Canadian Math Kangaroo Contest

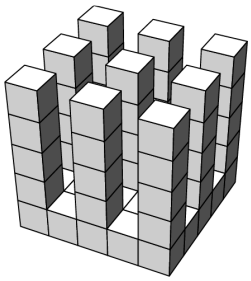
2014 Grade 11 and 12 Questions and Answers

YEAR 2014

Canadian Math Kangaroo Contest
PROBLEMS

Part A: Each correct answer is worth 3 points

1. If you take a number of $1 \times 1 \times 1$ cubes out of a $5 \times 5 \times 5$ cube, you end up with a solid figure consisting of columns of the same height, which stand on the same ground plate (see figure).



How many small cubes have been taken out?

(A) 56 (B) 60 (C) 64 (D) 68 (E) 80

2. Carla, Emilie and Lilia have a birthday today. The sum of their ages is now 44. What will the sum of their ages be the next time it is a two-digit number with two equal digits?

(A) 55 (B) 66 (C) 77 (D) 88 (E) 99

3. The positive integers a and b satisfy the equation $a^2 + b^2 = 100$. What is the value of $a + b$?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

4. If $a^b = \frac{1}{2}$ what is the value of a^{-3b} ?

(A) $\frac{1}{8}$ (B) 8 (C) -8 (D) 6 (E) $\frac{1}{6}$

5. There are 48 balls placed into three baskets of different sizes. The smallest and the largest baskets together contain twice the number of balls the medium-sized basket contains. The smallest basket contains half the number of balls the medium-sized basket has. How many balls are there in the largest basket?

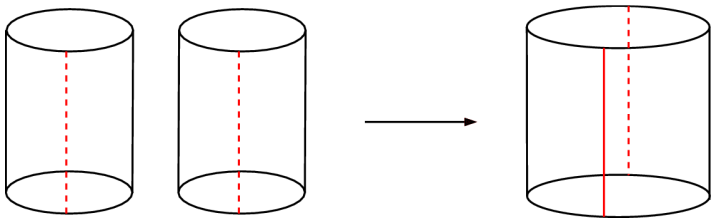
(A) 16 (B) 20 (C) 24 (D) 30 (E) 32

6. How many digits long is the result of the multiplication: $(2^{22})^5 \cdot (5^{55})^2$?

(A) 22 (B) 55 (C) 77 (D) 110 (E) 111

7. Handsome Harry has a secret email account that only four friends know. Today he received 8 emails in that account. Which of the following is certainly true?
(A) Harry received two emails from each friend.
(B) Harry cannot have received eight emails from one of his friends.
(C) Harry received at least one email from each friend.
(D) Harry received at least two emails from one of his friends.
(E) Harry received at least two emails from 2 different friends.

8. Two identical cylinders are cut open along the dotted lines and glued together to form one bigger cylinder – see the figure. What can you say about the volume of the big cylinder compared to the volume of one small cylinder?



(A) It has twice the volume. (B) It has 3 times the volume. (C) It has π times the volume.
(D) It has 4 times the volume. (E) It has 8 times the volume.

9. In the number 2014 the digits are different and the last digit is greater than the sum of the other three digits. How many years ago did this occur the last time?
(A) 5 (B) 215 (C) 305 (D) 395 (E) 485

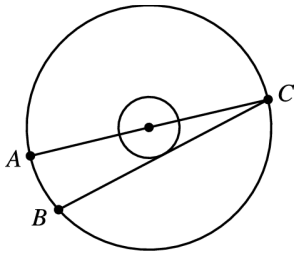
10. An isosceles triangle has two sides of length 2. What is the largest possible area of such a triangle?
(A) 1 (B) 2 (C) $\sqrt{3}$ (D) $2\sqrt{2}$ (E) $2\sqrt{3}$

Part B: Each correct answer is worth 4 points

11. The size of a rectangular box is $a \times b \times c$, with $a < b < c$. If you increase a or b or c by a given positive value, the volume of the box also increases. In which of the following cases is the increase of the volume of the box the greatest?
(A) If you increase a. (B) If you increase b. (C) If you increase c.
(D) The increase of the volume is the same in A), B), C). (E) It depends on the values of a, b, c.

12. In a soccer match, the winner gets 3 points, the loser gets 0 points, while in the case of a draw each team gets 1 point. Four teams, A, B, C, D, take part in a soccer tournament. Each team plays three games: one against each other team. At the end of the tournament team A has 7 points and teams B and C have 4 points each. How many points does team D have?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

13. The radii of two concentric circles are in proportion 1 : 3.



AC is a diameter of the big circle; BC is a chord of the big circle which is tangent to the smaller circle; and the length of the segment AB is 12. What is the radius of the big circle?

(A) 13 (B) 18 (C) 21 (D) 24 (E) 26

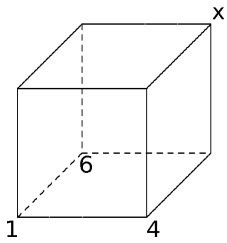
14. How many triples (a, b, c) of integers with $a > b > c > 1$ satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > 1$?

