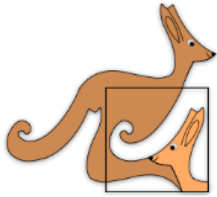# For training purposes only!

INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2020

INSTRUCTIONS
GRADE 11-12

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, decide which answer is correct and fill in (blacken) the oval that has the same letter as the appropriate answer. If you fill in (blacken) more than one oval for a question, your response will be marked as wrong.

1. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.

2. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.

3. The use of external material or aid of any kind is **not permitted**.

4. The figures *are not* drawn to scale. They should be used only for illustration purposes.

5. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.

6. At the end of the allotted time, please **give the response form to the contest supervisor**.

7. Do not forget to pick up your Certificate of Participation on your way out!

Good luck!

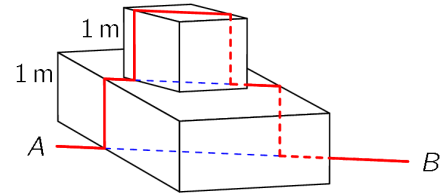*Canadian Math Kangaroo Contest team*

mathkangaroo.ca

CANADIAN MATH KANGAROO CONTEST PROBLEMS

PART A: EACH CORRECT ANSWER IS WORTH 3 POINTS

1. How many real numbers x are there, which satisfy the equation $\frac{x^2}{3} = \frac{3}{x^2}$?
   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

2. An ant walked every day on a straight horizontal line path from A to B, which are 5 m apart. One day humans placed on its path two strange obstacles of height 1 m each. Now the ant walks along or above the same straight line except that it now has to climb up and down vertically over both the two obstacles, as in the picture. How long is its path now?
   (A) 7 m   (B) 9 m   (C) $5 + 4\sqrt{2}$ m   (D) $9 - 2\sqrt{2}$ m
   (E) the length depends on the angles the obstacles are situated along the path

3. Rene marked two points a and b as accurately as possible on the number line. Which of the points p, q, r, s, t on the number line best represents their product ab?
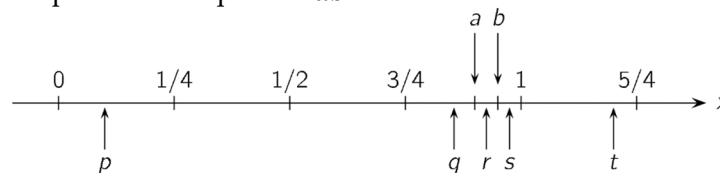
   (A) p (B) q (C) r (D) s (E) t

4. The sum of five three-digit numbers is 2664 as shown on the board. What is the value of $A + B + C + D + E$?
   (A) 4   (B) 14   (C) 24   (D) 34   (E) 44

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    A B C
  + B C D
  + C D E
  + D E A
  + E A B
  -------
    2 6 6 4
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5. What is the value of $\frac{1010^2 + 2020^2 + 3030^2}{2020}$?
   (A) 2020   (B) 3030   (C) 4040   (D) 6060   (E) 7070

6. Let a, b and c be integers satisfying $1 \leq a \leq b \leq c$ and $abc = 1\,000\,000$. What is the largest possible value of b?
   (A) 100   (B) 250   (C) 500   (D) 1000   (E) 2000

7. If D dogs weigh K kilograms and E elephants weigh the same as M dogs, how many kilograms does one elephant weigh?
   (A) $DKEM$ (B) $\frac{DK}{EM}$ (C) $\frac{KE}{DM}$ (D) $\frac{KM}{DE}$ (E) $\frac{DM}{KE}$

8. There are two dice. Each one has two red faces, two blue faces and two white faces. If we roll both dice together, what is the probability that both show the same color?
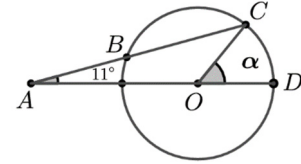   (A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) $\frac{1}{6}$ (D) $\frac{2}{9}$ (E) $\frac{1}{3}$

9. Which of the following numbers is not divisible by 3 for any integer n?
   (A) $5n + 1$ (B) n^2 (C) $n(n+1)$ (D) $6n - 1$ (E) $n^3 - 2$

