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### Solving Word Problems using Schemas: A Review of the Literature

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#### **Abstract**

Solving word problems is a difficult task for students at-risk for or with learning disabilities (LD). One instructional approach that has emerged as a valid method for helping students at-risk for or with LD to become more proficient at word-problem solving is using schemas. A schema is a framework for solving a problem. With a schema, students are taught to recognize problems as falling within word-problem types and to apply a problem solution method that matches that problem type. This review highlights two schema approaches for 2<sup>nd</sup>- and 3<sup>rd</sup>-grade students at-risk for or with LD: schema-based instruction and schema-broadening instruction. A total of 12 schema studies were reviewed and synthesized. Both types of schema approaches enhanced the word-problem skill of students at-risk for or with LD. Based on the review, suggestions are provided for incorporating word-problem instruction using schemas.

Since Pólya (1945) introduced four steps for solving word problems (understand the question, devise a plan, carry out the plan, and look back and check), teachers have been encouraged to provide more systematic instruction on problem solving in mathematics. Word-problem instruction has become vital for students. High-stakes standardized tests like the National Assessment of Educational Progress (NAEP; National Assessment Governing Board, 2009) place heavy emphasis on mathematics word problems and national educational organizations like the National Council of Teachers of Mathematics (NCTM; 2000) heavily value the teaching of problem solving across grades K through 12. Many researchers have investigated methods for teaching problem solving to general-education students (Marshall, 1995; Schoenfeld, 1992; Shavelson, Webb, Stasz, & McArthur, 1988) and, in more recent years, to students with learning disabilities (LD) (e.g., Case, Harris, & Graham, 1992; Mastropieri, Scruggs, & Shiah, 1997; Miller & Mercer, 1993).

Over the last two decades, a sizeable literature has begun to accumulate with an emphasis on helping students develop schemas to solve word problems in mathematics (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Fuchs, Seethaler, et al., 2008; Griffin & Jitendra, 2009; Jitendra & Hoff, 1996; Willis & Fuson, 1988). A *schema* is a framework, outline, or plan for solving a problem (Marshall, 1995). In mathematics, students can use schemas to organize information from a word problem in ways that represent the underlying structure of a problem type. Pictures or diagrams, as well as number sentences or equations, can be used to represent schemas.

Often, word problems can be differentiated into *types* of problems. The problem *type* is determined by what is happening in the word-problem narrative. For example, students may be given the following information: *There are 7 blue birds and 4 red birds sitting on a tree*. If, however, students are asked, *How many birds are on the tree?*, the problem type is

combining or totaling (the birds). If students are asked, *Five blue birds flew away, how many blue birds are left sitting in the tree?*, the problem type is finding the change (in the number of blue birds). Combining or totaling is different from finding a change in that the examples represent two distinct problem types. Two distinct schemas can be used to solve the problems. Once students determine the problem type, they can apply a schema (i.e., diagram, equation, or plan) to assist in solving the word problem. In the elementary grades, most word problems can be sorted into only a few types (Riley & Greeno, 1988). If students know a schema for each type, and understand how to sort problems into the problem types and apply the solution method for each schema, then students should be able to solve most word problems (Cooper & Sweller, 1987).

The first purpose of the present paper was to review and synthesize the literature on schemas within word-problem instruction to determine (a) what schemas were taught to elementary students at-risk for or with LD, (b) how these schemas were taught, and (c) what effects were associated with solving word problems using schemas. The second purpose was to provide suggestions for classroom teachers on how to teach students to use schemas to solve word problems.

#### **Word-Problem Difficulty**

Students at-risk for or with LD often struggle with word-problem solving (Parmar, Cawley, & Frazita, 1996). For example, Wilson and Sindelar (1991) worked with second-through fifth-graders with LD. On a test of addition and subtraction word problems, these students performed significantly below third-grade students without LD. At second- and fourthgrade, Englert, Culatta, and Horn (1987) tested 24 students with LD on 16 addition word problems. When compared to grade-level peers, the students with LD demonstrated significantly lower accuracy on word-problem solutions. More recently, Jordan and Hanich (2000) administered 14 addition and subtraction word problems to 20 second-grade students at-risk for or with mathematics LD and 29 second-grade students without LD. Students atrisk for or with LD answered fewer word problems correct than students without LD and employed less efficient strategies. These findings were corroborated with a larger group of second-grade (Hanich, Jordan, Kaplan, & Dick, 2001) and third-grade students (Jordan & Montani, 1997). Moving beyond simple word problems, Fuchs and Fuchs (2002) administered 10 word problems comprising four types (i.e., shopping list, buying bags, half, and pictograph) and 10 multi-step word problems with tables and graphs to fourth-grade students. The performance of 40 students with LD was compared to normative data collected from typical fourth-grade students. On both word problem sets, students with LD scored significantly lower than students in the normative group with effect sizes (ESs) ranging from 0.49 to 1.10 favoring the normative group.

Word problems may pose a challenge for students at-risk for or with LD because numerous steps and skills are necessary to solve a word problem (Parmar et al., 1996). Additionally, students may struggle with comprehension of the text of the word problem (Cummins, Kintsch, Reusser, & Weimer, 1988). Many students with LD struggle with mathematics and reading difficulty; therefore, embedding mathematics within a linguistic context may challenge students who also have reading deficits (Fuchs, Fuchs, Stuebing, et al., 2008). To solve a word problem, students must use the text to identify missing information, derive a plan for solving for the missing information, and perform a calculation to find the missing information. Even when complex calculations are not required, students with LD struggle with problem solving compared to their average-performing peers (Pellegrino & Goldman, 1987).

#### Instruction for Students with LD

To help students at-risk for or with LD become more efficient and accurate word-problem solvers, explicit word-problem instruction may be warranted (Parmar et al., 1996). For example, Kroesbergen, Van Luit, and Maas (2004) randomly assigned 265 8- to 11-year-old students with LD or behavior disorders to receive constructivist multiplication instruction, explicit multiplication instruction, or regular classroom instruction (control). Students in the constructivist and explicit conditions received 30 lessons over 4 to 5 months. Intervention focused on multiplication automaticity and problem solving. In the constructivist condition, students were encouraged to discuss different approaches to solving a multiplication problem and then determine whether they could use one of these approaches to solve the problem. In the explicit condition, students were told how a problem should be solved and were provided examples of good problem-solving strategies. Students were always told which strategies to use. In the control condition, students followed the school's regular mathematics curriculum which included instruction on multiplication. At posttest, explicit instruction students significantly outperformed constructivist and control students on a computation multiplication measure and a measure of multiplication word problems. Kroesbergen et al. concluded that explicit or direct mathematics instruction, but not discovery or constructivist learning, may benefit lower-performing students.

Several explicit approaches exist for teaching students at-risk for or with LD to solve word problems (Jitendra & Xin, 1997). These include diagramming or drawing the word problem (van Garderen, 2007); identifying key words in a word problem and solving the problem based on the key word; utilizing computer-assisted instruction with explicit step-by-step work (Mastropieri et al., 1997); using a mnemonic device to guide word-problem solving (Miller & Mercer, 1993); learning metacognitive strategies to monitor word problem-solving progress (Case et al., 1992); and using a checklist of steps to solve word problems along with monitoring work with metacognitive strategies (Montague, Warger, & Morgan, 2000). An additional approach to teaching word-problem solving to students at-risk for or with LD, which has been developed over the last 20 years, is using schemas to solve word problems (e.g., Fuchs, Fuchs, Finelli, et al., 2004; Jitendra & Hoff, 1996). Word-problem instruction using schemas differs from typical word-problem instruction (e.g., key words, checklist of steps) because students first identify a word problem as belonging to a problem type and then use a specific problem-type schema to solve the problem. In conventional wordproblem instruction, students may organize word-problem information or follow a mnemonic device to work step-by-step through the problem; however, students are not taught to determine a problem type and solve word problems according to a problem-type schema. In this paper, the research conducted to evaluate using schemas in word-problem solving was reviewed and synthesized.

