Journal of Learning Disabilities
Volume 41 Number 6
November/December 2008 498-513

10.1177/0022219408315638
http://journaloflearningdisabilities.sagepub.com
hosted at
http://online.sagepub.com

Mental Arithmetic in Children With Mathematics Learning Disabilities

The Adaptive Use of Approximate Calculation in an Addition Verification Task

Laurence Rousselle
Marie-Pascale Noël
Catholic University of Louvain, Belgium

The adaptive use of approximate calculation was examined using a verification task with 18 third graders with mathematics learning disabilities, 22 typically achieving third graders, and 21 typically achieving second graders. Participants were asked to make true-false decisions on simple and complex addition problems while the distance between the proposed and the correct answer was manipulated. Both typically achieving groups were sensitive to answer plausibility on simple problems, were faster at rejecting extremely incorrect results than at accepting correct answers on complex addition problems, and showed a reduction of the complexity effect on implausible problems, attesting to the use of approximate calculation. Conversely, children with mathematics disabilities were unaffected by answer plausibility on simple addition problems, processed implausible and correct sums with equal speed on complex problems, and exhibited a smaller reduction of the complexity effect on implausible problems. They also made more errors on implausible problems. Different hypotheses are discussed to account for these results.

Keywords: *mathematical disabilities; arithmetic development; number magnitude; problem-solving strategies; approximate calculation*

Being able to understand numbers, to count, and to calculate are basic but necessary abilities to participate adaptively in modern societies. Therefore, over the past decade, a growing number of studies have been conducted on mathematical disability (MD), a disorder of numerical and/or arithmetical processing that affects 3.5% to 13.8% of school-age children of normal intelligence, depending on the country and on the criteria used to define MD (Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974; Lewis, Hitch, & Walker, 1994). One of the difficulties in studying learning disability in mathematics is the complexity of the field. Mathematics is like a house of cards: Each mathematical ability (e.g., arithmetic, geometry) requires the coordination of lower level interrelated skills (e.g., reading numbers; understanding the base-10 number system; knowing arithmetic facts, carrying, borrowing, measuring), each of which is itself grounded on very basic conceptual and procedural knowledge (e.g., understanding the meaning of numbers or counting). In theory, MD could result from difficulty in learning any single or any combination of these low-level skills, and consequently, children with MD can present very different and changing patterns of weaknesses in the course of their development.

In spite of this diversity, some signs are widely acknowledged in the literature to be common characteristics of MD. In particular, children with MD are known to experience difficulties in executing arithmetical procedures and in learning and/or retrieving arithmetic facts from memory, which entails the persistence of immature problem-solving strategies (Geary, 1993; Geary, Bow-Thomas, & Yao, 1992; Geary, Brown, & Samaranayake, 1991; Geary, Hamson, & Hoard, 2000; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, Hoard, & Hamson, 1999; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2003; Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997; Temple & Sherwood, 2002).

Authors' Note: Please address correspondence to Marie-Pascale Noël, Université Catholique de Louvain, Pl. Cardinal Mercier, 10, 1348 Louvain-la-Neuve, Belgium; e-mail: marie-pascale.noel@uclouvain.be.

Two Dissociable Problem-Solving Approaches

A striking feature in the development of arithmetical skills is that children use an increasing number of strategies for solving arithmetic problems (Ashcraft & Fierman, 1982; Geary et al., 1991, 1992, 2000; Groen & Parkman, 1972; LeFevre, Greenham, & Waheed, 1993; Lemaire & Lecacheur, 2002; Shrager & Siegler, 1998; Siegler, 1988; Siegler & Shrager, 1984). Although very different, all these strategies aim at computing the correct answer to a given problem and, therefore, can be called *exact calculation strategies*. In some situations, however, children were also found to use strategies that do not lead to the exact result but to an approximate answer. For instance, when they make a procedural mistake leading to a grossly incorrect result (e.g., 26 + 57 = 713), they reject the very implausible answer and start calculating again. This suggests that they use *approximate calculation strategies* that enable them to estimate a plausible range of answers for a given problem on the basis of the number magnitudes, with no need to calculate the exact answer.

The distinction between exact and approximate calculation approaches finds its origins in the theoretical framework of Dehaene and his collaborators (Dehaene & Cohen, 1995; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999), who provide converging and conclusive evidence in support of the dissociation between exact and approximate arithmetic in adults. The clearest illustration of this dissociation probably comes from the case of N. A. U. (Dehaene & Cohen, 1991), a patient with an extended left posterior lesion who had lost the ability to read Arabic numbers and to perform even the simplest calculations (e.g., 2 + 2). In contrast, N. A. U. was still able to process number magnitude: He could compare single- and two-digit numbers, could posit two-digit numbers on a scale from 1 to 100, and could reject gross calculation errors such as 2 + 2 = 9, although he was unable to tell if the correct result was 3, 4, or 5 (for case reports of double dissociation between exact and approximate arithmetic capacities, see Dehaene & Cohen, 1997, and Lemer, Dehaene, Spelke, & Cohen, 2003). This neuropsychological evidence, along with behavioral (Spelke & Tsivkin, 2001) and brain-imaging (Stanescu-Cosson et al., 2000; for a review, see Dehaene, Piazza, Pinel, & Cohen, 2003) studies conducted in adults, suggests that the retrieval of basic arithmetic facts involves language-specific representations and relies on a network of left-lateralized language-related areas (including the angular gyrus and the inferior prefrontal cortex), whereas approximate arithmetic depends on language-independent magnitude representations located in the bilateral intraparietal sulci.

Exact and Approximate Calculation Abilities in Children With Mathematics Disabilities

Exact calculation skills have been extensively investigated in children with MD. Numerous studies have shown that they are slower and less accurate in the execution of counting and retrieval strategies and use less-efficient counting procedures than their typically achieving (TA) peers when computing the correct result of a given problem (Geary et al., 1991, 1992, 1999, 2000; Hanich et al., 2001; Jordan & Hanich, 2003; Jordan et al., 2003; Jordan & Montani, 1997). Recent investigations suggest that children with MD could be simply delayed in their development of exact calculation skills, as they were found to apply the same strategies with the same frequency, speed, accuracy, and adaptiveness as younger children matched on mathematical level (Torbeyns, Verschaffel, & Ghesquière, 2004). However, other researchers have suggested that the computational and retrieval deficits identified in these children exhibit divergent evolution (Geary, Widaman, Little, & Cormier, 1987; for reviews, see Geary, 1993, 1994): The computational deficit seems to disappear in the course of development and, therefore, might reflect a developmental delay in the acquisition of counting-based solving procedures (i.e., frequent computational errors and use of immature counting algorithms), whereas the arithmetic fact retrieval deficit (i.e., lower retrieval frequency and speed and high frequency of retrieval errors) points toward a more fundamental impairment persisting throughout the elementary school years.

In contrast to exact arithmetic, few studies have been conducted on the relationships between mathematical disabilities and approximate calculation. Russell and Ginsburg (1984) reported that children with MD who were impaired in a mental addition and subtraction task were still able to judge in an approximate addition task whether a proposed answer was close in magnitude to the correct sum. However, only a small number of problems (i.e., six) were presented, and the stimuli used probably failed to assess the target approximation process because half of the far proposed sums were also logically impossible (i.e., the proposed sum was smaller than one of the addends, e.g., 210 + 530 = 300). Other studies found that children with MD were less accurate than their peers when asked to select the result closest to the correct answer from two incorrect ones (e.g., 4 + 9 = 12 or 19), suggesting an impairment of the approximate calculation approach (Hanich et al., 2001; Jordan & Hanich, 2003; Jordan et al., 2003). However, once again, only a small number of items were presented (i.e., 10 addition and 10 subtraction problems), and reaction times were not recorded. More important, the validity of this task has frequently been questioned, as nothing prevents participants, dyscalculic or not, from calculating the correct result and comparing it with the presented answers instead of estimating an approximate result (see Note 1). If this was in fact occurring, the poorer performance of children with MD in the approximate calculation task would reflect their deficit in exact calculation or comparison processes (Landerl, Bevan, & Butterworth, 2004; Rousselle & Noël, 2007) rather than a specific impairment in approximate calculation skills.