(A) none (B) 1 (C) 2 (D) 3 (E) infinitely many

15. Six weeks is $n!$ ($n! = 1 \times 2 \times 3 \times 4 \times \ldots \times n$) seconds. What is the value of n?

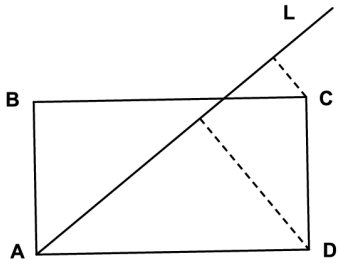
(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

16. The vertices of a cube are numbered 1 to 8 in such a way that the result of adding the four numbers of the vertices of a face is the same for all faces. Numbers 1, 4 and 6 are already set on some vertices as shown. What is the value of x?



(A) 2 (B) 3 (C) 5 (D) 7 (E) 8

17. Line L passes through the vertex A of a rectangle $ABCD$. The distance from point C to L is 2, and the distance from point D to L is 6. If AD is twice AB, find AD.



(A) 10 (B) 12 (C) 14 (D) 16 (E) $4\sqrt{3}$

18. The function $f(x) = ax + b$ satisfies the equalities $f(f(f(1))) = 29$ and $f(f(f(0))) = 2$. What is the value of a?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

19. The number of intersection points of four distinct straight lines on the plane cannot be equal to

(A) 6. (B) 5. (C) 4. (D) 3. (E) 2.

20. Which of the following numbers is divisible by 5?

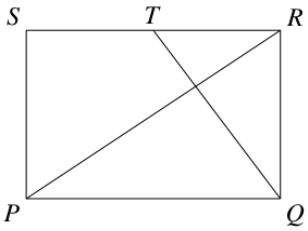
(A) $2^{100}+1$ (B) $2^{101}+1$ (C) $2^{102}+1$ (D) $2^{103}+1$ (E) $2^{104}+1$

Part C: Each correct answer is worth 5 points

21. The label on a package of cream cheese reads: 24% total fat. The same label also reads: 64% fat in dry matter. What is the percentage of water in this cheese?

(A) 88% (B) 62.5% (C) 49% (D) 42% (E) 37.5%

22. $PQRS$ is a rectangle. T is the midpoint of RS. QT is perpendicular to the diagonal PR. What is the ratio $PQ:QR$?

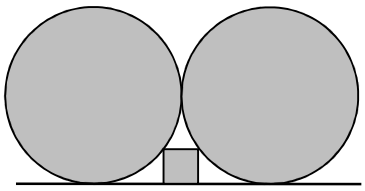


(A) 2 : 1 (B) $\sqrt{3}:1$ (C) 3 : 2 (D) $\sqrt{2}:1$ (E) 5 : 4

23. There are 9 kangaroos called Greatkangs. Their fur is either silver or gold in color. When 3 Greatkangs happen to meet, there is a two in three chance that none of them is silver. How many Greatkangs are gold?

(A) 1 (B) 3 (C) 5 (D) 6 (E) 8

24. A square fits snugly between the horizontal line and two touching circles of radius 1.



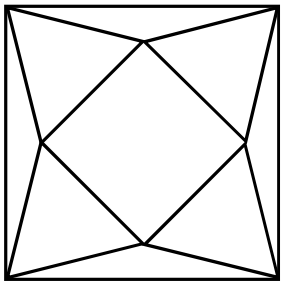
What is its side length?

(A) $\frac{2}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$

25. Tom wants to write several distinct positive integers, none of them exceeding 100. Their product should not be divisible by 54. At most how many integers can he write?
(A) 8 (B) 17 (C) 68 (D) 69 (E) 90

26. There are three boxes, each with 1 white, 1 black, 1 red and 1 blue balls. Charles takes out 1 ball from each box, blindfolded. What is the probability that exactly two of the balls taken out are red?
(A) 1/8 (B) 1/9 (C) 3/16 (D) 3/64 (E) 9/64

27. The figure shows four equilateral triangles and two squares.



Each equilateral triangle has two vertices which coincide with the vertices of the inner square, and the third vertex coincides with a vertex of the outer square. The area of the inner square is 1. What is the area of the outer square?
(A) $2\sqrt{3}$ (B) $2+\sqrt{3}$ (C) 4 (D) $3+\sqrt{2}$ (E) $1+\sqrt{2}+\sqrt{3}$

28. There are 10 different positive integers, exactly 5 of them are divisible by 5 and exactly 7 of them are divisible by 7. Let M be the greatest of these 10 numbers. What is the minimum possible value of M?
(A) 105 (B) 77 (C) 75 (D) 63 (E) none of these

29. The equalities $k = (2014 + m)^{\frac{1}{n}} = 1024^{\frac{1}{n}} + 1$ are given for positive integers k, m, n. How many different values can the number m take?
(A) None (B) 1 (C) 2 (D) 3 (E) Infinitely many

30. Two regular polygons of side length 1 lie on opposite sides of their common side AB. One of them is a 15-gon $ABCD$ ... and the other is an n-gon $ABZY$ .... What value of n makes the distance CZ equal to 1?
(A) 10 (B) 12 (C) 15 (D) 16 (E) 18