



Structural balance in real-world social networks: incorporating direction and transitivity in measuring partial balance

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Abstract

Structural balance theory predicts that triads in networks gravitate towards stable configurations. This theory has been verified for undirected graphs. Since real-world networks are often directed, we introduce a novel method for considering both transitivity and sign consistency for evaluating partial balance in signed digraphs. We test our approach on graphs constructed by using different methods for identifying edge signs: natural language processing to infer signs from underlying text data, and self-reported survey data. Our results show that for various social contexts and edge sign detection methods, partial balance of these digraphs is moderately high, ranging from 61 to 96%. Our approach not only enhances the theoretical framework of structural balance but also provides practical insights into the stability of social networks, enabling a deeper understanding of interpersonal and group dynamics across different communication platforms.

Keywords Structural balance · Partial balance · Signed directed networks · Organizational communication

1 Introduction

Social and communication networks in the real world are composed of intricate and constantly changing interactions between individuals or groups. Network scholars have examined core principles that explain patterns of social interactions at various levels of analysis, i.e., node level (Borgatti

and Everett 2006), dyadic level (Block 2015), triadic level (Cartwright and Harary 1956; Johnsen 1986), subgroup level (Riley and Eckenrode 1986; Talaga et al. 2023), and the graph level (Kilduff and Tsai 2003). In this study, we focus on a fundamental unit of analysis, namely triads. Triads matter as there are patterns of tie formations between three actors that cannot be explained at any other levels of analysis (Simmel 1950; Flament 1963; Taylor 1970). Empirical evidence has shown that balance at the triad-level has implications for balance at higher levels of analysis, for purposes such as clustering (Talaga et al., 2023) and understanding the stability of larger social structures (Talaga and Nowak 2022; Chiang and Tao 2019). Two fundamental theories that explain interactions at the triadic level are structural balance (Heider, 1946) and transitivity (Feld and Elmore, 1982), which both model how stable relationships emerge in groups of three nodes. The theories of structural balance and transitivity have been applied to examine various social processes of opinion formation (Altafini, 2012), cooperation in groups (He et al., 2018), and maintenance of social status in organizations (Dong et al., 2015). There is considerable empirical evidence in support of these theories (Chiang and Tao 2019; Diesner and Evans 2015; Aref et al. 2020), as well as evidence that falsifies the premises of these theories (Doreian and Krackhardt 2001; Doreian and Mrvar 2014; Leskovec et al. 2010). What these studies have in common

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is that they highlight the need to consider directionality in the evaluation of balance for networks where relationships are directed. Representing truly directed data as undirected graphs for balance evaluation removes relevant information about balance (Song and Meyer 2015; Leskovec et al. 2010) and influence between nodes (Şimşek and Meyerhenke 2020; Uribe-Leon et al. 2020). Thus, we consider both the sign and direction of edges as graph properties to evaluate balance for ten real-world social networks, and empirically assess whether the theory of structural balance can be verified or not. Specifically, we assess partial balance (Aref and Wilson, 2018) for different types of signed directed relations, namely sentiment, morality, trust, friendship (preference), and alliance (a strategic partnership between organizations where one supports another against a third competitor). Examining networks with various edge types of signed and directed connections allows us to determine if equilibrium is maintained within and among different types of social relationships.

Our empirical assessment considers both transitivity and structural balance as two explanatory mechanisms for partial balance in signed and directed networks. This approach stems from the original formulation of balance (Heider 1946, 1958), which states that transitivity is a crucial property of balance, as transitivity explains how directed and signed ties are oriented in ways that are consistent with balance. We capture both transitive and balanced configurations by leveraging the triad census (Holland and Leinhardt, 1971) and generalized balance model (Cartwright and Harary, 1956) to extract all transitive triples within any given triad and calculate the overall balance for the proportions of balanced triples within that triad. We also operationalize structural balance as partial balance, given that complete balance in real-world networks is rare (Aref and Wilson 2019, 2018). To compute partial balance, we consider the relative frequencies of positive and negative triples within each triad, capturing the extent to which a network tends towards balance. Altogether, we assess structural balance with transitivity and partial balance considered across the aforementioned signed relations and find that partial balance ratios vary, with an average balance ratio of 0.835.

In comparison to results obtained by using the traditional method for determining balance, which utilizes an undirected approach, our findings indicate a more nuanced understanding of network dynamics. By incorporating directionality, we reveal hidden patterns of relationships and influences not captured in undirected models. Moreover, this directed approach helps to identify asymmetric relationships, such as dominance or influence, which are critical for understanding social hierarchies and power dynamics (Block 2015; Shizuka and McDonald 2012; McMillan et al. 2022). Relatedly, our approach allows us to retain and analyze these asymmetric or conflicting edges within a triad,

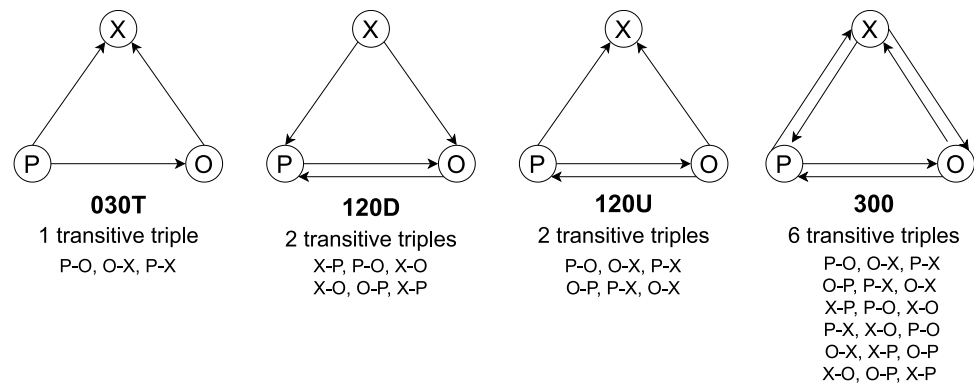
based on the transitive configurations permitted by the original formulation of structural balance theory. We also observe that different types of relationships are related to distinct patterns of balance and transitivity, indicating the importance of context in network analysis. For instance, we found that dominance relations, indicated by the presence of asymmetric and transitive balanced relations, occur more frequently in friendship preferences and email communication compared to trust relations. Our study contributes to a more comprehensive understanding of network theory by demonstrating the significance of considering both sign and direction in assessing the structural balance of real-world networks. This approach has potential applications in various domains, such as organizational behavior, social media analytics, and political science, where understanding the dynamics of complex relationships is crucial.