#### **Literature Search**

The studies selected for this literature review met four criteria. First, the implemented treatments incorporated explicit instruction on solving a word problem though a schema. Second, studies needed to include, but not necessarily be limited to, students at-risk for or with LD. Third, study participants comprised students in second or third grade. These were the target grades because this is often when identification of students with LD occurs (Fletcher, Lyon, Fuchs, & Barnes, 2006) and when written word-problem solving is a major focus of the curriculum as opposed to less formal, oral problems presented in kindergarten and first grade. Fourth, studies needed to be published in a peer-reviewed journal. I conducted searches in electronic databases including ERIC, PsycInfo, and ProQuest using the following terms: *schema, word problem, story problem,* and *problem solving*. Then, I read the titles and abstracts of articles to identify studies that fit the four criteria resulting in

12 word-problem solving studies. In all 12 studies, instruction focused on addition and subtraction word problems, which are the two operations most commonly found in second-and third-grade instruction and on standardized tests (Hudson & Miller, 2006).

#### **Overview of Included Studies**

Each schema study reviewed in this paper is outlined in Table 1. Study publication dates ranged from 1996 to 2009. Across studies, almost 4000 students were included. Of these students, 411 were at-risk for LD and 173 were identified as receiving special education services. Jitendra and colleagues tended to work with students with LD whereas Fuchs and colleagues generally worked with students at-risk for LD. The researchers utilized a variety of experimental designs: single subject (1), group teaching without assignment to treatment conditions (1), student random assignment (4), matched pairs random assignment (2), and classroom random assignment (5). Instruction occurred during school hours in individual settings (3), in small groups (2), and in large groups (8). Assessments for determining instructional effects were experimenter-designed in all 12 studies. Two of the studies included standardized assessments as well as experimenter-designed measures in the testing battery. In ten of the studies, more than one assessment was administered.

#### Two Approaches to Schema Instruction

In this section, two approaches to schema instruction are discussed. The first, referred to as schema-based instruction, teaches students to use schematic diagrams to solve addition and subtraction word problems (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra & Hoff, 1996). The student reads a word problem, selects a schema diagram into which the word problem fits, and uses the structure of the diagram to solve the problem. In more recent studies, students are taught to use a mathematical equation (i.e., 4 + ? = 7), after filling in a schematic diagram, to solve the problem (Griffin & Jitendra, 2009). The work by Jitendra and colleagues uses schema-based instruction. By contrast, Fuchs et al. (2003) uses a second approach to schema instruction, schema-broadening instruction. Schemabroadening instruction is similar to schema-based instruction in that students read the word problem and select a schema (from the taught schema) to solve word problems. Schemabroadening instruction differs from schema-based instruction because students are taught to transfer their knowledge of problem types to recognize problems with novel features (e.g., different format, additional question, irrelevant information, unfamiliar vocabulary, or information presented in charts, graphs, or pictures) as belonging to a problem type for which they know a solution. As with Jitendra and colleagues, Fuchs and colleagues also teach students to set up and solve mathematical equations (e.g., X - 3 = 7) representing the structure of problem types (Fuchs et al., 2009).

In terms of schema instruction, the schema-based instruction of Jitendra and colleagues differs from the schema-broadening instruction of Fuchs and colleagues in one primary way. With schema-broadening (but not schema-based) instruction, students receive explicit instruction on transfer to novel problems. The schemas that Jitendra and colleagues used rely on diagrams for organizing word-problem work. (See Figure 1 for an example.) Fuchs and colleagues, in contrast, teach students to organize word-problem information in sections or in mathematical equations. (See Figures 2 and 3 for examples.)

#### **Schema-Based Instruction**

To understand how schema-based instruction may benefit students with LD, Jitendra and Hoff (1996) worked with three third- and fourth-grade students with LD. During 13 to 16 days of intervention, students learned to recognize defining features of addition and subtraction word-problem types, classify problems in terms of problem types, map the word

problem information onto the schema's diagram, and use the diagram to solve the problem. Jitendra and Hoff taught three schemas: change, group, and compare. All three students demonstrated positive growth as the study progressed and maintained skills 2 to 3 weeks after the final intervention session, with only a slight decline in scores. Through this multiple-baseline, single-subject design, Jitendra and Hoff demonstrated the possible benefit of using schemas for teaching word-problem solving to students with LD.

Working with a larger number of students, Jitendra et al. (1998) recruited 34 second-to fifth-grade students who performed below the 60<sup>th</sup> percentile on a word-problem measure. Students were randomly assigned to receive small-group schema instruction or small-group traditional instruction during 17 to 20 sessions. Schema instruction focused on change, group, and compare problems. Students learned how to identify the schema for a word problem and to use a schema diagram to organize the problem's information. The traditional instruction followed a basal mathematics program focused on general mathematics skills and was implemented to control for tutoring time. At posttest, students participating in schema tutoring outperformed students in the traditional tutoring on experimenter-designed measures of word problems. A delayed posttest, administered one week after tutoring commenced, continued to favor schema students over traditional students. Jitendra et al. also recruited 24 average-performing third graders to serve as a normative sample. At posttest, the schema-tutoring students performed comparably to students in the normative sample, whereas traditional-tutoring students did not. These results favoring schema instruction led Jitendra et al. to conclude that word-problem instruction using schemas is more advantageous to students at-risk for LD than traditional word-problem instruction.

In the next phase of this research program, Jitendra moved from small-group schema intervention to whole-class schema-based instruction. Jitendra, Griffin, Deatline-Buchman, et al. (2007) provided schema-based instruction similar to Jitendra and Hoff (1996) with students receiving instruction on using schematic diagrams to solve change, combine or group, and compare problems. Students were taught to fill word-problem information into a problem type's corresponding schematic diagram and then generate a mathematical equation (i.e., a number sentence with missing information) to help solve the problem. A question mark was used to mark the missing information (i.e., ? + 5 = 10). Across three classrooms, 38 lower-performing third-grade students, 9 of whom were identified with LD, received schema-based instruction. Instruction lasted 15 weeks with three 30-min sessions per week. On two experimenter-designed word-problem posttests, students in the three classrooms demonstrated improvement from pretest although the improvement was not significant. Jitendra, Griffin, Deatline-Buchman, et al. concluded that lower-performing students and students with LD need and benefit from explicit word problem-solving instruction focused around schemas. With the absence of control classrooms for comparison purposes or significant growth from pre- to posttest, Jitendra, Griffin, Deatline-Buchman, et al. indicated, but did not verify, that schema instruction may be beneficial for students at-risk for or with LD.

