INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2017

INSTRUCTIONS
GRADE 11-12

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.
2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred here by the end of the contest.
3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.
4. Calculators and graph paper are not permitted. You are allowed to use rough paper for draft work.
5. The figures are not drawn to scale. They should be used only for illustration.
6. Remember, you have about 2-3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to the other problems.
7. At the end of the allotted time, please **submit the response form to the contest supervisor**. Please do not forget to pick up your Certificate of Participation!

Good luck! *Canadian Math Kangaroo Contest team*

2017 CMKC locations: Algoma University; Bishop's University; Brandon University; Brock University; Carlton University; Concordia University; Concordia University of Edmonton; Coquitlam City Library; Dalhousie University; Evergreen Park School; F.H. Sherman Recreation & Learning Centre; GAD Elementary School; Grande Prairie Regional College; Humber College; Lakehead University (Orillia and Thunder Bay); Laurentian University; MacEwan University; Memorial University of Newfoundland; Mount Allison University; Mount Royal University; Nipissing University; St. Mary's University (Calgary); St. Peter's College; The Renert School at Royal Vista; Trent University; University of Alberta-Augustana Campus; University of British Columbia (Okanagan); University of Guelph; University of Lethbridge; University of New Brunswick; University of Prince Edward Island; University of Quebec at Chicoutimi; University of Quebec at Rimouski; University of Regina; University of Toronto Mississauga; University of Toronto Scarborough; University of Toronto St. George; University of Windsor; The University of Western Ontario; University of Winnipeg; Vancouver Island University; Walter Murray Collegiate, Wilfrid Laurier University; YES Education Centre; York University; Yukon College.

2017 CMKC supporters: Laurentian University; Canadian Mathematical Society; IEEE; PIMS.

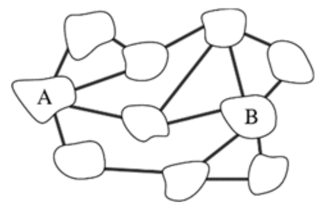
Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. The greatest common divisor of two integer numbers is 6. Which of the following cannot be the sum of the two integer numbers?
(A) 318 (B) 272 (C) 186 (D) 462 (E) 906

2. The H0 version of Ben's railroad is built in the 1:87 ratio. It contains a 2cm high model (reproduction) of his brother. In the real world, how tall is his brother?
(A) 1.74 m (B) 1.62 m (C) 1.86 m (D) 1.94 m (E) 1.70 m

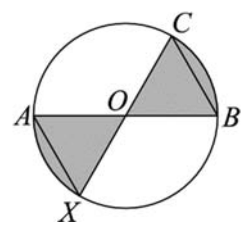
3. In the figure, we see 10 islands that are connected by 15 bridges. What is the smallest number of bridges that can be eliminated to make it impossible to get from island A to island B by bridge?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

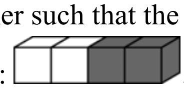
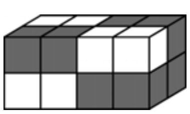
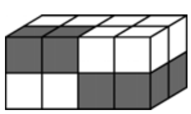
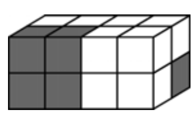
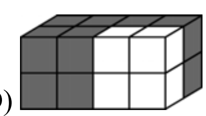
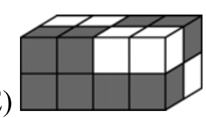


4. Two positive numbers a and b are such that 75% of a equals 40% of b. Which of the following statements is true?
(A) $15a = 8b$ (B) $7a = 8b$ (C) $3a = 2b$ (D) $5a = 12b$ (E) $8a = 15b$

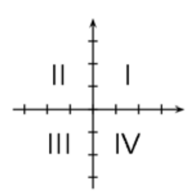
5. What is the probability that a natural number not divisible by 9 is divisible by 3?
(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{3}{8}$ (E) $\frac{1}{4}$

6. Given a circle with centre O and diameters AB and CX such that $OB = BC$. What portion of the area of the circle is shaded?
(A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{7}$ (D) $\frac{3}{8}$ (E) $\frac{4}{11}$



7. A bar consists of two white and two grey cubes glued together such that the result is a 4×1×1 bar with two white cubes on one end and two grey cubes on the other end: .
Which figure can be built from four bars?
(A)  (B)  (C)  (D)  (E) 

8. Which quadrant contains no points of the graph of the linear function $f(x) = -3.5x + 7$?
(A) I (B) II (C) III (D) IV
(E) All quadrants contain points.



