

# The Effects of a Tier 3 Intervention on the Mathematics Performance of Second Grade Students With Severe Mathematics Difficulties

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## Abstract

The purpose of this study was to determine the effectiveness of a systematic, explicit, intensive Tier 3 (tertiary) intervention on the mathematics performance of students in second grade with severe mathematics difficulties. A multiple-baseline design across groups of participants showed improved mathematics performance on number and operations concepts and procedures, which are the foundation for later mathematics success. In the previous year, 12 participants had experienced two doses (first and second semesters) of a Tier 2 intervention. In second grade, the participants continued to demonstrate low performance, falling below the 10th percentile on a researcher-designed universal screener and below the 16th percentile on a distal measure, thus qualifying for the intensive intervention. A project interventionist, who met with the students 5 days a week for 10 weeks (9 weeks for one group), conducted the intensive intervention. The intervention employed more intensive instructional design features than the previous Tier 2 secondary instruction, and also included weekly games to reinforce concepts and skills from the lessons. Spring results showed significantly improved mathematics performance (scoring at or above the 25th percentile) for most of the students, thus making them eligible to exit the Tier 3 intervention.

## Keywords

intensive intervention, number, operation, algebraic thinking, severe mathematics difficulties, tertiary intervention, Tier 3

Studies have shown 5% to 10% of the school-aged population is classified as having persistent low achievement in mathematics (Berch & Mazzocco, 2007; Geary, 2011; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), including students who have been identified as having a mathematics learning disability (MLD; Murphy, Mazzocco, Hanich, & Early, 2007). Unfortunately, according to National Assessment of Educational Progress (2013) findings on mathematics achievement, students who struggle in mathematics continue to demonstrate the greatest lag, as evidenced by low levels of performance. For example, fourth grade findings indicated a persistent problem of underachievement (i.e., scoring slightly at or below the basic level) for students who are chronically low achievers in mathematics. This is alarming because of the hierarchical nature of mathematics curriculum and the need to learn foundational knowledge in the primary grades. Thus, because a significant number of students demonstrate poor mathematics performance (Swanson & Jerman, 2006),

which is pervasive with potentially long-term consequences in later mathematic performance (Bryant et al., 2008; Geary, 2004, 2011; Murphy et al., 2007), early intervention is necessary for students who demonstrate the greatest need. It is important to examine the difficulties manifested by younger students in foundational concepts and skills so that appropriate, relevant interventions can be developed and implemented to bolster mathematical knowledge (National Mathematics Advisory Panel [NMAP], 2008).

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## **Mathematics Cognition and Young Students With Mathematics Difficulties**

Findings from empirical research on mathematics cognition (e.g., L. Fuchs et al., 2010; Geary, 1993, 2004, 2011; Geary et al., 2009; Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, & Hanich, 2002) have informed the field of the difficulties primary level students (e.g., students who scored below the 25th percentile on a mathematics measure) exhibit in mathematical competencies related to foundational concepts and skills that are necessary for later mathematics success including, for example, numerical knowledge and rapid retrieval of basic facts (L. Fuchs et al., 2005; Geary, 2004, 2011; Jordan et al., 2003).

Numerical knowledge competencies refer to “a core system of interrelated quantitative competencies” (Geary, 2011, p. 253) related to the number sense system (Butterworth & Reigosa, 2007). “Number sense” is defined as “moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value” (National Council of Teachers of Mathematics [NCTM], 2000, p. 79). Numerical knowledge competencies are viewed as a general cognitive skill composed of discerning (i.e., larger and smaller quantities) and ordering quantities, counting, and solving simple addition and subtraction facts (e.g.,  $2 + 3 = ?$ ,  $4 - 2 = ?$ ; Clements, 2008; Geary, 2011; Jordan, Kaplan, Olah, & Locuniak, 2006). Overall, research findings have shown that cognitive development problems are manifested in difficulties with understanding number knowledge and relationships in young students. Butterworth and Reigosa (2007) and Geary (2011) hypothesized that students may have developmental delays in number systems (e.g., understanding numerical magnitude), which may contribute to deficits in number processing fluency, rapid retrieval of basic facts, and understanding the base 10 system (Geary, 2011; Geary et al., 2007; Jordan et al., 2003; Jordan et al., 2006).

Research findings on basic facts have shown that primary aged students with mathematics difficulties exhibit procedural errors and immature counting strategies at a higher rate than typically achieving students (Geary, 2011). According to Siegler and Shrager (1984), children with low levels of numerical knowledge or number sense make significantly more errors using strategies (e.g., counting all, counting on, and decomposition) than typically performing students. Geary (2004) also found evidence for the use of developmentally immature counting strategies, such as using fingers to solve simple facts, as compared to typically achieving peers. Evidence supports a relationship “in part to the intrusion of related but task-irrelevant information into working memory” when students are solving basic facts (Geary, 2011, p. 256). Thus, persistent problems with rapid retrieval of basic facts are identified as a hallmark of math-

ematics difficulties and characteristic of developmental differences (Geary, 2004; Gersten, Jordan, & Flojo, 2005).

In sum, young students with persistent mathematics difficulties demonstrate developmental delays in their numerical knowledge and abilities with basic facts. For a group of these students, within a response to intervention (RtI) multitiered system of instruction, they continue to demonstrate poor mathematics performance despite primary intervention (Tier 1) and intensive, secondary intervention (Tier 2). These are the students with the most severe difficulties, which warrant increased intensive, tertiary intervention (Tier 3; D. Fuchs, McMaster, Fuchs, & Al Otaiba, (2013).