In TA children, however, the pattern of latencies reported by Hamann and Ashcraft (1985) in an addition verification task clearly invalidates this exact-calculation/comparison hypothesis. Classically, the verification task requires participants to make a true-false decision for problems while the difficulty of the problem and the distance between the proposed and the correct answer are manipulated. Children from first grade were found to need less time to reject an extremely incorrect answer (i.e., a result wrong by 12 or 13 units) than to accept a correct sum, suggesting that they did not calculate the exact answer before rejecting grossly incorrect results (for similar results in adults, see Ashcraft & Stazyk, 1981, and Faust, Ashcraft, & Fleck, 1996). Consistent with Dehaene's work, this particular result led us to speculate about the use of a parallel global magnitude evaluation process competing with exact computation in TA children. In cases of extreme mismatch between the proposed and the estimated sum, this global evaluation process would "short-circuit" the computation of the correct sum, triggering a faster decision prior to the completion of the exact calculation process. However, there is currently no data available for children with MD that would enable the exact-calculation/comparison hypothesis to be excluded.

To sum up, the literature on mathematics learning disabilities provides plentiful evidence for impairments in exact calculation tasks. However, little is known about how MD affects approximate calculation abilities. At present, the few existing data suggesting an approximate calculation deficiency in children with MD do not allow the question of whether these children used an approximate calculation approach or an exact-calculation/comparison process to be disentangled.

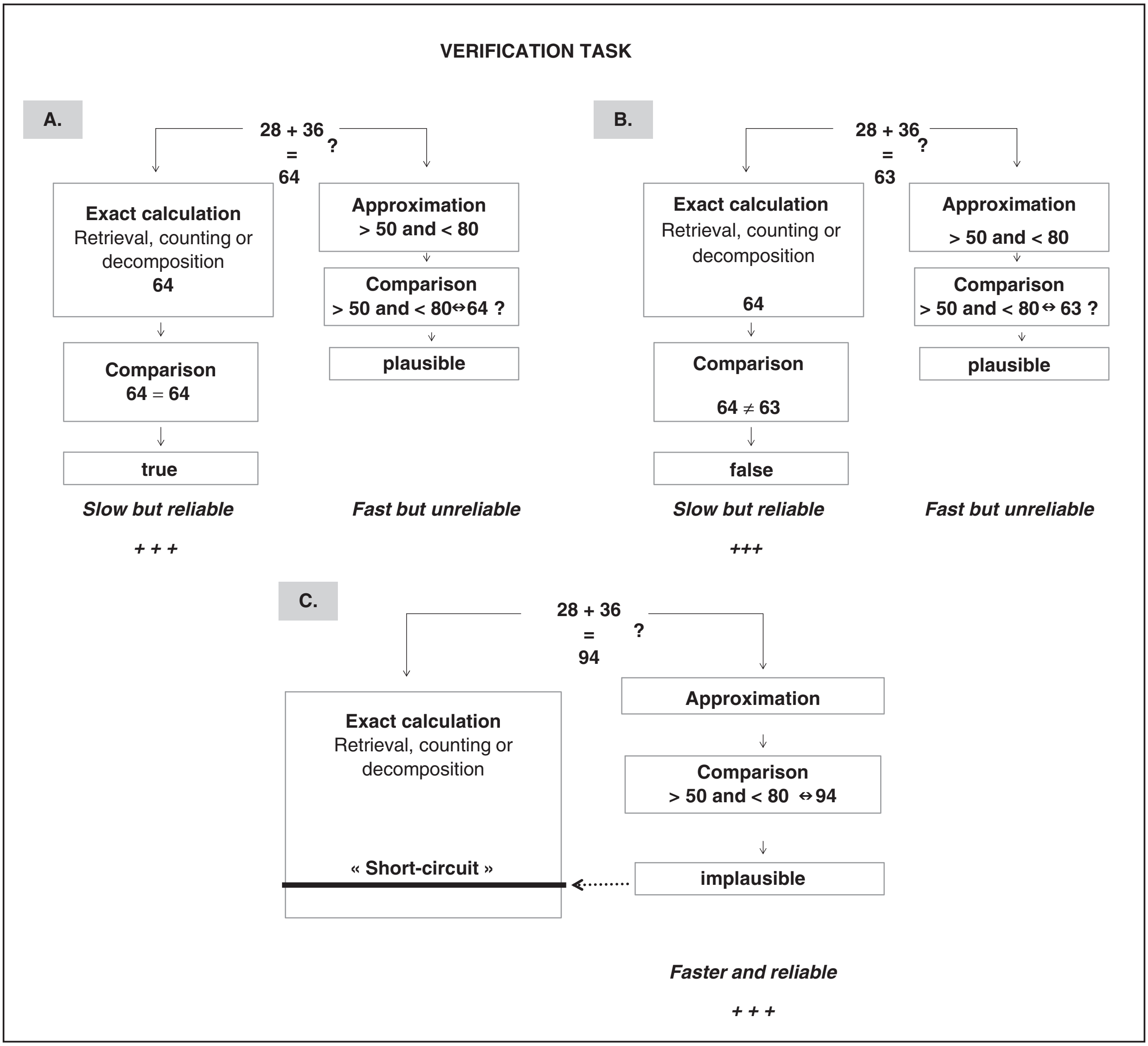
The Present Study

The main issue examined in this study is the ability of children with MD to use an approximate calculation approach in an addition verification task. Unlike other tasks that are designed to force, not necessarily with success, the use of approximation (i.e., presenting two incorrect results for a given equation), the verification task enables the spontaneous use of an approximate calculation approach to be examined. Previous studies suggest the existence of a "horse-race competition" between exact computation strategies and the global magnitude evaluation process. In this framework, choosing between exact and approximate calculation approaches depends not only on strategy speed but also on reliability, that is, on the probability that the approach will provide an accurate answer (Ashcraft & Stazyk, 1981; Hamann & Ashcraft, 1985; LeFevre, Greenham, & Waheed, 1993). In production and verification tasks, the exact calculation approach is usually the most efficient method for obtaining a reliable answer (see Figures 1A and 1B). However, in a verification task, when the proposed result is extremely false, a fast and reliable answer can be obtained with an approximate approach, before the calculation of the exact result (see Figure 1C).

To vary the speed and reliability of each approach, the complexity of the problems and the difference between the proposed and the correct sum were manipulated. The complexity of the problem is known to have a strong effect on exact calculation latencies but a lesser impact on approximate calculations (Ashcraft & Stazyk, 1981; Molko et al., 2003; Stanescu-Cosson et al., 2000). On the other hand, the distance between the correct and the proposed answer should have only minor or no impact on the exact calculation approach but should directly influence the reliability of the approximate calculation approach, with large-split problems being estimated more reliably.

In line with previous findings, TA children were expected to show three typical signs associated with the use of an approximate calculation approach. First, they should be faster at processing large-split problems than small-split problems, demonstrating their sensitivity to the plausibility of the proposed sum (Ashcraft & Fierman, 1982; Hamann & Ashcraft, 1985). However, the presence of a distance effect alone does not constitute sufficient evidence for the use of approximation. Indeed, as noted earlier, children may simply have calculated and then compared the correct result with the presented answers without having done any estimation. In this case, the distance effect would be an artifact of the comparison process and would not reflect approximation. To rule out this exact-calculation/comparison hypothesis, children in the TA group should also be faster at processing large-split problems than true equations, as in Hamann and Ashcraft's (1985) study, providing a second and more compelling sign of the use of an approximate calculation approach. Finally, approximate calculation has been

Figure 1
Schematic Depiction of the "Horse-Race Competition" Between the Exact Calculation Approach and the Global Magnitude Evaluation Process in a Verification Task



Note: Strategy selection depends on the speed and the reliability of each approach.

shown to be less influenced by the problem size than the exact calculation approach is (Molko et al., 2003; Stanescu-Cosson et al., 2000). Accordingly, if children in the TA group use approximation on large-split problems, the complexity effect should be reduced on these problems in comparison with true problems and small-split problems.

By contrast, children with MD were expected to have lower performance than normally achieving children on equations requiring an exact calculation, that is, on correct equations and small-split problems (Geary et al., 1991, 1992, 1999, 2000; Hanich et al., 2001; Jordan et al., 2003; Jordan & Hanich, 2003; Jordan & Montani, 1997). In addition, because approximate calculation is assumed to place great emphasis on the manipulation of magnitude representations (Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1991, 1997; Dehaene et al., 1999; Hamann & Ashcraft, 1985; Stanescu-Cosson et al., 2000), a basic deficit in the ability to process numerosities (Landerl et al., 2004) or to connect a number to its quantitative meaning (Rousselle & Noël, 2007) should negatively affect the speed and/or the reliability of the global evaluation process and, consequently, the use or discovery of the approximate calculation approach in children with MD. As a result, children with MD are expected to make less use of the approximate calculation approach than their peers do and should differ from children in the TA group with respect to the three typical signs associated with the use of approximation. To determine whether the observed deficit (or deficits) reflect a developmental delay or a more specific deficit of the underlying processes, third-grade children with MD were compared to normally achieving third and second graders.

