International Contest-Game MATH KANGAROO

Part A: Each correct answer is worth 3 points.

1. The number 200013-2013 is not divisible	by
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(A) 2

(B) 3

(C)5

(D) 7

(E) 11

2. The eight semicircles built inside the square are congruent and the side of this square has length 4. What is the area of the non-shadowed part of the square?

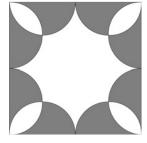


(B) 8

(C) $6 + \pi$

(D) $3\pi - 2$

(E) 3π



3. Mrs. Margareth bought 4 cobs of corn for each person in her family, which has four members. She got the discount shown in the sign. How much did she pay?

Corn sale

1 cob 20 cents every sixth cob is free

(A) 0.80 EUR

(B) 1.20 EUR

(C) 2.80 EUR

(D) 3.20 EUR

(E) 80 EUR

4. Three of the numbers 2, 4, 16, 25, 50, 125 have a product 1000. What is the sum of these three numbers?

(A) 70

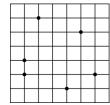
(B) 77

(C) 131

(D) 143

(E) 145

5. Six points are marked on a square grid with cells of size 1×1 , as shown.



Kanga wants to choose three of the marked points to be the vertices of a triangle. What is the smallest possible area of such a triangle?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(c) $\frac{3}{2}$

(D) 1

(E) 2

6. Which of the following is equal to $4^{15} + 8^{10}$?

(A) 2^{31}

(B) 2^{30}

(C) 2^{20}

(D) 2^{15}

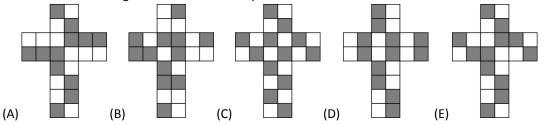
(E) 2^{10}



7. The outside of a cube is painted with grey and white squares in such a way that it appears as if it was built using smaller grey cubes and white cubes, as shown.



Which of the following could be a net of the painted cube?



8. The number n is the largest positive integer for which 4n is a three-digit number, and m is the smallest positive integer for which 4m is a three-digit number. What is the value of 4n - 4m?

(A) 900

(B) 899

(C)896

(D) 225

(E) 224

9. The plan of the Scotts' family apartment is made in scale 1:50 and has a rectangular shape with dimensions 20 cm by 30 cm. What is the area of the apartment?

(A) 12 m²

(B) 150 m²

(C) 300 m²

(D) 450 m²

(E) 600 m²

10. Which of the following has the largest value?

(A) $\sqrt{20} \cdot \sqrt{13}$ (B) $\sqrt{20} \cdot 13$

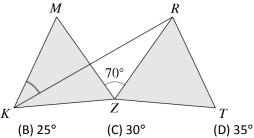
(c) $20 \cdot \sqrt{13}$

(D) $\sqrt{201} \cdot 3$

(E) $\sqrt{2013}$

Part B: Each correct answer is worth 4 points.

11. Triangle RZT is the image of the equilateral triangle KZM upon a rotation clockwise around Z so that $\angle MZR = 70^{\circ}$. What is the measure of $\angle RKM$?



(A) 20°

12. The diagram shows a zigzag shape made from six squares, each measuring 1 cm by 1 cm.



The shape has perimeter of length 14 cm. The zigzag shape is continued until it has 2013 squares. What is the length of the perimeter of the new shape, in cm?

(A) 2022 cm

(B) 4028 cm

(C) 4032 cm

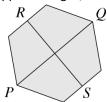
(D) 6038 cm

(E) 8050 cm

 $(E) 40^{\circ}$



13. The points P and Q are opposite vertices of a regular hexagon and the points P and S are midpoints of opposite edges, as shown.

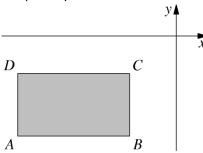


The area of the hexagon is 60 cm^2 . What is the product of the lengths of PQ and RS?

- (A) 40 cm²
- (B) 50 cm²
- (C) 60 cm²
- (D) 80 cm^2
- (E) 100 cm²
- **14.** A class of students had a test. If each boy had obtained 3 points more on the test, then the average result of the class would have been 1.2 points higher.

What percentage of the class are girls?

- (A) 60
- (B) 50
- (C) 40
- (D) 30
- (E) 20
- **15.** The rectangle *ABCD* lies below the *x*-axis, and to the left of the *y*-axis. The edges of the rectangle are parallel to the coordinate axes. For each point *A*, *B*, *C*, *D*, the *y*-coordinate is divided by the *x*-coordinate. Which of the points yields the least value from this calculation?



