Imperial Oil Foundation

Canadian Mathematical Society

IEEE Canada Northern Canada Section

Pacific Institute for the Mathematical Sciences

Banff International Research Station for Mathematical Innovation and Discovery

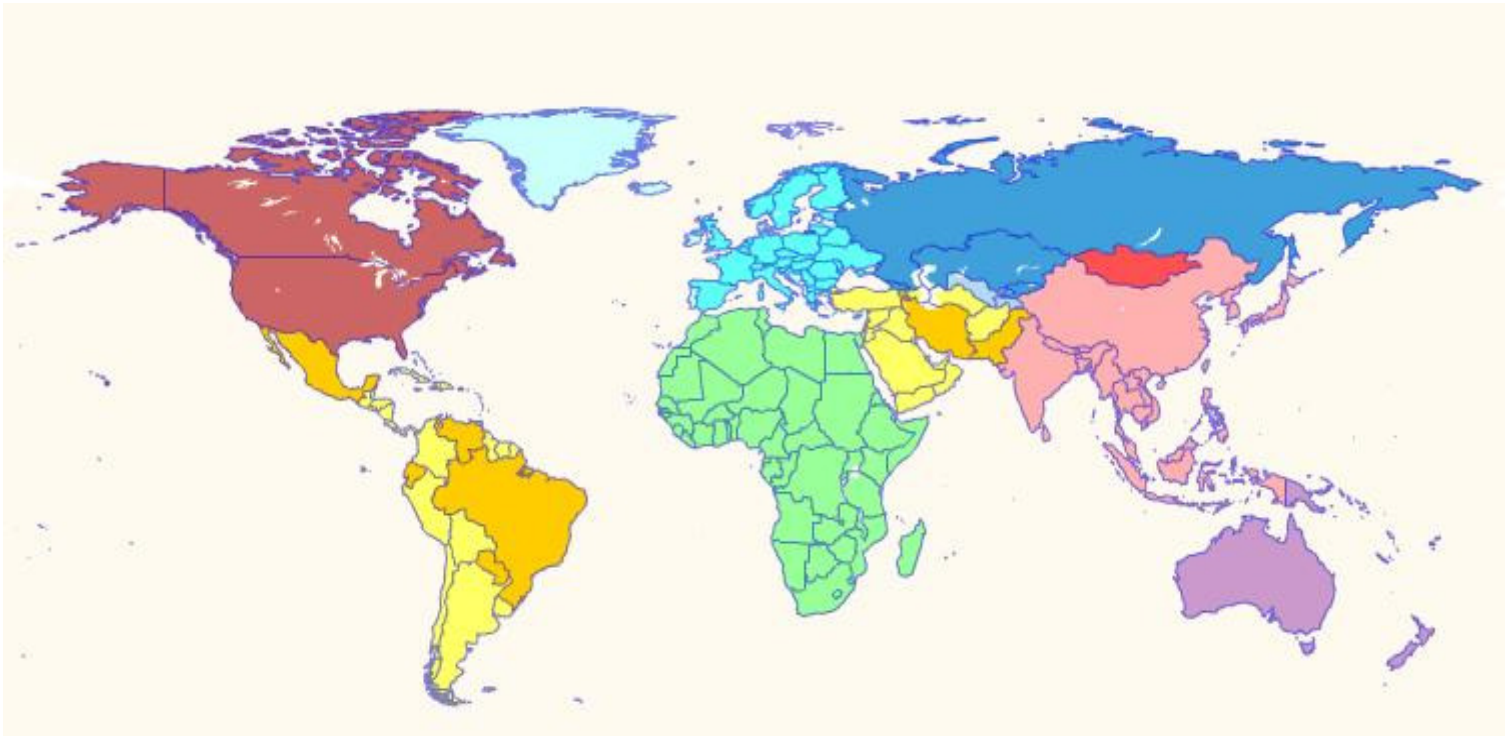
International Math Kangaroo Contest

Canada, 2011

Grade 5 and 6

CONCORDIA UNIVERSITY COLLEGE OF ALBERTA

The International Math Kangaroo Contest-Game



International Facts

- The contest started in 1991 and runs every year at the end of March
- Open for students of age from 8 to 18
- Currently, there are more than 46 countries in the International Association "Kangaroo without Borders"
- Approximately 5,500,000 students participated in 2010
- Problems are selected by the International Jury at the Annual Meeting of the Association "Kangaroo without Borders"

Contest Information

- **Date**: March 27, 2011 (Sunday)
- **Who can write**: students in grades 3-12

- The Contest-Game Math Kangaroo originated in France in 1991 and quickly became very popular among students.

- The first Canadian edition of the Math Kangaroo was in 2001, in Ottawa.

- In 2011, it is available in Edmonton, Ottawa, The Greater Toronto Area, Calgary, St. John's, Montreal, Winnipeg, Sudbury, Langley.