Our research marks a substantial advancement by empirically evaluating structural balance in a diverse range of real-world relationships (edge types). We examine five principal types of social relationships (trust, alliance, friendship preference, morality, and sentiment), offering a comprehensive view of partial balance in various social contexts. In addition, our findings reveal a critical insight: the coexistence and concurrent impact of sign consistency and transitivity in shaping the partial balance of networks. This finding highlights the intricate interplay between these mechanisms across different types of relationships, underscoring their pivotal role in network dynamics.

2 Related work

The central tenet of social network analysis is to understand the structures of relations between sets of nodes, such as individuals, groups, or organizations (Wasserman and Faust, 1994). Heider's balance theory (Heider, 1946) was the first formulation of how relationships form between three entities, or between a pair of individuals and their perceptions of or attitudes toward a common object. Heider initially examined *POX* triads, where *P* is a focal individual, *O* is a second individual, and *X* is either an individual or an object in common. He asserted that the relationship between these three entities is 'balanced' if *P* liked *O*, *O* liked *X* and *P* also liked *X*. On the other hand, 'imbalance' would occur if *P* liked *O*, *O* liked *X*, but *P* did not like *X*. Heider's primary claim was that balance is a state of equilibrium, and that individuals in networks strive to move towards and maintain that equilibrium or balance. Cartwright and Harary (1956) brought balance theory into the context of signed and directed networks, where relationships between pairs of nodes can be represented as either a (+) or (−). They further formalized that a triad within a network is balanced if the product of the signs of its edges is positive. While Cartwright and Harary's

Fig. 1 Triad census (include only triads which are transitive and balanced). Example of MAN census label - 030T: Mutual=0, Asymmetric=3, Null=0, Letter Label: T=Transitive, D=Down, U=Up



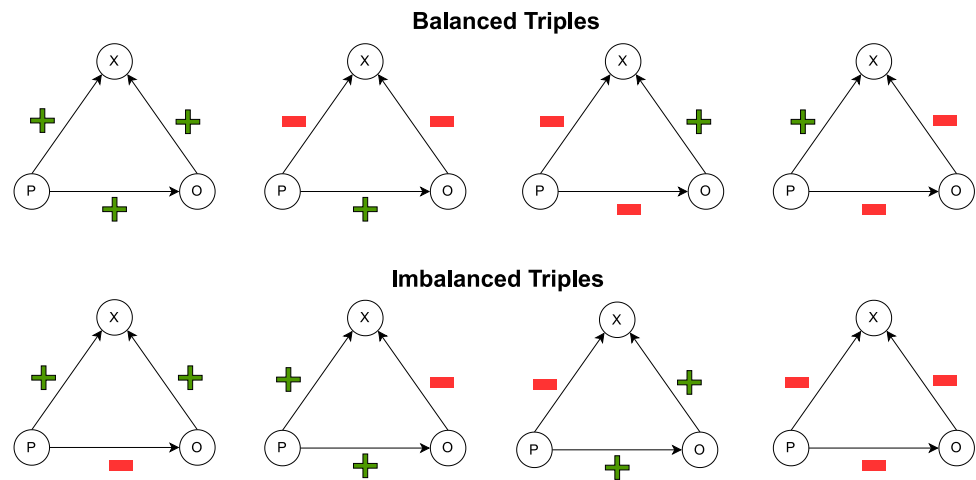
generalization focuses on clusterability of networks, this study leverages the generalization to evaluate the balance of signed and directed triads. Subsequently, Holland and Leinhardt (1971) introduced the transitivity model, which asserted that transitivity is an important structural property of social relationships in general, and balance in particular. Specifically, Holland and Leinhardt proposed eight possible types of triads for balance assessment, with transitivity considered (as shown in Fig. 1).

The relationship between transitivity and structural balance is explicitly stated in Heider's formulation as follows: "three positive relations may be considered psychologically transitive." This means that "*P* tends to like *X* if *PRO* and *ORX* are valid at the same time" (*R* represents positive relation between two nodes) (Heider, 1946), p. 109). Thus, Heider suggested that beyond cognitive consistency as a primary theoretical mechanism, transitivity may also co-exist in a triad, though his formulation was at the level of an individual's perception. Holland and Leinhardt (1971) generalized this conjecture further to the context of a social arrangement with three individuals, represented as a triad of transitive relations (or a "t-graph"). Feld and Elmore (1982), followed by Doreian and Krackhardt (2001), found that differential popularity was another process, beyond cognitive consistency highlighted in Heider, that explained the presence of transitivity in balance. Doreian and Krackhardt (2001) found that transitivity and balance were both present, but rival processes, in the Newcomb fraternity dataset (Newcomb, 1968). Hummon and Doreian (2003) found the complexity of the balance process to involve multiple operating mechanisms, namely transitivity, balance, and reciprocity. Davis, Holland, and Leinhardt Davis (1979) in their studies of positive relations in triads found that transitivity is a pervasive property in balanced (all positive) triads. Stix (1974) also asserted that transitivity is an important property of balance, and that "the transitivity of any structure corresponds to its sensitivity to imbalance" (p. 447). Altogether, prior literature shows that the relationship between transitivity and balance is complex; it is still uncertain whether the two are complementary or rivaling processes. All we know is that they are distinctive

but often co-existing in social relations (Stix 1974; Hummon and Doreian 2003; Holland and Leinhardt 1971).