- (A) A
- (B) B
- (C) C
- (D) D
- (E) It depends on the size of the rectangle.
- **16.** On John's birthday this year, 2013, he multiplied his age by his son's age and correctly obtained the answer 2013. In which year was John born?
 - (A) 1981
- (B) 1982
- (C) 1953
- (D) 1952
- (E) More information is needed.
- **17.** In the picture the areas of the triangles are 5 and 10, as shown, and the lines, a b and c are parallel. If the distance between the lines a and c is 6, what is the length of PQ?



- (B) 5
- (C) 6

- (D) $\frac{36}{5}$
- (E) $\frac{13}{2}$

- b = P $c = \frac{10}{5}$ Q
- **18.** Ivana wants to write down five consecutive positive integers with the property that three of them have the same sum as the other two. How many different sets of five numbers can she write down?
 - (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4



- 19. Roo wants to find a six-digit number, the sum of whose digits is even, and the product of whose digits is odd. Which of the following statements about such a number is correct?
 - (A) Either two or four of the digits are even.
- (B) Such
 - a number cannot
- exist.

- (C) The number of odd digits is odd.
- (E) None of (A), (B), (C), (D) is correct.

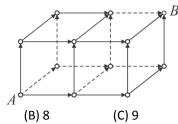
(D) All six digits can be different.

- (A) 0
- (B) 1
- (C)6
- (D) 7
- (E) 11

Part C: Each correct answer is worth 5 points.

21. How many different paths are there between points A and B, only travelling along the edges in the direction of the arrows shown?

20. How many scalene triangles are there with integer side lengths and with a perimeter less than 11?



(A)6

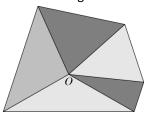
(D) 12

(E) 15

22. A gardener wants to plant 100 trees (oaks and birches) along one side of a trail in the park. The number of trees between any two oaks must not be equal to five.

Of these 100 trees, what is the greatest number of oaks that the gardener can plant?

- (A) 48
- (B) 50
- (C) 52
- (D) 60
- (E) 80
- **23.** Several non-overlapping isosceles triangles have vertex O in common. Every triangle shares an edge with each immediate neighbour. The smallest angle of a triangle at O has size m° , where m is a positive integer. The other triangles have angles at O of size $2m^{\circ}$, $3m^{\circ}$, $4m^{\circ}$, and so on. The diagram shows an arrangement of five such triangles.



What is the smallest value of m, for which such a set of triangles exists?

- (A) 1
- (B) 2
- (C) 3
- (D) 6
- (E) 8
- 24. Julio creates a procedure for turning a set of three numbers into a new set of three numbers: each number is replaced by the sum of the other two. For example, {3,4,6} becomes {10,9,7}, which becomes {16,17,19}, and so on. How many times must Julio apply this procedure starting with the set {1,2,3} before he first obtains a set containing the number 2013?
 - (A) 8
- (B) 9
- (C) 10
- (D) more than 10

(E) 2013 will never appear

25. On each face of the cube a positive integer is printed. At each vertex of the cube there is the product of the numbers, which are on the sides adjacent to that vertex. The sum of all numbers at the vertices is equal to 1001. What can the sum of all the numbers on the faces be equal to?

(A) 1003

(B) 151

(C) 103

(D) 91

26. Using the whole numbers from 1 to 22 inclusive, Horatio wants to form eleven fractions by choosing one number as the numerator, and one number as the denominator. Every number will be used exactly once. What is the maximum number of Horatio's fractions that could have an integer value?

(A) 7

(B) 8

(C)9

(D) 10

(E) 11

27. A regular 13-sided polygon is inscribed in a circle with centre O. Triangles can be formed by joining three vertices of this polygon. How many of the triangles that can be formed in this way will contain the point *O* in their interior?

(A) 72

(B) 85

(C)91

(D) 100

(E) other value

28. A car left point S and drove along a straight road at a speed of 50 km/h. Then every hour another car left S; each car was 1 km/h faster than the previous one. The last car (at a speed of 100 km/h) left 50 hours after the first one. What was the speed of the car which was in front of all the other cars 100 hours after the first car left S?

(A) 50 km/h

(B) 66 km/h

(C) 75 km/h

(D) 84 km/h

(E) 100 km/h

29. The diagram shows a rectangle, which is divided into four pieces by cutting along the straight lines shown. The four pieces are then rearranged to form a square. What is the length of the perimeter of the square?

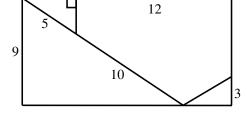


(B) 48

(C) 52

(D) 56

(E) 60



30. Yurko saw a tractor slowly pulling a long pipe down the road. Yurko walked along beside the pipe in the same direction as the tractor, and counted 140 paces to get from one end to the other. He then turned around and walked back to the other end, taking only 20 paces. The tractor and Yurko kept to a uniform speed, and Yurko's paces were all 1 m long. How long was the pipe?

(A) 30 m

(B) 35 m

(C) 40 m

(D) 48 m

(E) 80 m