## **Conceptual Framework for Intensifying Instruction for Young Students With Severe Mathematics Difficulties**

For students who repeatedly fail to respond to evidence-based intervention, a tertiary level of mathematics intervention, which intensifies empirically based instructional design and delivery features is essential (Bryant et al., 2011; Dyson, Jordan, & Glutting, 2013; L. Fuchs et al., 2008).

Coyne, Kame'enui, and Carnine (2011) identified evidence-based principles of explicit, systematic instruction that should frame intervention work for struggling students. For example, focusing on key content, making learning strategies obvious, scaffolding instruction, and purposefully crafting review opportunities are exemplars of carefully designed interventions. Moreover, providing multiple opportunities for students to respond to appropriately paced instruction supports active engagement. These instructional practices are well grounded in the literature as critical for struggling students (Baker, Gersten, & Lee, 2002; Gersten, Beckmann, et al., 2009; Gersten, Chard, et al., 2009; NMAP, 2008).

Yet researchers are challenged by those students who continue to show minimal response to Tier 2 interventions as well designed as they may be. Consequently, studies have been conducted on examining the effects of intensifying Tier 2 variables for low responders. For example, Wanzek and Vaughn (2007) found in their review of early reading interventions that increasing the duration and/or length of the intervention and teaching in smaller groups were characteristic of Tier 3 interventions for students with severe reading difficulties (reading disabilities). Regarding mathematics, however, a search of the literature revealed no early numeracy studies that focused specifically on Tier 3 identified students (i.e., student percentile rankings that was indicative of severe, unresponsive achievement). Although there have been intervention studies on young students with mathematics difficulties (e.g., Doabler et al., in press; Dyson et al., 2013; L. Fuchs et al., 2006), typically the cut

score for identification purposes was such that a range of low performance (e.g., below the 25th or 35th percentile) was evident rather than focusing exclusively on the “lowest of low” students and Tier 2 was the focus of the study. Thus, we turned our attention to recommendations for structuring Tier 3 interventions.

D. Fuchs et al. (2013) suggested that Tier 3 not only should focus on “quantitatively” different approaches but also should include features that would be “qualitatively” different (e.g., instructional design and delivery, response opportunities, skills-based instruction with cognitive training). For example, with instructional design, instructional tasks could be broken down into smaller units and carefully sequenced, thus controlling for task difficulty and reducing “cognitive load,” much like in reading, as a means for optimizing cognitive capacity for learning (Sweller, 2005). Moreover, activities to activate prior mathematical understandings from previous lessons and using a concrete–semi-concrete–abstract instructional routine, conceivably, could facilitate important connections across mathematical ideas and provide an even more structured approach, beyond Tier 2, for intervention, respectively (Gersten, Chard, et al., 2009; Miller & Mercer, 1993; Swanson, Hoskyn, & Lee, 1999).

The type of intervention, which is employed, is another consideration. That is, the determination must be made whether to administer a standard protocol, which is common for Tier 2, or to utilize an individualized approach, which allows for more manipulation of instructional variables in response to student learning. For instance, Vaughn et al. (2011) in their research on the effects of a standardized versus individualized approach to Tier 3 intervention for students with severe reading difficulties (reading disabilities) found stronger treatment effects for the standardized approach. Although these results reside in the reading domain, these features of instructional design and delivery are relevant across domains (i.e., mathematics).

Finally, Swanson et al. (1999), in their meta-analysis of mathematics intervention studies for students with learning disabilities, identified the positive contribution not only of explicit instruction but also of strategic instruction (e.g., cognitive strategy training), which focuses on the teaching of rules and the process of learning including metacognitive cues and the use of mnemonics for memory retention and retrieval (Swanson & Sachse-Lee, 2001). Moreover, Swanson et al. found that a combined model, that is, utilizing the features from both explicit and strategic instruction, “increase[d] the predictive power of treatment effectiveness beyond what can be predicted by variations in methodology and age” (p. 218).

Thus, collectively, the purpose of this study was to determine the effects of an intensive, Tier 3 intervention for students who had previously received two rounds of Tier 2 intervention resulting in inadequate mathematics performance. These students had severe mathematics difficulties

according to their performance on mathematics measures and were considered to be eligible for Tier 3 intervention. In some states, Tier 3 is synonymous for special education; that was not the case in this present study. However, because these students had significant learning problems, intensified instructional design and delivery features, taken from learning the special education (e.g., Swanson et al., 1999), were employed. This study was guided by the following research question: What are the effects of a systematic, explicit, strategic Tier 3 (tertiary) intervention on the mathematics performance of students in the second grade who have identified severe mathematics difficulties?

## Method

### Participants

Twelve second grade students (3 boys and 9 girls) from three schools with similar demographics in a suburban school district in central Texas participated in this study. As first-graders, all of these students had received two 10-week rounds of Tier 2 standard protocol (scripted) secondary intervention (see Bryant et al., 2008). Yet, despite this intervention intensity (i.e., duration of treatment) in first grade, all of the students continued to fall at or below the 10th percentile on a proximal measure and below the 16th percentile on a distal measure. These percentile rankings are indicative of chronically severe unresponsive performance, which places the students at risk for further mathematics difficulties as the curriculum advances in the upper elementary and middle grades.

In second grade, these students were assigned to groups based on homogeneity of instructional need and availability according to the general classroom schedule. Demographic information for the 12 students is provided in Table 1. Information on performance on mathematics measures, which are described in the Measures section, is in Table 2.