Comparing schema-based instruction to another word-problem solving approach, Jitendra, Griffin, Haria, et al. (2007) randomly assigned 88 third-grade students to two conditions: schema-based instruction and general-strategy instruction. Four of the 88 participants were identified with LD. Schema-based instruction focused on the change, combine, and compare problem types as in Jitendra, Griffin, Deatline-Buchman, et al., (2007) whereas students receiving general-strategy instruction were taught four steps to solve a word problem (i.e., read and understand, plan, solve, and check) along with four strategies to assist in solving a word problem (i.e., use manipulatives, act it out or draw a diagram, write a number sentence, and use information from a graph). Similar to Jitendra, Griffin, Deatline-Buchman, et al., students receiving schema instruction learned to identify the schema of a word problem, fill

word-problem information into a schematic diagram, and then generate an equation to help solve the word problem. Students used different schematic diagrams for each of the three problem types, and the use of schematic diagrams was faded toward the end of instruction on each problem type. Many students, however, continued to draw schematic diagrams independently. After all three problem types were introduced, tutors taught the students to solve two-step problems that combined two schemas. All students received 41 lessons, each lasting approximately 25 min. From pre- to posttest, students in the schema-based condition outperformed students in the general-strategy condition on an experimenter-designed word-problem measure, with an ES of 0.52. The same measure, administered six weeks after posttest, again showed students in the schema-based condition outperforming general-strategy condition students (ES = 0.69). The number of students with LD was small (n = 4), so results for students with disabilities were not presented by Jitendra, Griffin, Deatline-Buchman, et al. separate from the main analysis. Therefore, conclusions about the benefit of schema-based instruction for students with LD could not be inferred.

Interestingly, Griffin and Jitendra (2009) also compared schema-based instruction to general-strategy instruction with third-grade students but did not replicate the results from Jitendra, Griffin, Haria, et al. (2007). Students from three classrooms (n = 60; 5 with LD) were matched based on performance on a standardized mathematics test and then the pairs were randomly assigned to schema-based or general-strategy instruction. Schema-based and general-strategy instruction were similar to that provided in Jitendra, Griffin, Haria, et al., except that instruction was provided in 20 lessons lasting 100 min each. Schema instruction included completing schematic diagrams and generating equations. The final four lessons comprised instruction on two-step problems where tutors taught students to solve problems using two schemas. On an experimenter-designed word-problem measure, there were no significant differences between the two groups at posttest or at 12-week maintenance (even though both groups demonstrated growth from pretest to posttest to maintenance). On a measure of word-problem solving fluency administered three times throughout instruction, there were significant differences favoring schema-based instruction at the beginning of treatment. These effects, however, faded over the course of the study: At posttest, schemabased and general-strategy groups performed similarly. Griffin and Jitendra attributed the inconsistency of this finding to the fact that instruction was provided in 100-min sessions and once a week rather than shorter sessions occurring several times a week.

Jitendra and colleagues' program of research on schema-based instruction is impressive and demonstrates that students at-risk for or with LD may benefit from explicit schema instruction. These researchers taught students to use three schemas (i.e., change, combine or group, and compare) on different types of word problems with two operations (i.e., addition and subtraction). Even though the specific nature of schema-based instruction varied in small ways from study to study, the majority of students benefitted from learning about different schema and applying the schema to solve word problems. Across studies, two instructional design features were consistently incorporated within schema-based instruction. First, interventions were of long duration (13 to 45 lessons), and second, explicit instruction focused on recognizing a problem's schema, using a diagram based on the schema, and solving the problem. The research by Jitendra and colleagues offers a solid foundation for future schema-based investigations and provides strategies that teachers can use to enhance the performance of their students with LD on word problems.

#### **Schema-Broadening Instruction**

As in Jitendra and colleagues' schema-based instruction, schema-broadening instruction relies on schemas for conceptualizing word problems. Some of Fuchs and colleagues' schema-broadening instruction comprises problem types (i.e., shopping list, half, buying bags, pictograph) that are notably different from the problem types used by Jitendra and

colleagues. Other schema-broadening problem types of Fuchs and colleagues (i.e., total, difference, and change) are similar to the combine, compare, and change problem types of Jitendra and colleagues. Schema-broadening instruction includes a focus on transfer features to help students expand their conceptualization of the schema. Thus, schema-broadening instruction helps students recognize a novel problem (with unfamiliar problem features such as different format, additional question, irrelevant information, unfamiliar vocabulary, or information presented in charts, graphs, or pictures) as belonging to the schema for which they know a problem solution strategy.

To pinpoint the effects of explicit transfer instruction within schema-broadening instruction, Fuchs et al. (2003) randomly assigned 24 third-grade classrooms (n = 375) to four conditions: problem-solution instruction, partial-problem-solution-with-transfer instruction (to control for instructional time), full problem-solution-with-transfer-instruction, or control, business-as-usual instruction with a 6-lesson introductory general-problem solving unit that all 24 classrooms received. Students receiving special education services (n = 23) were distributed across the four conditions. After this introductory unit, problem-solution instruction was presented over the next 20 lessons, in which students were explicitly taught to understand and recognize four schema (i.e., shopping list, half, buying bags, and pictograph) and to apply rules for solving problems for each schema. Students in the partialproblem-solution-plus-transfer condition received only 10 solution lessons but also received 10 transfer lessons. The transfer lessons included explicit instruction on the meaning of transfer and instruction to broaden schema to address problems with different formats, unfamiliar vocabulary, additional questions, and broader problem-solving contexts. Students in the full-problem-solution-plus-transfer condition received all 20 solution lessons and all 10 transfer lessons. In terms of classroom performance from pre- to posttest, students in the problem-solution, partial-problem-solution-with-transfer, and full-problem-solution-withtransfer classrooms outperformed control classrooms on an experimenter-designed immediate-transfer measure (ESs = 2.61, 2.15, and 1.82, respectively). On a far-transfer measure, students who received the partial- or full-solution-plus-transfer-instruction significantly outperformed control classrooms. Additionally, classrooms that received the full-solution-plus-transfer instruction improved more than classrooms that received the solution instruction alone. For students with disabilities, however, the results were not as promising. In the partial-problem-solution condition, 60-80% of the students were unresponsive to treatment. Students in the problem-solution and full-problem-solution-withtransfer conditions demonstrated greater levels of response. This study, as well as a similar study conducted with 24 classrooms of 366 students by Fuchs, Fuchs, Prentice, et al. (2004), demonstrated the added value of schema instruction with an explicit focus on transfer schemas. Interestingly, in Fuchs, Fuchs, Prentice, et al. students in special education demonstrated significant gains over control students with ESs of 0.87 to 1.96.

To further extend this research program on schema-broadening instruction, Fuchs, Finelli, et al. (2004) randomly assigned 24 classrooms, with 351 students, to three conditions: schema-broadening instruction that addressed three transfer features, schema-broadening instruction that addressed six transfer features, and business-as-usual control. Twenty-nine students received special education services. All classrooms received six sessions about generic word problem-solving steps. Schema-broadening classrooms also received 28 lessons focused on the four schemas taught in Fuchs et al. (2003). The schema-broadening instruction condition addressed three transfer features (i.e., different format, different question, or different vocabulary). The six-feature schema-broadening instruction condition addressed different format, different question, different vocabulary, irrelevant information, combined problem types, and mixing of transfer features. On experimenter-designed measures with the shortest transfer distance (unfamiliar problems but without novel features), students participating in both schema-broadening instruction conditions

performed comparably but significantly better than control (ESs = 3.69 and 3.72, respectively). On measures assessing word problems with medium transfer distance (i.e., different format, question, or vocabulary transfer features), again there were no significant differences between the two schema-broadening instruction conditions, which outperformed the control group (ESs = 1.98 and 2.71, respectively). However, on the measure assessing the greatest transfer distance (i.e., involving all six transfer features), students in the schema-broadening instruction condition that incorporated all six transfer features demonstrated a significant advantage with an ES of 2.71 over control students and an ES of 0.72 over students in the narrower schema-broadening instruction treatment. Students with disabilities demonstrated similar gains to those of students without disabilities. Fuchs, Fuchs, Finelli, et al. demonstrated that students benefit from explicit schema-broadening instruction focused on a wide variety of transfer features.