10. In the figure shown (diagram not to scale), the angle $\angle CAD$ measures 11°.
    If the length AB is the same as the radius of the circle with centre O, what is
    the measure of the angle $\angle COD$?
    (A) 22°   (B) 30°   (C) 33°
    (D) 44°   (E) 60°

PART B: EACH CORRECT ANSWER IS WORTH 4 POINTS

11. Mary has in her hand a deck of 8 cards. On each card, there is one of the letters K, A, N, G, A, R, O, O. She lays
    down the cards on the table in the following way: top card goes to the bottom of the cards in hand, then the top
    card goes on the table, then the top card goes to the bottom of the cards in hand, then the top card goes on the
    table (to the right of the cards that are already there), etc. until all the cards are on the table. The cards, from
    left to right form the word $KANGAROO$. What was the order of the cards (top to bottom) in the deck that Mary
    started with?
    (A) $OKANGARO$ (B) $RKOOANGA$ (C) $OORAGNAK$ (D) $OKAAONRG$ (E) $AKGNRAOO$

12. Five coins are lying on a table with the "heads" side up. At each step you must turn over exactly three of the
    coins. What is the least number of steps required to have all the coins have lying with the "tails" side up?
    (A) 2   (B) 3   (C) 4   (D) 5
    (E) It is not possible to have all the coins with their "tails" side up.

13. Four identical boxes are glued together to make the shape shown in the picture.
    One litre of paint is needed to paint the outside of one such box. How many
    litres of paint are needed to paint the outside of the glued construction?
    (A) 2.5   (B) 3   (C) 3.25   (D) 3.5   (E) 4

14. Let a, b and c be integers. Which of the following is certainly NOT equal to $(a - b)^2 + (b - c)^2 + (c - a)^2$?
    (A) 0   (B) 1   (C) 2   (D) 6   (E) 8

15. The first two digits of a 100-digit integer are 2 and 9.
    How many digits does the square of this number have?
    (A) 101   (B) 199   (C) 200   (D) 201
    (E) It cannot be determined

16. A polyhedron P has for its vertices the mid-points of each edge of a cube. What is the sum of the number of
    edges and the number of faces of P?
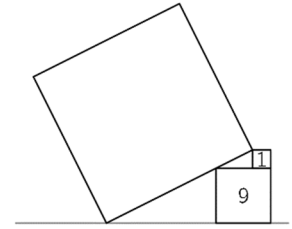   (A) 38   (B) 46   (C) 18   (D) 36
    (E) none of these
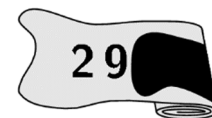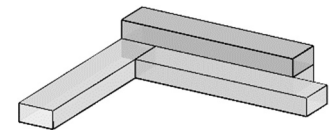
17. A large square touches two other squares, as shown in the diagram. The numbers in the small squares represent their areas. What is the area of the large square?
(A) 49 (B) 80 (C) 81
(D) 82 (E) 100

18. The sequence f_n is given by $f_1 = 1$, $f_2 = 3$ and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$. How many of the first 2020 elements of the sequence are even?
(A) 673 (B) 674 (C) 1010 (D) 1011 (E) 1347

19. A circle and a rectangle have been drawn in such a way that the circle touches two of the sides of the rectangle and passes through one of its vertices. The distances of the two vertices of the rectangle from one of the points where the circle touches the rectangle are 5 and 4, as shown. What is the area of the rectangle?
(A) 27π (B) 25π (C) 72 (D) 63
(E) none of the previous
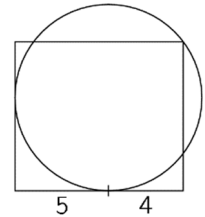
20. Three rectangular prisms are arranged to make a larger rectangular prism as in the figure. The width of one of them is 6 and the areas of some of their faces are $14, 21, 16, 30$, as shown. What is the area of the face with the question mark?
(A) 18 (B) 24 (C) 28 (D) 30
(E) cannot be determined
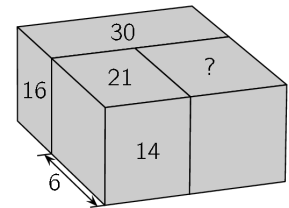
PART C: EACH CORRECT ANSWER IS WORTH 5 POINTS