### Research Design

Although many single case design studies provide data on individual students, data also may be reported for small student groups (Horner et al., 2005), which is the approach taken in this study. A multiple-baseline across-subjects design was employed; the number of phase repetitions was within recommended best practice (Kratochwill et al., 2010).

As baseline probes, the four subtests of the *Texas Early Mathematics Inventory–Aim Checks* (TEMI-ACs; University of Texas System/Texas Education Agency, 2009; see description in the Measures section) were administered until a stable baseline was established (Horner et al., 2005). Intermittent data collection occurred in the baseline phase. In School 1, after the first four data points, data were collected twice more (total of 8 days). After the first four data

**Table 1.** Student Demographic Data.

School	Student	Age <sup>a</sup>	Gender	Ethnicity	Free or Reduced Lunch	English Language Learner
1	1	7, 4	Female	White	Yes	No
1	2	8, 0	Male	White	No	No
2	1	7, 5	Male	White	Yes	No
2	2	7, 3	Female	Black	No	No
2	3	7, 7	Female	Black	Yes	No
2	4	7, 6	Female	Hispanic	No	No
2	5	8, 4	Female	Hispanic	Yes	No
2	6	7, 3	Female	Black	Yes	Yes
3	1	7, 6	Male	Black	Yes	No
3	2	7, 2	Female	Black	Yes	No
3	3	7, 5	Female	Hispanic	Yes	Yes
3	4	7, 8	Female	Black	No	No

<sup>a</sup>Presented as years, months.

**Table 2.** Student Performance on Mathematics Measures.

School	Student	TEMI-PM Fall	KeyMath-3 Total: Pre	KeyMath-3 Application: Pre	KeyMath-3 Total: Post	KeyMath-3 Application: Post
1	1	81	82	80	106	102
1	2	81	70	71	96	95
2	1	79	75	82	101	95
2	2	78	71	75	91	92
2	3	76	74	78	75	75
2	4	76	69	70	82	80
2	5	69	75	69	86	86
2	6	65	76	77	94	97
3	1	65	80	82	104	100
3	2	74	77	82	96	97
3	3	72	83	90	96	92
3	4	65	80	82	94	95

Note. TEMI-PM = *Texas Early Mathematics Inventory–Progress Monitoring*.

points were collected at School 2, data were gathered three more times across 12 days. Following School 3's first four data points, data were collected four more times across a total of 18 school days.

During the baseline phase, data were collected to estimate trends and related patterns within and between groups. The intervention lasted 10 weeks for Schools 1 and 2 and 9 weeks for School 3. The TEMI-AC was administered weekly on Tuesday and Friday during the intervention phase. A maintenance phase lasted 2 weeks for each school. The TEMI-AC was administered each Tuesday, and a final Aim Check was given 2 weeks after the maintenance phase. Generalization was examined by administering a distal measure of mathematics following the maintenance phase.

## Measures

Several measures were administered as part of this study (see Table 2). All students were given the *Texas Early Mathematics Inventory–Progress Monitoring* (TEMI-PM; University of Texas System/Texas Education Agency, 2008a) and the *KeyMath-3* (Connolly, 2008) to determine whether they met the criteria for participation. The *KeyMath-3* also served as a generalization measure. The TEMI-AC (University of Texas System/Texas Education Agency, 2009) served as the dependent variable for the study (Horner et al., 2005) and was administered during the baseline, intervention, and maintenance phases of the study.

**TEMI-PM.** The TEMI-PM (University of Texas System/Texas Education Agency, 2008a) is a researcher-devised

measure that served as the universal screener for this study. The test was normed on more than 2,000 students across Texas, and its raw scores were converted to percentile ranks. The TEMI-PM has three alternate forms to allow for fall, winter, and spring testing. Alternate-form reliability of the total score exceeds .90. As a universal screener, the TEMI-PM has demonstrated sensitivity and specificity, and meets rigorous area under the curve standards for technical adequacy (University of Texas System/Texas Education Agency, 2008b).

**KeyMath-3 Diagnostic Assessment.** The *KeyMath-3* Diagnostic Assessment (DA; Connelly, 2007) is a norm-referenced measure of mathematical concepts and skills. It covers three broad content areas (Basic Concepts, Operations, and Applications) and contains 10 subtests. Three types of reliability estimates were reported for *KeyMath-3* DA: the internal consistency reliabilities were mostly in the .80s, the alternate-form reliability coefficients ranged from .74 to .92, and the test-retest reliability coefficients were in the mid-.90s.

**TEMI-AC.** The TEMI-AC (University of Texas System/Texas Education Agency, 2009), the dependent variable in this study is a researcher-devised measure that contains four 2-min fluency measures assessing Magnitude Comparisons (circle, from two numbers shown, the number that is less or circle both numbers if they are equal), Number Sequences (write the number that is missing from a three-number sequence), Place Value (write how many hundreds, tens, and ones pictorially depicted), and Addition-Subtraction Combinations (solve basic addition and subtraction facts). Knowledge of these number and operations skills and concepts helps students form a mathematics foundation of number sense, which is critical for later mathematics success (NCTM, 2008). The raw scores of the four measures are summed, yielding a total score that can be used to monitor student progress. The TEMI-AC has five alternate forms; alternate-form reliability of the Total Score exceeds .80 across all forms.