In an expansion of Fuchs, Fuchs, Finelli, et al. (2004), Fuchs and colleagues tested how reallife problem-solving skills might provide added benefit to schema-broadening instruction (Fuchs et al., 2006). From 30 classrooms, 445 third-grade students (34 of whom received special education services) were randomly assigned by classroom to schema-broadening instruction, schema-broadening and real-life instruction, or business-as-usual control. All 30 classrooms received six 40-min sessions on general problem-solving strategies. Both schema-broadening treatments received an additional 30 sessions on four problem types. Additionally, schema-broadening plus real-life instruction classrooms received explicit instruction via video on real-life problem solving skills (i.e., review the problem, determine extra steps necessary for solving the problem, find important information without number, figure out important information not provided within the problem, reread, and ignore irrelevant information). On experimenter-designed measures of immediate and medium word-problem transfer, both schema-broadening treatments outperformed control classrooms with ESs ranging from 3.59 to 6.84. On a far transfer task, the added benefit of explicit real-life problem solving emerged on an open-ended question about what the student could buy. Students could use information from a pictograph, a price chart, or their own experiences to answer the question. On this question, the schema-broadening plus real-life students outperformed schema-broadening students (ES = 1.83). In this way, Fuchs et al. (2006) demonstrated how the combination of schema-broadening and real-life problemsolving instruction is beneficial for solving word problems. Results for students with disabilities, however, were not disaggregated from the entire sample, thus it was unclear if these students performed in a similar manner.

To investigate the effect of schema-broadening instruction for students at-risk for LD, Fuchs, Fuchs, Craddock, et al. (2008) randomly assigned 119 classrooms to receive schemabroadening instruction or to participate in a business-as-usual control group. Then, within each whole-class condition, 243 students at-risk for or with LD were randomly assigned to receive small-group schema-broadening tutoring or to remain in their whole-class condition without tutoring. In this way, 28 students received business-as-usual whole-class instruction and no schema-broadening tutoring, 51 students received whole-class schema-broadening instruction but no schema-broadening tutoring, 56 students received business-as-usual whole-class instruction with schema-broadening tutoring, and 108 students received wholeclass schema-broadening instruction plus schema-broadening tutoring. The schemabroadening instruction at the classroom level provided explicit instruction on solving the four problem types (i.e., shopping list, half, buying bags, and pictograph) over 16 weeks. Tutoring occurred 3 times a week for 13 weeks following completion of three weeks of whole-class instruction. Tutoring sessions lasted 20 to 30 min in small groups of two to four students. For students who received whole-class schema-broadening instruction, tutored students outperformed students who did not receive tutoring on experimenter-designed measures (ES = 1.13). In a similar way, for students in business-as-usual classrooms, tutored

students outperformed students who did not receive tutoring (ES = 1.34). Importantly, students who received two tiers of schema-broadening instruction (whole class and small-group tutoring) significantly outperformed students who received schema-broadening tutoring without whole-class schema-broadening instruction. This finding suggests that the combination of whole-class instruction and small-group tutoring provided the best outcome for students struggling with word problems. Whole-class instruction was beneficial alone as was small-group tutoring; however, the combination proved better than one or the other.

Two other studies in the Fuchs's program of research (Fuchs et al., 2009; Fuchs, Seethaler, et al., 2008) rely on schema-broadening instruction but with problem types (i.e., change, total, and difference) that parallel those used by Jitendra and colleagues (i.e., change, combine, and compare). In these tutoring studies (conducted on a one-to-one basis), however, students were also explicitly taught to set up and solve mathematical equations that represent the underlying schema of the word problems similar to Griffin and Jitendra (2009), Jitendra, Griffin, Deatline-Buchman, et al. (2007), and Jitendra, Griffin, Haria, et al. (2009). In a pilot study, Fuchs, Seethaler, et al. (2008) randomly assigned 35 third-grade students at-risk for or with LD to two conditions: schema-broadening instruction tutoring with mathematical equations or no-tutoring control. All students performed below the 26<sup>th</sup> percentile on global math and reading tests. Students in the schema-broadening condition received individual instruction over 12 weeks with sessions conducted 3 times a week, 30 min per session. Instruction focused on the three problem types with three transfer features (irrelevant information, important information embedded within charts, graphs, or pictures, and doubledigit numbers). First, students learned to understand and identify the three schemas (i.e., problem types), to set up an equation to represent each schema (i.e., 3 + X = 9), and to solve equations. Then, explicit instruction to broaden schema to the three transfer features occurred. Students receiving schema-broadening tutoring demonstrated significantly better growth than control students on an experimenter-designed test of word problems (ES = 1.80) and on a test of word problems designed by a research team not affiliated with the study (ES = 0.69). On a standardized test of problem solving, however, there were no significant differences.

Expanding the pilot study to focus on the effects of treatment as a function of difficulty subtype (i.e., students at-risk for or with mathematics LD alone versus students at-risk for or with mathematics and reading LD) and controlling for tutoring time with a contrasting math tutoring condition, Fuchs et al. (2009) randomly assigned 133 third-grade students, blocking by difficulty subtype and by site (i.e., Nashville vs. Houston) to three conditions: number combinations tutoring, schema-broadening word-problem tutoring, or no-tutoring control. Students in the two tutoring conditions received individual tutoring on word problems or on number combinations 3 times a week for 15 weeks, each time for 20 to 30 min. Wordproblem tutoring relied on schema-broadening instruction with mathematical equations similar to Fuchs, Seethaler, et al. (2008). Growth from pre- to posttest on an experimenterdesigned word-problem measure, including problems that required transfer, indicated that students in word-problem tutoring significantly outperformed students in numbercombinations tutoring and in the control group (ESs = 0.83 and 0.79, respectively). On a standardized test of problem solving, students in word-problem tutoring significantly outperformed students in the control group (ES = 0.28). Additionally, difficulty subtype did not moderate the effect of schema-broadening instruction with equations. That is, students at-risk for or with mathematics and reading LD and students at-risk for mathematics without reading LD responded comparably well to the treatments.

Expanding beyond whole-class, schema-broadening instruction to incorporate mathematical equations, the research conducted by Fuchs et al. (2009) and Fuchs, Seethaler, et al. (2008) revealed how students at-risk for or with LD may benefit from tutoring that combines

schema-broadening instruction with instruction on setting up and solving addition and subtraction mathematical equations. Because students did not receive concurrent whole-class instruction and individual word-problem tutoring as in Fuchs, Fuchs, Craddock, et al. (2008), future research may investigate the added value of such a combination with schema-broadening plus mathematical equations instruction provided at the whole-class and small-group or individual tutoring levels.