Empirical studies on structural balance in real-world social networks offer mixed evidence for balance theory. Several studies have found evidence that structural balance is present in email communication networks (Diesner and Evans, 2015), sentiment networks of individuals from the same (and different) nationalities (Chiang and Tao, 2019), and ten real-world benchmark network datasets (Aref et al., 2020) based on several relationship types such as friendship, trust, and alliance relations. There are also several studies that did not find evidence for balance theory (Doreian and Mrvar 2015; Doreian and Krackhardt 2001). Doreian and Mrvar (2015) in their structural examination of signed and directed relations between countries from 1946 to 1999 found that the networks did not gravitate towards balance, and that balance was not achieved in the network (contrary to Aref and Wilson (2019)'s finding that this network achieves a high degree of balance (≈ 0.86)). Rawlings and Friedkin (2017) analyzed signed directed networks of sentiment among members in the same housing community, and found that some statements of balance hold true (e.g., a friend of a friend is a friend), while others are violated (e.g., an enemy of an enemy is a friend). While evidence for structural balance has been mixed, these studies show that real-world relationships are complex, where properties such as signs and direction co-exist.

In this study, we revisit the longstanding literature on transitivity and balance to calculate structural balance for networks of different signed relations. Specifically, we calculate balance, with transitivity considered, using Holland and Leinhardt's triad census (1971) and Estrada (2019) approach for analyzing triples. The triad census includes 16 classes of MAN (Mutual, Asymmetric, Null) triads, representing all possible combinations of directed ties between three nodes. Specifically, each configuration contains different combinations of edges, either mutual (*P* likes *O* and *O* likes *P*), asymmetric (*P* likes *O* but *O* does not like *P*), or null (*P* and *O* do not like each other). Additionally, a focal node *P* in one triad can be an *O* or *X* in another triad with a

Fig. 2 Balanced and imbalanced triples

different focal node P , allowing for the analysis of nodes that assume multiple roles across various triads. The triad census is especially useful for our calculation, as it contains four triad types that are driven by both transitivity and balance (as shown in Fig. 1). Furthermore, the triad census includes all the possible triples (containing directed ties) embedded in each triad type, allowing us to conveniently extract transitive triples and exclude intransitive (cyclic) triples in our analyses. Another reason to exclude cyclic triples, besides the fact that they violate the transitivity rule, is that cycles contained limited information on the process of influence among relationships (Rank et al. 2010; Veenstra et al. 2013; Evmenova and Gromov 2020). In the next section, we show that these four configurations are relevant for our operationalization of balanced triads with transitivity as a co-existing operating mechanism.

3 Balance evaluation for signed directed networks with transitivity considered

In this section, we present our approach to calculating partial balance in signed directed networks with respect to transitive triples (typologies of balanced and imbalanced transitive triples are shown in Fig. 2).

3.1 Problem definition

Definition 1 Let G be a signed digraph where $G = (D, \sigma)$. D is a digraph underlying G , where $D = (V, E)$ and sign function $\sigma : E \rightarrow \{+, -\}$. A *triad* T in G is a set of three distinct nodes with (at least) one directed edge between each two of them.

A *triple* S in signed directed T is a set of three directed edges that starts from a vertex V , follows the direction of edges, and does not return to the same vertex. In other words, S is *transitive* and *non-cyclic*. Every *triple* S in a

signed digraph G must be *transitive* to be considered for our balance analysis. Based on triad census (Holland and Leinhardt 1971), we recognize four types of triads as transitive: (030T, 120D, 120U, and 300). These specific triad types are deemed transitive due to the presence of only transitive triples within them. Specifically, the 030T triad contains one transitive triple, while the 120D and 120U triads each contain two transitive triples, and the 300 triad contains a total of six transitive triples.

For simplicity of notation, we define $T_{(i)}$ as the set of all transitive triads of type i , where $i = 1, 2, 3, 4$ refers to 030T, 120D, 120U, and 300, respectively.

Proposition We define T a *completely balanced triad* if and only if every transitive triple in T is balanced. A *transitive triple* is *positive* if it contains an even number of negative directed edges. Furthermore, we define T a *partially balanced triad* if it contains at least one negative triple. Finally, T is *completely imbalanced* if every transitive triple in T is imbalanced (or negative thereof).

To evaluate balance, we first calculate the weighted balance ratio for each triad of type i . The *overall balance ratio* of our digraph G is determined by averaging the balance ratios of all triads across the network. A signed digraph $G = (D, \sigma)$ is considered balanced if all triads T in G are balanced. We denote the overall balance of the graph as $B_{\text{Avg}(G)}$.

Definition 2 We compare our method for determining structural balance with the conventional approach, where networks are assumed to be and represented as undirected graphs. We consider G' an undirected signed network where $G' = (D', \sigma)$, and D' is a graph underlying G' , where $D' = (V', E')$ and sign function $\sigma : E' \rightarrow \{+, -\}$. A *triad* T' in G' is a set of three distinct nodes with (at least) one edge between each two of them. The sign of T' is the product of the signs of its edges. A T' is balanced if its sign is positive

and unbalanced otherwise. After calculating the balance score for each T' , we compute the average balance ratio in G' . The *overall balance ratio of G'* ($B_{\text{Avg}(G')}$) is calculated by averaging the balance ratio of all triads across a network. A signed graph $G' = (D', \sigma)$ is balanced if all triads T' in G' are balanced.

4 Empirical analysis

4.1 Data

Several methods such as interviews, surveys, observations, and data mining were employed to gather social network data, enabling the construction and analysis of social connections and interactions among entities. Moreover, real-world communication, whether verbal or nonverbal, written or visual, encompasses a range of explicit and implicit relationships, including those characterized by like or dislike, trust or distrust.