## Procedures

**Instructional content.** A Tier 3 intervention served as the independent variable (Horner et al., 2005) for this study. The intervention was adapted from a Tier 2 intervention (Bryant et al., 2008) that focused on number and operation skills and concepts. Booster lesson content focused on early numeracy concepts and procedures that are considered fundamental to later mathematics success (NCTM, 2008). The lessons included counting activities (e.g., counting sequence, counting principles) and activities that focused on comparing the magnitude of numbers and quantity and sequencing numbers. Also, activities were included to

develop an understanding of the base 10 system (e.g., partitioning and grouping of hundreds, tens, and ones; composing and decomposing numbers). Finally, activities (e.g., number families, part-part-whole relationships) were designed to help students develop a conceptual understanding of addition and related subtraction facts and the mathematical properties that can be used to solve these facts along; fluency-building activities were also contained as part of the total intervention.

**Instructional components.** Each lesson consisted of the following components: a Warm-Up (activating background knowledge by reviewing prerequisite concepts and skills and previously taught basic facts), Preview (providing an advance organizer), Modeled Practice (teaching the concepts and procedures while engaging students during instruction), Guided Practice (practicing as a group [choral and individual responding] with the interventionist), and Daily Check (for progress monitoring purposes; see Cuillos, SoRelle, Kim, Seo, & Bryant, 2011), which assessed the content in each lesson. At the beginning of the Daily Check activity, the teacher provided a review statement to help students make connections between the lesson and the Daily Check, then gave students 2 min to complete the items independently. In all, for Schools 1 and 2, 70 lessons were taught and 20 TEMI-AC items were administered during the 10-week intervention phase. For School 3, students were taught 63 lessons over 9 weeks and administered 18 TEMI-AC.

**Instructional design and delivery.** The booster lessons were standardized (scripted) based on features that are identified as critical for students who have identified mathematics difficulties; moreover, features were added to temporarily reduce the “cognitive load” (Sweller, 2005). Adaptations to the Tier 2 intervention, in which the students previously participated, to increase intensity occurred during the first semester of the school year; comparisons are explained between the Tier 2 and Tier 3 intervention. Dosage (i.e., duration [number of days each week] and increased rates of responding), group size, design and delivery, and progress monitoring features were targeted as features to intensify instruction.

The following features were incorporated into the lessons. First, in terms of dosage, the intervention was conducted second semester for a duration of 30-min sessions, 5 days per week for 10 weeks (9 weeks for School 3 because it was the end of the school year) by a certified, experienced mathematics interventionist. The previous Tier 2 intervention occurred 4 days per week for 20 min per session across 23 weeks. Second, grouping structures consisted of interventionist/student ratios of 1:2 or 1:3 at each school as opposed to previous grouping arrangements of 1:5.

Third, a combined instructional approach employing explicit, systematic teaching procedures and strategic instruction was implemented (Swanson et al., 1999). The Tier 2 lesson design was modified to include a review of previous material, increased checks for understanding, immediate error correction, and “purposeful” practice with both individual and choral responding to increase the dose and accuracy of responding (i.e., increased response rate). Practice was purposefully designed with examples and non-examples of concepts and concrete and visual representations to support learning. Also, strategies for learning number facts were transparent and taught explicitly and systematically with practice opportunities during the lesson and part of the review to facilitate acquisition and fluency. Finally, “teacher talk” was greatly reduced to simple questions after modeling focusing on activities such as how to use a strategy or how to use based-ten models to build numbers. Learning a strategy for addition involved teacher talk such as “What fact?” ( $+2 = 7 + 2 = 9$ ) or “What strategy?” (count on; 7 in my head, 8, 9;  $7 + 2 = 9$ ). Teacher talk for teaching place value focused on quantity, counting groups, and reading and writing numbers (e.g., “How many tens?” “Count it,” “Write it,” “What number?”).

Third, to reduce cognitive load, the instructional content was controlled over a 2-week instructional time period. Within each instructional 2-week time period, booster lessons focused on concepts and procedures related to number concepts, the base 10 system, and addition and subtraction facts. Particular emphasis was placed on those number concepts that proved problematic for students with severe mathematics difficulties (e.g., teen numbers, 0 as a place holder). Also, within each 2-week instructional period, a restricted number range was targeted, which represented a smaller “chunk” for second grade. For instance, numbers 0 to 999 composed the curriculum, but the instructional number range for a 2-week period included numbers 1 to 50. Number facts were sequenced from easier (e.g., addition facts  $+0$ ,  $+1$ ,  $+2$ ; subtraction facts  $-0$ ,  $-1$ ) to more difficult combinations (e.g., addition facts doubles  $+1$  [ $6 + 7$ ], make 10  $+ \text{more}$  [ $9 + 7$ ]; subtraction inverse facts). Addition and subtraction facts were taught separately with the easier combinations for both preceding the more difficult ones.

Fourth, the concrete–semiconcrete–abstract approach (Butler, Miller, Crehan, Babbit, & Pierce, 2003) was used to teach number concept and procedures, the base 10 system, and addition and related subtraction facts. This sequence, although utilized in the Tier 2 intervention, focused longer on the concrete–semiconcrete stages by providing carefully chosen materials for modeling for the concepts. For example, at the concrete level, base 10 models and counters were used, and for the semiconcrete level number lines and 100s charts were employed; models were withdrawn during the abstract/symbolic state. Finally, the stages were mixed so that students could manipulate base 10 models to represent

numbers, then read and write the numbers for the concrete model representations.

