The Probability Approach

Haavelmo (1944)

Data: $X = (X_1, \ldots, X_n)$

Probability Model: Θ

Supply and demand,

Asset Pricing: CAPM Heath-Jarrow-Morton

Macro Model: DSGE

Game theory

Contract theory

Mapping model to data

1. Avoid quantitative comparison
   - "Analogy" (Gilboa et al 2014)
   - "Rhetoric" (McCloskey 1998)
   - "Fables" (Rubinstein 2012)
   - "Narrative" (Shiller 2019? 2020?)

2. Approximation
Define a metric between data and model

3. Expand models to allow variability

Tool for 2 and 3 in this class is probability

Model

$\Theta : \{\theta \in \Theta : \text{parameters}\}$

P_θ : probability

$\hat{\Theta}$: Statistics, "estimator"

$X = \{x_1, \dots, x_n\}$

For all $\theta \in \Theta$: $P_\theta(d(\hat{\theta}, \theta) < \text{small})$ is large

Want $\hat{\theta}$ to be "probably approximately correct"

Asymptotics: as dataset size $n \to \infty$

then approximate the probability

Results will be eventually approximately probably approximately correct

Alternatives: Keynes, Knight, David Blackwell

Probability

Probability triple, Probability space

$$(\Omega, \mathcal{F}, \mathbb{P})$$

Ω: Space

$\mathcal{F}$: sigma field

$\mathbb{P}$: probability measure

Ω: a set, $\{0, 1\}$, {integers}

{"red", "blue", "Green"}

$[0, 1]$

<u>Event</u>: $A \subseteq \Omega$ such that
$A \in \mathcal{F}$

$\mathcal{F}$: sigma field, sigma algebra, information set

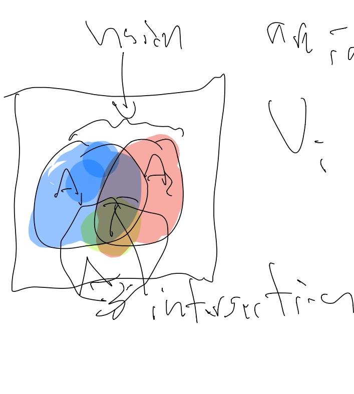
collection of subsets of Ω s.t.

1. $\underbrace{\emptyset}_{\text{empty set}} \in \mathcal{F}$, $\Omega \in \mathcal{F}$

2. $A \in \mathcal{F}$ implies $A^c \in \mathcal{F}$

   $A^c := \{\omega \in \Omega : \omega \notin A\}$

3. If $A_1, A_2, \ldots A_\infty$ is a countable collection of events in $\mathcal{F}$ Then their unions and intersections are in $\mathcal{F}$



$\bigcup_i A_i \in \mathcal{F}$ $\qquad$ $\bigcup_i A_i := \{\omega \in \Omega : \omega \in A_i \text{ for some } i\}$

$\cap_i A_i \in \mathcal{F}$ $\quad \cap_i A_i = \{\omega \in \Omega : \omega \in A_i \text{ for all } i\}$

$\Omega = \{0, 1, 2\}$

$A = \{\{0\}, \{1, 2\}, \{0, 1, 2\}, \{\emptyset\}\} = \mathcal{F}$

$\Omega = [0, 1]$ or $\mathbb{R}$

For any Ω which has a topology, a collection of open sets, eg (a, b)

Can define an $\mathcal{F}$ called the "Borel" sigma field $\mathcal{B}$ which contains all open sets

$(-\infty, a] \cup [b, \infty) \in \mathcal{B}$

$[b, c) \in \mathcal{B}$

$[a, b] \in \mathcal{B}$

$\{a\} \in \mathcal{B}$

$(\Omega, \mathcal{F}, \mu)$: measure space

Measure μ: map from $\mathcal{F}$ to $\mathbb{R}^+$

Axioms/Definition

1. $0 \le \mu A < \infty \quad \forall A \in \mathcal{F}$

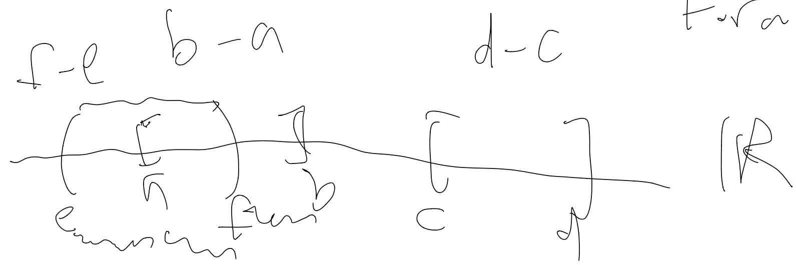
   ↑ forall

2. $\mu \emptyset = 0$

3. If $A_1, A_2, \dots A_\infty$ is a countable collection of disjoint sets in $\mathcal{F}$, then

$$\mu \bigcup_{i=1}^{\infty} A_i = \sum_{i=1}^{\infty} \mu A_i$$

$\{A_i\}_{i=1}^{\infty}$ are disjoint if $A_i \cap A_j = \emptyset$ forall $i \neq j$



Probability Measure: $\mathbb{P}$

is a measure such that $\mathbb{P}\Omega = 1$

Random Variable: X or $(X(\omega))$

on a probability space Ω is a function $\Omega \to \underbrace{\mathbb{X}}_{\text{a set}}$

for each $\omega \in \Omega$, $X(\omega) \in \mathbb{X}$

e.g.: $\Omega = \{\text{heads}, \text{tails}\}$

$$X = \begin{cases} 1 & \text{if } \omega = \text{heads} \\ 0 & \text{if } \omega = \text{tails} \end{cases}$$

Define a sigma field on $\mathbb{X}$, call it $\mathcal{X}$

$\left\{ \begin{array}{l} X \text{ is a map such that for all } B \in \mathcal{X} \\ X^{-1}(B) := \{\omega \in \Omega : X(\omega) \in B\} \in \mathcal{F} \end{array} \right.$

say X is <u>measurable</u>

Failure case: $\mathcal{F} = \{\emptyset, \{1,2\}, \{0,1,2\}, \{0\}\}$

$$X = \begin{cases} 1 & w=0 \\ 2 & w=1 \\ 3 & w=2 \end{cases} \qquad X = w+1$$

X = All subsets of $\{1,2,3\}$

not measurable

$$X = \begin{cases} 1 & w=0 \\ 2 & w=1 \text{ or } 2 \end{cases}$$

Measurable