For this study, we intentionally selected ten datasets of different sizes (Table 1) to robustly validate our proposed method for calculating balance in directed triads. Our selected datasets cover a wide range of data collection techniques and portray multiple types of relationships in the networks; namely sentiment, morality, trust, friendship preference, and alliance. The following section provides more details about each dataset.

Highland Tribes The Highland tribes dataset represents an alliance structure among tribal groups using the Gahuku-Gama system of the Eastern Central Highlands of New Guinea. The dataset consists of two parts: (1) GAMAPOS representing alliance (“rova”) relations, and (2) GAMANEG for antagonistic (“hina”) relations (Read 1954; Hage et al. 1983).

College House The college A, B, and C datasets are constructed with respect to the relationships between three different groups of college girls at an Eastern college in the United States. Each group consists of approximately 20 members who lived together for at least four months. The girls in each house were asked to provide evaluations of other girls in the same house based on a range of behavioral characteristics. These data are then converted into matrices of choice and rejection for each house (Lemann and Solomon, 1952).

Bitcoin The Bitcoin OTC and the Bitcoin Alpha datasets both represent trust between Bitcoin traders on two platforms, called Bitcoin OTC and Bitcoin Alpha. The transactions on these platforms are anonymous. However, to minimize fraudulent behavior and maintain reputation, members of OTC and Alpha rate others in a scale of -10 (total distrust) to $+10$ (total trust) in steps of 1 (Kumar et al., 2016).

Table 1 Descriptive network measures for all networks

Network measures	Relational type	Bitcoin-OTC		Bitcoin-alpha		Highland tribes		College House A		College House B		College House C		Enron		Avocado	
		Trust		Trust		Alliance		Preference		Preference		Preference		Morality	Sentiment	Morality	Sentiment
# of nodes		5881		3783		16		21		17		20		517		557	
# of edges		35,592		24,186		116		94		83		81		7605		22,479	
# of components		4		5		1		1		1		1		1		19	
Giant component statistics																	
# of nodes		5047		3381		16		21		17		20		494		452	
# of edges		34,759		23,786		116		94		83		80		7520		22,953	
Transitivity		0.05		0.07		0.53		0.39		0.40		0.27		0.21		0.5	
Density		0.001		0.002		0.48		0.22		0.31		0.21		0.031		0.11	
Average path length		3.44		3.49		1.54		2.21		1.95		2.34		2.53		1.7	
Clustering coefficient		0.21		0.20		0.54		0.35		0.37		0.25		0.46		0.62	

Enron The Enron email data is a large-scale, temporal dataset that provides real-world organizational communication data of a global, U.S. based, former energy brokerage that went bankrupt in 2001 over a span of 3.5 years. The communication (email) dataset of 158 employees was released in 2002 by the FERC (Diesner et al. 2005; Investigation, FERC Western Energy Markets-Enron 2004). The original dataset went through various edits and modifications over the years. In this study, we use the latest release of the dataset from 2015 (<https://www.cs.cmu.edu/~enron/>).

We disambiguated the email addresses of 558 Enron employees (Diesner et al., 2005), (Diesner and Evans, 2015). Furthermore, we used natural language processing techniques to analyze emails and label each tie (email) based on their valence for moral values (Graham et al. 2013; Rezapour et al. 2019) and sentiment (Wiebe and Riloff 2005; Jiang et al. 2020). More details on text analysis methods are provided in §A.

Avocado-IT The Avocado Research Email Collection (Oard et al., 2015) is provided by the Linguistic Data Consortium (<https://catalog.ldc.upenn.edu/LDC2015T03>) and consists of emails between 279 accounts of a defunct information technology company referred to as “AvocadoIT”, a pseudonym assigned for anonymity. The dataset consists of calendars, attachments, contacts, reports, and emails. In this study, we only used the email communication for studying balance.

For the Avocado dataset, we considered emails that were sent to or from corporate email addresses (emails ending to @avocadoit.com). Similar to the Enron dataset, we further used text mining techniques to label each email based on their valence for moral values (Graham et al. 2013; Rezapour et al. 2019) and sentiment (Wiebe and Riloff, 2005). §A provides more details regarding the methods used for analyzing emails.

4.2 Balance analysis based on edge signs and transitivity

To analyze balance in triads, we follow the steps explained in §3.1. We used *NetworkX*, a *Python* library, to extract instances of four transitive triads (030T, 120D, 120U, and 300) and analyze balance within each triad with respect to their triples. For networks with several components, we first selected the giant component and removed self-loops, pendants, and edges with neutral (zero) scores from the networks to make the analysis more efficient.¹

¹ All datasets (if permitted), code, and results are available in our GitHub repository: https://github.com/social-nlplab/Network_Balance.

5 Results

5.1 Descriptive network measures

Table 1 shows the structural characteristics of the ten networks in terms of network size, transitivity, density, average path length, and clustering coefficient. We focus on the giant component of each network, meaning that pendants and isolates are excluded from analysis. The transitivity score is the highest for the Highland tribes network (0.53) and lowest for the Bitcoin-OTC network (0.05). High transitivity means the network has a sizable proportion of triads for balance analysis, whereas low transitivity suggests the network contains other configurations beyond the triad (e.g., cycles of length > 3). Density captures the occurrences of dyads in the network, and we find a close connection between density and transitivity scores. Highland tribes has the highest density (0.48). This reveals the connection between the number of dyads and triads present in the network and how they allude to the interconnectedness between nodes. Measures of average path length and clustering coefficient reveal the topological characteristic of a network with respect to the presence of subgroups and communities. Most networks are found to have low average path length and mid-range clustering coefficient, meaning that they are exhibiting some characteristic of a small-world topology (low average path length, high clustering coefficient), though this needs to be statistically evaluated. Highland tribes network, Enron-morality, Avocado-morality, and Avocado-sentiment are three networks that most clearly exhibit the small-world effect.