Questions

- 24 multiple-choice questions, arranged in increasing difficulty, to be answered in 60 minutes.
- Calculators are not permitted!
- A correct answer for questions 1-8 is worth **three** points, for questions 9-16 is worth **four** points, and for questions 17-24 is worth **five** points.
- There is a penalty of one point for each incorrect answer. If the question is left unanswered, it is worth 0 points. The maximum score attained is 120 points. To avoid negative scores, each student begins with 24 points. Hence, the minimum score attainable is 0 points.

Example 1

Kanga can solve 6 math problems in 12 minutes. At this rate, how many problems can she solve in 60 minutes?

A) 60 B) 30 C) 20 D) 10 E) 120

Solving 6 problems in 12 minutes means she solves (6/6) question in (12/6) minutes! That is 1 question in 2 minutes.

To get to 60 minutes we multiply both sides by 30 giving:
She would solve (1×30) questions in (2×30) minutes

This means 30 questions in 60 minutes

Answer: B)

Example 2

Minous has 16 cards: 4 spades (♠), 4 clubs (♣), 4 diamonds (♦) and 4 hearts (♥). She wants to arrange them in the square below, in such a way that every row and every column has a card of each sort. In the square below you can see how she started. What sort must be put in the square marked by the question mark?

♠		?	♥
♣	♠		
	♦		
	♥		

A) ♠ B) ♣ C) ♦ D) ♥
B) E) Impossible to decide

Solution of Example 2

Minous has 16 cards: 4 spades (♠), 4 clubs (♣), 4 diamonds (♦) and 4 hearts (♥). She wants to arrange them in the square below, in such a way that every row and every column has a card of each sort. In the square below you can see how she started. What sort must be put in the square marked by the question mark?

A) ♠ B) ♣ C) ♦ D) ♥

E) Impossible to decide

Column two needs a ♣ in the first row.

Then, row one must contain a ♦ in the third square.

The answer is C) ♦

♠	♣	♦	♥
♣	♠		
	♦		
	♥		

Answer: C)

Example 3

Which of the following expressions is equal to the value of

$(10 \times 100) \times (20 \times 80)$?

A) 20000×80000 B) 2000×8000 C) 2000×80000

D) 20000×8000 E) 2000×800

Notice that all the signs are multiplication signs!!! So we can regroup the numbers as we see fit!

(100 × 20) × (10 × 80) is a possible rearrangement which yields

2000 × 800 as an equivalent expression.

Answer: E)

Example 4

Vesna chose a whole number and multiplied it by three. Which of the following numbers could not be her answer?

A) 103 B) 105 C) 204 D) 444 E) 47988

Since she multiplied the chosen number by 3, her answer must be divisible by 3! The fastest way to check if a number is divisible by 3 is to add up its digits and check if the sum is divisible by 3!

103 => 1 + 0 + 3 = 4 {NOT DIVISIBLE BY 3}

105 => 1 + 0 + 5 = 6 {DIVISIBLE BY 3}

204 => 2 + 0 + 4 = 6 {DIVISIBLE BY 3}

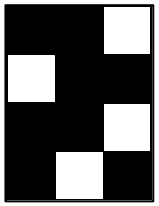
444 => 4 + 4 + 4 = 12 {DIVISIBLE BY 3}

47988 => 4 + 7 + 9 + 8 + 8 = 36 {DIVISIBLE BY 3}

Answer: A)

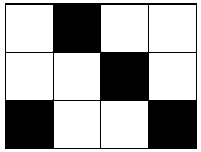
Example 5

Which of the rectangles (A) to (E) can be covered by the pattern

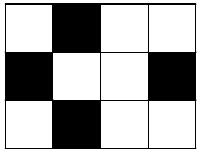


to become a completely black or a completely white rectangle?

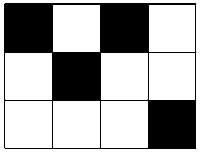
A)



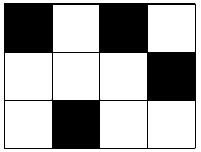
B)



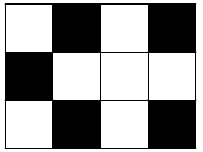
C)



D)

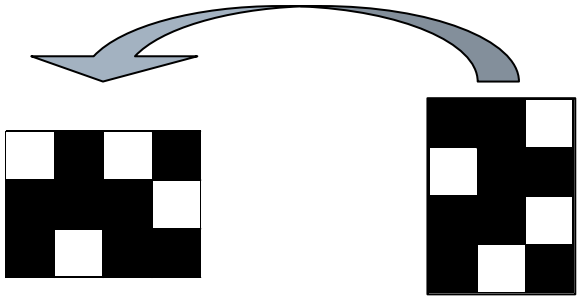


E)



