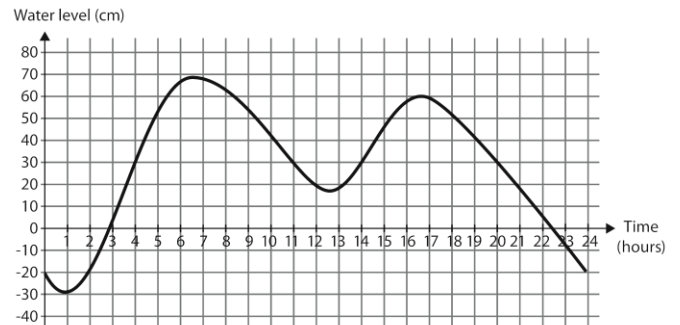**International Contest-Game**
MATH KANGAROO

Part A: Each correct answer is worth 3 points.

1. The water level in a port city rises and falls on a certain day as shown in the figure. For how many hours was the water level above 30 cm on that day?

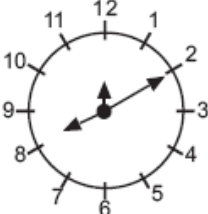(A) 5   (B) 6   (C) 7
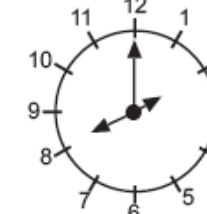(D) 9   (E) 13
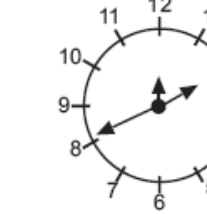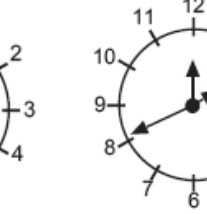

Water level (cm) vs Time (hours)

2. In a list of five numbers, the first number is 2 and the last number is 12. The product of the first three numbers is 30, the product of the three in the middle is 90 and the product of the last three numbers is 360. Which number is in the centre of the list?

2				12
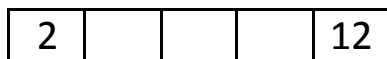
(A) 3   (B) 4   (C) 5   (D) 6   (E) 10

3. A clock has 3 hands of different length (for hours, for minutes, and for seconds). We do not know which hand is which, but we know that the clock runs correctly. At 12:55:30 the hands were in the positions shown. Which of the pictures shows this clock at 8:10:00?
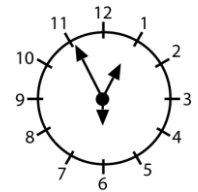

(A)   (B)   (C)   (D)   (E)

4. The cube of the number a equals 2012^{12}. What is the product of the number a and the square of the number 2012^{11} ?
(A) 2012^{58} (B) 2012^{26} (C) 2012^{88} (D) 2012^{15} (E) 2012^{12}

5. A rectangular piece of paper $ABCD$ measuring 4 cm × 16 cm is folded along the line MN so that vertex C coincides with vertex A, as shown in the picture. What is the area of quadrilateral $ANMD'$?
(A) 28 cm^2 (B) 30 cm^2 (C) 32 cm^2
(D) 48 cm^2 (E) 56 cm^2
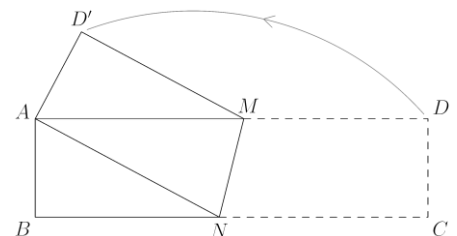
6. Each morning, five polite policewomen Anna, Bea, Clara, Dana and Ella walk to the police station in a line in exactly this order. On the way to the changing room on the far end of the building there are 8 doors. Each door is held open by the first policewoman in the line, who walks at the line after the others have passed. Who opens the last door?
(A) Anna   (B) Bea   (C) Clara   (D) Dana   (E) Ella

7. The maximum value of the integer n, for which $n^{200} < 5^{300}$, is equal to:
(A) 5   (B) 6   (C) 8   (D) 11   (E) 12

8. A chain of operations is displayed in the diagram. How many different real values can the number N take?

$$\boxed{1} \xrightarrow{+N} \square \xrightarrow{\times N} \square \xrightarrow{-N} \square \xrightarrow{:N} 5$$
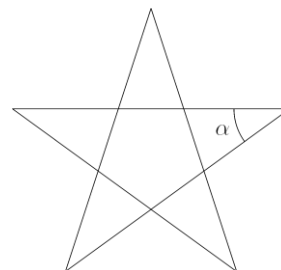
(A) 0   (B) 1   (C) 2   (D) 4   (E) infinitely many

9. Which of the following functions satisfies the equation $f\left(\dfrac{1}{x}\right) = \dfrac{1}{f(x)}$?

(A) $f(x) = \dfrac{2}{x}$ (B) $f(x) = \dfrac{1}{x+1}$ (C) $f(x) = 1 + \dfrac{1}{x}$ (D) $f(x) = \dfrac{1}{x}$ (E) $f(x) = x + \dfrac{1}{x}$

10. What is the size of the angle α in the regular 5-pointed star?

(A) 24°   (B) 30°   (C) 36°
(D) 45°   (E) 72°

Part B: Each correct answer is worth 4 points.