5.2 Partial balance analysis

The average partial balance ratio across all networks (shown in Table 2) is 0.835 (min = 0.61, max = 0.96, s.d = 0.10). The network with the highest partial balance is College House C (0.96), and the network with the lowest partial balance is College House B (0.61). We observe that high partial ratios persist across the different network sizes, with the smallest network having a balance ratio of 0.87 (Highland tribes), and the largest network having a balance ratio of 0.88 (Bitcoin OTC). We also examine the proportions of triads with respect to their balance ratios, as shown in Table 2. For five of the networks (Bitcoin-OTC, Bitcoin-Alpha, Highland tribes, Avocado-morality, Avocado-sentiment), triad 300 is the most frequently occurring, and often most balanced when compared to the other triad types. The prevalence of balanced triads 300 s shows that balance is often present in situations where individuals initiate and reciprocate the connection, specifically in the cases of trust (Bitcoin networks) and email communication (Avocado networks). For the other networks (College Houses A, B, C, Enron-morality,

Table 2 Balance counts for different triad types across all networks

Type	Balance ratio (and counts of triads) for each triad type									
	Bitcoin-OTC	Bitcoin-alpha	Highland tribes	College House A	College House B	College House C	Enron-morality	Enron-sentiment	Avocado-morality	Avocado-sentiment
030T	0.91 (3,706)	0.82 (974)	0.00 (0)	0.67 (27)	0.43 (21)	0.82 (17)	0.91 (4,514)	0.67 (4,238)	0.81 (8,787)	0.76 (8,577)
120D	0.85 (2,096)	0.82 (1,142)	0.00 (0)	0.85 (13)	0.33 (6)	1.00 (8)	0.92 (2,390)	0.68 (2,384)	0.86 (14,111)	0.81 (14,276)
120U	0.83 (2,910)	0.87 (1,780)	0.00 (0)	1.00 (13)	0.80 (15)	1.00 (3)	0.92 (3,615)	0.64 (3,513)	0.87 (26,165)	0.83 (28,615)
300	0.93 (13,752)	0.92 (9,894)	0.87 (68)	1.00 (4)	0.88 (4)	1.00 (1)	0.94 (3,056)	0.70 (3,056)	0.93 (124,371)	0.90 (144,865)
Average balance (Directed)	0.88 (22,464)	0.86 (13,790)	0.87 (68)	0.88 (57)	0.61 (46)	0.96 (29)	0.92 (13,575)	0.68 (13,191)	0.87 (173,434)	0.82 (196,333)

Enron-sentiment), triad type 030T is the most frequently occurring. The frequent presence of 030T in these networks shows that some relationships are bounded by a “local hierarchy” - in the context of Enron Sentiment, individual P sends an email with a positive valance to O , O replies to X negatively, thus P replies to X negatively based on the prior interaction between O and X . The prevalence of 030T triads in these particular networks also shows that reciprocity is lower for certain types of relationships, specifically friendship preferences, and email communication). On the other hand, trust relations are observed for the Bitcoin networks to have a higher frequency of reciprocation within triads.

5.3 Sign analysis of triples

All networks in our dataset contain higher proportions of positive than negative edges in both directed and undirected networks (results shown in Table 3). For the majority of directed networks, all-positive triples are most prevalent, indicating the dominance of strong balance within the network. This result is consistent with Leskovec et al. (2010)’s finding that real-world social networks contain 70–87% of all-positive triples. The prevalence of strongly-balanced triples (+ + +) also provides evidence in support of prior work by Doreian and Krackhardt (2001) and Davis (1979), who all found transitivity to be a pre-condition for balance when both $P \rightarrow O$ and $O \rightarrow X$ are positive. Our findings also show two possible outcomes for the pre-transitive condition; there is a higher likelihood that $P \rightarrow X$ will be positive, but there is also a possibility for $P \rightarrow O$ to be negative. For the remaining four networks (Highland Tribes, College House A, B, and C), there is a higher frequency of (+ - -) triples, showing empirical evidence of signed transitivity (Doreian and Krackhardt, 2001) in several real-world networks. These four networks are all noticeably smaller than other networks, which may suggest that the prominence of (+ - -) triples may be specific to these social contexts, as opposed to being generalizable to other networks with the same relational type.

For the imbalanced triples, we find that in six out of ten networks, (+ + -) triples are more prevalent than (- - -) triples. This finding is in contrast with Szell et al. (2010)’s results, which found that (+ + -) triples are the least prominent in real-world networks, followed by (- - -) triples. Leskovec et al. (2010) also empirically observed the rarity of (+ + -) triples in real networks. However, in the case of directed networks, where edges are defined in terms of social status theory, (+ + -) triples are over-represented beyond what is expected by chance. Easley and Kleinberg (2010) also suggested that these two types of tensions are unique, and suggest that (+ + -) triples are more likely to be resolved over time. In the context of College House friendship preference, for example, where A likes B , B likes C , but

Table 3 Types of signed triples for all networks

	Directed				Undirected			
	+++	+-	+-	---	+++	+-	+-	---
Bitcoin- OTC	0.85	0.07	0.07	0.004	0.71	0.16	0.12	0.01
Bitcoin-Alpha	0.87	0.04	0.08	0.003	0.78	0.08	0.14	0.01
Highland Tribes	0.28	0.59	0.03	0.10	0.28	0.59	0.03	0.10
College House A	0.42	0.45	0.05	0.08	0.23	0.48	0.14	0.15
College House B	0.20	0.48	0.15	0.17	0.08	0.44	0.23	0.26
College House C	0.33	0.60	0.02	0.05	0.19	0.69	0.04	0.08
Enron Morality	0.92	0.01	0.07	0.0003	0.93	0.00	0.06	0.00
Enron Sentiment	0.58	0.11	0.30	0.01	0.39	0.39	0.21	0.01
Avocado Morality	0.92	0.01	0.07	0.0002	0.89	0.01	0.10	0.00
Avocado Sentiment	0.89	0.01	0.10	0.004	0.85	0.01	0.13	0.00