#### A Framework for Teaching Word Problems in the Primary Grades

Across the two lines of work applying schema theory to word-problem solving, Jitendra and colleagues and Fuchs and colleagues provide evidence that students, including those at-risk for or with LD, may benefit from this explicit approach to word-problem instruction at the classroom and tutoring levels. In the schema-based instruction of Jitendra and colleagues, students learned to use schematic diagrams to solve word problems. In Jitendra's more recent research, students also learned to set up and solve an equation to find the word-problem answer after filling in a schematic diagram. The schema-broadening instruction of Fuchs and colleagues incorporated explicit schema instruction about word problem transfer features so students could learn how to recognize novel problems as belonging to the schemas they learned, but without reliance on schematic diagrams. Additionally, in the schema-broadening instruction of Fuchs et al. (2009) and Fuchs, Seethaler, et al. (2008), students learned to use mathematical equations to represent the structure of a word problem.

#### Limitations

Before proceeding, it is important to discuss a few limitations across the two lines of schema work. First, and perhaps most importantly, many of the measures used to determine treatment effects were designed by the experimenters conducting the research. Some measures included word problems almost identical to those presented to students during instruction, perhaps raising questions about the generalizability of the word-problem instruction. When standardized tests of problem solving were administered, effects were either not significant or not as large as on the experimenter-designed measures. Second, a few of the studies, especially those by Fuchs and colleagues, included students at-risk for LD and not necessarily students with identified LD. Some of these students at-risk for LD received special education services; most did not. Within these studies for students at-risk for LD, the results for students with disabilities were not disaggregated from the primary sample. The same holds true for some of the other studies conducted by both Fuchs and colleagues and Jitendra and colleagues (Fuchs et al., 2006; Fuchs et al., 2009; Fuchs, Fuchs, Craddock, et al., 2008; Fuchs, Seethaler, et al., 2008; Griffin & Jitendra, 2009). Therefore, it remains unclear if the interventions benefit students at-risk for LD, students with LD, or both.

#### Implications for Practice

Other schema investigations (e.g., Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Hoff, & Beck, 1999; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009) suggest using schema to solve word problems in the intermediate grades. That aside, the focus of the present literature review was on the primary grades, where the literature provides the basis for conceptualizing a framework for teaching word problems to students at-risk for or with LD that comprises the following features. First, instruction should be explicit. Across the two lines of schema work, schema were introduced in an explicit manner, and teachers or tutors often modeled or provided worked examples of word problems using each schema. It is not surprising that students at-risk for or with LD benefitted from explicit instruction, given that other mathematics researchers not focused on schema instruction (e.g., Kroesbergen et al., 2004; Mercer, Jordan, & Miller, 1996) have demonstrated the benefits of

explicit instruction for students at-risk for or with LD. Across all the schema studies at the primary grades, students learned one word-problem schema at a time and had adequate practice (i.e., for days or weeks) on the schema before learning another schema.

Next, word-problem instruction should be organized. Students with LD profited from organizing word problems via schemas and having an explicit method for conceptualizing their solutions for each schema. This solution method could be a schematic diagram (Jitendra et al., 1998), a mathematical equation (Fuchs et al., 2009; Griffin & Jitendra, 2009), or a way of organizing information (Fuchs, Fuchs, Finelli, et al., 2004). Because methods that work for one student may not work for another, it is important that teachers familiarize themselves with various explicit methods for helping students learn schema approaches to word problems so they can best help their students.

A schematic diagram may help some students organize their word-problem work, as in Figure 1. With a schematic diagram, students fill in the relevant numbers from the word problem. The area of the schematic diagram that represents the question of the word problem (i.e., the word-problem solution) is left blank or filled in with a question mark. Students then learn how to solve for the blank space in the diagram to solve the word problem by calculating the answer. For example, the following is a typical elementary-school word problem: A classroom has 15 students. If 6 of the students are boys, how many students are girls? Using the schemas employed by Jitendra et al. (1998), this word problem falls under the "group" schema because there is a larger set (i.e., the classroom) with smaller sets (i.e., boys and girls) within the larger set. The larger set (15) and one of the smaller sets (6) are defined within the text of the word problem. The other smaller set is the missing information needed to answer the word-problem question. After a student selects the word-problem schema (i.e., group), they fill in a schematic diagram. The schematic diagrams assist in organizing the word-problem information in pictorial fashion, which, as Jitendra and colleagues have demonstrated, may be beneficial for students at-risk for or with LD.

Another approach that may assist students in organizing word-problem work is to decide on a problem's schema and then use a mathematical equation to represent the underlying structure of the schema. Working with the word problem just discussed: A classroom has 15 students. If 6 of the students are boys, how many students are girls?, Fuchs et al. (2009) categorized this problem as falling within a "total" schema. In a "total" schema, parts are put together for a total. Instead of using a schematic diagram, students are taught a mathematical equation (i.e., P1 + P2 = T) that represents the two parts (i.e., P1, P2) put together for a total (i.e., T). After students decide the word problem's schema, they write the mathematical equation to help organize their word-problem work. (See Figure 2 for a worked example.) Students fill in the relevant numbers from the word problem and write an X for the missing part of the equation. Students then solve for X (i.e., solve the word-problem question). As demonstrated by Fuchs and Jitendra, using an equation to represent the schema is also an effective approach for strengthening the word-problem skill of students at-risk for or with LD.

For more difficult word problems, Fuchs and colleagues (e.g., Fuchs, Fuchs, Finelli, et al., 2004) have demonstrated how students can organize their word-problem work without a schematic diagram or mathematical equation, while relying on their knowledge of the problem schema. The word problem, *Maya wants to buy 2 bags of pencils for \$3 each, 4 notebooks for \$2 each, and 6 folders for \$1 each. How much will Maya spend?*, would fall under the "shopping list" schema because multiple items of various prices are purchased. Instead of using a schematic diagram or equation for a "shopping list" problem, students draw vertical lines on their paper to organize their work. (See Figure 3 for a worked example.) Students calculate one part or step of the shopping list problem (i.e., pencils,

notebooks, folders) in each section and calculate the overall cost of items in the right-most section. The vertical lines assist students in organizing the word-problem information and their work, but drawing the lines is not a necessity. That is, students could use their knowledge of schemas to solve the word problem with or without the lines.

Several other dimensions of a word-problem teaching framework using schema theory also emerge across the two lines of schema. These include practice in sorting word problems into schema, many instructional sessions, and multiple settings (i.e., whole class, small group, individual) for schema instruction to occur. With regard to sorting word problems into schema, both Jitendra and colleagues and Fuchs and colleagues made a point of mixing word-problem types so students had to differentiate what problems belonged to which schema. Some of this practice was explicit via flash cards (Fuchs, Seethaler, et al., 2008). Some of the schema identification practice was embedded within the lesson, whereby teachers and tutors presented students with word problems, students had to decide which schema represented the word problem, and then use the structure of the schema to solve the word problem.

Across schema studies, the number of instructional sessions varied from 13 to 45 sessions. For all studies but one, students were taught or tutored multiple times each week, and students demonstrated significant gains from the schema instruction. Only the results of Griffin and Jitendra (2009) proved disappointing, with the absence of significant differences between the schema-based instruction and a comparison group. The authors attributed this lack of significance to the fact that instruction was provided once a week instead of multiple times each week. The significant results from the other 11 studies highlighted in this review suggest that instruction should be of sufficient duration (i.e., weeks and months, not days) and occur multiple times each week.