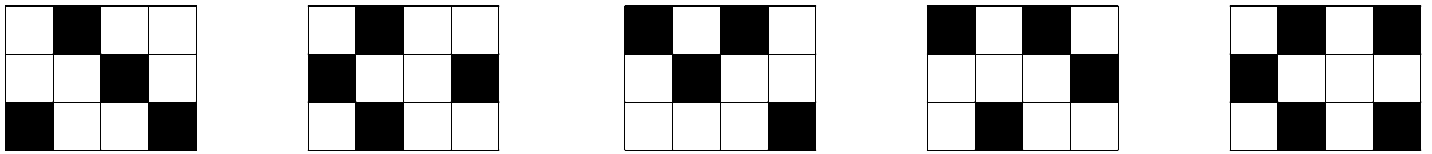
Solution of Example 5

Turning the pattern in the following way



helps us see that it can cover the rectangle D) to become a completely black rectangle.

A) B) C) D) E)



Answer: D)

Example 6

In one month, 5 Sundays occurred. This month could not have

A) 5 Saturdays B) 5 Fridays C) 5 Tuesdays

D) 5 Thursdays E) 5 Mondays

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25
29	30	31	

If Sundays started on the 1st, then we can have 5 Mondays and 5 Tuesdays.

If Sundays started on the 2nd, then we can have 5 Mondays and 5 Saturdays.

If Sundays started on the 3rd, then we can have 5 Saturdays and 5 Fridays.

The only ones of the options we cannot get in any way are 5 Thursdays!!

Answer: D)

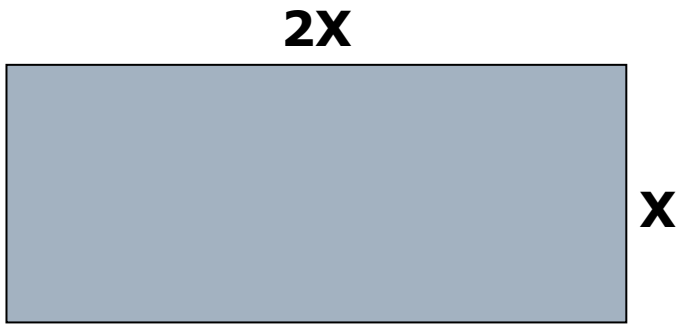
Example 7

A rectangle has a perimeter of 24cm and one side is twice as long as another. What, in square centimetres, is the rectangle's area?

A) 12 B) 16 C) 20 D) 24 E) 32

Let the short side be X.

Then the long side is 2X.



The perimeter is the distance around the figure, or

$24\text{cm} = X + 2X + X + 2X$

$24\text{cm} = 6X$

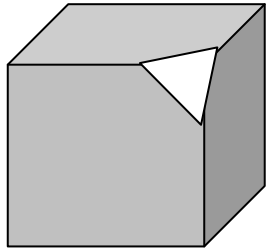
$4\text{cm} = X$

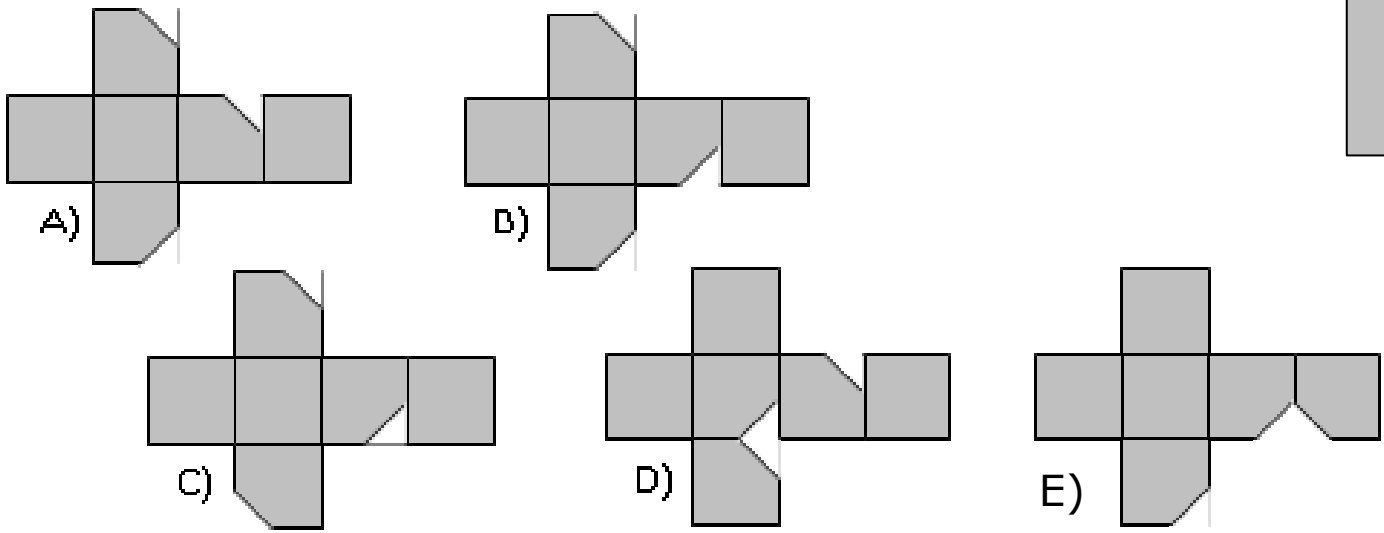
So the short side is 4cm and the long side is 8cm, therefore the area is **4cm x 8cm** which is **32cm²**