Method

Participants

Three groups of children participated in the study: 18 third-grade children with mathematics learning disabilities (MD group; mean age = 8 years 7 months), 22 third-grade children with normal achievement in mathematics (third-grade TA group; mean age = 8 years 10 months), and 21 second-grade children with no known difficulties in mathematics (second-grade TA group; mean age = 7 years 7 months).

Participant selection procedure. Third-grade participants (MD and TA groups) were a subset of those who took part in an earlier study (Rousselle & Noël, 2007; see Note 2). They were selected 1 year earlier, in the fall of their second grade, from a screening pool of 427 children drawn from 12 state schools in the area of Louvain-la-Neuve, Belgium. These children were screened as part of a large-scale follow-up study of mathematical disabilities. All of them still attended general education classes, and only children whose parents gave their informed consent for the extended research program were included in the final experimental groups.

Original participant screening included mathematics, reading, and IQ assessments (for an extended description, see Rousselle & Noël, 2007). Mathematics achievement level was assessed using a composite test battery that was administered collectively in the classroom. This battery comprised a number-processing subscale (including Arabic number writing, Arabic number comparison, and a task assessing base-10 understanding in Arabic numeral coding) and an arithmetic subscale (including an untimed addition and subtraction test and a timed addition test). Reading achievement level was examined with two French-speaking reading subtests: the LUM subtest of the LMC-R battery (speed-word reading; Khomsi, 1998) and the L3 subtest of the ORLEC battery (reading comprehension; Lobrot, 1980; recent normative data appear in Mousty & Leybaert, 1999). Finally, IQ was estimated with the Similarity and Picture Completion subtests of the *Wechsler Intelligence Scale for Children–Third Edition* (Wechsler, 1992).

Participants with low intellectual efficiency (i.e., mean standard note of the two subtests equal to or less than 7; $M = 10$, $SD = 3$) were excluded as were children who were not native French speakers. Participants with reading difficulties (i.e., mean standardized reading score more than 1 standard deviation below the grade-level norms) were also discarded from the present sample to avoid multicausal interpretations. Indeed, these children are known to experience phonological processing difficulties suspected to interfere with exact calculation development between second and fifth grade and to account for a great part of the frequent association between mathematical and reading disabilities (Hecht, Torgesen, Wagner, & Rashotte, 2001; Robinson, Menchetti, & Torgesen, 2002). This phonological deficit might account for the differential pattern of performance on exact calculation according to whether MD is associated with reading disability or not (Hanich et al., 2001)

Children with a mathematics composite score below the 15th percentile and designated as having learning difficulties in mathematics by their teachers were classified as the MD group. The chosen cutoff was more conservative than the criteria often used in studies on mathematical disabilities (35th percentile in Hanich et al., 2001; Jordan et al., 2003; Jordan, Kaplan, & Hanich, 2002; Geary et al., 2000; 30th percentile in Geary et al., 1999, 2004; 25th percentile in Jordan et al., 2003). This criterion was used to reduce the number of false positives to ensure that all children in our sample really had MD (Mazzocco & Myers, 2003). Recently, Geary and his collaborators used the same selection cutoff to distinguish children with mathematics learning disabilities (below the 15th percentile) from low-achieving children (below the 39th percentile; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). With the present selection criteria, 6.7% of the original sample was included with the MD group (10.5% including children with reading disabilities), a percentage consistent with epidemiological data (Barbaresi et al., 2005; Gross-Tsur et al., 1996). Children with mathematics composite scores above the 50th percentile and designated as having no learning difficulties by their teachers were considered normally achieving children and classified into the TA group. The TA group was constituted to match the MD group as closely as possible for gender within their class group.

Table 1 reports descriptive information and mean mathematics percentile scores for MD and TA third graders. There was no group difference on the performance and

Table 1
Descriptive Information and Mathematics Percentile Scores for MD and TA Third Graders

				IQ[b]			
Group	n	Gender (boys/girls)	Age[a]	Verbal	Performance	Subtest Mean Score	Mathematics Percentile Score
Third-grade MD	18	6/12	102.67 (3.16)	11.33 (2.72)	11.11 (2.47)	11.22 (2.14)	8.06 (3.37)
Third-grade TA	22	8/14	105.32 (3.00)	11.61 (2.15)	9.84 (2.91)	10.73 (1.77)	77.82 (14.33)

Note: Standard deviations are shown in parentheses. MD = mathematical disability; TA = typically achieving.
a. Age is expressed in months.
b. IQ estimated with the *Wechsler Intelligence Scale for Children–Third Edition*: standard note, $M = 10$, $SD = 3$.

verbal IQ subtests nor on the subtest mean standard note: Picture Completion, $F(1, 39) = 2.17$, and Similarity and subtest mean standard note, $F(1, 39) < 1$, ps $> .10$. However, the 3-month age difference between the two groups was significant, $F(1, 39) = 7.37$, $p = .01$. Finally, as expected, children with MD had a lower mathematics achievement level than their peers, as attested by the high discrepancy between the mean mathematics percentile scores of the two groups in Table 1, $F(1, 39) = 406.66$, $p < .001$.

For this study, 21 second-grade children (13 boys and 8 girls; mean age = 91.19 months, SD = 3.72, range = 83–96 months) were randomly selected from one school of the third-grade participants'. All the children in the second-grade group were designated by their teacher as having no learning difficulties. The proportion of boys and girls in the three experimental groups did not differ, $\chi^2(2) = 4.08$, $p > .10$.

Materials

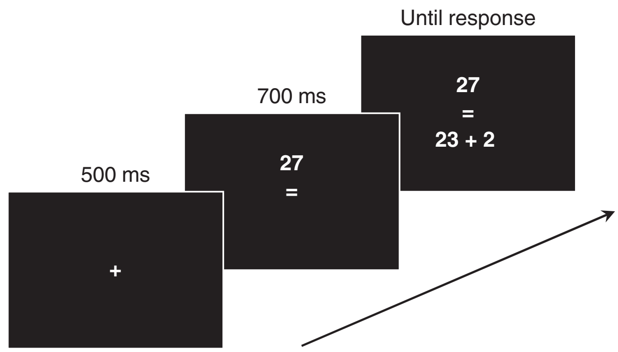
Stimuli were presented with the E-Prime 1.1 software on a personal computer screen. Responses and latencies were collected with 1 ms precision using the E-prime SR-box. The stimuli consisted of 60 addition problems presented with either a correct or an incorrect answer. The complexity of the problems and the distance between the correct and the presented sum (i.e., the split) were manipulated. Half of the problems were simple additions involving no carry operation, and half were complex additions that required carrying to reach the exact result. For both levels of complexity, one third of the proposed sums were correct (split 0, true problems); one third were incorrect but plausible, as the proposed answer was only 2 units greater than the correct one (split 2); and one third were incorrect and implausible, as the proposed answer was 14 units more than the correct sum (split 14). As both the splits chosen for the incorrect equations were even, the parity of the correct and the proposed answers always matched.

To avoid direct recognition of basic facts, the correct sums were always greater than 10. Moreover, several constraints were imposed on stimulus selection to avoid potential confounds (for reviews, see Geary, 1994, and Ashcraft, 1995). First, the correct sums were never equal to a multiple of 10. Second, tie problems (e.g., 9 + 9) and related addition problems (e.g., 19 + 9) were excluded from the stimulus set. Third, to prevent associative confusions (e.g., 8 + 3 = 24), none of the incorrect proposed sums equaled the products of the addends. Finally, the digit 1 and multiples of 10 were never used as addends to prevent the use of shortcut strategies that could bypass the calculation process. For example, solving $n + 1$ only requires the number word coming after n in the counting sequence to be found. Likewise, 10 + 6 can be solved by replacing the 0 by the 6, and 10 + 13 can be solved by adding the 10s only.