A does not like C. Person A may be more likely to reconcile this tension by liking C. However, if there are all negative relations (---) between A, B, and C, Davis (1967) argues that it is less likely for two enemies to like each other. Instead, the three enemies are likely to just stay enemies, and not be involved in the triadic relationship at all. In the remaining four networks (Highland tribes, College Houses A, B, and C), we find slightly higher occurrences of one-negative than all-negative triads. This finding signals the difficulty for the triads to reconcile the tensions and thus may be less likely to become more balanced over time. This is a particularly interesting finding for the case of Highland Tribes, showing that the alliance structure is rigid and less likely to change over time. In the context of the three College House friendship networks, friendship preference is not likely to change, even though the change could reduce tension for the network as a whole.

Szell et al.'s (2010) study brings out an important distinction between strong and weak structural balance. Weak structural balance (Davis, 1967) posits that the (++-) triple is the only configuration that is (weakly) imbalanced, while the (---) is considered (weakly) balanced because empirical observations have found that “an enemy of one’s enemy can also be an enemy.” Additionally, the all-negative triple is more likely to become balanced, compared to the one-negative triple, because it is more likely that two enemies will become friends (or at least form a strategic alliance) because they have a mutual enemy (“an enemy of my enemy is my friend”). However, as alluded to in Easley and Kleinberg (2010), the presence of the one-negative triple (++-) can also signal the possibility of resolving tension among a friend of a friend. For instance, in the Enron email network, we observe that P may send a positive email to X in the future, driven by the positive connection that P has with O, and knowing that O sent a positive email to X. Differing findings from the literature suggest that the sign configurations of triples are complex, especially when directed edges are considered (Leskovec et al., 2010). In our analysis of

these signed and directed triples, we find that weak balance is not prominent in relationships that are transitive. In other words, “an enemy of an enemy is an enemy” is not as likely to happen as a “a friend of a friend is an enemy” connection. It would be worthwhile to examine the temporal changes of these two triple types over time, and whether it is more likely that an enemy would become a friend in the (---) case, or that (1) an enemy would become a friend, or (2) a friend becomes an enemy in the (++-) triple. From this perspective, there are two possibilities for the (++-) to obtain balance, as opposed to (---) triple, which can only be resolved if an edge becomes positive.

Ultimately, the differences in these results show that the composition of triples differs (1) across different real-world contexts and relations, and (2) across directed and undirected representations of the networks.

5.4 Utility of directed networks

The overall trend observed across all networks in our study is that the undirected versions of the network inflate the count of edges, triples, and consequently, the triads available for analysis (Table 4). This inflation occurs because undirected networks do not require transitivity, meaning that not all connections must follow a certain pattern of relationships, making more configurations available for analysis. For instance, in the College House A and College House C networks, the undirected versions included a notable number of extra triples, which subsequently influenced the balance score. In particular, the undirected version of College House A contains 48 extra triples compared to the directed version, in which 24 triples are balanced as per sign products, and 24 triples are imbalanced. Hence, the resulting balance score is influenced and dropped to 0.72 as opposed to 0.88 in the directed network. The College House C network (Fig. 3) shows similar patterns as the College House A network in terms of the inflated number of triples in the undirected network. There are 19 additional triples not presented in

the directed network, 16 of which are balanced and 3 are imbalanced. For instance, there is a triad between nodes V4, V9, and V10 in the undirected network but not the directed network because this particular triad violates the transitivity assumption. In particular, this triad is a cycle, whereby $V4 \rightarrow V9$ (+ sign), $V9 \rightarrow V10$ (-), and $V10 \rightarrow V4$ (+). Similar situation occurred with triad V3, V6, V12 where the triad contains one cycle ($V3 \rightarrow V6$ (-), $V6 \rightarrow V12$ (-), $V12 \rightarrow V3$ (+)). This particular triad, however, also contains one triad that satisfies both transitivity and balance conditions ($V3 \rightarrow V6$ (-), $V6 \rightarrow V12$ (-), $V3 \rightarrow V12$ (+)), but the presence of a violating cycle influences the overall transitivity of the triad.

Undirected versions also remove edges when there is a discrepancy in the signs of the incoming and outgoing edges between two impacted nodes, as shown in Fig. 4. In the College House B network, we observe the loss of three particular edges that in turn influence the final balance score of the undirected realization. As shown in Fig. 5, two edges $V2 \rightarrow V15$ (-), and $V2 \rightarrow V17$ (-) from the *V2 egonet* are present in the directed network but not the undirected network, as they were canceled out due to the mismatch in edge signs. The directed realizations hence preserve at least an edge between V2 and V15, for instance, because they are involved in multiple triadic configurations that satisfy the transitivity and balance assumptions in the directed network (e.g., V15, V11, V2 as 120D triad; V3, V15, V2 as 120U triad). There is an exception to $V2 \rightarrow V7$ edge being present in the undirected network but not the directed network. This is because, in the directed network, this edge is not part of any triads that meet the transitivity requirement. On the other hand, the undirected network keeps this edge, but this is problematic because there is only an edge between $V2 \rightarrow V7$, but not $V7 \rightarrow V2$ (Fig. 5).

The Highland Tribes network stands out as the only case where the directed and undirected networks share identical triad composition and overall balance scores. This is

because the network is completely connected, hence there is a maximal number of triads that can exist in both network realizations. Such a network structure is, however, rare in real-world networks. Our case study of College Houses A, B, and C demonstrates that even the slightest discrepancy in edge configurations and their signs that result from switching the network from directed to undirected can substantially impact the networks' overall balance score.