Answer: E)

Example 8

One corner of a cube is cut off, as shown on the picture. Then, the resulting solid is unfolded. Which of the nets below is the net of this solid?

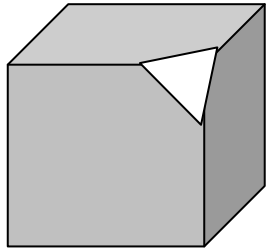


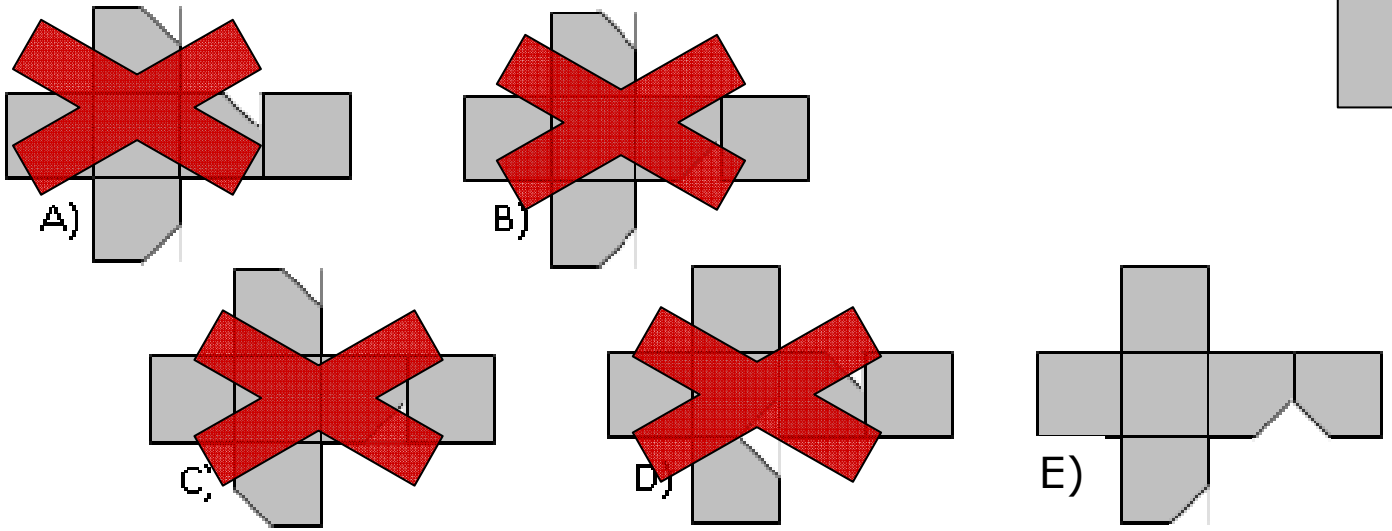


Solution of Example 8

One corner of a cube is cut off, as shown on the picture. Then, the resulting solid is unfolded. Which of the nets below is the net of this solid?

We solve this problem by eliminating nets that cannot be the result (please think and answer why):





Answer: E)

Example 9

After three games of the soccer championship, Platypus United has scored three goals and received one. They get three points for a win, one point for a draw and no points for a loss. How many points can they not have right now?

A) 7 B) 6 C) 5 D) 4 E) 3

It is possible to obtain the following scores:

- 7 points: 2 wins, 1 draw $= 2 \times 3 + 1 = 7$;
- 6 points: 2 wins, 1 loss $= 2 \times 3 + 0 = 6$;
- 5 points: 1 win, 2 draws $= 1 \times 3 + 2 \times 1 = 5$;
- 4 points: 1 win, 1 draw, 1 loss $= 3 + 1 + 0 = 4$.

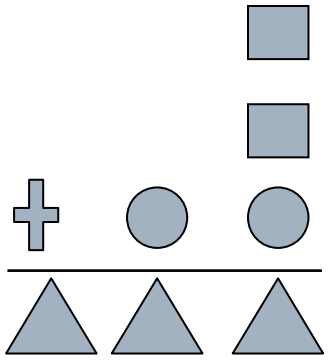
There is no way to have 3 points after 3 games if they scored 3 goals and received 1.