21. In a game, there are 10 cards: one is red and the rest are black. The dealer shuffles the cards and deals them, face up, one by one. The player can choose when to stop. If the player stops when there are only black cards on the table, she gets 1 coin per card. However, if she draws a red card, she wins nothing. The game is played 100 times with the same number of cards drawn each time.
What number of cards drawn is the most likely to maximize her earnings?
(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

22. The figure shows a section of the parabola with equation $y = ax^2 + bx + c$.
Which of the following numbers is positive?
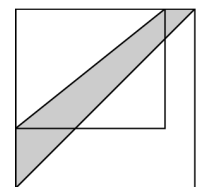(A) c (B) $b + c$ (C) ac (D) bc (E) ab
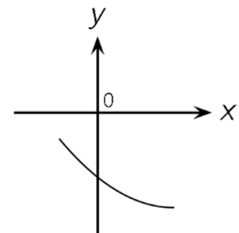
23. Numbers $a = 60^{99}, a_1 = 4a, a_2 = 8a, a_3 = 10a, a_4 = 25a$ are given.
Which two numbers have the same number of positive divisors?
(A) a_1 and a_3 (B) a_1 and a_4 (C) a_2 and a_3 (D) a_2 and a_4 (E) a_3 and a_4

24. The length of one of the sides of a rectangular garden is increased by 20% and the other side is increased by 50%. The new garden is shown in the diagram. The shaded area between the diagonal of the new garden and the diagonal of the original garden is 30 m². What was the area of the original garden?
(A) 60 m² (B) 65 m² (C) 70 m² (D) 75 m² (E) 80 m²
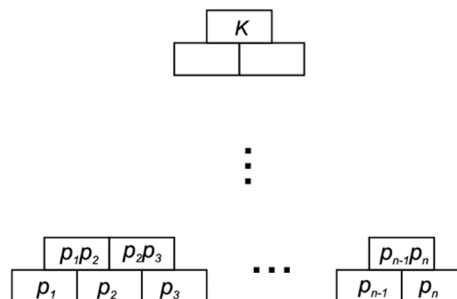
25. A large integer N is divisible by all except two of the integers from 2 to 11. Which of the following pairs of integers could be these exceptions?

(A) 2 and 3   (B) 4 and 5   (C) 6 and 7   (D) 7 and 8   (E) 10 and 11

26. In the morning, the ice-cream shop offers 16 flavours. Anna wants to choose a 2-flavour ice cream. In the evening several flavours are sold out and Bella wants to choose a 3-flavour ice cream from those flavours left. Both Anna and Bella can choose from the same number of possible combinations.
How many flavours were sold out?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

27. Tony has 71 marbles at his disposal in a box. He is allowed to take out exactly 30 marbles from the box or to return exactly 18 marbles to it. Tony is allowed to apply each operation as many times as he wishes. What is the smallest number of marbles than can be in the box?

(A) 1   (B) 3   (C) 5   (D) 7   (E) 11

28. Wajda took a square piece of paper of side 1 and folded two of its sides to the diagonal, as shown in the diagram, to make a quadrilateral. What is the area of this quadrilateral?
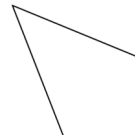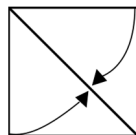
(A) $2 - \sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\sqrt{2} - 1$ (D) $\frac{7}{10}$ (E) $\frac{3}{5}$

29. An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Only three edges of the cube are partially visible over the water. The visible parts of these edges are 24 m, 25 m and 27 m. How long is an edge of the cube?

(A) 30 m   (B) 33 m   (C) 34 m   (D) 35 m   (E) 39 m

30. There are n different prime numbers p_1 to p_n written from left to right in the bottom row of the diagram shown. The product of two numbers next to each other in the same row is written in the box exactly above them. The number $K = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_n^{\alpha_n}$ is written in the box in top row.

If $\alpha_2 = 8$, how many numbers are divisible by the number p_4?

(A) 4   (B) 16   (C) 24   (D) 28   (E) 36