Fifth, scaffolds were built into the lessons and gradually faded. For example, when teaching a strategy to solve  $+3$  facts (e.g.,  $8 + 3 = 11$ ), three dots were placed next to the numeral three as a cue to count on 3. Another scaffold was tapping or making tallies three times to signify the count of 3. To ensure students were learning the facts, number sequencing, or place value, after guided practice the interventionist would say, “Teach me how to do this.” This technique served as a means to determine whether students could articulate their conceptual understanding of the process or procedure.

Sixth, increased progress monitoring (Daily Checks) was conducted from once to twice a week coupled with lessons. On Monday, Wednesday, and Thursday, two lessons were taught. On Tuesday, one lesson was taught, followed by a TEMI-AC administration; on Friday, students played a game designed to reinforce previously taught concepts. After the game, students were administered another TEMI-AC. Students were given 2 min to respond to four either oral or written problems to determine their understanding of the instruction on each booster lesson. The majority of students in the group had to demonstrate accuracy on three out of four of the problems to consider the lessons as successful for each day.

Seventh, on the last day of each week, additional practice was built into the Tier 3 intervention in a game format, which was designed to reinforce concepts taught each week. Studies have shown that a game-like format (i.e., board games) is effective in improving low performing students’ abilities in early numeracy concepts and skills (Siegler, 2009). Games occurred for 20 to 25 min; students were awarded prizes (motivational aspect of the instruction) based on the number of correct answers they gave to the mathematics problems on the game cards.

Finally, all of the students continued to receive core mathematics instruction, which consisted of textbook instruction along with working in groups. Instruction tended to be more of a student-centered, inquiry approach and minimal differentiation.

### *Fidelity of Implementation*

The interventionist was observed for four sessions during the 10-week intervention to assess the quality (i.e., fidelity) of implementation. Quality of implementation (QoI) indicators included the degree to which the interventionist followed scripted procedures throughout the lesson (i.e., Warm Up, Preview, Modeled Practice/Guided Practice, and Daily Check). Also, the QoI indicators examined elements of instruction (e.g., maintaining appropriate pacing, distributing opportunities among students to respond) and monitoring/managing (e.g., using a behavior management system

or plan, transitioning effectively from one lesson/activity to another).

Performance indicators were rated on a 4-point scale (1 = *poor*, 4 = *excellent*). In all cases, results on the QoI showed a rating of 4 for fidelity, which is a very high degree of fidelity in the implementation of the Tier 3 lessons.

### Social Validity

Cooper, Heron, and Heward (1987) described social validity as “behavioral analysis efforts that are effective in changing an individual’s life in a socially important way” (p. 56). Other researchers (Tawney & Gast, 1984; Wolf, 1978) suggested that there are three levels of social validation, goals, procedures, and effects that present three important questions.

1. Is a desired behavior change important in the social environment?
2. Is the intervention appropriate, humane, and the most efficient and least intrusive method to produce the desired outcome?
3. If a major change occurs, is life appreciably changed?

Regarding Item 1, there is little doubt that the importance of mathematical knowledge is unquestionable. For Item 2, the intervention targeted specific skills shown by testing to be in need of remediation and included validated instructional practices. All participating students and teachers were treated in accordance with university research policy regarding humane treatment of participants. Also, although the intervention focused on a “pull-out” model rather than the less intrusive “pull-in” approach, the “pull-out” practice has been found to be useful in intervention work (Bryant et al., 2008, 2011; L. Fuchs et al., 2008). Finally, for Item 3, given the strong correlation between early and later success, it can be predicted with a fair amount of confidence that successful skill attainment at a young age can positively influence later mathematics performance (Jordan, Kaplan, Ramineni, & Locuniak, 2009).

### Results

Typically, single case results are interpreted via a combination of visual inspection (Cooper et al., 1987; Parsonson & Baer, 1978) consisting of an analysis of factors including level, trend, variability, immediacy of effect, and overlap (Kratochwill et al., 2010) and effect size (Parker, Vannest, Davis, & Sauber, 2011). This section is organized in terms of findings from the visual analysis of the data (Figure 1), posttest of the distal measure, clinical effects analysis, and effect size analysis. Explanations of experimental control and external validity are also provided.

### Visual Analysis

Figure 1 displays the total scores averaged for each group on the TEMI-AC for School 1, School 2, and School 3. School 1’s scores (Figure 1, top panel) were initially low during baseline and continued at low and stable levels throughout the baseline condition ( $M = 63$ ; range = 61–65.5). School 1’s scores immediately increased on the implementation of the intervention ( $M = 109.8$ ; range = 81.5–138.5) and continued at high and increasing levels throughout the remainder of the intervention condition. Responding remained high following the removal of the intervention during maintenance ( $M = 146.3$ ; range = 144–149.5).

School 2’s scores (Figure 1, middle panel) were initially low during baseline and continued at low and relatively stable levels throughout the baseline condition ( $M = 40.9$ ; range = 35.5–47). School 2’s scores immediately increased on the implementation of the intervention ( $M = 90$ ; range = 53–125.2) and continued at relatively high and increasing levels throughout the remainder of the intervention condition. Responding remained high following the removal of the intervention during maintenance including maintenance ( $M = 121.3$ ; range = 119–124.5).