6 Discussion and conclusion

In this paper, we use an empirical framework for calculating balance in signed and transitive digraphs, which is essential to appropriately model and study balance in real-world networks where links might be asymmetric. We applied this framework to ten social network datasets with five different types of relationships (trust, alliance, friendship preference, morality, and sentiment). Our rationale for testing our approach on different networks was to determine whether the assumptions of structural balance and transitivity hold true across different social and socio-technical contexts. Moreover, prior research has mainly examined structural balance in signed and undirected graphs. Our study provided an actionable solution to measure structural balance in signed digraphs, using principles of transitivity to evaluate the directionality between edges.

Overall, our findings show that the amount to which a network is balanced is strongly impacted by choices of measuring social relations. When the direction of edges is taken into account, along with sign consistency, we expected that the overall balance ratio may be different from findings where only sign consistency was considered (Diesner and Evans, 2015; Leskovec et al., 2010). This particular finding has implications for higher-order network analysis, as it highlights the importance of considering directionality for

Table 4 Comparing Partial and Non-Partial Balance

	Directed_Partial			Directed_Non-Partial			Undirected		
	BR	# BT	# IT	BR	# BT	# IT	BR	# BT	# IT
College House A	0.88	46	11	0.8	46	11	0.74	29	10
College House B	0.61	29	17	0.52	24	22	0.44	14	18
College House C	0.96	26	3	0.89	26	3	0.92	25	2
Bitcoin-Alpha	0.86	13069	721	0.84	11649	2141	0.84	2930	546
Bitcoin-OTC	0.88	21192	1272	0.86	19447	2969	0.91	3814	360
Highland Tribes_sym	0.87	59	9	0.87	59	9	0.92	24	2
Enron sentiment	0.68	10444	2747	0.53	6045	7165	0.69	2104	949
Avocado_sentiment	0.82	185832	10501	0.46	105222	91505	0.85	8846	1495
Avocado_morality	0.87	165639	7795	0.63	117515	67781	0.84	8060	1544
Enron morality	0.92	12856	719	0.79	10908	2813	0.93	2831	226

BR: balance ratio, BT: balance triangles, IT: imbalance triangles

Fig. 3 Case study: College House C network as a directed network (left) and undirected network (right)

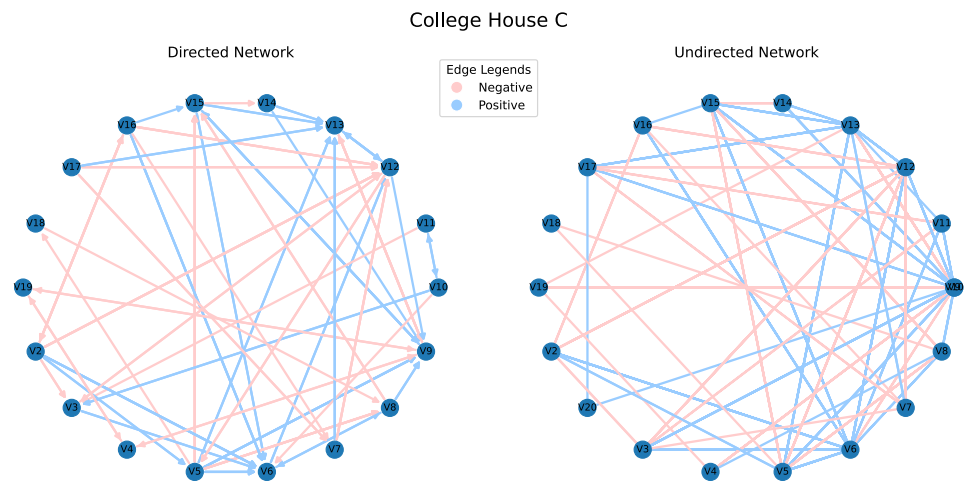


Fig. 4 Examining differences in network analysis outcomes with and without considering directionality in real-world data

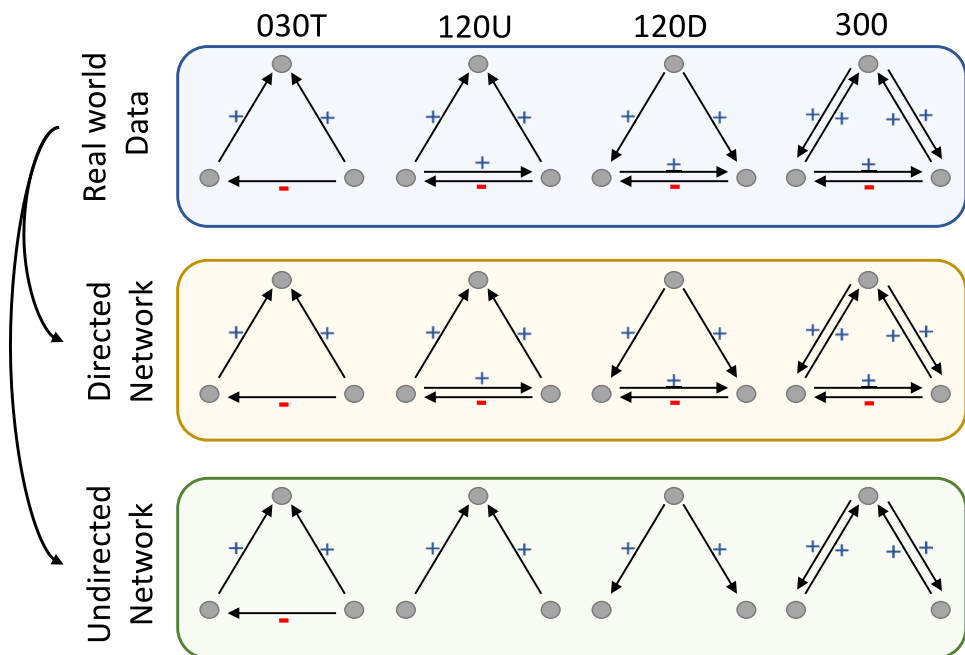
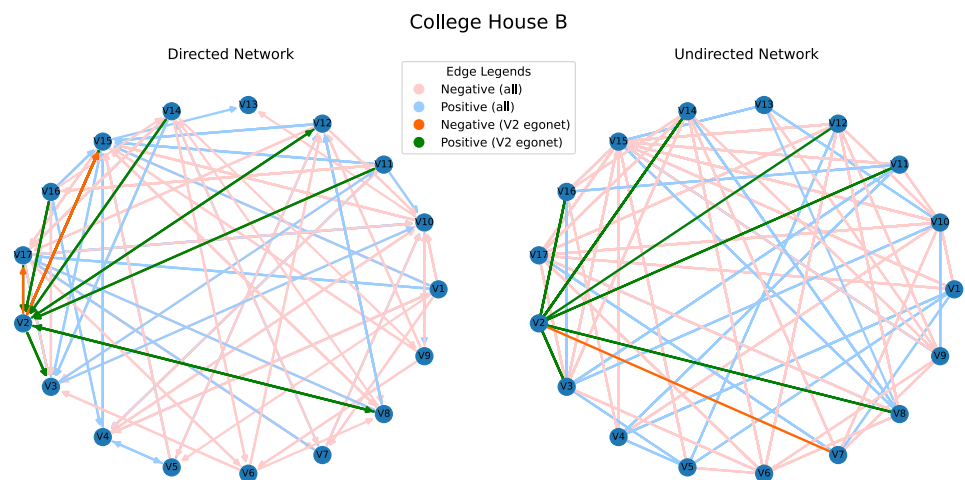