Answer: E)

Example 10

In the addition shown on the figure the digits of the numbers are replaced by symbols. Different symbols represent different digits, and equal symbols represent equal digits. Find the digit replaced by the square.

A) 9 B) 8 C) 7 D) 6 E) 5



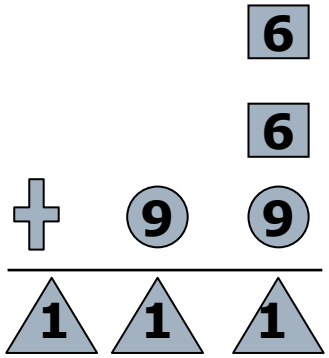
Solution of Example 10

In the addition shown on the figure the digits of the numbers are replaced by symbols. Different symbols represent different digits, and equal symbols represent equal digits. Find the digit replaced by the square.

A) 9 B) 8 C) 7 D) 6 E) 5

Since the triangle is in the leftmost position, it appears rather obvious that the triangles represent 1's.

The circle must be 9.



Answer: A)

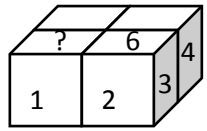
Example 11

In a regular die, the faces are numbered by the numbers 1 to 6 and the sum of the numbers on any two opposite faces is 7. Nick composed a rectangular prism $2 \times 2 \times 1$ using four identical regular dice, with the numbers on any two touching faces of the dice being equal (see the figure). The numbers on some faces are shown.

Which number must be written on the face denoted by the (**?**)?

A) 5 B) 6 C) 2 D) 3

E) not enough information



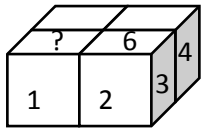
Solution of Example 11

In a regular die, the faces are numbered by the numbers 1 to 6 and the sum of the numbers on any two opposite faces is 7. Nick composed a rectangular prism $2 \times 2 \times 1$ using four identical regular dice, with the numbers on any two touching faces of the dice being equal (see the figure). The numbers on some faces are shown.

Which number must be written on the face denoted by the (?)?

A) 5 B) 6 C) 2 D) 3

E) not enough information



Using the information that the numbers on any two opposite sides have a sum of 7, it is easy to see that the top and the bottom faces of the die that contains the "?" cannot be 6 and 1 or 3 and 4, thus, the "?" can only be 5 or 2.

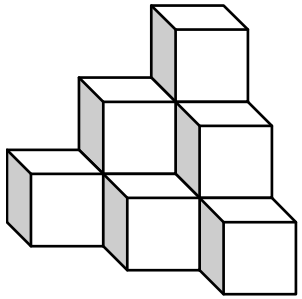
Taking into consideration the position of 2 and 3 relative to each other and to the rest of the digits, as well as the fact that all dice are identical, we obtain that "?" must be 5.

Answer: A)

Example 12

The structure shown on the picture is glued together from 10 cubes. Ryan painted the entire structure, including the bottom. How many faces of the cubes are painted?

A) 18 B) 24 C) 30 D) 36 E) 42

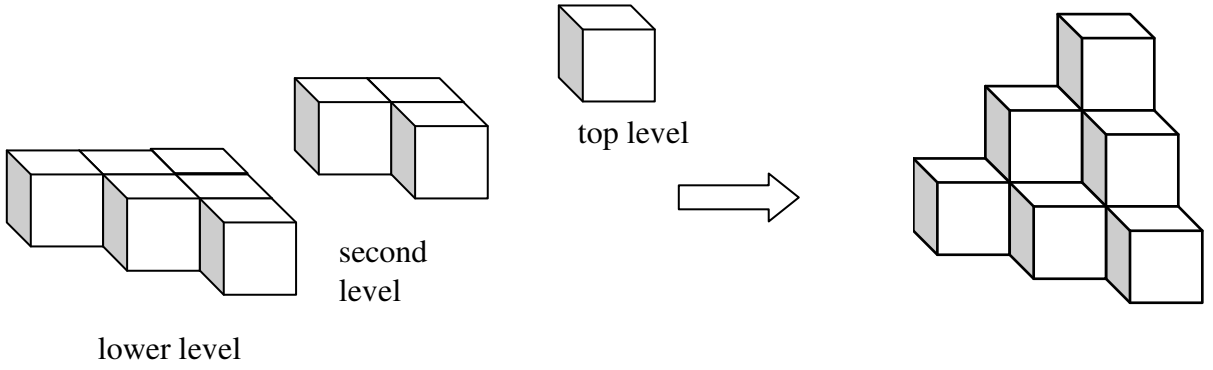


Solution of Example 12

The structure shown on the picture is glued together from 10 cubes. Ryan painted the entire structure, including the bottom. How many faces of the cubes are painted?