Ten different equations were selected in each experimental condition, giving a total of 60 problems to be solved (10 problems × 2 complexity levels × 3 splits; see the appendix). Resulting from the selection constraints and from the experimental design, correct sums fell either between 11 and 17 or between 21 and 27. All the simple addition problems involved one two-digit and one single-digit addend (between 2 and 5), whereas complex addition problems involved either two single-digit addends (both between 5 and 9) or one two-digit and one single-digit addend (between 5 and 9). The two-digit addend was always presented in the first position (e.g., 21 + 3).

Forty-four filler problems were integrated in the experimental stimulus set in order to equalize the number of true and false problems. These included 32 correct fillers added to the 20 true equations and 12 incorrect fillers added to the 40 false equations, giving a total of 52 true and 52 false addition problems to be solved. The fillers were also used to balance the frequency of the proposed sums to the true and false problems so as to avoid some answers (in particular those > 30) being systematically associated with a false response. The fillers were selected so that the proposed answers occurred

Figure 2
Schematic Depiction of the Stimuli Disposition and of the Succession of Events During a Trial



with the same frequency for false and true problems among the 104 stimuli of the experimental set (e.g., 19 + 8 = 41 = false; corresponding filler: 40 + 1 = 41 = true). As the data on the fillers were not used in the statistical analyses, filler problems were made as easy as possible (e.g., they did not involve carrying) so as not to tire the children unnecessarily.

The 104 verification problems were presented twice in two separate sessions, for a total of 208 trials (120 test stimuli and 88 fillers). The stimuli were sequenced in a fixed pseudorandom order with the constraint that no more than three consecutive trials should require the same response or have the same split. A different order was used for each testing session.

The equations were presented in an unusual way, as the proposed sum was presented before the onset of the addition problem. This procedure was adopted so that children could get an idea of the magnitude of the sum before starting to calculate. The aim was to encourage children to evaluate the plausibility of the proposed sum instead of rigidly engaging in a fixed calculation process. As shown in Figure 2, each trial started with the presentation of a fixation cross for 500 ms. Then, the proposed sum appeared on the screen followed by the addition problem 700 ms later. Both the addition and the proposed sum remained on screen until the child responded.

Procedure

Children were tested individually in a quiet room in their school. They were asked to check whether or not the proposed sum for an addition problem was correct and to press the left button for the false equations and the right button for the true ones. "False" and "True" labels were placed on the top left and right sides of the screen, respectively, and on the response box as clues for response key assignment. Children were informed that problems are presented in a odd way, as the proposed sum appears first, a little before the calculation. Two examples were shown to illustrate this mode of presentation. Furthermore, children were told that sometimes the proposed sum was so incorrect that they would not have to calculate to answer. Two additional examples were given to illustrate implausible problems. Equal emphasis was placed on speed and accuracy.

The two testing sessions were separated by a short break and lasted approximately 25 minutes each. Twenty practice trials were administered before the first session so that children could familiarize themselves with the task and with the response keys.

Results

Statistical analyses were carried out on median reaction times (RTs, based on correct responses only) and number of correct responses. However, RTs and accuracy data did not fit some of the assumptions of the repeated measures ANOVA statistical model, in particular, the variance homogeneity and the sphericity assumptions. In the RT analyses, a logarithmic transformation was applied to median RTs in order to eliminate variance heterogeneity. These log-transformed RTs were then analyzed using multivariate analyses of variance for repeated measures (MANOVA, Wilks's Lambda test statistic), which do not have a sphericity assumption (Howell, 2001, p. 519). When multiple pairwise comparisons were necessary, they were adjusted using the Bonferroni correction. In accuracy analyses, however, the percentage of correct responses was high (between 75.3% and 97.5% in the MD group, 83.3% and 98.8% in the third-grade TA group, and 71.4% and 96.9% in the second-grade TA group), and no transformation could improve the fit between the accuracy data and the assumptions for ANOVA. Accordingly, the accuracy data were analyzed using nonparametric statistics.

RT Analysis

Six children with MD, 5 second-grade children, and 3 third-grade children in the TA group scored at or below chance level (i.e., $\leq 10/20$) in one of the complex problem conditions (only on splits 0 and 2). These children were removed from the RT analyses that included complex problems in order to avoid bringing noise into the statistics, giving a total of 12 children with MD, 16 second-grade TA children, and 19 third-grade TA children (see Note 3). So, the log-transformed RTs entered in the analysis were always based on a significant number of correct responses (no fewer than 11). Nevertheless, as all of the 14 children excluded from RT analyses performed at the better-than-chance level on simple problems, the analyses conducted on simple problems only were also carried out including all children (i.e., 18 children with MD, 21 second-grade TA children, and 22 third-grade TA children) and were then reported in footnotes.

Mazzocco and Myers (2003) argued against the use of a one-time assessment to identify MD because children's weaknesses can shift over time as a function of their individual development. So, before exploring children's ability to use approximation, the persistence of mathematical disability in the MD group was assessed by examining their performance on true problems that require the correct sum to be computed. In this group, children's RTs on true problems were converted into a standard score using the mean and the standard deviation of the TA group. MD group mean RT was on average 2.81 standard deviations above the TA group mean on true problems (SD = 1.39, minimum = 1.00, maximum = 4.98; simple problems, M = 8.02, SD = 4.75, minimum = 0.52, maximum = 14.83; complex problems, M = 2.02, SD = 1.35, minimum = 0.13, maximum = 4.67). All children in the MD group still had mean RTs superior to 1 standard deviation above the control mean, confirming their slower calculation speed when computing the correct sum of a given problem. The pattern of group differences on true problems was further explored in a 2 × 3 repeated measures MANOVA with problem complexity (simple, complex) as within-subject factor and group (MD, third-grade TA, second-grade TA) as between-subjects factor. As expected, RTs were faster for complex than for simple problems, F (1, 44) = 129.92, p < .001, and differed between groups, F (2, 44) = 22.86, p < .001. Third-grade children in the TA group were faster than children in the MD and second-grade TA groups, ps < .001, but RTs of the latter two groups did not differ, ps > .10. There was no significant interaction, F (2, 44) = 1.23, p > .10. These results indicate that children with MD and second graders exhibited very similar calculation speeds (Note 4) and that all groups exhibited the comparable complexity effect when solving true equations.

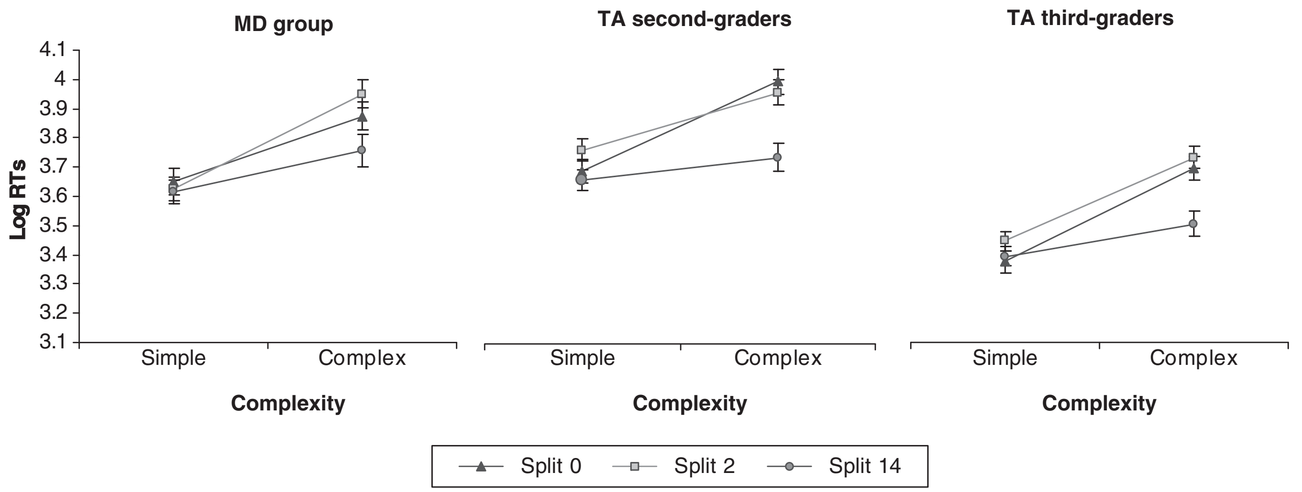
To examine the use of approximate calculation, a 2 × 3 × 3 repeated measures MANOVA with problem complexity (simple, complex) and split (0, 2, 14) as within-subject factors and group (MD, third-grade TA, second-grade TA) as between-subjects factor was conducted on the log-transformed RTs. All the main effects were significant: RTs increased with problem complexity, F (1, 44) = 159.13, p < .001, were affected by the split between the proposed and the correct sum, F (2, 43) = 54.59, p < .001, and differed between groups, F (2, 44) = 19.85, p < .001. Third-grade children in the TA group were on average faster than children in the MD and second-grade TA groups (ps < .001), but these two groups exhibited similar RTs (p = 1). In addition, all split differences were significant, ps < .005. The time required to verify an equation was the shorter for implausible sums (split 14) and was longer for plausible sums (split 2) than for correct sums (split 0). More interesting, there were significant Split × Complexity interactions, F (2, 43) = 27.69, p < .001, and Split × Complexity × Group interactions, F (4, 86) = 4.12, p < .005. Other interactions were not significant, Fs < 1.64, ps > .10.

