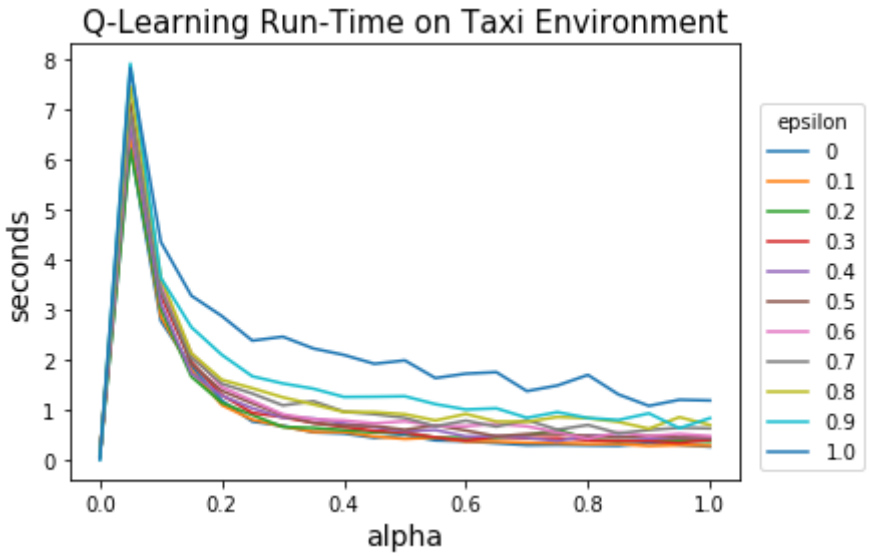
Out[311]:
```
Text(0,0.5,'average reward')
```

In [349]:
```
data = [d for d in taxi_data if d['epsilon']==1]
data[8:16]
```

Out[349]:
```
[{'alpha': 0.4,
  'epsilon': 1.0,
  'trainScore': 7.241,
  'trainTime': 2.1023571491241455},
 {'alpha': 0.45,
  'epsilon': 1.0,
  'trainScore': 7.403,
  'trainTime': 1.9249579906463623},
 {'alpha': 0.5,
  'epsilon': 1.0,
  'trainScore': 8.141,
  'trainTime': 1.9925189018249512},
 {'alpha': 0.55,
  'epsilon': 1.0,
  'trainScore': 7.237,
  'trainTime': 1.6398530006408691},
 {'alpha': 0.6,
  'epsilon': 1.0,
  'trainScore': 6.869,
  'trainTime': 1.7275810241699219},
 {'alpha': 0.65,
  'epsilon': 1.0,
  'trainScore': 8.494,
  'trainTime': 1.7579970359802246},
 {'alpha': 0.7,
  'epsilon': 1.0,
  'trainScore': 4.416,
  'trainTime': 1.3789749145507812},
 {'alpha': 0.75,
  'epsilon': 1.0,
  'trainScore': 6.007,
  'trainTime': 1.490821123123169}]
```

In [306]:
```
for ep in [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]:
    data = [d for d in taxi_data if d['epsilon']==ep]
    pyplot.plot([d['alpha'] for d in data], [d['trainTime'] for d in dat
a])
pyplot.legend([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0], loc
=(1.03, 0.02), title="epsilon")
pyplot.title("Q-Learning Run-Time on Taxi Environment", fontsize=15)
pyplot.xlabel("alpha", fontsize=14)
pyplot.ylabel("seconds", fontsize=14)
```

Out[306]:
```
Text(0,0.5,'seconds')
```



In [ ]: