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## Early Numerical Competencies and Students with Mathematics Difficulty

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Children start elementary school with variable mathematics skills. Some children understand the fundamentals of numbers and mathematics, while others struggle with basic counting, number recognition, understanding of symbols, quantity discrimination, and concepts of addition and subtraction. Often, this set of early numerical competencies is referred to as *number sense* or *early numeracy competencies*. Students need to establish and understand these competencies before moving on to more complex mathematical tasks. This article describes important early numerical competencies and provides a description of how these competencies can be taught to students who struggle with mathematics.

### EARLY NUMERICAL COMPETENCIES AND STUDENTS WITH MATHEMATICS DIFFICULTY

Before solving algebra, geometry, fractions, and computation problems, students must have a solid understanding of numbers (Malofeeva, Day, Saco, Young, & Ciancio, 2004). Sometimes this is called *number sense* (e.g., Jordan, 2007; Kaminski, 2002; Wagner & Davis, 2010) or *early numeracy* (e.g., Aunio, Hautamaki, Sajaniemi, & Van Luit, 2009; Bryant et al., 2011; VanDerHeyden et al., 2011). Regardless of the term used, the construct refers to the early numerical competencies that are foundational to building competence in mathematics. In this article, we refer to this collection of skills as *early numerical competencies*.

#### What Are Early Numerical Competencies?

Although there is no one definition of early numerical competencies, several researchers have identified early numerical competencies that are important for young students (Berch, 2005; Bryant et al., 2011; Gersten & Chard, 1999; Griffin & Case, 1997; Kaufmann, Handl, & Thony, 2003; Lago & DiPerna, 2010). See Figure 1 for a diagram of early numerical competencies. Some topics (i.e., place value or basic number combinations) require prerequisite knowledge of other topics (i.e., number recognition or comparing numbers). We present these early numerical competencies as a collection because students' development of early numerical competencies is not always linear, and students differ in the timeline by which they acquire these skills.

#### Importance of Early Numerical Competencies

Children start school (i.e., kindergarten) with a wide array of early numerical competencies. Some children already appreciate quantities, know their number names, and are able to solve simple addition and subtraction problems; others struggle to identify numbers and count from 1 to 10 (Lembke & Foegen, 2009). Exposure to early numerical activities at home, in preschool, or in daycare plays an important role in the establishment of early numerical

competencies for kindergarten students (Baroody & Benson, 2001 ; Jung, 2011 ; Skwarchuk, 2009). The more exposure to early numerical competencies students receive through games, stories, or play before formal schooling begins, the more they understand the building blocks of mathematics (Ramani & Siegler, 2008).

One indication that these early numerical competencies are important is that they predict later mathematics achievement. For example, Locuniak and Jordan (2008) tested 198 students in the spring of kindergarten on early numerical measures and again in the winter of second grade on a calculation fluency measure. Students scoring below the 25th percentile at beginning of kindergarten were designated at risk of poor mathematics development. The early numerical measures included items about counting, knowledge of numbers, non-verbal calculation, number combinations, and story problems. The calculation fluency measure consisted of 25 addition and 25 subtraction number combinations. Early numerical competency measured in kindergarten was a significant predictor of calculation fluency at second grade. Over 50% of the at-risk students (identified in kindergarten) still performed below the 25th percentile in second grade, and 25% of at-risk students performed between the 25th and 50th percentiles. Locuniak and Jordan's findings indicate that many students with weaker early numerical skills in kindergarten will continue to demonstrate lower mathematics performance after kindergarten. Jordan, Kaplan, Locuniak, and Ramineni (2007) found a similar pattern with 277 students from kindergarten to first grade. Number sense performance in the fall of kindergarten accounted for 66% of the variance on tests of mathematics calculation and problem solving administered at the end of first grade. Other studies (Duncan et al., 2007; Jordan, Glutting, Ramineni, & Watkins, 2010) also indicate that early numerical skills predict mathematics achievement in later grades.

### **Difficulties with Early Numerical Competencies**

Many young students struggle with early numerical competencies (Lembke & Foegen, 2009; Lloyd, Irwin, & Hertzman, 2009). In the United States, differences emerge at the onset of schooling: Some children come to school with an established set of early numerical competencies: others demonstrate much lower performance on early numerical tasks (Jordan et al., 2007). For example, Jordan, Kaplan, Ramineni, and Locuniak (2009) administered early numerical measures of counting, number recognition, comparison, number combinations, and story problems in kindergarten. Lower income students in their sample demonstrated significantly lower early numerical scores than middle-income peers. While low income may not be the only factor contributing to differences in early numerical competencies, Jordan et al. (2009) demonstrated that kindergarten students display varying levels of early numerical skills. The same trend holds true for students in other countries (Ee, Wong, & Aunio, 2006; Lloyd et al., 2009). For example, Finnish students between 5 and 7 years old with special needs (i.e., attention deficit disorder, language difficulties, or developmental difficulties) demonstrated significantly lower early numerical performance than students without special needs (Aunio et al., 2009).

Because students who perform lower on early numerical tasks often demonstrate lower mathematics competence in later elementary and middle school (Duncan et al., 2007), early identification and early intervention are key (Dowker, 2005). While identification of struggling students may be difficult because of inadequate assessments (Mazzocco, 2005), with some students misidentified as struggling in mathematics (Locuniak & Jordan, 2008), research indicates that early intervention can help students with their early numerical skills (Berch, 2005; Bryant et al., 2011 ; Fuchs et al., 2005a).

## Early Numerical Instruction

Based on experimental work with students who struggle with mathematics, Fuchs et al. (2008) provided several recommendations for important components of mathematics instruction. Instruction should be explicit with a focus on conceptual and procedural knowledge. Instruction should be designed in a meaningful way that minimizes challenges, and practice and review should be a part of any instructional program. Fuchs et al. also emphasized the use of motivational tools embedded within instruction to help students with on-task behavior and to monitor academic progress. Monitoring student progress is important so that teachers have objective indicators of when student response to the current instructional program is inadequate and unlikely to produce goal attainment. When a student's data indicate inadequate response, the teacher adjusts the student's instructional program.

Gersten et al. (2009) highlighted explicit instruction, use of strategies, student verbalizations, use of visual representations, progress monitoring, and using a variety of examples as important instructional practices for students who struggle with mathematics. Adding to these points, Gersten and Chard (1999) suggested working on mathematics fluency to integrate instruction on concepts and procedures with sufficient practice. These recommendations are especially important for students with mathematics difficulties, and the following examples demonstrate how these important instructional recommendations, when used to teach early numerical competencies, are beneficial to students with mathematics difficulties.

For example, Bryant et al. (2011) worked with first-grade students ( $N = 224$ ) who performed below the 35th percentile on an early numerical competencies assessment. Some students ( $n = 151$ ) were assigned to an early numerical program, whereas other students ( $n = 73$ ) remained in their regular education classroom for mathematics instruction. Small-group tutoring for the early numerical program students lasted 22 weeks, four sessions per week, 25 minutes per session. Students participated in explicit instruction with guided and independent practice on the procedural and conceptual ideas of counting, number relationships, sets of 10, number combinations, and place value. At posttest, students who participated in the early numerical program performed significantly higher than students in the control group, with effect sizes (ES) of 0.18 on magnitude comparison, 0.47 on number sequences, 0.39 on place value, and 0.55 on addition and subtraction number combinations.

Fuchs et al. (2005a) also provided early numerical tutoring to first-grade students ( $N = 127$ ) who were struggling with mathematics. Students were randomly assigned to receive early numerical tutoring ( $n = 64$ ) or to participate in their regular mathematics instruction without tutoring ( $n = 63$ ). Students received tutoring for 16 weeks, three times a week, 40 minutes a session. Tutoring focused on early numerical competencies such as number identification and writing, symbol use, counting, place value, and addition and subtraction combinations. After tutoring ended, students who received tutoring outperformed students without tutoring on tests of addition facts ( $ES = 0.40$ ), subtraction facts ( $ES = 0.14$ ), calculation ( $ES = 0.57$ ), concepts and applications ( $ES = 0.67$ ), and story problems ( $ES = 0.70$ ).

In other countries, early numerical programs have also been shown to strengthen the mathematics performance of struggling students. Kaufmann et al. (2003) worked with six students with mathematics difficulties. These students participated in an early numerical program for 6 months, three times a week, 25 minutes a session. Students learned about counting, symbols, facts equal to 10, addition and subtraction facts, and place value through explicit instruction and working from the concrete (i.e., manipulatives) to the abstract (i.e., solving problems with numbers and symbols). The six students enjoyed strong growth over the course of the program compared to peers without mathematics difficulties. Kaufmann,

Delazer, Pohl, Semenza, and Dowker (2005) expanded this work by comparing an early numerical program focused on procedural and conceptual learning versus a program focused on training of basic skills. Students who participated in the procedural and conceptual program demonstrated significant gains on measures of counting, cardinality, comparisons, and calculations over students who participated in the basic skills program. Van Luit and Schopman (2000) worked with kindergarten students ( $N = 124$ ) who performed below the 25th percentile on an early numerical measure. Half the students were assigned to receive early numerical instruction; the other half participated in their regular classroom program. The early numerical instruction focused on counting skills, and teaching was explicit and interactive and followed a sequence of concrete to representational to abstract (Hudson & Miller, 2006). After twenty 30-minute sessions, students who participated in the early numerical program outperformed control students on early numerical measures of comparing numbers, counting, and understanding the meaning of numbers.

These results from early numerical investigations in the United States and abroad indicate that struggling mathematics students benefit from programs focused on early numerical competencies. All the instruction in these programs was explicit and focused on teaching students the meaning (i.e., concepts) behind early numerical competencies along with the procedures to solve mathematics problems.

## EARLY NUMERICAL COMPETENCIES

For this article, we highlight four main categories of early numerical competencies: counting, comparing numbers, understanding symbols, and addition and subtraction concepts. In this section, we describe each of these categories and how students may struggle with skills in that category. We then present an example of an intervention to help students who are struggling with these early numerical competencies. Finally, we provide suggestions for practitioners.

### Counting

There is more to counting than reciting, “1, 2, 3, 4, 5...” Students can often count to 10, but they may not understand what the numbers represent (Bermejo, Morales, & deOsuna, 2004; Bruce & Threlfall, 2004). For example, students may not assign meaning to their counting or realize that the number words map onto counted items. Counting encompasses five principles: stable order, one-to-one correspondence, cardinality, abstraction, and order irrelevance (Gelman & Gallistel, 1978). Students can struggle with one or more of these principles (Bruce & Threlfall, 2004). These principles are often combined (i.e., students say number names and point to each counted object), and, because of this, these principles should be practiced together (Camos, Barrouillet, & Fayol, 2001).

Many students develop counting skills before entering kindergarten (Gelman & Gallistel, 1978). Some students, however, come to school with inadequate counting skills or a lack of understanding of the principles of counting. For example, many students can count to five without difficulty, but they may struggle with counting larger sets (i.e., sets larger than 5 or 6), make more errors, and not understand how to use counting to identify the number of items in a set (Carrasumada, Vendrell, Ribera, & Montserrat, 2006). Counting skills, however, can be taught and improve with instruction and practice (Camos et al., 2001; Xin & Holmdal, 2003). Often, a helpful way to understand whether students understand counting principles is to demonstrate counting and miscounting with a puppet (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Muldoon, Lewis, & Francis, 2007). Recommendations for counting instruction can be determined by awareness (or lack of) of the puppet’s counting skills. For example, if a student says that it is incorrect for a puppet to count from

the student's right to left, then the student should receive instruction on the counting principle of order irrelevance.

For counting, students should know the number words in order (Slusser & Sarnecka, 2011), a concept called *stable order*. These words are usually recited in forward order (e.g., "one, two, three, four, five"), and the sequence of these counting words needs to be used consistently (Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989). Stable order is often learned and practiced through songs, chants, or stories.

Also, when counting, students must count each item only once (Van De Walle, Karp, & Bay-Williams, 2010). This is called *one-to-one correspondence*. When practicing one-to-one correspondence, it is easier for students to keep track of items in a row or items that have been tagged and partitioned rather than items that are counted randomly (Potter & Levy, 1968). To count using one-to-one correspondence, students must know number names, appreciate stable order, and understand the relationship between counts and number names (Potter & Levy, 1968). One-to-one correspondence is often practiced by passing out items (like cookies) and ensuring that each student receives one cookie (Van De Walle et al., 2010).

By combining stable order and one-to-one correspondence, students start to count sets of objects to determine the number in the set (i.e., the *cardinality* principle). When counting a set of items, the final count (e.g., "4" after counting four dinosaurs) represents the set. Cardinality refers to understanding that the final or last count represents the total of items counted (Bermejo et al., 2004). Often this is practiced by asking students to count a set of objects and then asking them to answer the question, "How many?" (Muldoon, Lewis, & Freeman, 2003).

While the counting principle of *abstraction* is not a necessity for counting, it is helpful for students to understand that any objects can comprise a set (Frye et al., 1989). For example, the counting set does not have to contain only frogs. The counting set can contain frogs, toads, trucks, and pencils. Counting can be applied to any set of items no matter how abstract those items may be. Similar to abstraction, *order irrelevance* is not as important as other counting principles (Kamawar et al., 2010). The principle of order irrelevance dictates that the order in which items are counted does not matter as long as each item is counted only once (i.e., one-to-one correspondence). Many students count from left to right and top to bottom because that is how they read in English, so it may be confusing to these students that counting does not have to occur in a linear fashion.

Students should move from counting items one by one to subitizing (Bruce & Threlfall, 2004; Hannula, Rasanen, & Lehtinen, 2007). *Subitizing* is the ability to instantly recognize how many items are in a group. See Figure 2 for subitizing examples. Students should be able to look at each of the examples and instantly recognize that there are four boxes, three circles, one hexagon, and six squares. Often, students who struggle with mathematics struggle with subitizing (Schleifer & Landerl, 2011), but practice can help to improve their skills (Clements, 1999; Fischer, Köngeter, & Hartnegg, 2008). Subitizing is often viewed as a central component of early numerical competency, and we mention it here because students may subitize (instead of count) to compare amounts and work with addition and subtraction.

### Appreciating Quantity

Subitizing is related to children's appreciation of quantity, a related early numerical competency. This is sometimes referred to as quantity discrimination, magnitude, or comparing numbers. On the most basic level, students look at two numbers (e.g., 4 and 9)



and answer the question, “Which is more?” (9) or “Which is less?” (4). Students can use manipulatives or pictures to aid in discriminating between the two quantities. Students have an easier time discriminating between quantities that are much farther apart (e.g., 9 and 2) than those that are closer in magnitude (e.g., 9 and 8; Murray & Mayer, 1988). When comparing larger, two-digit numbers, students have an easier time discriminating between numbers where the tens place is different than when the decade is the same but the ones place differs (Ganor-Stern, Pinhas, & Tzelgov, 2009).