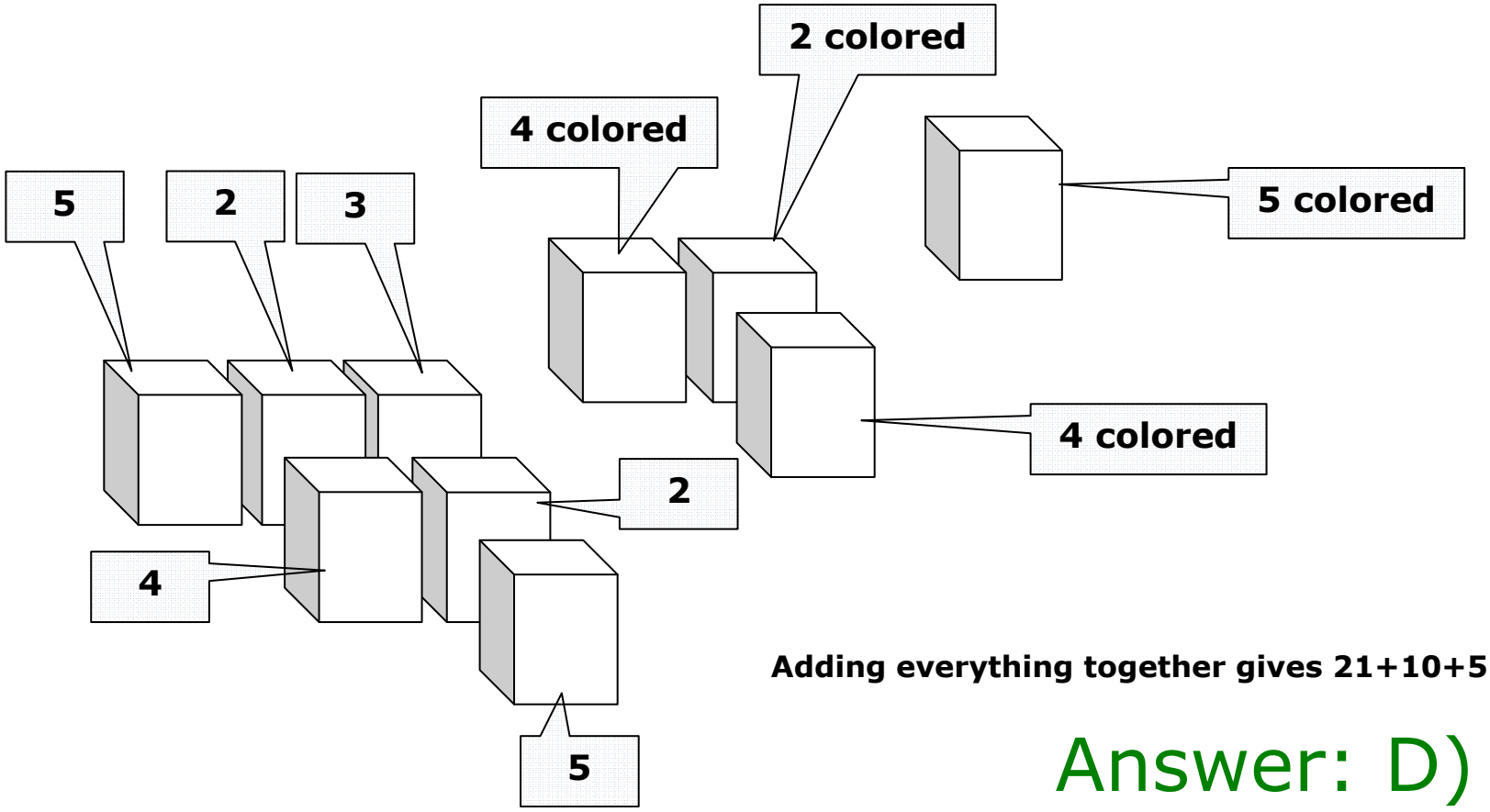
A) 18 B) 24 C) 30 D) 36 E) 42

Let us first make sure that we know how the structure is built. Although from the given picture we don't see all of the cubes that were used to build the structure, we know that the lower level consists of 6 cubes, the second level – 3 cubes, and the top level – only one cube.



Otherwise, the cubes cannot stay in the way they are shown on the picture if there are no other cubes below them for support.

Solution of Example 12



Adding everything together gives 21+10+5 =36

Answer: D)

Example 13

The total number of participants in a math club is between 50 and 100. The club teacher wanted to divide them in teams. He tried to group the students in 5, or 6, or 12 per team, and noticed that there were always 3 students left. How many students are there in the math club?