For the present purposes, the most important result was the significant three-way interaction between split, complexity, and group, which is represented in Figure 3. Although it might seem redundant, the nature of this interaction was further explored in two different ways in order to detect the three typical signs associated with use of the approximate calculation approach.

First, the modulation of the split effect across groups was examined by running separate analyses for simple and complex addition problems. On simple addition problems (see Note 5), separate repeated measures MANOVAs carried out in each group revealed a significant effect of split in the second-grade TA group, F(2, 14) = 10.40, p = .002, and the third-grade TA group, F(2, 17) = 16.71, p < .001, but not in the MD group, F(2, 10) = 1.04, p > .10. In both TA groups, RTs to reject false answers were longer for plausible than for implausible sums, ps < .005, and true problems were processed faster than plausible ones, ps < .05. However, the difference between RTs for true and implausible equations was not significant, ps > .10. It is worth noting that the presence of a divergent pattern of split effect across groups was unrelated to the smaller sample size of the MD group. Indeed, the same pattern of split differences emerged when entering a restricted sample of second- and third-grade children in the TA group in the analyses in order to equate sample size across groups (n = 12 in all groups; see Note 6).

On complex addition problems, repeated measures MANOVAs carried out separately in each group showed that the main effect of split was significant in all three groups: MD group, F (2, 10) = 8.21, p < .01; third-grade TA group, F (2, 17) = 54.52, p < .001; and second-grade TA group, F (2, 14) = 12.35, p < .005. In all groups, RTs for true and plausible problems did not differ, ps > .10, and the time required to reject a false answer was longer for plausible than for implausible sums, ps < .01. However, children in the MD and the TA groups exhibited divergent patterns of latencies, as second- and third-grade children in the TA groups needed significantly less time to reject implausible equations than to accept a correct sum, ps < .001, whereas third-grade children in the MD group did not, p > .10. As for simple equations, the same pattern of split differences emerged when entering a

Figure 3
Log-Transformed Reaction Times and Standard Errors by Split and Problem Complexity for Each Group



Note: MD = mathematical disability; TA = typically achieving.

restricted sample of second- and third-grade children in the TA group in the analyses ($n = 12$ in all groups). The presence of a divergent pattern of split differences across groups could thus not be accounted for by the smaller sample size of the MD group.

This first set of analyses indicates that children with MD differ from second graders and third graders in the TA groups in two of the three typical signs associated with the use of approximate calculation: (a) On simple problems, children in the MD group were not sensitive to the plausibility of the proposed sum, contrary to children in the TA group; (b) on complex addition problems, although children in the MD group exhibited a sensitivity to answer plausibility, they processed implausible and correct sums with equal speed, contrary to children in the TA group who were faster at rejecting extremely incorrect results than at accepting correct answers. A second set of analyses was conducted to examine the third sign of the use of approximate calculation, that is, the reduction of the complexity effect on implausible equations (split 14) in comparison with true and small-split problems. Detecting the presence of this sign required examining the modulation of the complexity by split interaction across groups and thus decomposing the three-way interaction differently.

First of all, we examined how this interaction was modulated by introducing groups two by two in the global analysis: 2 (complexity) × 3 (split) × 2 (group) repeated measures MANOVAs. The three-way interaction was significant when including the MD and the third-grade TA groups, F (2, 28) = 4.30, $p < .05$, or the MD and the second-grade TA groups, F (2, 25) = 7.12, $p < .005$, but not when introducing the two TA groups together in the analysis, F (2, 32) < 1. This identified the MD group as the source of the significant three-way interaction.

The second step was then to test the presence of a Complexity × Split interaction in each group separately. The 2 (complexity) × 3 (split) repeated measures MANOVA showed a significant interaction in all groups: MD group, F (2, 10) = 8.99, $p < .01$; TA third graders, F (2, 17) = 23.91, $p < .001$; and TA second graders, F (2, 14) = 10.85, $p < .001$. Although these results did not help to account for the three-way interaction, they indicated that all groups showed a significant reduction of when the split increases.

Finally, we tracked the reduction of the complexity effect on implausible equations in each group by introducing only two split levels at the same time in the analysis. For the sake of simplicity, we focused on the contrasts with implausible problems that are supposed to be processed using approximation. When introducing true and implausible sums, the 2 (complexity) × 2 (split 0, 14) repeated measures MANOVA yielded a significant interaction in TA third graders, F (1, 18) = 42.84, $p < .001$, and second graders, F (1, 15) = 22.57, $p < .001$, but not in the MD group, F (1, 11) = 1.96, $p > .10$. When introducing plausible and implausible sums in the analysis (split 2, 14), results showed a significant interaction in all three groups: MD group, F (1, 11) = 16.19, $p < .01$; TA third graders,

$F(1, 18) = 38.87$, $p < .001$; and TA second graders, $F(1, 15) = 7.19$, $p < .05$. As in the first set of analyses, it is worth noting that the same pattern of results came out when entering a restricted sample of second- and third-grade children from the TA groups in the analyses ($n = 12$ in all groups). The presence of a nonsignificant result in the MD group could thus not be explained by its smaller sample size.

This second set of analyses clearly established that the two TA groups showed a reduction of the complexity effect for implausible sums in comparison with true and plausible equations. In this respect, however, results in the MD group were less clear-cut. Although there was no reduction of the complexity effect on implausible sums when contrasted with true problems, the reduction was significant when contrasted with plausible problems. As true and plausible addition problems are supposed to be both processed by exact calculation, this contrasting pattern of results is difficult to interpret. Previous analyses indeed demonstrated that true and plausible equations did not differ in the MD group (neither on simple nor on complex problems). Given that the MD group was previously identified as the source of the three-way interaction, results of the analysis contrasting true and implausible sums either way indicated that the reduction of the complexity effect according to the split in the MD group was not as important as in the two TA groups (see Figure 3).

To sum up, RT analyses suggest that children with MD differ from second- and third-grade TA children in the way they verify simple and complex addition problems. The group differences in the verification task cannot be accounted for simply by differences in calculation skills, as children with MD and second graders exhibited different patterns of latencies on this task despite their very similar calculation speeds.

Accuracy Analysis

One child in the third-grade TA group was removed from the accuracy analysis because he scored more than 2.5 standard deviations below his group mean in four out of the six experimental conditions. Thus, 18 children with MD and 21 second graders and 21 third graders in the TA groups were entered in the subsequent nonparametric statistics. Table 2 shows the mean number of correct responses (maximum = 20) and standard deviations according to the split and the problem complexity for each group.

As in RT analysis, the persistence of mathematical disability in the MD group was assessed by converting individual mean number of correct responses on true problems into a standard score using the mean and the standard deviation of the TA group. Children in the MD group performed on average –0.8 standard deviation below the TA group mean on true problems ($SD = 2.04$, minimum = –5.26, maximum = +1.60; simple problems, $M = -0.51$, $SD = 2.12$, minimum = –7.27, maximum = +0.84; complex problems, $M = -0.74$, $SD = 2.02$, minimum = –5.82, maximum = +1.53). In the MD group, 28% of the children performed more than 2.5 standard deviations below the TA group mean, whereas others had a standard score superior to –0.42 standard deviation around the control mean.

A first series of nonparametric statistics was carried out to analyze problem complexity and split effects in each experimental group. As expected, all groups had a higher number of correct responses for simple than for complex problems, Wilcoxon's Zs < –3.52, ps < .001, and showed a significant effect of split, all Friedman's $\chi^2(2) > 12.83$, ps < .005. Children performed more accurately on implausible sums than on true and plausible equations, Wilcoxon's Zs < –2.77, ps < .01. Second-grade children in the TA group also had a higher number of correct responses on plausible than on true sums, Wilcoxon's Zs = –2.28, $p < .05$, but this difference was not significant in the two other groups, Wilcoxon's Zs = –.48 and –.63 in the MD and third-grade TA groups, respectively, ps > .10.