Students with difficulty in mathematics often struggle with comparing numbers and perform lower on comparing tasks than peers without mathematics difficulties (De Smedt & Gilmore, 2011; Holloway & Ansari, 2009). It is interesting that students may perform better on number magnitude tasks that do not involve the number symbols (Rousselle & Noel, 2007). For example, when presented with a group of six candies and four candies, students are able to determine that six is more than four. When students have to compare two number symbols (e.g., “6” and “4”), this generally is more difficult (De Smedt & Gilmore, 2011).

Preschool students, when presented with two sets for comparison, often do not count and use the principal of cardinality for comparing the two sets. Typically, students instead rely on visual (i.e., nonsymbolic) inspection (Zhou, 2002). Relying on visual scanning can help students only for a while, generally when numbers are between 1 and 3. Therefore, providing instruction on counting to determine differences between sets can be helpful (Muldoon et al., 2003). Often students do not realize that counting can be used for comparison because teachers generally ask, “How many?” with every counting task instead of questions like, “How many fewer?” or “Which has more?”

### Mathematical Symbols

With the early numerical skill of counting, eventually students will associate counts (e.g., one, two, three) with the number symbols (e.g., 1, 2, 3). Students can often repeat the numbers words in stable order, use one-to-one correspondence, and understand cardinality without using number symbols. Students can also compare amounts without using number symbols (i.e., when provided with visual representations of two sets). Once students begin kindergarten, however, most activities involving counting and comparing numbers require students to have knowledge of number symbols and the meaning of these symbols to complete mathematical tasks. Mathematical symbols are important because most of mathematics is represented using symbols.

Ten number symbols (i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) can be used alone or combined to represent any number (e.g., 14,597). Besides the ten number symbols, early elementary students learn two operation symbols: the plus sign (+) for addition and the minus sign (−) for subtraction. Students also use the equal sign (=) in number sentences. Students might also use the inequality symbols for greater than (>) and less than (<) when comparing amounts. Students usually learn the number symbols before any other symbols (Zhou, Wang, Wang, & Wang, 2006).

Students need to learn how to write and interpret the symbols because they do not automatically assign meaning to symbols. Meaning of the symbols develops over time and with practice. For example, students learn that “three” or \* \* \* or three manipulative bears can be represented by the written symbol 3 and vice versa. Students must learn to put items together when they see a plus (+) symbol and to take an item away or find the difference when they see the minus (−) symbol. Many students understand the operations indicated by the plus and minus signs, but fewer students correctly interpret the equal sign and the inequality symbols (e.g., Hattikudur & Alibali, 2010; Matthews & Rittle-Johnson, 2009; McNeil, 2008). The equal sign should be understood as a relational symbol, indicating that a

balanced relationship exists between numbers on the two sides of the equal sign (=) symbol (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). The inequality symbols (< and >) should also be understood as relational, with one side of the symbol representing a larger or smaller quantity.

Unfortunately, students come to misunderstand symbols as teachers provide instruction or practice that does not promote full understanding of the symbol (Capraro, Ding, Matteson, Capraro, & Li, 2007; McNeil, 2008). For example, students often practice hundreds of equations like  $2 + 3 = \_$ , which require little understanding of the equal sign in a relational manner (Powell, in press). By contrast, students, even those who struggle with mathematics, learn to interpret the equal sign relationally with appropriate instruction and practice (Powell & Fuchs, 2010). Without adequate instruction and practice, however, students continue to misuse or misinterpret symbols well into middle and high school (Knuth, Alibali, Hattikudur, McNeil, & Stephens, 2008; Rowntree, 2009; Verikios & Farmaki, 2010).

### Addition and Subtraction Concepts

Learning the concepts of addition and subtraction does not necessarily follow mastery of counting, comparing numbers, and mathematical symbols. Children can often solve simple addition and subtraction problems presented without symbols (i.e., presented orally and/or solved with manipulatives or counting; Cobb, 1987; Sherman & Bisanz, 2009). Adequate counting, comparing, and symbol knowledge skills, however, are necessary to carry out most addition and subtraction problems presented to students in early elementary school.

When starting to learn about the addition and subtraction number combinations (i.e., basic facts), students often work on simple problems with manipulatives. With practice, students rely less on manipulatives and more on their fingers for counting (Groen & Resniek, 1977). Because counting is often involved in solving addition and subtraction number combinations, counting skills are important (Baroody, Bajwa, & Eiland, 2009). Most of the time, young students use counting by ones as their default counting mechanism. Counting by twos or other increments or utilizing subitizing skills is not prevalent until second grade or later (Camos, 2003). Students then move from counting to solving number combinations using reasoning strategies or from memory. Mastery and fluency, of course, is the end goal of number combinations. Typically, students should know all 100 addition and 100 subtraction number combinations by the end of first grade (Baroody et al., 2009).

When beginning addition and subtraction, students often solve addition problems more successfully than subtraction problems (Shinsky, Chan, Coleman, Moxom, & Yamamoto, 2009). This is related to the fact that students learn counting forward well before they succeed in counting backward. The addition skills of students, even students who are struggling with mathematics, are generally stronger than their subtraction skills. This is apparent in that many students solve subtraction problems more efficiently when they use addition skills (Torbeys, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). For example, when presented with the problem  $14 - 9 = \_$ , many students find it easier to think, “What can I add to 9 to make 14?” and a counting forwards strategy can be employed.

While students may understand the principle of subtraction, they often lag in their ability to understand that subtraction is the inverse of addition (Baroody, Lai, Li, & Baroody, 2009). Because students do not automatically understand the inverse relationship between addition and subtraction, this concept should be made more explicit through instruction and practice (Baroody, 1999). Students who understand the relationship between addition and subtraction (i.e., addition is the inverse of subtraction and vice versa) demonstrate better conceptual knowledge and better subtraction performance than students who do not understand this relationship (Gilmore & Papadatou-Pastou, 2009).

Counting strategies (i.e., counting to find the answer to an addition or subtraction number combination) are an aid to help students solve combinations. Not all students, however, use a counting strategy (Saxton & Cakir, 2006). Some students simply guess. For many students, especially those who struggle with early numerical competencies, counting strategies to solve number combinations are helpful and can be learned with relative ease. There are several counting strategies that students might employ when solving addition and subtraction number combinations. See Figure 3 for diagrams. With counting all, students count out the first addend, count out the second addend, and then count both addends together starting at 1. This is usually the first counting strategy for addition that students employ (Fuson & Secada, 1986). The counting all strategy is not terribly efficient and, given the numbers of counts required, it often results in incorrect answers. Students generally abandon use of counting all in favor of the more advanced “counting up” or “counting on” strategy (Fuson & Secada, 1986). Counting up can be carried out in two ways: Start with the larger addend and count up the smaller addend (i.e., the “min” strategy because the student counts the minimum amount) or vice versa (i.e., the “max” strategy because the student counts the maximum amount). Before students learn the commutative property of addition (i.e., the order of addition does not make a difference to the sum), they often start with the first addend in the number sentence (e.g., 4 of  $4 + 9 = \_$ ) without realizing the greater efficiency of starting with the larger addend and counting up the smaller addend (Groen & Parkman, 1972). For example, if presented with  $5 + 9 = \_$ , students start at 9 and count 5 more: “10, 11, 12, 13, 14.” Students often develop this counting strategy through experience and practice (Weiland, 2007), but it may be necessary, especially for students who struggle with mathematics, to provide explicit instruction on this more efficient counting strategy (Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009).