A) 51 B) 61 C) 63 D) 75

E) Not enough information

The total number of students is a number that leaves a remainder of 3 when divided by 5, or 6, or 12.

Therefore, if we subtract 3 from it, the number must be a common multiple of 5, 6, and 12.

As LCM (5, 6, 12)=60, all such numbers are multiples of 60. In the range from 50 to 100, only 60 is such a number.

Therefore, the students in the math club are 63.

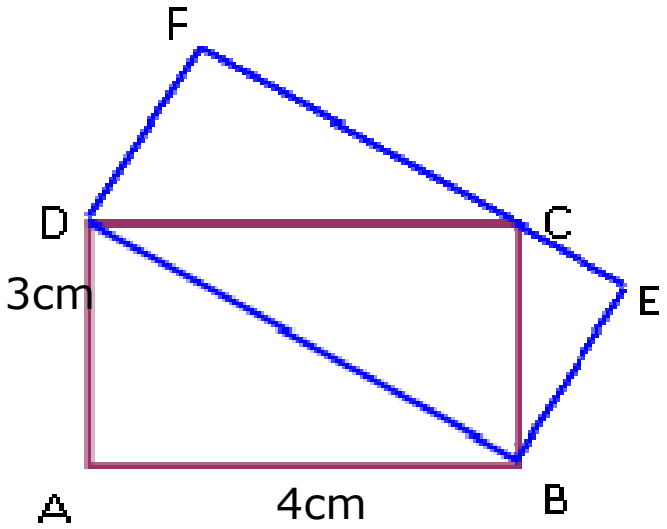
Answer: C)

Example 14

Two rectangles, ABCD and DBEF are shown in the diagram. The length of AD is 3cm; the length of AB is 4cm. What is the area of rectangle DBEF?

A) $10cm^2$ B) $12cm^2$ C) $13cm^2$

D) $14cm^2$ E) $16cm^2$



Solution of Example 14

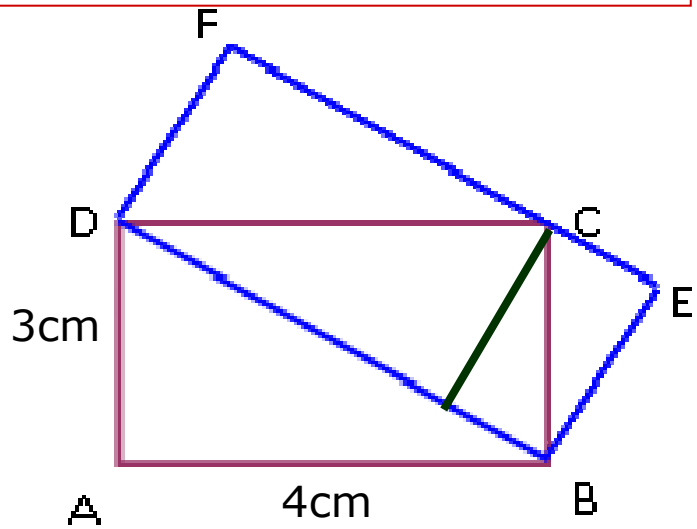
Two rectangles, ABCD and DBEF are shown in the diagram. The length of AD is 3 cm; the length of AB is 4 cm. What is the area of rectangle DBEF?

A) $10cm^2$ B) $12cm^2$ C) $13cm^2$ D) $14cm^2$ E) $16cm^2$

The area of the purple rectangle is $3cm \times 4cm = 12cm^2$

The portion (DCB) of this rectangle that is within the blue rectangle is half of this which is $6cm^2$

Now consider this imaginary green line in the blue rectangle.



Observing the four triangles now in the blue rectangle, we can tell that the triangle (DCB) from before is half of the blue rectangle.

This means the area of the blue rectangle is $2 \times 6cm^2 = 12cm^2$

Answer: B)

Example 15

If you read the date 21.02 (the 21st of February) from right to left, you get 20.12, and this is also some date (the 20th of December). The date 10.09 does not possess this property (there is no 90th of January). How many dates (in the year) possess this property?

A) 183 B) 182 C) 34 D) 35

E) depends on the year

Solution of Example 15

If you read the date 21.02 (the 21st of February) from right to left, you get 20.12, and this is also some date (the 20th of December). The date 10.09 does not possess this property (there is no 90th of January). How many dates (in the year) possess this property?

First, we observe the set of days of a month we should be interested in. Just those days which when flipped would produce a valid month. For example, 20.XY would become YX.02 which is permitted.

The good days are:
10.XY => YX.01
20.XY => YX.02
30.XY => YX.03
01.XY => YX.10
11.XY => YX.11
21.XY => YX.12

Next, we observe the months and ensure that we are attentive to special properties

<u>January</u> AB.01 => 10.BA
The 10th day is valid in every month and January can have all the days we saw on the left so there are 6 dates in January.