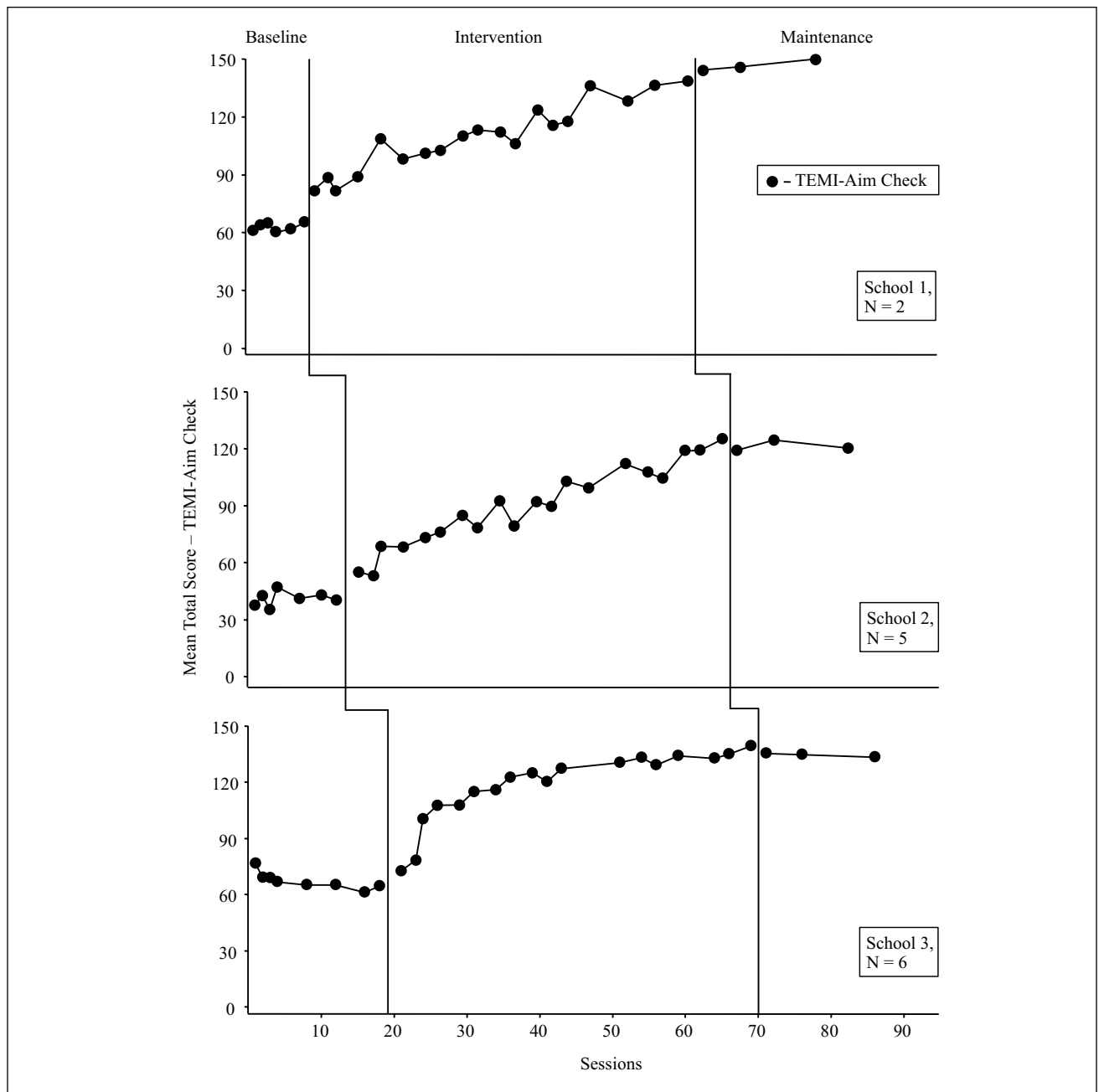
School 3’s scores (Figure 1, bottom panel) also were initially low during baseline and continued at low and, like scores from Schools 1 and 2, remained relatively stable throughout the baseline condition ( $M = 67.3$ ; range = 61.3–76.8). Again like Schools 1 and 2, School 3’s scores immediately increased on the implementation of the intervention ( $M = 118.2$ ; range = 72.5–139.5) and continued at relatively high and increasing levels throughout the remainder of the intervention condition.

To address maintenance, the TEMI-AC was administered three times over 4 weeks following intervention. As shown in Figure 1, Scores remained high following the removal of the intervention during maintenance including maintenance ( $M = 134.7$ ; range = 133.5–135.5).

Taken together, these results suggest that the intervention was effective and had a positive effect on participants’ performance on the TEMI-AC. Specifically, with regard to factors of visual analysis, with all three groups, low and stable levels of responding were observed during baseline, an immediate intervention effect was observed, and stable and increasing trends in responding were observed during the intervention. Thus, based on the factors of visual analysis, these results provide clear evidence of a positive intervention effect (see Kratochwill et al., 2010) with each of the three groups.

### Generalization to the Distal Measure

Typically in single-subject designs, generalization of learning to new behaviors, activities, and settings is desirable (Gillis & Butler, 2007; Gliner & Morgan, 2000). Cooper et



**Figure 1.** Average Texas Early Mathematics Inventory–Progress Monitoring total scores across baseline, intervention, and maintenance for each group of students across the three schools.

al. (1987) used the term *response generalization* to “indicate the development of behaviors not directly trained” (p. 555). Response generalization was determined by the extent to which performance in number and operations generalized across broader mathematics behaviors as measured by the *KeyMath-3* (Connolly, 2008). That is, the three subtests of the *KeyMath-3* Applications area—Foundations of Problem Solving, Time & Money, and Applied Problem

Solving—measure broad-based mathematics Common Core Standards, Mathematics (CCSM) domains that were not assessed by the TEMI-AC, which simply assessed number and operations concepts and skills. The *KeyMath-3* was readministered after the intervention and maintenance phases were completed. The last two columns of Table 2 provide each student’s scores for the posttest administration. As can be seen, most of the students made



considerable progress in Applications (Student 3 in School 2 did not), an important area that was not taught as part of our instruction.