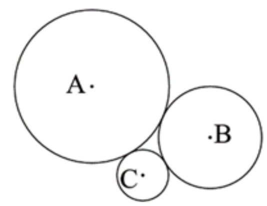
9. Each of the following five boxes is filled with red and blue balls as labelled. Ben wants to take one ball out of a box without looking. From which box should he take the ball to have the highest probability to get a blue ball?

(A) 10 blue, 8 red (B) 6 blue, 4 red (C) 8 blue, 6 red
(D) 7 blue, 7 red (E) 12 blue, 9 red

10. The graph of which of the following functions has the most points in common with the graph of the function $f(x) = x$?
(A) $g_1(x) = x^2$ (B) $g_2(x) = x^3$ (C) $g_3(x) = x^4$
(D) $g_4(x) = -x^4$ (E) $g_5(x) = -x$

Part B: Each correct answer is worth 4 points

11. Three mutually tangent circles with centres A, B, C have the radii 3, 2 and 1, respectively. What is the area of the triangle ABC?



(A) 6 (B) $4\sqrt{3}$ (C) $3\sqrt{2}$ (D) 9 (E) $2\sqrt{6}$

12. The positive number p is less than 1, and the number q is greater than 1. Which of the following numbers is the largest?
(A) $p \cdot q$ (B) $p + q$ (C) $\frac{p}{q}$ (D) p (E) q

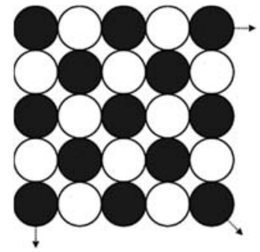
13. Two right cylinders A and B have the same volume. The radius of the base of B is 10% larger than that of A. By what percentage is the altitude of A larger than that of B?
(A) 5% (B) 10% (C) 11% (D) 20% (E) 21%

14. I look at our old church clock at 9 o'clock. How many full minutes must go past until the minute hand has just overtaken the hour hand?
(A) 46 (B) 47 (C) 48 (D) 49 (E) 50

15. We have four tetrahedral dice, perfectly balanced, with their faces numbered 2, 0, 1 and 7. If we roll all four of these dice, what is the probability that we can compose the number 2017 using exactly one of the three visible numbers from each die?
(A) $\frac{1}{256}$ (B) $\frac{63}{64}$ (C) $\frac{81}{256}$ (D) $\frac{3}{32}$ (E) $\frac{29}{32}$

16. The lengths of three different sides of a rectangular prism are the three different roots of the equation $x^3 - 9x^2 + 26x - 24 = 0$. What is the total surface area of the solid?
(A) 36 (B) 48 (C) 26 (D) 52 (E) 24

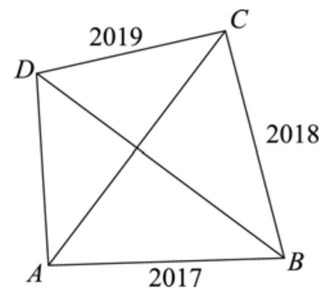
17. Julia has 2017 chips: 1009 of them are black and the rest are white. She places them in a square pattern as shown, beginning with a black chip in the upper left hand corner, alternating colours in each row and each column. How many chips of each colour are left after she has completed the largest possible square?



(A) None (B) 40 of each (C) 40 black ones and 41 white ones
(D) 41 of each (E) 40 white ones and 41 black ones

18. Two consecutive numbers are such that the sums of the digits of each of them are multiples of 7. At least how many digits does the smaller number have?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. In a convex quadrilateral $ABCD$ the diagonals are perpendicular. The sides have lengths $|AB| = 2017$, $|BC| = 2018$ and $|CD| = 2019$ (figure not to scale). What is the length of AD?