11. My age is a two-digit integer, which is a power of 5, and my cousin's age is a two-digit integer, which is a power of 2. The sum of the digits of our ages is an odd number. What is the product of the digits of our ages?
(A) 240   (B) 2010   (C) 60   (D) 50   (E) 300

12. A travel agency organized four optional tours of Sicily for a group of tourists. Each tour had a participation rate of 80%. What is the smallest possible percentage of tourists who took part in all four tours?
(A) 80%   (B) 60%   (C) 40%   (D) 20%   (E) 16%

13. The picture shows the plan of a rose bed. White roses grow in the two equal squares, and red roses grow in the third square. Yellow roses grow in the right-angled triangle. Both the length and width of the bed are 16 m. What is the area of the rose bed?

(A) 114 cm²   (B) 130 cm²   (C) 144 cm²
(D) 160 cm²   (E) 186 cm²
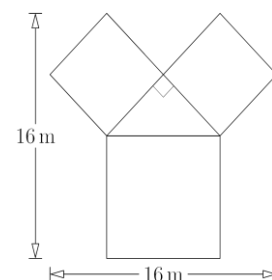
14. Given a regular 2012-gon. How many equilateral triangles with vertices among the vertices of the polygon are there?

(A) 1   (B) 1006   (C) 2012   (D) $\dfrac{2012 \times 2011}{2}$ (E) other answer

15. School marks in Slovakia are divided into five degrees, from 1 (the best) to 5. In one Slovak school, a test didn't turn out very well in the 4th class. The average mark was 4. The boys did a little better; their average mark was 3.6 while the average mark of the girls was 4.2. Which of the following statements about the class is correct?
(A) There are twice as many boys as girls.   (B) There are 4 times as many boys as girls.
(C) There are twice as many girls as boys.   (D) There are 4 times as many girls as boys.
(E) There are as many girls as boys.

16. All the tickets for the first row in a cinema were sold. The seats are numbered consecutively starting with 1. An extra ticket was sold for one seat by mistake. The sum of the seat numbers on all tickets sold for that row is equal to 857. What is the number of the seat for which two tickets were sold?
(A) 4   (B) 16   (C) 25   (D) 37   (E) 42

17. Circles C_1 and C_2 are centered in O_1 and O_2, respectively. AB is a diameter of C_1; C is a point on C_1; C_2 is tangent to the lines AC and BC, and $\angle BAC = x$. What is the measure of the angle O_2CO_1?

(A) $180°$

(B) $180° - \dfrac{\alpha}{2}$

(C) $135° + \alpha$

(D) $135° - \alpha$

(E) It depends on the radii of the circles
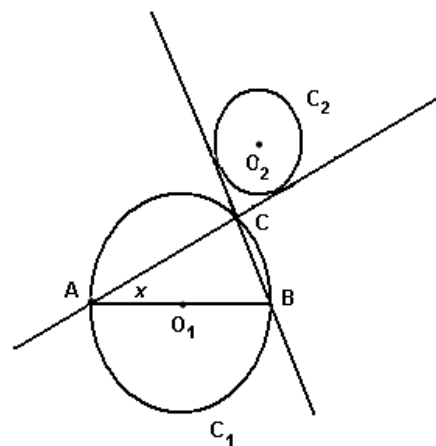
18. A kangaroo wants to build a row of standard dice (for a standard die, each pair of opposite faces has a total of 7 dots). He can glue two faces together if they have the same number of dots. He would like the total number of dots on the outer faces of the dice in the row to be 2012. How many dice does he need?
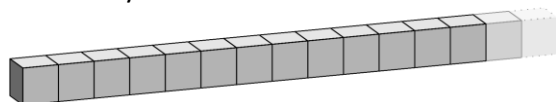
(A) 70　　　　(B) 71　　　　(C) 142　　　　(D) 143　　　　(E) A total of 2012 dots is impossible

19. A square $ABCD$ has sides of length 2. E and F are the midpoints of the sides AB and AD respectively. G is a point on CF such that $3CG=2GF$. What is the area of triangle BEG?

(A) $\dfrac{7}{10}$　　　　(B) $\dfrac{4}{5}$　　　　(C) $\dfrac{8}{5}$　　　　(D) $\dfrac{3}{5}$　　　　(E) $\dfrac{6}{5}$

20. The clock in the picture is rectangular in shape, but each hand moves at a constant rate, like a normal clock. The distance between the numbers 8 and 10 on the dial is 12 cm and the distance between 1 and 2 is x cm. What is the value of x?
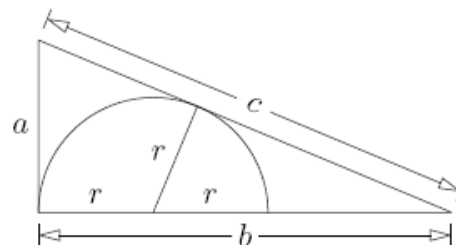
(A) $3\sqrt{3}$　　　　(B) $2\sqrt{3}$　　　　(C) $4\sqrt{3}$

(D) $2+\sqrt{3}$　　　　(E) $12-3\sqrt{3}$
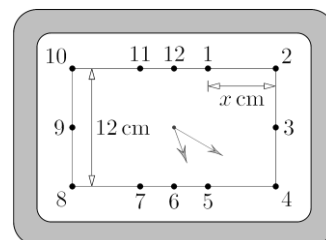
Part C: Each correct answer is worth 5 points.