The second step was to examine the split effect in each group on simple and complex problems separately, in the same way as in the RT analyses. None of the experimental groups exhibited a significant effect of split on simple equations, all Friedman's $\chi^2(2) < 2.8$, ps > .10, but all did on complex problems, all Friedman's $\chi^2(2) > 14.88$, ps < .001. In all three groups, the number of correct responses on complex problems was higher for implausible sums than it was for true or plausible sums, Wilcoxon's Zs < –2.7, ps < .01. Moreover, plausible problems tended to be solved more accurately than true ones, but this trend was significant only in the second-grade TA group, Wilcoxon's $Z = -2.44$, $p < .05$.

Finally, Kruskal-Wallis tests were carried out to track the presence of a group effect on the number of correct responses. Group differences were not significant for any level of split, χ^2s(2) < 3.54, ps > .10, or problem complexity, χ^2s(2) < 3.83, ps > .10, or for any split-by-complexity combination, χ^2s(2) < 3.70, ps > .10, excepted on complex implausible problems, $\chi^2(2) < 6.02$, $p = .05$. Indeed, children with MD made more errors on these problems ($M = 18.50/20$, $SD = 2.09$) than did second- and third-grade children in the TA group ($M = 19.24/20$, $SD = 2.02$, Mann-Whitney $U = 133$, $Z = -1.88$, $p = .06$; $M = 19.76/20$, $SD = .44$, Mann-Whitney $U = 122$, $Z = -2.21$, $p < .05$, respectively). Individual profiles revealed that one half of the children in the MD group made a higher

Table 2
Mean Number of Correct Responses and Standard Deviations by Split and Problem Complexity for Each Group

		Simple Problems		Complex Problems		All Problems	
Group	Split	*M*	*SD*	*M*	*SD*	*M*	*SD*
Third-grade MD	0	19.00	1.57	15.06	4.40	17.03	2.53
	2	19.28	1.07	15.61	5.23	17.44	2.91
	14	19.50	1.04	18.50	2.09	19.00	1.38
Total		19.26	0.94	16.39	2.77	17.82	1.74
Third-grade TA	0	19.38	0.74	16.67	2.18	18.02	1.24
	2	19.29	1.27	17.14	4.02	18.21	2.21
	14	19.62	0.59	19.76	0.44	19.69	0.40
Total		19.43	0.50	17.86	1.82	18.64	1.00
Second-grade TA	0	19.10	1.04	14.29	4.06	16.69	2.34
	2	18.90	1.18	17.05	3.34	17.98	2.05
	14	19.38	1.16	19.24	2.02	19.31	1.42
Total		19.13	0.87	16.86	2.27	17.99	1.52

Note: Maximum number of correct responses = 20. MD = mathematics disabilities; TA = typically achieving.

number of errors on complex implausible sums in comparison to both TA group means. By contrast, both TA groups performed equally well in this condition (Mann-Whitney $U = 217.5$, $Z = -0.11$, $p > .10$).

Discussion

This study aimed to examine the adaptive use of an approximate calculation approach in children with mathematical disabilities by using a verification task. On simple addition problems, second- and third-grade children in the TA group took longer to reject plausible than implausible equations. Moreover, in keeping with Hamann and Ashcraft's (1985) study, they needed less time to reject complex implausible sums than to accept complex true equations. Finally, they showed a reduction of the complexity effect on implausible sums, compared with true and plausible equations. By contrast, children with MD were equally fast at processing plausible and implausible sums on simple problems and implausible and correct sums on complex problems. In addition, the reduction of the complexity effect on implausible sums was smaller than in the two TA groups. Finally, they made more errors on complex implausible problems than their normally achieving peers and than younger children with similar levels of exact calculation skills. Taken together, these contrasting results indicate the presence of processing peculiarities in children's ability to use approximate calculation in the MD group.

From a longitudinal perspective, the children with MD in this sample, who had been selected 1 year earlier for their number-processing and arithmetical disabilities, are still struggling with arithmetic a year later. All these children still need more time to compute the correct answer of a given problem, with RTs equal or superior to more than 1 standard deviation above the TA group mean in exact calculation (i.e., below the 15th percentile). Some of them even continue to make more errors than their normally achieving peers. These results confirm the persistence of mathematical difficulties across the elementary school years (Geary & Hoard, 2005; Jordan & Hanich, 2003; Jordan et al., 2003; Shalev, Manor, & Gross-Tsur, 2005). The absence of significant difference between the MD and second-grade TA groups on true problems is in accordance with previous reports indicating the existence of a developmental delay in the exact calculation abilities of children with MD (Geary, 1993; Geary et al., 1987; Torbeyns et al., 2004).

Despite having similar calculation skills, children with MD exhibited a different pattern of latencies than those of younger children in the addition verification task: They exhibited no split effect on simple problems, processed implausible and true complex problems with the same speed, and showed a smaller reduction of the complexity effect on implausible sums. These results might suggest the existence of a more fundamental deficit impeding the use of an approximate calculation approach.

In children in the TA group, the pattern of results provides compelling evidence for the use of an approximate calculation approach. Indeed, not only were they sensitive to the plausibility of the stated sums on simple problems, but they also rejected complex implausible sums more quickly than they accepted complex true equations, confirming that they were able to reject grossly incorrect results before the completion of the exact calculation process. In keeping with this, they consistently showed a smaller complexity effect on implausible problems, a sign expected to occur when using approximate calculation.

By contrast, the absence of a split effect on simple problems for children with MD clearly indicates that they did not use an approximate calculation approach to solve simple addition problems, as they did not exhibit any sensitivity to the plausibility of the proposed answer. Accordingly, it seems likely that they processed all simple addition problems using an exact-calculation/comparison procedure. If so, the absence of a split effect could be explained in at least two complementary ways. First, the great variability of the exact calculation process could have absorbed the small effect related to the comparison stage, that is, the effect of split. Second, the absence of a split effect could reflect a deficit in the comparison stage itself. When comparing Arabic digits, children with MD have been found to exhibit not only slower response times than their TA peers but also a reduced distance effect (Rousselle & Noël, 2007).

On the other hand, children with MD proved to be sensitive to the plausibility of the proposed sums on complex problems. However, unlike second- and third-grade children in the TA group, implausible sums were not processed faster than true ones. Does this mean that children with MD also used an exact-calculation/comparison procedure on complex problems? Given their pattern of RTs on simple addition problems, this possibility seems very unlikely. Indeed, if children in the MD group processed all equations using an exact-calculation/comparison procedure, there is no reason to expect the difference between plausible and implausible sums to be larger on complex problems than on simple ones. On the contrary, in this case, we would expect greater variability in exact calculation latencies on complex problems and thus smaller split differences in RTs, which is clearly not the case here. Given the pattern of performance on complex problems, the most likely hypothesis is that children in the MD group started to use an approximate calculation approach on these problems but not as efficiently or as frequently as children in the TA group did. The smaller reduction of the complexity effect on implausible problems in the MD group is consistent with this hypothesis.

Two alternative accounts could explain why children with MD are less efficient or less inclined to use an approximate calculation approach. First, assessing the magnitude of plausible results might be more time consuming for them than for second- and third-grade children in the TA group. As a result, the approximate calculation approach is too slow to short-circuit the calculation of the correct sum, at least with the split differences and complexity levels of the calculations used in the present study. Alternatively, their performance in this task might be influenced by children's individual uncontrolled factors, such as their confidence in their own calculations and estimations. Some authors speculate that each particular calculation strategy is associated with an internal confidence criterion (i.e., activation threshold) that must be exceeded for a retrieved answer to be produced (LeFevre et al., 1993; Siegler, 1988; Siegler & Shrager, 1984). This confidence criterion is assumed to vary from one individual to another. In this perspective, children with MD might be able to approximate, but they might use a more stringent confidence criterion for the less familiar and less reliable process of approximation than for the exact calculation strategies they are trained to use at school. If either of these interpretations is valid, then children with MD should manifest clearer signs of the use of an approximate calculation approach with larger split values.