To solve subtraction number combinations, students often count down. That is, they start with the minuend and count down the amount of the subtrahend. For  $9 - 4 = \_$ , students start with 9 and count back 4: “8, 7, 6, 5.” Counting down or counting backwards is difficult for students, especially students with mathematics difficulties, because fluency with counting backwards is limited compared to fluency counting forward (Passolunghi & Cornoldi, 2008). Students also tend to make many more mistakes counting backwards than counting forwards. A more efficient strategy for solving subtraction problems is counting up. Students start with the subtrahend and count up to the minuend. For  $9 - 4 = \_$ , students start with 4 and count, “5, 6, 7, 8, 9.” They count 5 fingers or make 5 counts, so  $9 - 4 = 5$ . This strategy makes use of students’ quick forward-counting skills and has shown to be a beneficial strategy for students with mathematics difficulties (Fuchs et al., 2009; Fuchs, Powell, et al., 2010). Using counting up for subtraction also underscores the fact that subtraction represents the difference between two amounts (i.e., the minuend and subtrahend).

With addition and subtraction, practice on using counting strategies and working towards fluency improves the performance of struggling students (Fuchs, Powell, et al., 2010). Students need to understand the concepts behind number combinations (Baroody, Lai, et al., 2009), but they also need to be presented with routine, even daily, practice to build fluency and help students make frequent, correct associations between problem stems and their answers (Fuchs, Powell, et al., 2010). This leads to students building representations in long-term memory and helps students rely on the most efficient strategy for solving addition and subtraction problems: automatic retrieval of answers (Fuchs et al., 2011). For this reason, students need practice on all the number combinations, especially the number combinations that involve double digits (e.g.,  $9 + 7 = 16$ ;  $14 - 8 = 6$ ) because they generally receive much more exposure to the easier number combinations (Hamann & Ashcraft, 1986).



## EXAMPLE OF AN EARLY NUMERICAL INTERVENTION FOR STRUGGLING STUDENTS

We discussed four early numerical competencies: counting, comparing numbers, understanding symbols, and concepts of addition and subtraction. While this is not an exhaustive list of early numerical competencies, these four are critical components of an effective early numerical program for struggling students. The four components are related to one another and build upon each another as students learn more and more mathematics in the early elementary grades. While students may struggle with one or more of these early numerical competencies, instruction and practice can improve the early numerical skill of students.

In this next section, we describe an early numerical program for first-grade students who struggle with mathematics. We describe this program to illustrate how teachers and parents of struggling students may incorporate four early numerical competencies discussed in this article into a successful instructional program for struggling students. This is not the only early numerical intervention available, so teachers should research options before choosing a program for their students. *Galaxy Math*, also referred to as *Number Rockets*, (Fuchs et al., 2011) was designed to help prevent long-term difficulty in mathematics by remediating early numerical skill deficits and promoting number knowledge and skill with number combinations and other key components of the first-grade mathematics curriculum. The program is called *Galaxy Math* because a space theme is used throughout the lessons to help motivate students. Tutors encourage students to “Blast off into the math galaxy!” and the students use mathematics manipulatives shaped as rockets. See Figure 4 for an example of the galaxy-themed motivation chart.

### Experimental Investigation of Galaxy Math

At the beginning of first grade, students with parental consent were screened to identify those at risk for inadequate mathematics development, although most students did not have a school-diagnosed learning disability. These students were randomly assigned to continue in their normal school program (i.e., the control group) or to one of two versions of *Galaxy Math*. In both versions of *Galaxy Math*, the main focus (25 minutes of each 30-minute tutoring session) is the types of early numerical competencies discussed in this article. One version of *Galaxy Math* (the standard version) added 5 minutes of practice at the end of each session; the other version added 5 minutes of games. In both tutoring conditions, *Galaxy Math* students received 48 individual sessions, three times a week.

See Table 1 for a list of *Galaxy Math* units and concepts. In Unit 1, students use manipulatives such as a number line, counting beans, and “Mr. Greater Gator” to learn magnitudes, practice counting, compare numbers, and learn symbols. See Figures 5 and 6 for example activities working on counting and learning about the terminology of *equal*. These activities are conducted during the first few lessons of *Galaxy Math*. See Figure 7 for a sample number line. The numbers on the number line increase in size as the numbers increase in value to help students understand the magnitude of the numbers. See Figure 8 for Mr. Greater Gator. This alligator has a mouth open wide with the inequality symbols (i.e., greater than or less than signs) superimposed over the open mouth. Students learn that the alligator is very hungry, and he wants to eat the larger number when presented with two amounts. The open mouth always faces the larger number. Also in Unit 1, the students learn the counting strategies of counting in (for addition) and counting up (for subtraction). With counting in, students hold up the smaller addend on their fingers and then count by folding in one finger at a time until no fingers remain (i.e., closed fist). For example, with  $3 + 6$  the student would hold up 3 fingers and then counts: “7” (folds 1 finger in), “8” (folds another

finger in), “9” (folds last finger in). The answer is the last number the student says (in this case, 9). Counting in is one version of the counting up strategy. Counting in was found useful for first-grade students because it helped them keep track of the counting amount. With counting up for subtraction, students start with a closed fist. They start with the subtrahend and use their fingers to count up to the minuend. For example, with  $9 - 3$  students count, “4, 5, 6, 7, 8, 9” (each time holding up another finger). Once students reaches the minuend, they count the number of fingers (in this case, 6) and 6 is recorded as the answer. In Unit 1, students also solve story problems with manipulatives, pictures, or actions. For example, when presented with the problem, “John has 4 apples in his grocery cart. He puts 1 more apple in the cart. How many apples are in John’s cart now?” students could draw apples or use manipulative blocks to solve the problem.

In Unit 2, students use beans and the number line to learn about doubles 0 through 6 (i.e.,  $0 + 0$ ,  $1 + 1$ ,  $2 + 2$ ,  $3 + 3$ ,  $4 + 4$ ,  $5 + 5$ ,  $6 + 6$ ,  $0 - 0$ ,  $2 - 1$ ,  $4 - 2$ ,  $6 - 3$ ,  $8 - 4$ ,  $10 - 5$ ,  $12 - 6$ ). Doubles are practiced early in the program because students usually have little difficulty memorizing doubles, and students can use doubles to solve other number combinations (Van De Walle et al., 2010).

In Unit 3, students begin learning number combinations in sets. Each set includes all number combinations with the sum and minuend as the target set number. For example, the 5 set consists of all number combinations with a sum of 5 or 5 as the minuend (i.e.,  $0 + 5$ ,  $1 + 4$ ,  $2 + 3$ ,  $3 + 2$ ,  $4 + 1$ ,  $5 + 0$ ,  $5 - 0$ ,  $5 - 1$ ,  $5 - 2$ ,  $5 - 3$ ,  $5 - 4$ ,  $5 - 5$ ). Tutors start with the 5 set and continue through the 12 set. While working on each set, the tutor conducts five activities with the student. First, the tutor and student use unifix cubes to see how cubes can be combined in different ways to make the addition and subtraction number combinations of the set. See Figure 9 for examples from the 5 set. With the manipulatives, students can also see how  $1 + 4$ ,  $4 + 1$ ,  $5 - 1$ , and  $5 - 4$  are related as a “family,” and the second activity of each lesson focuses on the families that constitute the relevant set. Third, students either answer number combination set problems on a worksheet or show all combinations in the number set with manipulative rockets. Next, the tutor and student work together to solve a story problem involving a number combination from the set. The student solves the problem and explains why the story problem is specific to the number set. The fifth activity each day is an oral review of previous number sets.