<u>February</u> AB.02 => 20.BA
The 20th day is valid in every month but February can have all the days we saw on the left except 30 so there are 5 dates in February.

<u>March</u> AB.03 => 30.BA
The 30th day is valid in every month except February so March can have all the days except the 20. There are 5 dates in March.

<u>November</u> AB.11 => 11.BA
The 11th day is valid in every month so there are 6 dates in November.

<u>October</u> AB.10 => 01.BA
The 1st day is valid in every month so there are 6 dates in October.

<u>December</u> AB.12 => 21.BA. The 21st day is valid in every month so there are 6 dates in December.

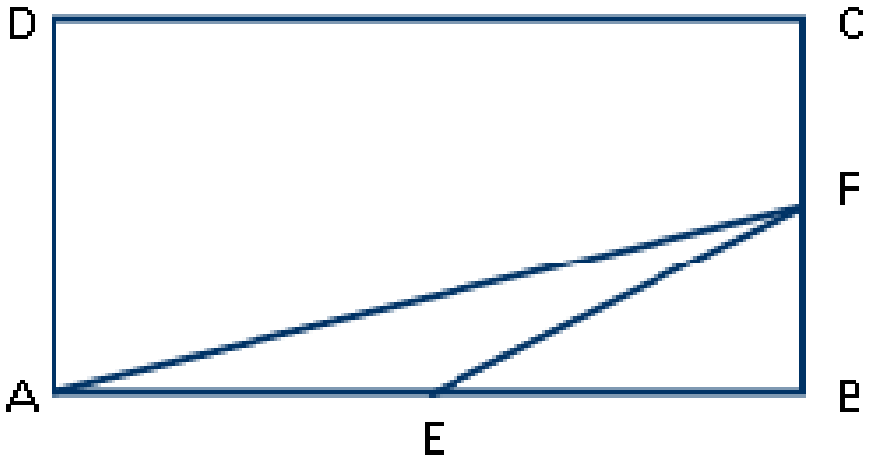
<u>April to September</u> are invalid because 04 to 09 when flipped give 40 to 90 and no months have more than 31 days.

In total, there are 6 + 5 + 5 + 6 + 6 + 6 = 34 dates.

Example 16

In the figure, ABCD is a rectangle; E is the midpoint of AB; F is the midpoint of BC. What is the ratio between the area of the rectangle ABCD and the area of the triangle AEF?

A) 4:1 B) 8:1 C) 16:1 D) 5:2 E) 3:2



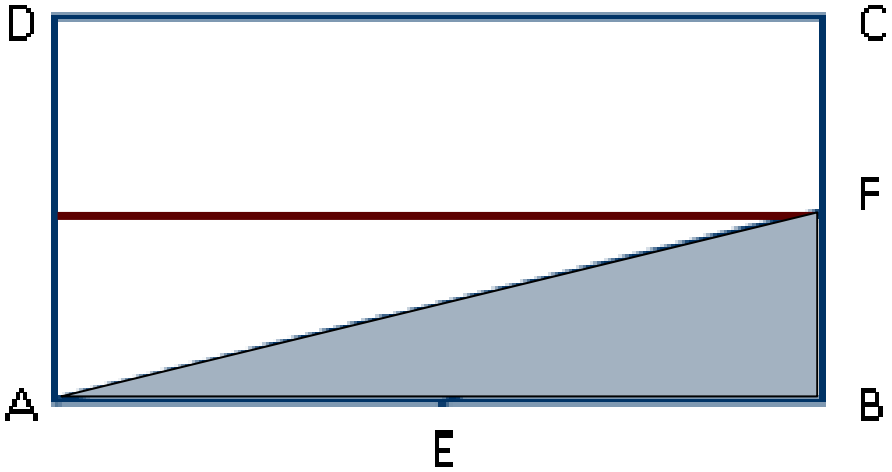
Solution of Example 16

In the figure, ABCD is a rectangle; E is the midpoint of AB; F is the midpoint of BC. What is the ratio between the area of the rectangle ABCD and the area of the triangle AEF?

First, let the area of the rectangle ABCD be 1.

Now we are going to add a few lines and make observations.

After we draw the brown line through F parallel to AB, we observe that the area of the triangle AFB is ¼ of the area of the rectangle ABCD. The shaded area is half of the area of the small rectangle which is half of the area of the big rectangle.



Solution of Example 16

After adding the green line, we observe that the area of the triangle EFB is 1/8 of the area of the big rectangle since it is 1/2 of the quarter of the rectangle.

The triangle AEF has an area that is the difference of the areas of triangles AFB and EFB, or, it is 1/4 – 1/8 = 1/8

So the ratio of the areas of the rectangle ABCD and the triangle AEF is 8:1.