### Clinical Effects

The clinical effects (Thompson, 2002) were determined by the percentage of students who moved out of the risk category, based on their end-of-year mathematics scores. By the end of second grade, 75% of treatment students (9 of 12; see Table 2) were no longer at risk for mathematics difficulties (scoring at or above the 25th percentile, a standard score of 90), as determined by the results on the spring *KeyMath-3* Total Score (Connolly, 2008).

### Effect Sizes

To examine effect sizes, we conducted two analyses. First, the percentage of nonoverlapping data (PND; Scruggs & Mastropieri, 1994) were identified across the baseline and intervention phases using aggregate student data for each group (i.e., students within a school). For Schools 1 and 2, 100% of the data were found to be nonoverlapping; for School 3, the figure was 94%.

The second analysis, Tau-U, has been recommended by Parker et al. (2011). Tau-U is a derivation of Kendall's rank correlation (a trend index) and the Mann-Whitney *U* test between groups (a nonoverlap index) and is based on non-overlap between phases that controls for confounding baseline trend. According to Parker et al., Tau-U offers several advantages over other effect size indexes. Tau-U

- Provides more statistical power than any other non-overlap index
- Is distribution-free and suitable for both ordinal and interval scaled scores
- Is consistent with visual analysis
- Avoids the 100% nonoverlap ceiling
- Indirectly controls autocorrelation with a "runs test" as an add-on
- Can include level and trend and control for preexisting trend in Phase A
- Can give a design-wide effect size by using an additive "bottom-up" method

Statistical significance for Tau-U values was determined using  $CI_{90}$  and  $CI_{95}$  ( $CI$  = confidence interval), which are standard for determining reliable change (Nunnally & Bernstein, 1994), indicating a reasonable change of 5% to 10% likelihood of error. Statistical significance between Tau-U values was determined by calculating  $CI_{83.4}$  to visually test for overlap of upper and lower limits between effect sizes. Visual comparison of two effect sizes with  $CI_{83.4}$  is the same as a  $p = .05$  or 95% confidence level test between the two scores (Parker et al., 2011). Results

provide support for the gains demonstrated by the three groups. Tau-U values for Schools 1, 2, and 3, respectively, were 1.0, 1.0, and .99, with  $p < .001$  for all schools ( $CI_{90}$  = 0.55, 1.45; 0.58, 1.43; and 0.58, 1.40). The result for the weighted average across all three schools was equally positive (Tau-U = .99,  $CI_{95}$  = 0.70, 1.29).

### Experimental Control

Documenting experimental control within multiple-baseline designs entails the staggered introduction of an intervention across participants (or groups of participants) and the staggered demonstration of positive effects of the intervention. Specifically, the demonstration of experimental control occurs when positive changes are observed only following the initiation of the intervention after consistently low levels are documented during baseline conditions across three or more data series (see Horner et al., 2005). Figure 1 presents a design that includes three series, with the introduction of the intervention at a different point in time for each series. Furthermore, positive changes were observed only following the initiation of the intervention following relatively low levels during baseline; and this effect was replicated across three groups of participants (see Horner et al., 2005). Thus, these results document experimental control by demonstrating a covariation between change in behavior patterns and introduction of the intervention within three different series at three different points in time (i.e., the effects of potential extraneous variables, or threats to internal validity, were controlled for).

### External Validity

External validity in single-subject research also is enhanced through operational description of (a) the participants, (b) the context in which the study is conducted, and (c) the factors influencing a participant's behavior prior to intervention (e.g., assessment and baseline response patterns; Horner et al., 2005). Based on the evidence provided in this article, it is clear that external validity has been controlled to a large extent. External validity will be further enhanced and established via replication of these results in future studies.

### Discussion

A single-subject, multiple-baseline design study was implemented to study the effects of a systematic, explicit, intensive Tier 3 intervention on the mathematics performance of students in the second grade who had serious mathematics difficulties. Teaching students with severe mathematics difficulties is not easy; however, results indicated that even the most struggling students can benefit from small group intervention that is intensive, strategic, and explicit.

Students entered the intervention because of minimal response to two rounds of Tier 2 intervention. It is encouraging that the majority of students qualified to exit at the end of treatment providing evidence for the need to intensify intervention not only in dosage (every day) and grouping (1:2 or 1:3) but also in instructional design and delivery. Moreover, breaking the learning sequence into smaller chunks, reducing “teacher talk” by increasing explicitness of instruction, and increasing response opportunities were pervasive throughout this Tier 3 intervention. In addition, carefully managing examples and instructional representations (e.g., base 10 models) seemed integral to the structure of the intervention.