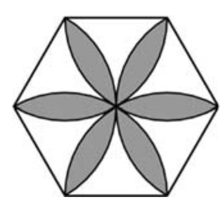
(A) 2016 (B) 2018 (C) $\sqrt{2020^2 - 4}$
(D) $\sqrt{2018^2 + 2}$ (E) 2020

20. Tytti tries to be a good little Kangaroo, but lying is too much fun. Therefore, every third thing she says is a lie and the rest is true. (Sometimes she starts with a lie and sometimes with one or two true statements.) Tytti is thinking of a 2-digit number and is telling her friend about it:
"One of its digits is 2."
"It is larger than 50."
"It is an even number."
"It is less than 30."
"It is divisible by 3."
"One of its digits is 7."
What is the sum of the digits of the number Tytti is thinking of?
(A) 9 (B) 12 (C) 13 (D) 15 (E) 17

Part C: Each correct answer is worth 5 points

21. How many positive integers have the property that the number obtained by deleting the last digit is equal to 1/14 of the original number?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. The picture shows a regular hexagon with side lengths equal to 1. The flower was constructed with sectors of circles of radius 1 and centres in the vertices of the hexagon. What is the area of the flower?



(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $2\sqrt{3} - \pi$ (D) $\frac{\pi}{2} + \sqrt{3}$ (E) $2\pi - 3\sqrt{3}$

23. Consider the sequence a_n with $a_1 = 2017$ and $a_{n+1} = \frac{a_n - 1}{a_n}$. What is the value of a_{2017}?

(A) -2017 (B) $\frac{-1}{2016}$ (C) $\frac{2016}{2017}$ (D) 1 (E) 2017

24. In a sequence of six numbers, the first number is 4 and the last number is 47. Each number after the second equals the sum of the previous two numbers. *S* is the sum of the six numbers in this sequence. In which interval does *S* lie?

(A) 51 to 90 (B) 91 to 100 (C) 101 to 110 (D) 111 to 120 (E) 121 to 160

25. The sum of the lengths of the three sides of a right-angled triangle is equal to 18 and the sum of the squares of the lengths of the three sides is equal to 128. What is the area of the triangle?

(A) 18 (B) 16 (C) 12 (D) 10 (E) 9

26. You are given 5 boxes, 5 black and 5 white balls. You choose how to put the balls in the boxes (each box must contain at least one ball). Your opponent comes into the room and randomly draws one ball from the box of his choice. He wins if he draws a white ball, otherwise you win. How should you arrange the balls in the boxes to have the best chance to win?

(A) You put one white and one black ball in each box.
(B) You arrange all the black balls in three boxes, and all the white balls in two boxes.
(C) You arrange all the black balls in four boxes, and all the white balls in one box.
(D) You put one black ball in every box, and add all the white balls in one box.
(E) You put one white ball in every box, and add all the black balls in one box.

27. Nine integers are written in the cells of a 3×3 table. The sum of the nine numbers is equal to 500. It is known that the numbers in any two neighbouring cells (that is cells sharing a common side) differ by 1. What is the number in the central cell?

(A) 50 (B) 54 (C) 55 (D) 56 (E) 57

	?	

28. If $|x| + x + y = 5$ and $x + |y| - y = 10$, what is the value of $x + y$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

29. If $f(xy) = f(x + y)$ and $f(11) = 22$, what is the value of $f(33)$?

(A) 11 (B) 22 (C) 33 (D) 44 (E) 66

30. Each of the 2017 people living on an island is either a liar (and always lies) or a truth-teller (and always tells the truth). More than one thousand of them take part in a banquet, all sitting together at a round table. Each of them says: "Of the two people beside me, one is a liar and the other one a truth-teller." What is the maximum number of truth-tellers on the island?

(A) 1683 (B) 668 (C) 670 (D) 1344 (E) 1343