For each set, students work for one to four lessons. After the first lesson in a set, each subsequent lesson begins with a flash card mastery test by which students can advance to the next set by answering cards correctly. Students have to answer within 3 seconds, with no more than one error. Students who achieve mastery prior to the full complement of four lessons within each set complete the 12 set. Other students complete the 10 set and then continue to Unit 4. This rule ensures appropriate content coverage.

In Unit 4, the focus is on place value: counting by tens to 100, showing and writing ones and tens, regrouping, and double-digit addition. Students also review the number sets during this unit. Unit 5 is designed for students who demonstrate mastery while working on the number sets. In this unit, students review the number sets and place value concepts.

In the last 5 minutes of each tutoring session, Fuchs et al. (2011) isolated the effects of providing practice. To do this, half the students in the study did systematic practice in the last 5 minutes; the other half played games. In the practice and games conditions, the content was the same: material related to that day’s lesson. Random assignment determined whether students participated in games or practice during the conclusion of each day’s lesson.

In the *games* condition, students play games with manipulative rockets to practice concepts. For example, with one game, students spin to learn how many rockets are called to the space

station, and they place that number of rockets on a game board. Then they spin again to see how many rockets are called away, back to earth, and they remove the corresponding number of rockets from the board. They then generate the number sentence representing this series of events. Throughout games, tutors encourage students to know the answer or to use fingers, beans, or number lines to calculate the answer. Tutors explain that “knowing the answer right off the bat” is the preferred strategy, if the student is sure of the answer.

In the *practice* condition, students practice content from the lessons with the “Meet or Beat Your Score Activity,” which relies on flash cards. For example, once addition/subtraction sets are introduced, students practice number combinations. Tutors encourage children to retrieve the combination from memory or, if they are unsure of the answer, use the counting strategy they learned in *Galaxy Math* to solve the combination. When the student answers correctly, the flash card is placed in a pile on the desk. When a student answers incorrectly, the tutor requires the students to use a counting strategy (i.e., counting in or counting up) to find the correct answer. The corrected card is then placed in the pile on the desk. At the end of 90 seconds, the student graphs the number of flash cards answered correctly. See Figure 10 for a sample flash card graph. Students then have two chances to meet or beat their first flash card score.

In both conditions, tutors promote on-task behavior and motivation to work hard (Fuchs et al., 2008) using a systematic reward program. Tutors teach students that on-task behavior means paying attention and trying hard to answer questions correctly. Students learn that on-task behavior is important for “blasting off into the math galaxy.” Students earn stickers for on-task behavior and hard, correct work. They place their stickers on the *Galaxy Math* chart (see Figure 4). Students earn a prize (i.e., small toy, sticker, or pencil) when they reach the sun on the galaxy chart.

In the study (Fuchs et al., 2011), results showed that students who participated in the *Galaxy Math* tutoring improved on number knowledge, simple arithmetic, more complex calculations, and word problems significantly better than control students. Students in the practice condition improved more than games students on simple arithmetic and more complex calculations, with no harm to their number knowledge or performance on word problems. This, along with other early numerical intervention studies (e.g., Bryant et al., 2011; Fuchs et al., 2005b), demonstrates positive outcomes for at-risk students when early numerical intervention occurs early and with intensity. The Fuchs et al. (2011) study also shows the specific importance of incorporating frequent, well-designed practice that supports correct responding.

## SUGGESTIONS FOR PRACTITIONERS

As students enter kindergarten and often progress to first grade with varying degrees of early numerical competency, practitioners should provide early assessment and early intervention to help students who struggle with the fundamentals of mathematics. For assessment, progress-monitoring measures (e.g., Lembke & Foegen, 2009; Seethaler & Fuchs, 2011) and screening measures that target specific mathematics skills (e.g., Geary et al., 2007; Jordan et al., 2009) can be used to determine which students warrant early numerical intervention.

After identifying students who struggle with early numerical competencies in kindergarten or first grade, practitioners need to evaluate early numerical programs and choose a program that best fits the needs of their students.

Based on experimental work with young students (e.g., Bryant et al., 2011; Fuchs et al., 2011), early numerical competencies programs should include the following: (a) explicit instruction focused on conceptual knowledge and procedural skill, (b) a sequence of

instruction that is meaningful and relevant, (c) review of topics previously taught, (d) practice of current topics, and (e) fluency work on addition and subtraction number combinations. The focus of instruction (i.e., counting skills, addition concepts) should be determined based on student needs. One program may not be the best for all students, so practitioners must monitor student progress while students participate in instruction to determine response. If students are not demonstrating appropriate learning, their instructional program should be modified to formatively develop a program tailored to the student's needs.

Because early numerical skill at kindergarten is predictive of mathematical achievement in later grades (Duncan et al., 2007; Jordan et al., 2010), providing timely intervention to students who lack skill in the early numerical competencies is vital. Researchers need to continue to refine early numerical assessments and interventions that will help schools in timely identification and effective intervention to ensure student competence with the basic building blocks of mathematics.

