Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. Which of the following numbers is closest to 20.15×51.02 ?
   (A) 100   (B) 1000   (C) 10000   (D) 100000   (E) 1000000

2. Mother did the laundry and hanged t-shirts in line on a clothing line. Then she asked her children to put a single sock between any two t-shirts. Now there are 29 pieces of clothing on the line. How many t-shirts are there on the line?
   (A) 10   (B) 11   (C) 13   (D) 14   (E) 15

3. The shaded part of the square with side a is bounded by a semicircle and two quarter arcs. What is its area?
   (A) $\frac{\pi a^2}{8}$ (B) $\frac{a^2}{2}$ (C) $\frac{\pi a^2}{2}$ (D) $\frac{a^2}{4}$ (E) $\frac{\pi a^2}{4}$

4. Three sisters, Ann, Beth and Cindy, bought a bag of 30 cookies together. Each of them received 10 cookies. Ann paid 80 cents, Beth paid 50 cents and Cindy paid 20 cents. If they had divided the cookies proportionally to the price each paid, how many more cookies would Ann have received?
   (A) 10   (B) 9   (C) 8   (D) 7   (E) 6

5. Mister Hide wants to dig up a treasure that he buried in his garden years ago. He can only remember that he buried the treasure at least 5 m away from the hedge and at most 5 m from the trunk of the old pear tree. Which of the following pictures shows the region where Mister Hide should look for the treasure?

   (A)   (B)   (C)   (D)   (E)
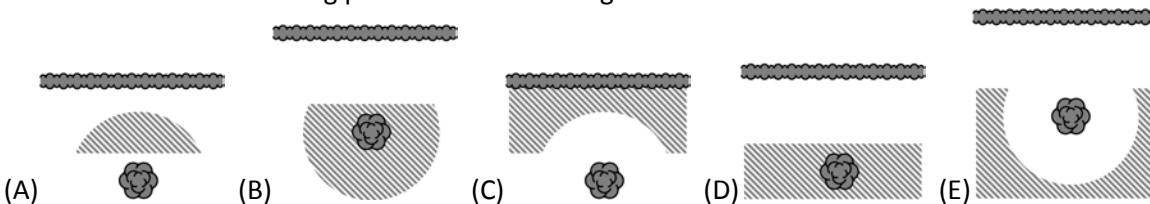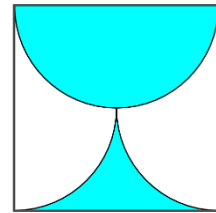
6. There are 30 problems in the MathQ competition for pupils. Two points are scored for each correct answer. If a pupil omits to solve a problem, he/she gets 0 points on it. One point is lost by a pupil for each wrong answer. Mark answered 25 problems and got 5 points. How many wrong answers did he have?
   (A) 25   (B) 20   (C) 15   (D) 10   (E) 5

7. There are 33 children in a class. When they were asked about their favourite subjects, only computer studies and/or physical education were among their answers. Three children liked both subjects. There were twice as many children who named only computer studies than those who named only physical education. How many children named computer studies as their favourite subject?
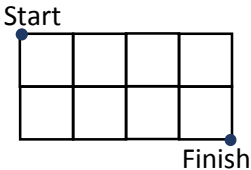   (A) 15   (B) 18   (C) 20   (D) 22   (E) 23

8. Which of the following is neither a square number nor a cube number?
   (A) 6^{13} (B) 5^{12} (C) 4^{11} (D) 3^{10} (E) 2^{9}

9. Mr Candle bought 100 candles. He burns one candle every day and always makes one new one from the remaining wax of seven burnt candles. After how many days will he have to go and buy new candles again?
   (A) 112   (B) 114   (C) 115   (D) 116   (E) 117

10. How many different triangles of area 1 can be drawn if each of their angles may only be equal to 40°, 50°, 100°, or 90°? (Congruent triangles are considered the same.)
    (A) 0   (B) 1   (C) 2   (D) 4   (E) 6

Part B: Each correct answer is worth 4 points

11. Which of the following is a counter-example to the statement
    'If n is prime then exactly one of the numbers $n - 2$ and $n + 2$ is prime'?
    (A) $n = 11$ (B) $n = 19$ (C) $n = 21$ (D) $n = 29$ (E) $n = 37$

12. The number of right angles in some convex pentagon is n. Which is the complete list of the possible values of n?
    (A) 1, 2, 3   (B) 0, 1, 2, 3, 4   (C) 0, 1, 2, 3   (D) 0, 1, 2   (E) 1, 2

13. The picture shows my decision die in two different positions.


    What is the probability to roll YES with this die?
    (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{5}{9}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

14. The length of a side of each square in the figure is 1. What is the smallest possible distance you can walk from "Start" to "Finish", if you are only able to move along the sides or diagonals of individual squares?