Findings from Wanzek and Vaughn (2007) were incorporated in terms of duration and grouping variables and maintained a highly explicitly scripted intervention. The design also included features identified by L. Fuchs et al. (2008) including “a strong conceptual basis, drill and practice, cumulative review, motivators to help students regulate their attention and behavior to work hard, and ongoing progress monitoring” (p. 85). Progress monitoring is a critical, nonnegotiable component of any RtI program (Office of Public Instruction, 2009; Shinn, 2010). In most Tier 2 interventions, progress-monitoring takes place once or twice a month, but for students who struggle mightily, as these students did, progress monitoring must occur much more frequently. Daily Checks and twice weekly progress monitoring measures were incorporated; of importance, students checked their work with the interventionist, which served as another opportunity to practice, correct errors, and see their success.

### Limitations

The intervention delivered by a project interventionist in a “pull-out” model could be viewed as a limitation of the study. As Stanton-Chapman, Denning, and Jamison (2012) noted, pull-out service delivery programs have been criticized because they isolate target children, who may find it difficult to generalize their new learning to core classroom instruction. The findings from this study show that generalization occurred to the distal measure. Certainly, the continuation of core instruction must be acknowledged as a contributing factor to improved performance on the distal measure.

A second limitation involves the risk status of these students in the fall of the following year. This was not possible. It is important to determine whether the effects of the preventative second-grade intervention for the “responders” to the intervention were maintained in subsequent years, as the demands of the mathematics curriculum increase (L. Fuchs et al., 2008).

### Future Research

Although considerable research on Tier 2 instruction has taken place in mathematics, there remains a dearth of research for students with severe mathematics difficulties in early numeracy concepts and skills. Further research needs to be conducted to identify how intensive intervention supports and generalizes to core instruction. Several directions for future research are necessary to better understand this group of young students.

First, because there is little consistency with who constitutes the severe mathematics difficulties population and how they are identified (Geary, 2011), research should focus on how students receiving intensive intervention differ, if they do, from students with learning disabilities, for example, or what characteristics separate lower responders from higher responders with groups of students with severe mathematics difficulties. Then, of course, it is critical to conduct research on differential diagnosis that can be used to identify which low-responding students with severe mathematics difficulties have LD and which do not.

Second, another direction for future research could involve further examinations concerning generalization; that is, bridging learning from intensive intervention to core instruction. Although most students in this study scored higher on the generalization measure, the Applications area of the *KeyMath-3*, one student did not (Student 3 in School 2). It is not surprising when students improve on measures that are closely aligned with concepts and skills taught as part of intervention. It was encouraging that most students showed considerable improvement on the Applications area of the *KeyMath-3*, a measure that assesses concepts and skills that were not directly taught by the intervention. Yet, it is interesting that Student 3, in this case, is an outlier, especially because the student’s pretest scores were higher than those of the other students who showed improvement. It has been well documented that foundational concepts and skills, such as those taught in the intervention, are critical prerequisites for more complex skills and concepts taught in second grade core. One hypothesis is that the other students’ gains in the fundamental concepts and skills taught in the intervention were beneficial in helping them to learn the core concepts and skills taught in their general education curriculum, which in turn helped them to improve in their understanding of concepts and skills assessed by the *KeyMath-3*. If this is the case, future research should examine what learner characteristics differentiate between those students who generalize mathematical learning and those who do not.

Finally, it is always of interest to examine the longitudinal effects of interventions. Maintenance effects in the current study were examined over a 4-week period. Findings showed, overall, that students maintained consistently good

performance after the termination of the intensive intervention. Of importance, future research should examine the effects over a 1-year period or longer as part of longitudinal studies. Obviously, such data take time to accumulate, and they are influenced by attrition that particularly affects single case study results; yet long-term effects provide an interesting perspective concerning intervention effectiveness, especially when data are examined on generalization measures as well.

### Implications for Practice

There are several implications for working with students with severe mathematics difficulties. Tier 3 intervention should occur every day in a small group “pull-out” model delivered by a trained interventionist who is knowledgeable about explicit, strategic interventions. Teachers must have ample materials for students to use to represent or model the mathematics both concretely and semiconcretely. Using reduced teacher talk and appropriately paced instruction to teach strategies engages students; increased response opportunities, both individual and choral, can also maintain engagement. Teachers should consider the use of scaffolds when students are first learning a concept or procedure and gradually fade these scaffolds. The idea of “Teach me how to do this” is powerful to help teachers continuously check student understanding. Finally, on going progress monitoring is critical; if students are not benefiting from instruction further adaptations may be necessary.

The findings from this study show promise regarding the features of Tier 3 (tertiary) intensive interventions for young students with severe mathematics difficulties. We offer a cautionary note. First, findings from this study must be replicated to build a stronger evidence base for the type of intensity for Tier 3 mathematics found in this study. Thus, the generalization of findings at this time point is cautionary at best. Second, although the majority of students showed improved mathematics performance making them eligible to move out of the intervention program, we strongly urge that students, when exited, be carefully monitored as slippage is possible if only Tier 1 is provided. Students were successful, in large part, because the intensity of the intervention, and most Tier 1 programs lack such intensity. Conceivably, moving from Tier 3 to Tier 2 is warranted for some students. Third, the results of this study may have implications for young students with learning disabilities (LD), but we urge caution here, as well. Although students in Tier 3 may share similar characteristics to students with MLD, they may or may not have been diagnosed as having MLD. As is known, LD is a heterogeneous condition, where students share some but not all characteristics. However, students with mathematics LD require intensive, iterative, relentless instruction (Hallahan, 2006), characteristics that are present to a large extent in the intervention studied here.

As teachers of students with MLD select from among myriad of interventions available, they might well consider the features of the intervention in this present study and the extent to which those features are present in other intervention materials.

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