It is important to note that these two hypotheses are not mutually exclusive: Approximate calculation in children with MD could be both more time consuming and could be associated with a more stringent confidence criterion to produce the pattern of performance reported in this study. The question then becomes what kind of factor or factors could have affected the speed and/or the confidence criterion associated with approximate calculations in children with MD? One such factor might be a basic number-processing deficit in these children. Be this a difficulty in processing number magnitudes (i.e., defective number module hypothesis) or in accessing number meaning from symbols (i.e., access deficit hypothesis), such a number-processing deficit would be expected to affect the use and the execution of strategies in proportion to their quantity-processing requirements. As approximate calculation is assumed to rely heavily on the computation of quantitative information conveyed by symbols (Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1991, 1997; Dehaene et al., 1999; Hamann & Ashcraft, 1985; Stanescu-Cosson et al., 2000), such basic deficits could affect the speed and/or the reliability of approximate calculations more than they affect exact calculations. In line with this view, children with MD showed a higher error rate on complex implausible problems, suggesting that their approximations, if they made any, might indeed be less reliable than those of children in the TA group.

In children with MD, the confidence criterion associated with approximate calculation might also be negatively influenced by affective components such as the children's confidence in their ability to estimate or to do mathematics, their tolerance of errors, and their level of math anxiety (Ashcraft & Ridley, 2005; LeFevre et al., 1993). Indeed, difficulties in mathematics or recurrent unpleasant experiences in math class are known to induce lower confidence in mathematical performance, math anxiety, and avoidance attitudes toward mathematics, all of which can contribute, in turn, to underachievement in mathematics (Ashcraft, Kirk, & Hopko, 1998; Butterworth, 2005). Ashcraft and Faust (1994) noticed that highly anxious individuals did not use alternative shortcut strategies to solve complex addition problems, whereas low-anxiety groups did. In particular, highly anxious participants did not take advantage of the opportunity to abort the exact calculation process by detecting an incorrect value in the units (e.g., $17 + 16 = 38$) and preferred to carry out the entire exact computation algorithm. In the present study, children with MD (who were probably aware of their calculation difficulties) might have experienced higher levels of anxiety than their TA peers. As a result, they might have responded as "perfectionists," feeling more confident when completing the exact calculation sequence than when relying on the less precise, albeit more rapid, approximate calculation approach even to detect implausible answers.

One final issue concerns the nature of the approximate calculation approach. What kind of strategies might be grouped under this label? Many authors take it for granted that approximate calculation relies on the computation of the approximate quantitative information conveyed by numbers (i.e., their magnitude), with no involvement of symbolic representations (Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1991, 1997; Dehaene et al., 1999; Hamann & Ashcraft, 1985; Lemer et al., 2003; Spelke & Tsivkin, 2001; Stanescu-Cosson et al., 2000). However, other strategies based on the computation of symbolic numerical representations enable approximate solutions to arithmetical problems to be found without calculating the exact result (LeFevre et al., 1993; Lemaire, Arnaud, & Lecacheur, 2004; Lemaire & Lecacheur, 2002; Lemaire, Lecacheur, & Farioli, 2000; Siegler & Booth, 2005). For example, to estimate the result of $43 + 56$, children and adults can round up or down one or both of the addends to the closest ten (e.g., $40 + 56 = 96$) or truncate one or both addends by neglecting the unit digits (e.g., $40 + 50 = 90$) and then, if necessary, add or subtract a small quantity to the sum in order to compensate for the distortions introduced by rounding or truncation. When the problem is complex, they can also decompose the problem into a series of subproblems before rounding or truncating (e.g., $153 + 286 = 100 + 200 + 50 + 90 = 440$). These partial calculation strategies, usually labeled *computational estimation strategies*, consist of performing an exact calculation algorithm on simplified symbolic representations to find an approximate answer. They are therefore expected to be more rapid, albeit less precise, than an exact calculation.

The present data do not allow us to determine whether the approximate calculation performed by children in the TA group relied on approximate number magnitudes or on exact but simplified symbolic numerical representations. Although very speculative, one hypothesis is that both kinds of approximate calculation processes coexist in children but do not operate in the same situation. In production tasks, where participants have to provide only one approximate solution for a given problem, children might be more inclined to use partial calculation strategies, giving rise to a single answer. Conversely, in verification tasks, computing the magnitudes of the addends might be sufficient to estimate the magnitude of plausible sums. Nevertheless, calculating a rough answer using rounding strategies should enable implausible answers to be rejected as well. However, a series of experiments conducted by Zbrodoff and Logan (1990) suggests that verification might rely on approximate number magnitudes rather than on symbolic numerical representations. Their results show that adults tend to evaluate an equation as a whole in verification tasks, rather than simply producing an answer (exact or approximate) and comparing it to the proposed result. Further research is needed to clarify the nature of approximate calculation strategies and to establish their conditions of use.

In summary, this study showed that children with MD did not use approximate calculation as frequently and as efficiently as their normally achieving peers and younger children with similar levels of calculation skills. Future work should aim at determining whether children with MD process larger split problems faster than true equations, which would provide clearer evidence for the use of an approximate problem-solving approach. If they do, this would support the view that approximate calculation is more time consuming and/or is associated with a more severe confidence criterion for children with MD than for children with normal achievement in mathematics. Whatever the origin of the divergences between children in the MD and the TA groups in the verification task, the fact is that children with MD did not exhibit the same level of adaptiveness as did TA children, who were able to flexibly adjust their problem-solving approaches to problem characteristics. In everyday life, being able to approximate may be very useful, for instance, to check the plausibility of calculation results or to solve complex problems rapidly when calculators are not available. Unfortunately, in spite of its large applicability, it must be admitted that the opportunities to use approximate calculation are rather sparse at school, which puts greater emphasis on exact calculation skills.

For the future, it is important to become aware of the essential role that the approximate calculation approach may play in mathematical development. Traditional teaching programs tend to focus only on the mastery of algorithms, neglecting the relationship between procedural and conceptual knowledge. Because it involves going beyond the rigid execution of procedures and applying mathematical knowledge adaptively, approximate calculation may contribute to developing a conceptual understanding of mathematics. It should be trained at school or in remediation to enhance what has been called *number sense* (Gersten & Chard, 1999; Gersten, Jordan, & Flojo, 2005; Lefevre et al., 1993; Resnick, 1989; Robinson et al., 2002). Thus, approximate calculation and estimation abilities in general should no longer be considered as isolated skills but rather as useful tools for promoting the development of mathematical thinking.

Appendix
Addition Problems According to the Complexity Level and the Split Between the Stated and the Correct Sum

Split	Simple Addition: Correct Sum Between 11 and 17	Simple Addition: Correct Sum Between 21 and 27	Complex Addition: Correct Sum Between 11 and 17	Complex Addition: Correct Sum Between 21 and 27
Split 0	11 + 2 = 13	21 + 2 = 23	5 + 8 = 13	16 + 7 = 23
	11 + 3 = 14	21 + 3 = 24	8 + 6 = 14	18 + 6 = 24
	12 + 3 = 15	22 + 3 = 25	8 + 7 = 15	17 + 8 = 25
	12 + 4 = 16	22 + 4 = 26	9 + 7 = 16	19 + 7 = 26
	13 + 4 = 17	22 + 5 = 27	9 + 8 = 17	18 + 9 = 27
Split 2	11 + 2 = 15	21 + 2 = 25	7 + 6 = 15	17 + 6 = 25
	11 + 3 = 16	21 + 3 = 26	6 + 8 = 16	15 + 9 = 26
	13 + 2 = 17	23 + 2 = 27	7 + 8 = 17	16 + 9 = 27
	14 + 2 = 18	21 + 5 = 28	7 + 9 = 18	17 + 9 = 28
	12 + 5 = 19	24 + 3 = 29	8 + 9 = 19	18 + 9 = 29
Split 14	11 + 2 = 27	21 + 2 = 37	8 + 5 = 27	15 + 8 = 37
	11 + 3 = 28	21 + 3 = 38	8 + 6 = 28	19 + 5 = 38
	11 + 4 = 29	21 + 4 = 39	8 + 7 = 29	18 + 7 = 39
	11 + 5 = 30	24 + 2 = 40	7 + 9 = 30	19 + 7 = 40
	14 + 3 = 31	25 + 2 = 41	9 + 8 = 31	19 + 8 = 41

Notes

1. Children were required only to give their answers within 5 seconds.

2. From the original sample—29 children with mathematical disability (MD) and 29 matched controls—8 children with MD and 7 typically achieving (TA) children moved to other schools, and 3 children with MD were retained in second grade.