    (A) $2\sqrt{5}$ (B) $\sqrt{10} + \sqrt{2}$ (C) $2 + 2\sqrt{2}$ (D) $4\sqrt{2}$ (E) 6

15. Every inhabitant of the Winger planet has at least two ears. Three inhabitants named Imi, Dimi and Trimi met in a crater. Imi said: "I can see 8 ears." Dimi: "I can see 7 ears." Trimi: "That's strange, I can only see five ears." None of them could see his own ears. How many ears does Trimi have?
    (A) 2   (B) 4   (C) 5   (D) 6   (E) 7

16. Today, the product of the ages (in integers) of father and son is 2015. What is the difference of their ages?
   (A) 26   (B) 29   (C) 31   (D) 34   (E) 36

17. Robert has three candy dispensers, each releasing one candy at a time. He cannot see inside the dispensers but he knows that one contains cherry-flavoured candies, another one contains lemon-flavoured candies, and the third contains candies of both flavours. He also knows that the labels of the dispensers have been swapped and they are all wrong. What is the minimum number of candies that Robert has to dispense, in order to find out how to re-label all the three dispensers correctly?

(A) 1   (B) 2   (C) 3   (D) 4   (E) All candies of 2 dispensers

18. If the two roots of the equation $x^2 - 85x + c = 0$ are prime numbers, what is the value of the sum of the digits of c?

(A) 12   (B) 13   (C) 14   (D) 15   (E) 21

19. How many three-digit positive integers are there in which any two adjacent digits differ by 3?

(A) 12   (B) 14   (C) 16   (D) 20   (E) 27

20. $ABCD$ is a square of side 1. A, B, C, D are centres of quarter-circles that intersect at P, Q, R and S, as shown in the drawing. What is the size of $\angle AQB$?
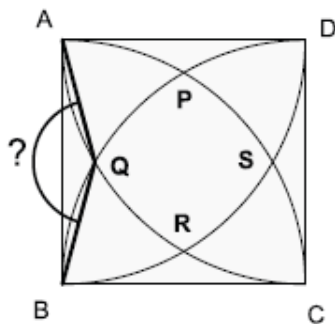
(A) 90°   (B) 105°   (C) 120°   (D) 135°   (E) 150°

Part C: Each correct answer is worth 5 points

21. The figure shows seven regions enclosed by three circles. A number is written in each region. It is known that the number in any region is equal to the sum of the numbers in all neighbouring regions. (We call two regions neighbouring if their boundaries have more than one common point.) Two of the numbers are known (see fig.). Which number is written in the central region?
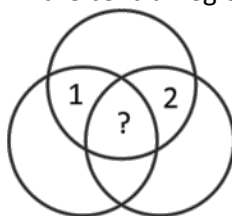

(A) 0   (B) −3   (C) 3   (D) −6   (E) 6

22. Petra has three different dictionaries and two different novels on a shelf. How many ways are there to arrange the books if she wants to keep the dictionaries together and the novels together?

(A) 12   (B) 24   (C) 30   (D) 60   (E) 120

23. How many 2-digit numbers can be written as the sum of exactly six different powers of 2, including 2^0?
  (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

24. In the triangle ABC, we can draw a line parallel to its base AC, through either point X or Y. The areas of the resulting shaded regions are the same. The ratio $BX{:}XA$ has the value $BX{:}XA = 4{:}1$. What is the value of the ratio $BY{:}YA$?
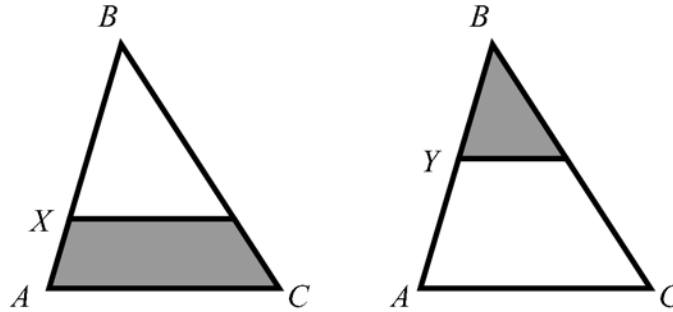
  (A) $1{:}1$ (B) $2{:}1$ (C) $3{:}1$ (D) $3{:}2$ (E) $4{:}3$

25. Ten different numbers are written down. Any number that is equal to the product of the other nine numbers is then underlined. How many numbers can be underlined at most?
  (A) 1   (B) 2   (C) 3   (D) 9   (E) 10

26. We are interested in sequences a_1, a_2, a_3, ... that satisfy the following property: $a_{n+2} = a_{n+1} + a_n$, where a_m is a positive integer for any m, and it is known that $a_7 = 2015$. How many such sequences exist?
  (A) 0   (B) 1   (C) 5   (D) 50   (E) Infinitely many

27. In a right triangle, the angle bisector of an acute angle divides the opposite side into segments of length 1 and 2.
  What is the length of the bisector?
  (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{4}$ (D) $\sqrt{5}$ (E) $\sqrt{6}$

28. When one of the numbers $1, 2, 3, ... , n-1, n$ was eliminated, the average of the remaining numbers was 4.75. Which number was eliminated?
  (A) 5   (B) 7   (C) 8   (D) 9   (E) This is impossible to determine.

29. Oyla the ant starts on one of the vertices of a cube whose edges have length 1. She wants to walk along every edge of the cube and return to her starting point, making the length of her journey as short as possible. What is the length of her journey?
  (A) 12   (B) 14   (C) 15   (D) 16   (E) 20

30. A safe in the Firm has six locks. Each member of the Board of Directors has three different keys to the safe, and no two directors have the same set of keys. Any two of the directors should never be able to open the safe due to missing at least one key between them. What is the highest possible number of members of the Firm's Board of Directors?
  (A) 8   (B) 9   (C) 10   (D) 12   (E) 20