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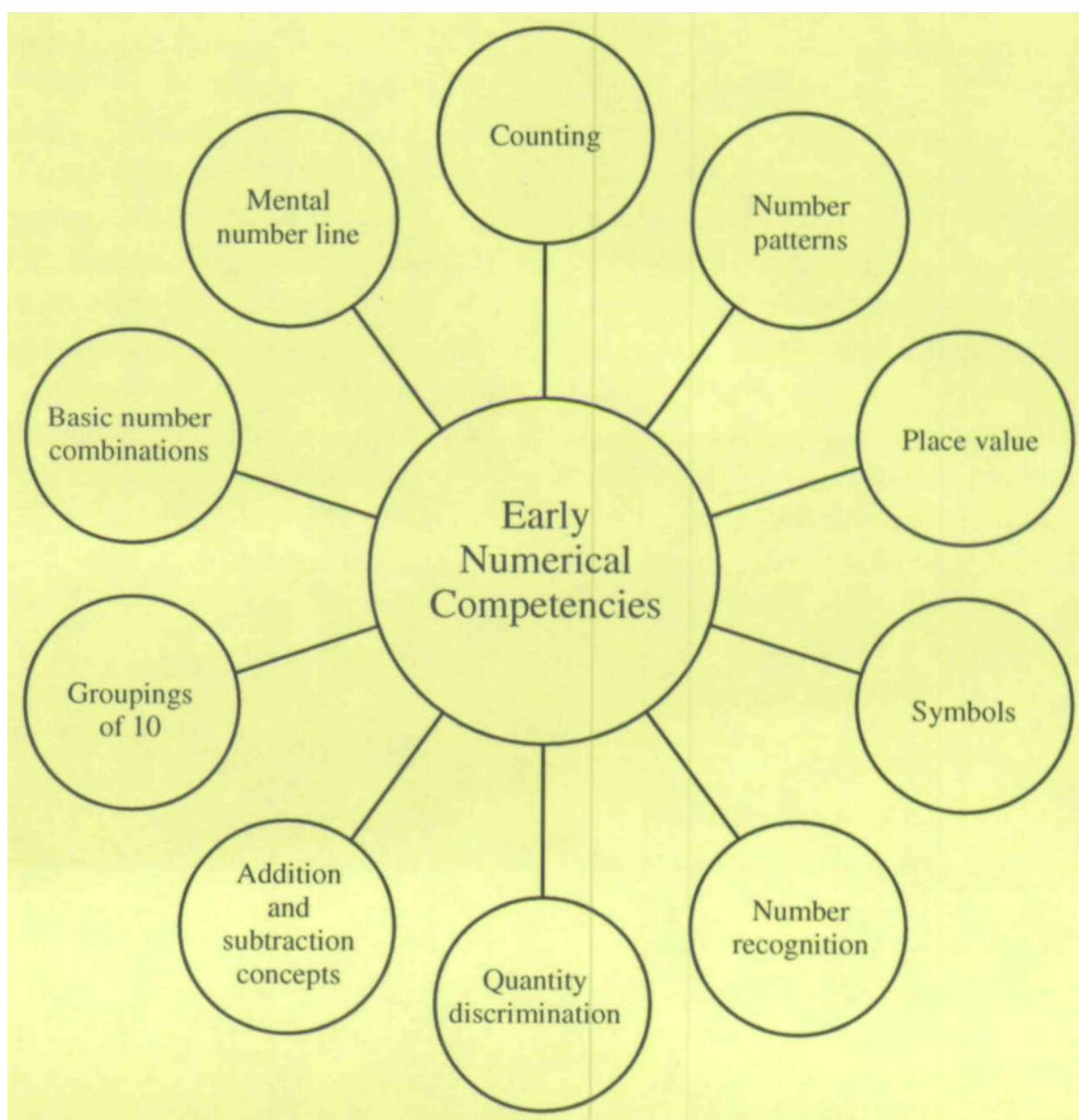
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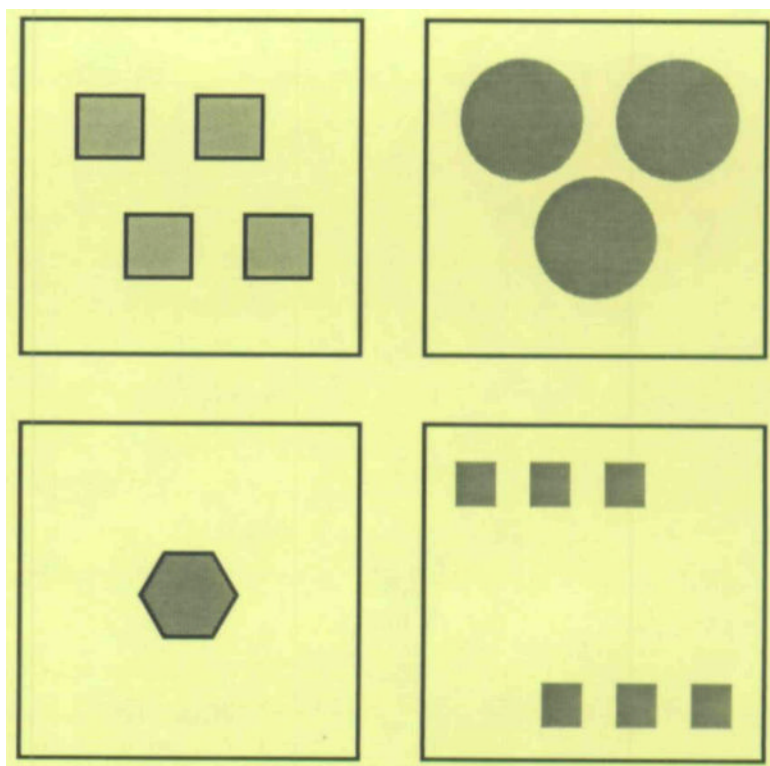
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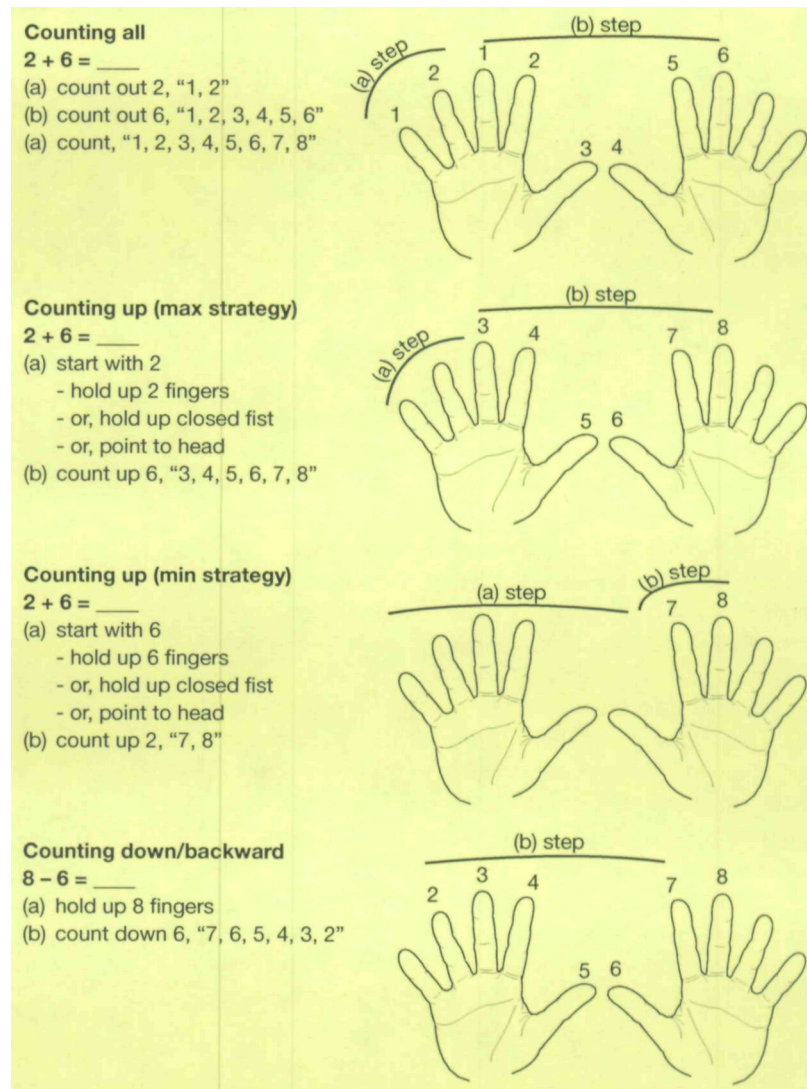