Finally, in the body of work reviewed in this paper, schema instruction occurred in whole-class, small-group tutoring, and individual tutoring settings. Students at-risk for or with LD benefitted from the schema instruction in all three of these settings. One study (Fuchs, Fuchs, Craddock, et al., 2008) isolated the effects of conducted schema instruction provided in whole-class arrangement versus small-group tutoring settings. They concluded that the combination of both whole-class teaching and small-group tutoring may optimally enhance outcomes for students at risk for LD and that tutoring was essential for promoting strong outcomes. Teachers, therefore, should be mindful that whole-class instruction may not be enough for students with or at risk for LD, and additional tutoring (i.e., Tier 2 or 3 within a Response to Intervention framework) may be necessary to improve the word-problem outcomes of students with LD.

The linking features of these two schema approaches require students to (a) read a word problem, (b) recognize the underlying structure of the word problem as belonging to a specific schema, and (c) solve the word problem using a solution method that represents a schema. Whether students use schematic diagrams, mathematical equations, or another method to help them apply their knowledge of the word-problem schema, the research conducted by Jitendra and colleagues and Fuchs and colleagues demonstrates that students at-risk for or with LD may benefit from explicit word-problem instruction that incorporates schemas.

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#### References

Case LP, Harris KR, Graham S. Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. Journal of Special Education. 1992; 26:1–19

- Cooper G, Sweller J. Effects of schema acquisition and rule automation on mathematical problem-solving transfer. Journal of Educational Psychology. 1987; 79:347–362.
- Cummins DD, Kintsch W, Reusser K, Weimer R. The role of understanding in word problems. Cognitive Psychology. 1988; 20:405–438.
- Englert CS, Culatta BE, Horn DG. Influence of irrelevant information in addition word problems on problem solving. Learning Disability Quarterly. 1987; 10:29–36.
- Fletcher, JM.; Lyon, GR.; Fuchs, LS.; Barnes, MA. Learning disabilities: From identification to intervention. New York, NY: Guilford Press; 2006.
- Fuchs LS, Fuchs D. Mathematical problem-solving profiles of students with mathematics disabilities with and without comorbid reading disabilities. Journal of Learning Disabilities. 2002; 35:563–573. [PubMed: 15493253]
- Fuchs LS, Fuchs D, Craddock C, Hollenbeck KN, Hamlett CL, Schatschneider C. Effects of small-group tutoring with and without validated classroom instruction on at-risk students' math problem solving: Are two tiers of preventions better than one? Journal of Educational Psychology. 2008; 100:491–509. [PubMed: 19122881]
- Fuchs LS, Fuchs D, Finelli R, Courey SJ, Hamlett CL. Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. American Educational Research Journal. 2004; 41:419–445.
- Fuchs LS, Fuchs D, Finelli R, Courey SJ, Hamlett CL, Sones EM, Hope SK. Teaching third graders about real-life mathematical problem solving: A randomized control trial. The Elementary School Journal. 2006; 106:293–311.
- Fuchs LS, Fuchs D, Prentice K, Burch M, Hamlett CL, Owen R, et al. Jancek D. Explicitly teaching for transfer: Effects on third-grade students' mathematical problem solving. Journal of Educational Psychology. 2003; 95:293–305.
- Fuchs LS, Fuchs D, Prentice K, Hamlett CL, Finelli R, Courey SJ. Enhancing mathematical problem solving among third-grade students with schema-based instruction. Journal of Educational Psychology. 2004; 96:635–647.
- Fuchs LS, Fuchs D, Stuebing K, Fletcher J, Hamlett CL, Lambert W. Problem-solving and computational skill: Are they shared or distinct aspects of mathematical cognition? Journal of Educational Psychology. 2008; 100:30–47. [PubMed: 20057912]
- Fuchs LS, Powell SR, Seethaler PM, Cirino PT, Fletcher JM, Fuchs D, et al. Zumeta RO. Remediating number combination and word problem deficits among students with mathematics difficulties: A randomized control trial. Journal of Educational Psychology. 2009; 101:561–576. [PubMed: 19865600]
- Fuchs LS, Seethaler PS, Powell SR, Fuchs D, Hamlett CL, Fletcher JM. Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. Exceptional Children. 2008; 74:155–173. [PubMed: 20209074]
- Griffin CC, Jitendra AK. Word problem-solving instruction in inclusive third-grade classrooms. The Journal of Educational Research. 2009; 102:187–201.
- Hanich LB, Jordan NC, Kaplan D, Dick J. Performance across different areas of mathematical cognition in children with learning difficulties. Journal of Educational Psychology. 2001; 93:615– 626.
- Hudson, P.; Miller, SP. Designing and implementing mathematics instruction for students with diverse learning needs. Boston, MA: Allyn & Bacon; 2006.
- Jitendra A, DiPipi CM, Perron-Jones N. An exploratory study of schema-based word-problem-solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. The Journal of Special Education. 2002; 36:23–38.
- Jitendra AK, Griffin CC, Deatline-Buchman A, Sczesniak E. Mathematical problem solving in third-grade classrooms. Journal of Educational Research. 2007; 100:283–302.

Jitendra AK, Griffin CC, Haria P, Leh J, Adams A, Kaduvettoor A. A comparison of single and multiple strategy instruction on third-grade students' mathematical problem solving. Journal of Educational Psychology. 2007; 99:115–127.

- Jitendra AK, Griffin CC, McGoey K, Gardill MC, Bhat P, Riley T. Effects of mathematical word problem solving by students at risk or with mild disabilities. The Journal of Educational Research. 1998; 91:345–355.
- Jitendra AK, Hoff K. The effects of schema-based instruction on mathematical word-problem-solving performance of students with learning disabilities. Journal of Learning Disabilities. 1996; 29:422–431. [PubMed: 8763557]
- Jitendra AK, Hoff K, Beck MM. Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. Remedial and Special Education. 1999; 20:50–64.
- Jitendra A, Xin YP. Mathematical word problem-solving instruction for students with mild disabilities and students at risk for math failure: A research synthesis. The Journal of Special Education. 1997; 30:412–438.
- Jordan NC, Hanich LB. Mathematical thinking in second-grade children with different forms of LD. Journal of Learning Disabilities. 2000; 33:567–578. [PubMed: 15495398]
- Jordan NC, Montani TO. Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. Journal of Learning Disabilities. 1997; 30:624–634. [PubMed: 9364900]
- Kroesbergen EH, Van Luit JEH, Maas CTM. Effectiveness of explicit and constructivist mathematics instruction for low-achieving students in the Netherlands. The Elementary School Journal. 2004; 104:233–251
- Marshall, SP. Schemas in Problem Solving. New York, NY: Cambridge University Press; 1995.
- Mastropieri MA, Scruggs TE, Shiah R. Can computers teach problem-solving strategies to students with mild mental retardation? Remedial and Special Education. 1997; 18:157–165.
- Mercer CD, Jordan L, Miller SP. Constructivistic math instruction for diverse learners. Learning Disabilities Research & Practice. 1996; 11:147–156.
- Miller SP, Mercer CD. Mnemonics: Enhancing the math performance of students with learning difficulties. Intervention in School and Clinic. 1993; 29:78–82.
- Montague M, Warger C, Morgan TH. Solve it! Strategy instruction to improve mathematical problem solving. Learning Disabilities Research & Practice. 2000; 15:110–116.
- National Assessment Governing Board. National assessment of educational progress. Washington, DC: Institute of Education Sciences; 2009.
- National Council of Teachers of Mathematics. Principles and standards for school mathematics. Reston, VA: Author; 2000.
- Parmar RS, Cawley JF, Frazita RR. Word problem-solving by students with and without mild disabilities. Exceptional Children. 1996; 62:415–429.
- Pellegrino JW, Goldman SR. Information processing and elementary mathematics. Journal of Learning Disabilities. 1987; 20:23–32. 57. [PubMed: 3805888]
- Pólya, G. How to solve it. Princeton, NJ: Princeton University Press; 1945.
- Riley MS, Greeno JG. Developmental analysis of understanding language about quantities and of solving problems. Cognition and Instruction. 1988; 5:49–101.
- Schoenfeld, AH. Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: Grouws, D., editor. Handbook for research on mathematics teaching and learning. New York, NY: MacMillan; 1992. p. 334-370.
- Shavelson, RJ.; Webb, NM.; Stasz, C.; McArthur, D. Teaching mathematical problem solving: Insights from teachers and tutors. In: Charles, RI.; Silver, EA., editors. The teaching and assessing of mathematical problem solving. Reston, VA: The National council of Teachers of Mathematics, Inc.; 1988. p. 203-231.
- van Garderen D. Teaching students with LD to use diagrams to solve mathematical word problems. Journal of Learning Disabilities. 2007; 40:540–553. [PubMed: 18064979]
- Willis GB, Fuson KC. Teaching children to use schematic drawings to solve addition and subtraction word problems. Journal of Educational Psychology. 1988; 80:192–201.