Fig. 5 Case study: College House B network as a directed network (left) and undirected network (right). Edge Legends: *all* indicates the whole network, and *V2 egonet* indicates the egonet of node V2



understanding the formation and stability of larger network structures, such as social hierarchies, influence patterns, and the potential for clustering (Talaga and Nowak 2022; Talaga et al. 2023; Chiang and Tao 2019). Choices of relational type, i.e., types of relation that are analyzed between the entities in a network, may also affect the overall partial balance: Our findings indicate that each type of relationship examined can capture a distinct aspect of the network, as evidenced by the varying balance ratios across different dimensions, including trust, friendship preferences, alliances, morality, and sentiments in email and online communications. While partial balance ratios for all relational types were about 0.82 and above (balance higher than imbalance), we found the compositions of triples that make up balanced triads to be different depending on the type of relationship. The patterns of partial balance that we discovered across the ten real-world networks offer implications for existing communication and social networks literature: First, we found that partial balance with transitivity considered provides important insights into the co-existence of hierarchy, influence, and reciprocity as mechanisms that influence balance. For instance, if we consider an undirected and balanced triad (ABC) where we know $A-B$ is positive, $B-C$ is positive, and $C-A$ is positive, the lack of edge direction does not explain the potential source(s) of balance. Whereas if we are presented with additional information, such as $A \rightarrow B$ is positive, $B \rightarrow C$ is positive, $A \rightarrow C$ is positive, we observe that A likely has a positive edge towards C because B has a positive edge for C , hence the process of social influence is captured. Secondly, our partial balance and sign analyses reveal that triad type 300 is most prominent in all empirical networks, and specifically this triad 300 is often composed of all-positive triples. This finding suggests that the processes of transitivity and balance are salient in all-positive triads, and this is consistent with Davis, Holland, and Leinhardt Davis (1979)'s observations for real-world small groups. We also found the occurrences of the one-negative triples to be higher than the all-negative triples, which is in contrast to several prior studies (Szell et al. 2010; Leskovec et al. 2010). Further investigations are needed on the presence of the one-negative triad, as the likelihood of resolving the tension is likely to be unique across different relational types.

Our findings are subject to several limitations. First, we only analyzed network structure and partial balance for the giant component of the networks in our sample. Our rationale for that choice was that only closed triads are considered for balance and transitivity analysis. Secondly, we consider partial balance only as a local property at the triad level. In future work, we will extend the measurement of partial balance and transitivity for longer cycles and examine if there are areas of tension detected in the network at other levels of analysis beyond the triad.

Analysis of email datasets

For the two email datasets, we leverage Natural Language Processing (NLP) methods to extract two types of edge signs from text data exchanged between the nodes (email writers) that form a dyad: moral values (virtue or vice) and sentiment (positive or negative). This approach is based on the premise that people's language use can reflect their cultural, economic, and ideological backgrounds (Triandis 1989). Language is one of the most powerful means through which people demonstrate their implicit or explicit beliefs, feelings, and moral values (Fiske 1993). Differences in people's feelings, opinions, and moral or personal values may be the sources of tension and conflict in relationships and groups. Therefore, extracting and analyzing these relationships from language exchanged between network participants can help to better understand the structure and balance in social networks.

To capture moral values in our email data sets, we used the Moral Foundations Theory (MFT) (Graham et al. 2013, 2009). MFT can help to capture people's spontaneous reactions and categorize human behavior into five basic principles (fairness/cheating, care/harm, authority/subversion, loyalty/betrayal, and purity/degradation) that are characterized by opposing values (virtues and vices). To extract moral values from our email data, we used an enhanced version of MFD² as developed, introduced, and validated in (Rezapour et al. 2019), (Rezapour and Diesner 2019). The second method we used for labeling links with signs is sentiment analysis; a technique commonly used for understanding people's opinions and affective state (Pang and Lee 2008). To identify the sentiment of each email, we domain adapted the Subjectivity Lexicon, a widely adopted and previously evaluated sentiment lexicon by Wiebe and Riloff (2005).

Declarations

Conflict of interest The authors declare no conflict of interest.

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² https://doi.org/10.13012/B2IDB-3805242_V1.1

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