**FIGURE 1.**  
Early Numerical Competencies

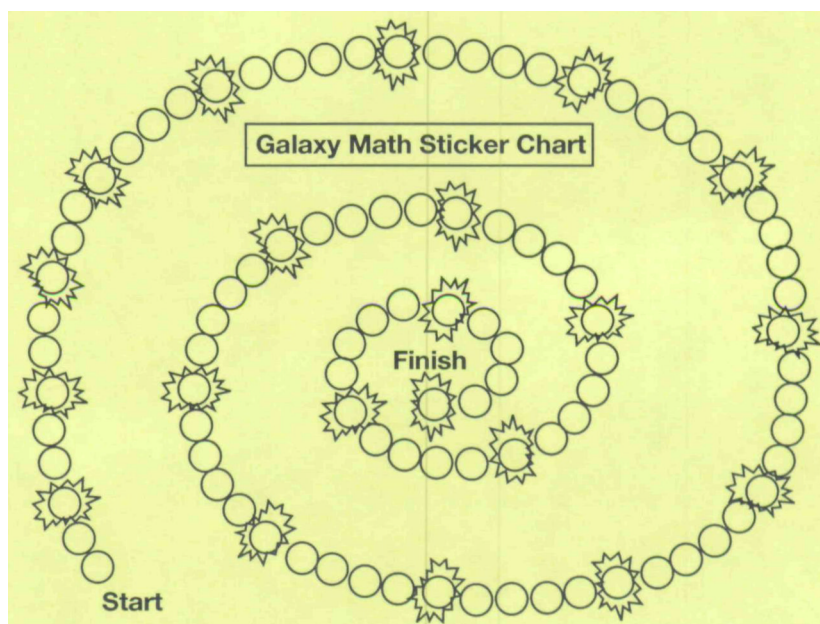


**FIGURE 2.**  
Subitizing Examples

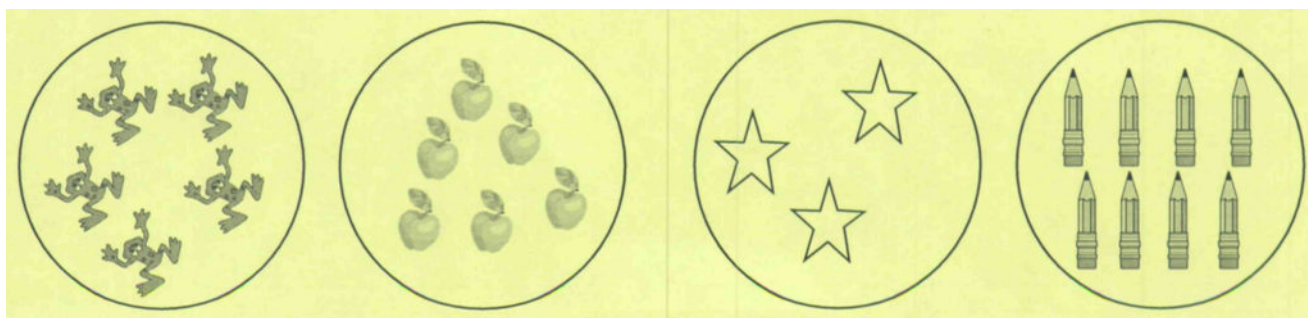


**FIGURE 3.****Counting Strategies**

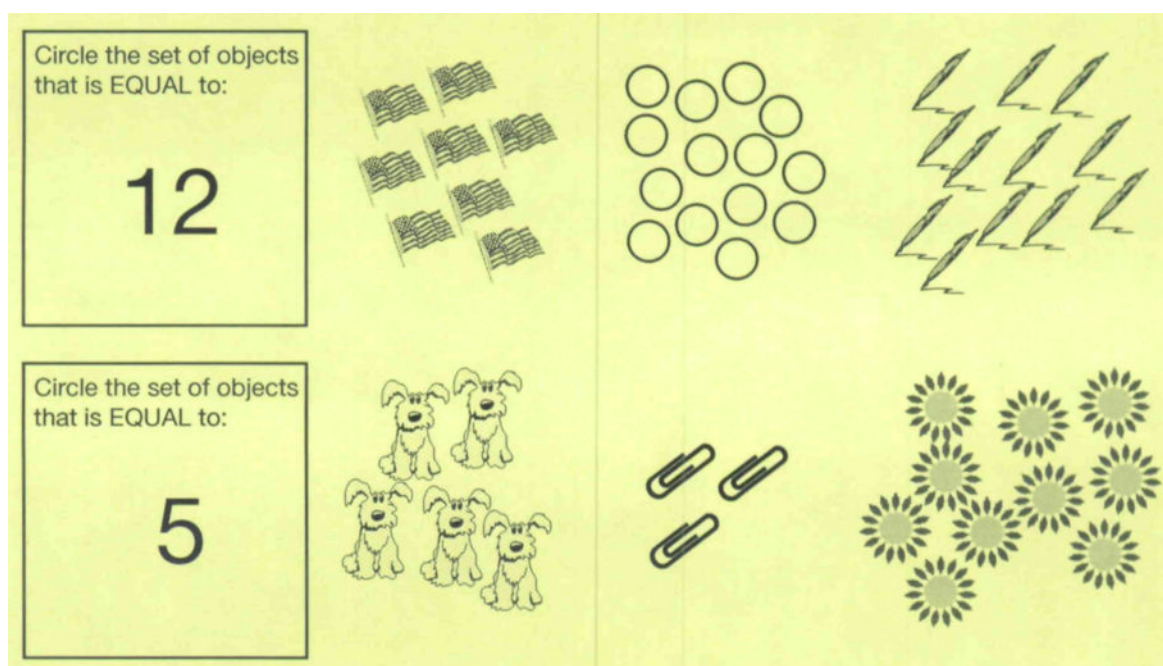
With each of these strategies, students may hold up fingers, fold in fingers, or tap fingers. Students may work with their palms facing toward or away from them. Also, students may count from left-to-right or right-to-left. They may start counting with their index finger or thumb or another finger.



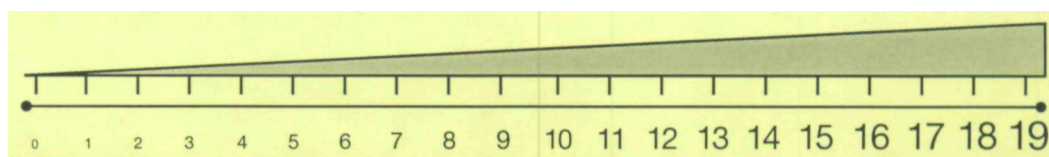
**FIGURE 4.**  
Galaxy *Math* Motivation Chart