Wilson CL, Sindelar PT. Direct instruction in math word problems: Students with learning disabilities. Exceptional Children. 1991; 57:512–519. [PubMed: 2070810]

- Xin YP, Jitendra AK, Deatline-Buchman A. Effects of mathematical word problem-solving instruction on middle-school students with learning problems. The Journal of Special Education. 2005; 39:181–192.
- Xin YP, Zhang D. Exploring a conceptual model-based approach to teaching situated word problems. The Journal of Educational Research. 2009; 102:427–441.

A classroom has 15 students. If 6 of the students are boys, how many students are girls?

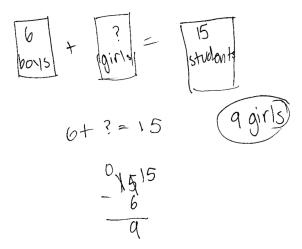


Figure 1. Worked example of group schema

A classroom has 15 students. If 6 of the students are boys, how many students are girls?

$$P \mid + P2 = T$$

$$b + x = 15$$

$$x = 9 \text{ girls}$$

Figure 2. Worked example of total schema

Maya wants to buy 2 bags of pencils for \$3 each, 4 notebooks for \$2 each, and 6 folders for \$1 each. How much will Maya spend?

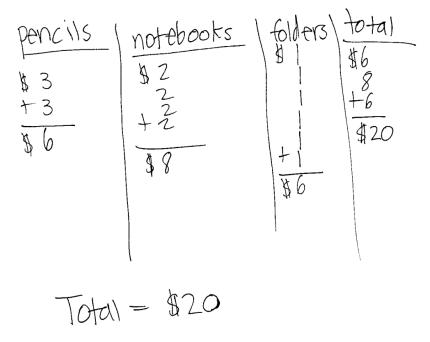


Figure 3. Worked example of shopping list schema

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## Table 1

# Description of Schema Studies

	ns: c > d (ES = 1.34) c > d (ES = 1.09) c > d (ES = 0.37)	disabilities: a > c (ES = 2.02); b > c (ES = 1.76) a > c (ES = 1.44); b > c (ES = 1.30) a > c (ES = 0.76); b > c (ES = 1.23) a > c (ES = 1.03); b > c (ES = 1.28) a > c (ES = 1.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c (ES = 1.45); b > a (ES = 0.03); b > c	a > c (ES = 5.97); b > c (ES = 5.33) a > c (ES = 6.84); b > c (ES = 3.59) a > c (ES = 0.90); b > c (ES = 2.40)	disabilities; nomesponsive; a. 20%, b. 60%, c. 9% nomesponsive; a. 60%, b. 60%, c. 18% nomesponsive; a. 20%, b. nomesponsive; a. 20%, c. 36%	n disabilities: a > c (ES = 1.55); b > c (ES = 1.79)
Outcomes	For al-risk students.	For students with disabilities:  2	For all students:	For students with disabilities.  1 60%, c. 3% 2 nonrespons 60%, c. 18 3 nonrespons b 80%, c. 18	For students with disabilities:  1 a > c (ES = 1.79)
Assessments	Experimenter-designed:  1 immediate transfer (10 problems)  2 near transfer (9 problems)  3 für transfer (4 problems)	Experimenter-designed:  1 transfer (10 problems)  2 transfer (7 problems)  3 transfer (2 problems)  4 transfer (1 problem)	Experimenter-designed:  I immediate transfer (10 problems)  2 near transfer (9 problems) 3 far transfer (4 problems)	Experimenter-designed:  Inmediate transfer (10 problems)  2 near transfer (7 problems)  3 far transfer (1 problem)	Experimenter designed:
Instruction	a. explicit schemu instruction     b. general problem-solving     strategies     c. explicit schema instruction     d. NA	a. explicit schema instruction b. treatment (a) + real-life problems c. NA	a. explicit schema instruction b. (a) treatment + real-life videos c. general problem-solving strategies frategies	a. explicit schema instruction b. part of (a) treatment + transfer instruction c. all of (a) treatment + transfer instruction d. NA	explicit schema instruction     treament (a) + practice     sorting problem types
Word-problem types Instr	shopping list half huying bags pictograph	shopping list half buying bags pictograph	shoping list half buying bags pictograph	shoping list half buying bags pictograph	shopping list half buying bags pictograph
Setting/Duration	Classtoom teaching:  a. 30 lessons: 30-40 min; 2 days/wk b. 6 lessons: 30-40 min; 2 days/wk Small-group tutoring: c. 35 lessons: 2 20-30 mins; 3 days/wk d none	Classroom teaching:  a. 34 lessons; 25-40 min; 2 days/wk b. 34 lessons; 25-40 min; 2 days/wk c. none	Classroom teaching:  a. 36 lessons; 2.5-40 mm; b. 36 lessons; 25-40 mm; 25-40 mm; c. 6 lessons; c. 6 lessons; days/wk days/wk days/wk	Classroom teaching:  a. 25 lessons: 25-40 min; 2 disys/wk b. 26 lessons: 25-50 min; 2 disys/wk c. 35 lessons: 25-40 min; 2 disys/wk disys/wk A. NA	Classroom teaching:
Grade	м	м	м	м	e
Special education participants	A risk (identified by researcher):  c $n = 164$ students  d $n = 79$ students	SPED (identified by school):  a. 7 LD; 0 MR; 1 BD; 1 other b. 5 LD; 0 MR; 0 BD; 2 other c. 10 LD; 1 MR; 0	SPED (identified by school):  a.  n = 10  b.  n = 12  c.  n = 12	SPED (identified by school)  a. $n = 5$ b. $n = 3$ c. $n = 9$ d. $n = 6$	SPED (identified by school):  a. 7LD; 4 Speech; 0 ADHD
Participants	N = 110 class grooms (2023 students)  a. $n = 79$ class grooms  b. $n = 40$ class frooms  c. $n = 164$ students  d. $n = 79$ students	N = 24 chastrooms (351 students)  a. n = 8 chastrooms b. n = 8 chastrooms c. n = 8 chastrooms	N = 30 classrooms (445 students)  a.	N = 24 chastrooms (375 students)  a. $n = 6$ chastrooms  b. $n = 6$ chastrooms  c. $n = 6$ chastrooms  d. $n = 6$ chastrooms	N = 24 classrooms (366 surdents)  a. $n = 8$
Study design	Classrooms randomly assigned:  a. schema instruction b. control Within classrooms, at-risk eundomly assigned: c. schema tutoring d. control	Classrooms randomly assigned:  a. schema schema instruction + 3 transfer features b. schema instruction + 6 transfer features c. control	Classrooms randomly assigned:  a. schema instruction  b. schema instruction tradiistruction real-life problems  c. control	Classrooms randomly assigned:  a. schema solution instruction b. partial solution plus transfer c. full solution plus transfer d. control	Classrooms randomly assigned to:  a. schema instruction
Use of equations	01	00	ou li	OI OI	по
Schema approach	broadening	broudening	broadening	broadening	broadening
Authors	Fuchs, Fuchs, Craddock, Hollenbeck, Shandert, & Shandert, & Cooks)	Fuchs, Fuchs, Freili, Courey, Freili, Courey, (2004)	Fuchs, Fuchs, Finelli, Courey, Hamlet, Sones, & Hope (2006)	Fuchs, Fuchs, Pentice, Burch, Hamlett, Overen, Hosp, & Janeck (2003)	Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey (2004)