3. The reduced MD group did not differ significantly from the original MD group on the selection measures (mean mathematics percentile score: original MD group = 8.06; reduced MD group = 8.67)

4. The effect of group on simple true problems was also examined including all children (i.e., 18 MD, 21 second-grade, and 22 third-grade TA children). With these larger samples, the group effect on simple problems was the same, $F(2, 58) = 23.71$, $p < .001$. Third-grade TA children were faster than MD and second-grade TA groups, ps < .001, which did not differ from each other, ps > .10.

5. The analyses on simple addition problems were also carried out including all children, that is, 18 MD, 21 second-grade TA children and 22 third-grade TA children. The outcomes of these analyses were perfectly the same as those described here.

6. In the third-grade TA group, the restricted sample included the subgroup of TA third graders matched to children with MD. On the other hand, the restricted sample of TA second graders included children whose mean log reaction times on true problems were the closest to the ones of children with MD.

References

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition, 1,* 3–34.

Ashcraft, M. H., & Faust, M. W. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition and Emotion, 8,* 97–125.

Ashcraft, M. H., & Fierman, B. A. (1982). Mental addition in third, fourth, and sixth graders. *Journal of Experimental Child Psychology, 33,* 216–234.

Ashcraft, M. H., Kirk, E. P., & Hopko, D. (1998). On the cognitive consequences of mathematics anxiety. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 175–196). Hove, UK: Psychology Press.

Ashcraft, M. H., & Ridley, K. S. (2005). Math anxiety and its cognitive consequences: A tutorial review. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 315–330). New York: Psychology Press.

Ashcraft, M. H., & Stazyk, E. H. (1981). Mental addition: A test of three verification models. *Memory and Cognition, 9,* 185–196.

Badian, N. A. (1983). Arithmetic and nonverbal learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. 5, pp. 235–264). New York: Grune and Stratton.

Barbaresi, W. J., Katusic, S. K, Colligan, R. C., Weaver, A. L., & Jacobsen, S. J. (2005). Math learning disorder: Incidence in a population-based birth cohort, 1976–82, Rochester, Minn. *Ambulatory Pediatrics, 5,* 281–289.

Butterworth, B. (2005). Developmental dyscalculia. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 455–467). New York: Psychology Press.

Dehaene, S., & Cohen, L. (1991) Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia, 29,* 1045–1054.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1,* 83–120.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantities knowledge of arithmetic. *Cortex, 33,* 219–250.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20,* 487–506.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284,* 970–974.

Faust, M. W., Ashcraft, M. H, & Fleck, D. E. (1996). Mathematics anxiety effects in simple and complex addition. *Mathematical Cognition, 2,* 25–62.

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological and genetic components. *Psychological Bulletin, 114,* 345–362.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

Geary, D. C., Bow-Thomas, C., & Yao, Y. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54,* 372–391.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27,* 787–797.

Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology, 77,* 236–263.

Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition (pp. 253–268).* New York: Psychology Press.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, C. M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88,* 121–151.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disabilities. *Child Development, 78,* 1343–1359.

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk of mathematical disability. *Journal of Experimental Child Psychology, 74,* 213–239.

Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development, 2,* 249–269.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33,* 18–28.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38,* 293–304.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79,* 329–343.

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38,* 25–33.

Hamann, M. S., & Ashcraft, M. H. (1985). Simple and complex addition across development. *Journal of Experimental Child Psychology, 40,* 49–72.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93,* 615–626.

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology, 79,* 192–227.

Jordan, N. C., & Hanich, L. B. (2003). Characteristics of children with moderate mathematics deficiencies: A longitudinal perspective. *Learning Disabilities Research and Practice, 18,* 213–221.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74,* 834–850.

Jordan, N. C., Kaplan, D., & Hanich, L. B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of Educational Psychology, 96,* 586–597.

Jordan, N. C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: A comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30,* 624–634.

Khomsi, A. (1998). *Epreuve d'évaluation de la compétence en lecture: Lecture de mots et compréhension–révisée* (LMC-R) [Evaluation tool of reading competence: Word reading and revised comprehension]. Paris: Editions du Centre de Psychologie Appliquée.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7,* 164–177.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition, 93,* 99–125.

LeFevre, J., Greenham, S. L., & Waheed, N. (1993). The development of procedural and conceptual knowledge in computational estimation. *Cognition and Instruction, 11,* 95–132.

Lemaire, P., Arnaud, L., & Lecacheur, M. (2004). Adults' age-related differences in adaptivity of strategy choices: Evidence from computational estimation. *Psychology and Aging, 10,* 467–481.

Lemaire, P., & Lecacheur, M. (2002). Children' strategies in computational estimation. *Journal of Experimental Child Psychology, 82,* 281–304.

Lemaire, P., Lecacheur, M., & Farioli, F. (2000). Children's strategy use in computational estimation. *Canadian Journal of Experimental Psychology, 54,* 141–148.

Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia, 41,* 1942–1958.

Lewis, C., Hitch, G. J., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9- to 10-year-old boys and girls. *Journal of Child Psychology and Psychiatry, 35,* 283–292.

Lobrot, M. (1980). *Lire avec épreuves pour évaluer la capacité de lecture* (D-ORLEC) [Proofs reading for assessing reading abilities]. Paris: Editions ESF.

Mazzocco, M. M., & Myers, G. F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia, 53,* 218–253.

Molko, N., Cachia, A., Rivière, D., Mangin, J. F., Bruandet, M., Le Bihan, D., et al. (2003). Functional and structural alterations of the intraparietal sulcus in a developmental dyscalculia of genetic origin. *Neuron, 40,* 847–858.

Mousty, P., & Leybaert, J. (1999). *Evaluation des habilités de lecture et d'orthographe au moyen de BELEC: Données longitudinales auprès d'enfants francophones testés en 2ème et 4ème années* [Evaluation of reading and orthographic abilities with the BELEC: Longitudinal data from French-speaking children tested in Grade 2 and 4]. *Revue Européenne de Psychologie Appliquée, 4,* 325–342.

Resnick, L. B. (1989). Developing mathematical knowledge. *American Psychologist, 44,* 162–169.

Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research and Practice, 17,* 81–89.

Rousselle, L., & Noël, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic versus non symbolic number magnitude processing. *Cognition, 102,* 361–395.

Russell, R. L., & Ginsburg, H. P. (1984). Cognitive analysis of children's mathematics difficulties. *Cognition and Instruction, 1,* 217–244.

Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: A prospective six-year follow up. *Developmental Medicine and Child Neurology, 47,* 121–125.

Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science, 9,* 405–410.

Siegler, R. S. (1988). Individual differences in strategy choices: Good students, not-so-good students, and perfectionists. *Child Development, 59,* 833–851.

Siegler, R. S., & Booth, J. L. (2005). Development of numerical estimation: A review. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 197–212). New York: Psychology Press.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills.* Hillsdale, NJ: Erlbaum.

Spelke, E. S., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition, 78,* 45–88.

Stanescu-Cosson, R., Pinel, P., van de Moortele, P. F., Le Bihan, D., Cohen, L., & Dehaene, S. (2000). Understanding dissociations in dyscalculia: A brain imaging study of the impact of number size on the cerebral networks for exact and approximate calculation. *Brain, 123,* 2240–2255.

Temple, C. M., & Sherwood, S. (2002). Representation and retrieval of arithmetical facts: Developmental difficulties. *Quarterly Journal of Experimental Psychology, 55A,* 733–752.

Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2004). Strategy development in children with mathematical disabilities: Insights from the choice/no-choice method and the chronological-age/ability-level- match design. *Journal of Learning Disabilities, 37,* 119–131.

Wechsler, D. (1991). *Echelle d'intelligence de Wechsler pour enfants, troisième édition* (WISC-III) [Wechsler Intelligence Scale for Children, third edition (WISC-III)]. Paris: Les Editions du Centre de Psychologie Appliquée.

Zbrodoff, N. J., & Logan, G. D. (1990). On the relation between production and verification tasks in the psychology of simple arithmetic. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 16,* 83–97.

Laurence Rousselle is a PhD student at the Catholic University of Louvain. Her thesis concerns the normal and pathological development of quantitative representations.

Marie-Pascale Noël, PhD, is a professor of child neuropsychology at the Catholic University of Louvain. She currently conducts research in developmental neuropsychology on numerical development and math learning disabilities.