**FIGURE 5.**  
Counting Activity

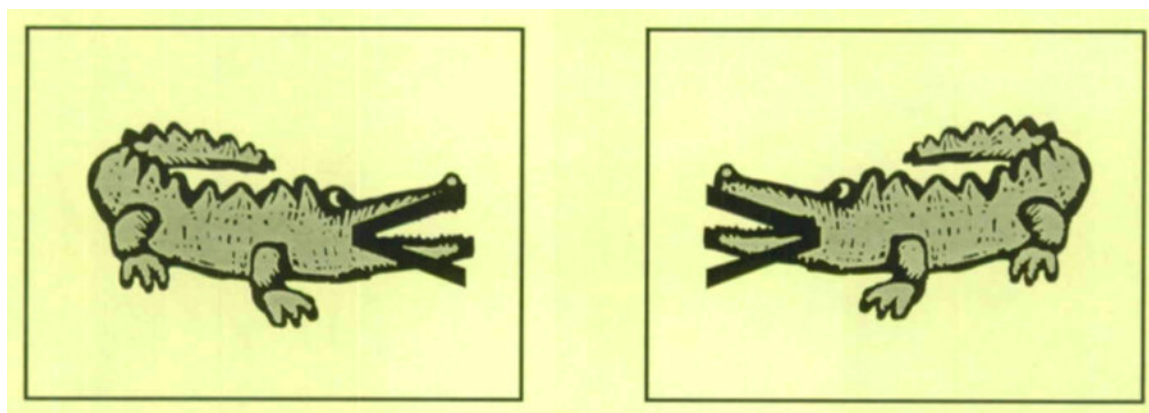


**FIGURE 6.**  
Equal Activity

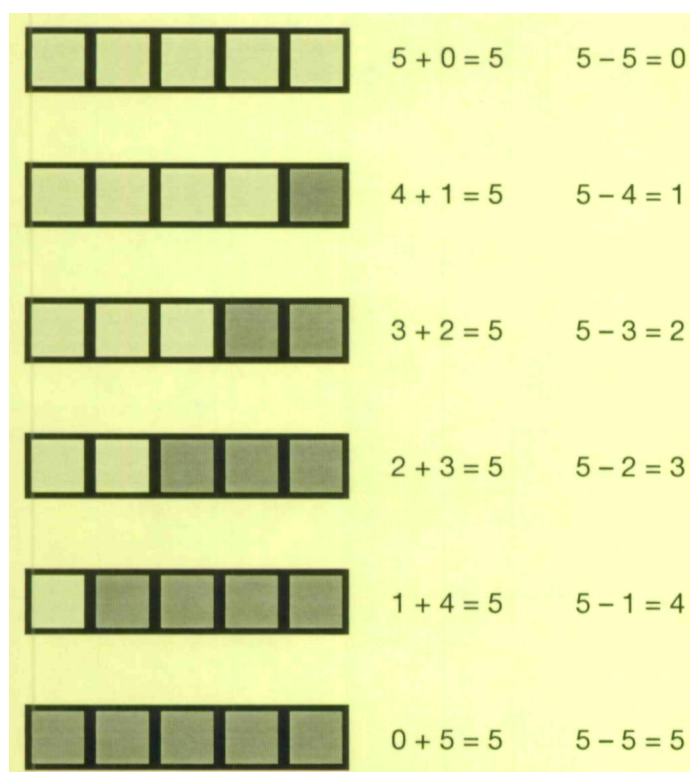


**FIGURE 7.**  
Number Line

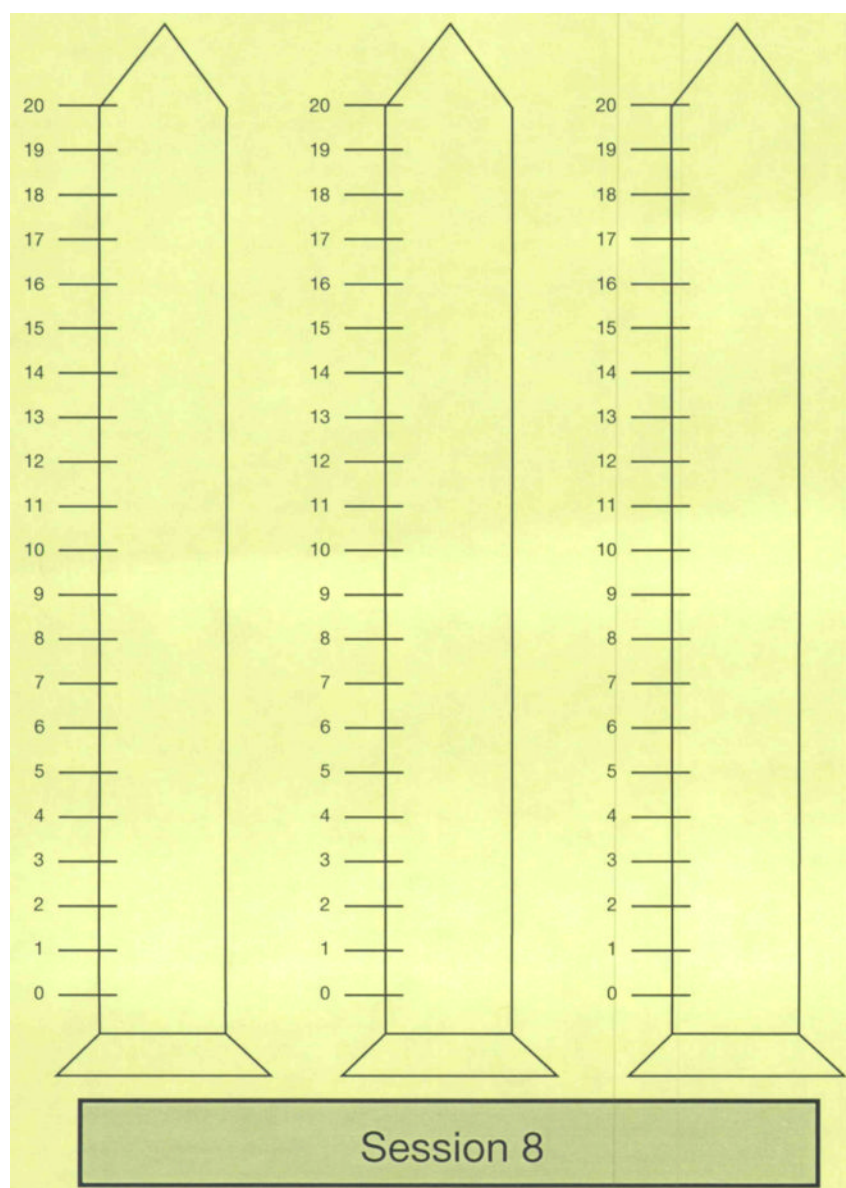




**FIGURE 8.**  
Mr. Greater Gator



**FIGURE 9.**  
Manipulative Examples of 5 Set



**FIGURE 10.**  
Practice Flash Card Graph

TABLE 1

## Galaxy Math Lessons

Unit	Lessons	Topics
1	1–3	Number line (0–9)
		Counting aloud
		Reading and writing numbers
		Counting objects
	4	Using hands to show numbers less than 10
		Using hands to show numbers less than 10
		Discussion of 0 and numbers 11–19
		Counting forwards
	5	Counting backwards
		Identifying largest and smallest numbers with number line
	6, 7	Comparing numbers with language and symbols
	8	Concept of addition
		Meaning of + and =
	9	Addition of 1 with number line
		Using manipulatives, pictures, and actions to solve story problems
	10	Addition of 0 and 1 with number line
		Using manipulatives, pictures, and actions to solve story problems
	11	Counting in
	12	Addition of 0, 1, and 2 with counting in
2	13	Concept of subtraction
		Meaning of - Review of counting in for addition
	14	Addition and subtraction of 1 with number line
		Using manipulatives, pictures, and actions to solve story problems
	15	Addition of 0 and 1 with number line
		Using manipulatives, pictures, and actions to solve story problems
	16	Counting up
	17	Addition and subtraction of 0, 1, and 2 with counting in/up
	18	Review of counting in/up
	19–20	Doubles
3	21–24	5 Set
	25–28	6 Set
	29–32	7 Set
	33–36	8 Set
	37–40	9 Set
	41–44	10 Set
	45–48	11 Set
	49–52	12 Set
4	53	Number line to 100
		Numbers 20–29

Unit	Lessons	Topics
	54–56	Number line to 100 Double-digit addition
	57	Counting by tens Using hands to represents tens and ones
	58	Introduction to place value
	59	Introduction to rods and cubes Regrouping 10 cubes into 1 rod
	60	Regrouping
	61	Representing one- and two-digit numbers with rods and cubes Meaning of 0 in place value
	62	Representing one- and two-digit numbers with rods and cubes
	63	Identifying larger and smaller numbers using place value and number line
	64–66	Practicing place value
5	67–74	Review