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	3; b > c 7; b > c	); a > b s) ifferences	9) ifferences	ifferences 5)	o 95%; 26	ifferences 1 non-LD	3)
	a> c (ES = 1.78); b> c (ES = 1.96) a> c (ES = 1.04)	a > c (ES = 0.79); a > b (ES = 0.83) a > c (ES = 0.28) no significant differences	a > b (ES = 1.80) a > b (ES = 0.69) no significant differences	no significant differences a > b (ES = 0.35)	Improvement from baseline to intervention: 1 20 to 90%; 31 to 95%; 26 to 95%	n disabilities: no significant differences from non-LD non significant differences from non-LD	a > b (ES = 0.52)
Outcomes	0 K	For all students:	For all surdens:	For all students:	Improvement froi	For students with disabilities.  1 from non-L  2 non significandificants of the significants of the significants of the significants of the significant of the signif	-
nents	immediate transfer (10 problems)  mear transfer (7 problems) far transfer (4 problems)	Experimenter-designed:  1 word problems (18 problems) (18 problems) 2 KeyMath Problem Solving 3 ITBS Problem Solving	Experimenter designed:  1 word problems (18 problems) (18 problems) designed: 2 word problems (14 problems) Standardized: 3 KeyMath Problem Solving	Experimenter designed:  1 word problems (16 problems)  2 word-problem fluency (8 problems)	Experimenter designed:  1 word-problem probes (15 problems)	Experimenter designed:  1 voord problems (25 problems)  2 word-problem fluency (16 problems)	Experimenter designed:  1 word problems (16 problems)
Assessments			Experimenter designed:  2  Sundardized: 3	Experin	Experin	Experin	Experim
Instruction	c. general problem-solving strategies	a. explicit schema instruction b. explicit number-combinations instruction c. NA	a. explicit schemu instruction b. NA	a. explicit schemu instruction     b. general problem-solving strategies	a. explicit schema instruction	a. explicit schemu instruction	explicit schema instruction     general problem-solving strategies
Word-problem types		total difference change	total difference change	change group compare	change group compare	change group compare	change group compare
Setting/Duration	a. 30 lessons; 25-40 min; 2 days/wk b. 30 lessons; 25-40 min; 2 days/wk c. 61 lessons; 30-40 min; 2 days/wk	Individual tutoring:  a. 48 lessons; 20-30 min; 3 20-30 min; 3 20-30 min; 3 20-30 min; 3 co. none	Individual tutoring:  a. 36 lessons; 30  min; 3 days/wk  b. none	Teaching in groups of 15:  a. 20 lessons; 100 min; 1 daywk b. 20 lessons; 100 min; 1 daywk	Individual instruction:  a. 13-16 lessons; 40-45 min	Classroom teaching:  a. 45 lessons; 30 mins; 3 days/	Teaching in groups of 15:  a. 411essons; 25 min; 5 days/wk
Grade		м	м	m	4,6	m	e
Special education participants	b. 71D;2 Speech; 0 ADHD c. 6 LD;4 Speech; 2 ADHD	MD (identified by researcher): $n = 73$ MNRD (identified by researcher): $n = 60$	MDRD (identified by researcher): $\mathbf{a}, \qquad n=16$ $\mathbf{b}, \qquad n=19$	LD (identified by school): $\mathbf{a}, \qquad n=3$ $\mathbf{b}, \qquad n=2$	LD (identified by school) a. $n=3$	LD (identified by school): $\mathbf{a.} \qquad n=9$	LD (identified by school): $\mathbf{a.} \qquad n=2$
Participants	20 00 	N = 133 students a. $n = 42$ b. $n = 44$ c. $n = 47$	N = 35 sudents a. $n = 16$ b. $n = 19$	N = 60 students a. n = 30 b. n = 30	N = 3 students	N = 38 students a. $n = 38$	N = 88 students a. $n = 45$
Study design	b. schema internetion + sorting c. control	Students randomly assigned:  a. schema instruction b. number: c. control	Students randomly assigned:  a. schema instruction  b. control	Matched pairs randomly assigned:  a. schema instruction  b. general-strategies instruction	Students selected: a. schema instruction	All studens participued: a. schema instruction	Matched pairs randomly assigned:  a. schema instruction
Use of equations		ye	yes	yes	ои	yes	yes
Schema approach		broadening	broadening	pased	based	based	based
Authors		Fachs, Powell, Sexblact, Crino, Fletcher, Fachs, Hamlet, & Zumeta (2009)	Fuchs, Seelnater, Pewell, Fuchs, Hamlett, & Fletcher (2008)	Griffin & Jitendra (2009)	Jitendra & Hoff (1996)	Jitendra, Griffin, Deatline- Buchman, & Sezesniak (2007)	Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor

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Authors	Schema approach	Schema approach Use of equations Study design		Participants	Special education participants	Grade	Setting/Duration	Word-problem types Instruction	Instruction		Assessments	Outcomes
			b, general-strategies instruction				b, 411essons; 25 min; 5 days/wk					
Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)	pasaq	по	Students randomly assigned:  a. schema instruction  b. general-strategies instruction	N = 34 sundents a. $n = 17$ b. $n = 17$	SPED (identified by school):  a. 8 LD; 2 MR; 2 ED; 5 arrisk b. 9 LD; 3 NR; 1 ED; 4 arrisk	5,3,4,	Smalt-group tutoring (3-6 students):  a. 17-20 kessons; 40-45 min  b. 17-20 kessons; 40-45 min	change group compare	ė ė	explicit schema instruction general problem-solving strategies	Experimenter designed:  1 problems)  2 dehydettet (15 problems)  3 maintennee test (15 problems)	For all students:

 $^{\it I}$  Only significant effects are reported.

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