21. We are given a right-angled triangle with sides of length a, b and c. What is the radius r of the inscribed semicircle shown in the figure?

(A) $\dfrac{a(c-a)}{2b}$　　　　(B) $\dfrac{ab}{a+b+c}$　　　　(C) $\dfrac{ab}{b+c}$

(D) $\dfrac{2ab}{a+b+c}$　　　　(E) $\dfrac{ab}{a+c}$

22. Consider two operations which can be performed on a fraction: 1) increase the numerator by 8; 2) increase the denominator by 7. Having performed a total number of n such operations in some order, starting with the fraction $\dfrac{7}{8}$ we obtain a fraction of equal value. What is the smallest possible value of n?

(A) 56　　　　(B) 81　　　　(C) 109　　　　(D) 113　　　　(E) This is impossible

23. Let $a > b$. If the ellipse shown in the picture is rotated around the x-axis one obtains the ellipsoid E_x with the volume $Vol(E_x)$. If the ellipse is rotated around the y-axis one obtains the ellipsoid E_y with the volume $Vol(E_y)$. Which of the following statements is true?

(A) $E_x = E_y$ and $Vol(E_x) = Vol(E_y)$

(B) $E_x \neq E_y$ and $Vol(E_x) > Vol(E_y)$

(C) $E_x = E_y$ but $Vol(E_x) \neq Vol(E_y)$

(D) $E_x \neq E_y$ and $Vol(E_x) < Vol(E_y)$

(E) $E_x \neq E_y$ but $Vol(E_x) = Vol(E_y)$
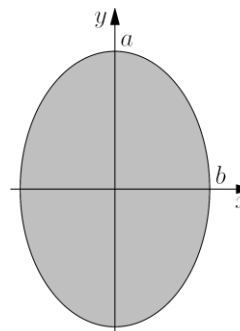
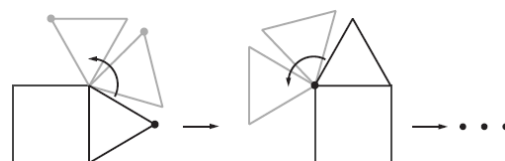24. How many solutions does the system of equations have?

$$|x| + |y| = 1$$
$$x^2 - y^2 = 1$$

(A) 1   (B) 2   (C) 4   (D) 8   (E) 0

25. An equilateral triangle rolls without slipping around a square with side length 1 (see picture). What is the length of the path that the marked point traces until the triangle and the point reach their starting positions the next time?

(A) 4π (B) $\dfrac{28}{3}\pi$ (C) 8π (D) $\dfrac{14}{3}\pi$ (E) $\dfrac{21}{2}\pi$

26. After an algebra lesson, the following was left on the blackboard: the graph of the function $y = x^2$ and 2012 lines parallel to the line $y = x$, each of which intersects the parabola in two points. What is the sum of the x-coordinates of the points of intersection of the lines and the parabola?

(A) 0   (B) 1   (C) 109   (D) 1006   (E) 2012

27. How many permutations (x_1, x_2, x_3, x_4) of the set of integers $\{1, 2, 3, 4\}$ have the property that the sum $x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1$ is divisible by 3?

(A) 8   (B) 12   (C) 14   (D) 16   (E) 24

28. A squirrel family lives 15m above the ground, in the lush crown of an old tree whose trunk has a circumference of 1m. Today the youngest squirrel Stig practices running down the tree trunk. Father squirrel shows Stig the sketch of the trunk and gives him the following instructions: "Keep constant speed, same angle to the ground, and make sure you make 8 revolutions around the trunk". This path seems to Stig so much longer than running down the trunk vertically. How much longer is it?

(A) 1m   (B) 2m   (C) 3m   (D) 4m   (E) 5m
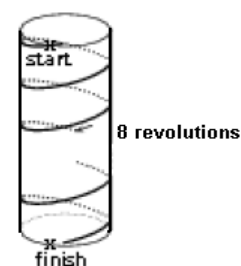
29. In the sequence 1, 1, 0, 1, -1,..., each of the first two terms a_1 and a_2 is 1. The third term is the difference of the preceding two terms, that is, $a_3 = a_1 - a_2$. The fourth is the sum of the two preceding terms, that is, $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on. What is the sum of the first 100 terms of this sequence?

(A) 0   (B) 3   (C) -21   (D) 100   (E) -1

30. Ioana picks out two numbers a and b from the set $\{1, 2, 3, ..., 26\}$. The product ab is equal to the sum of the remaining 24 numbers. What is the value of $|a - b|$?

(A) 10   (B) 9   (C) 7   (D) 2   (E) 6