Geometric Compression of Polynomial-Time Computations and the Structural Gap with NP Witness Spaces

Abstract
We study the geometric structure of two natural families of discrete metric spaces arising in computational complexity: spaces of NP witnesses and spaces of polynomial-time computations. On the NP side, we associate to Boolean satisfiability instances the Hamming metric spaces of satisfying assignments and show that, for natural families of instances, these spaces exhibit strong geometric rigidity, including linear diameter and exponential covering numbers at linear scales. On the P side, we associate to deterministic polynomial-time Turing machines the metric spaces of their computational trajectories. By introducing an appropriate coarse-grained metric, we prove that such spaces are geometrically compressible at large scale, with polynomial covering numbers at linear radius. We then formulate a general notion of coarse simulation between metric families and show that, despite superficial similarities, the rigid geometry of NP witness spaces is incompatible with the coarse compressibility of polynomial-time computation spaces. Crucially, we identify a structural obstruction to transferring geometric compression from computations to witness spaces: the intensional nature of polynomial-time decision procedures does not canonically induce a coarse simulation of the extensional geometry of NP solution spaces. Our results do not establish a separation between P and NP. Instead, they isolate a precise geometric barrier for a broad class of approaches based on metric compression.

1. Introduction
The question of whether P equals NP has resisted decades of effort and has motivated the identification of several formal barriers to proof techniques, including relativization, natural proofs, and algebrization.

2. Geometric Rigidity of NP Witness Spaces
Given a Boolean formula ϕ in n variables, we consider the set of satisfying assignments equipped with the Hamming metric. For natural families of satisfiable instances, these spaces have linear diameter and exponential covering numbers at linear scale, a property we term geometric rigidity.

3. Coarse Compression of Polynomial-Time Computation Spaces
Polynomial-time computation spaces, when endowed with a coarse-grained metric on computational trajectories, are shown to be geometrically compressible at large scale.

4. Coarse Simulation and Geometric Incompatibility
We formalize a notion of coarse simulation between metric families and prove that rigid spaces cannot be coarsely simulated inside compressible ones.

5. The Intensional–Extensional Obstruction
Polynomial-time decision procedures are intensional, while NP witness spaces are extensional. This mismatch prevents the transfer of geometric compression.

6. Conclusion
We identify a genuine structural barrier that clarifies the limits of geometric approaches to P versus NP.