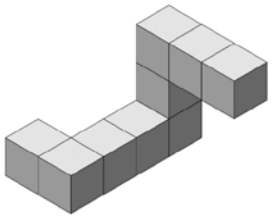
Canadian Math Kangaroo Contest

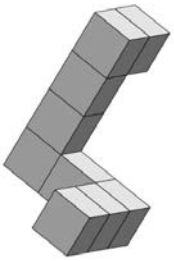
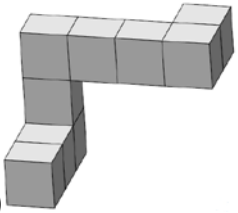
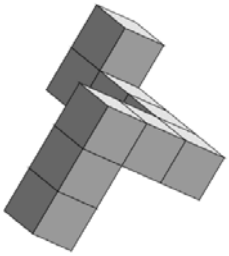
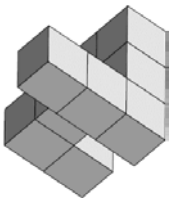
Part A: Each correct answer is worth 3 points

1. The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the age of the oldest one?

   (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

2. Anne the Kangaroo has glued some blocks together as shown on the right. She is rotating the construction in her paws to see it from different angles. Which of the following can she not see?

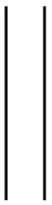


(A)  (B)  (C)  (D)  (E) 

3. Let a_n be a geometric progression with $a_{2015} = 2015!$ and $a_{2016} = 2016!$. What is the value of a_{2017}?

   (A) $2017!$ (B) $2016 \cdot 2016!$ (C) $2015!$ (D) 2017 (E) 2016

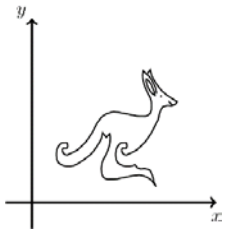
4. The Bear Construction Company is building a bridge across a river. The river has the interesting property that the shortest bridge across from any point on one bank to the other bank is always the same length. Which of these pictures cannot be the picture of the river?

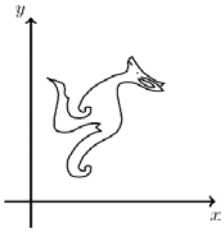
(A)  (B)  (C)  (D)  (E) 

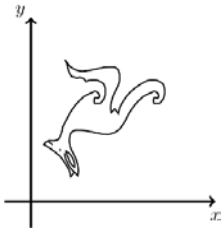
5. How many integers are greater than $2015 \cdot 2017$ and less than $2016 \cdot 2016$?

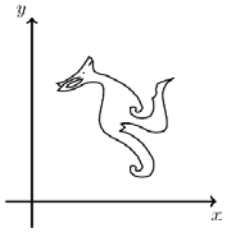
   (A) 0 (B) 1 (C) 2015 (D) 2016 (E) 2017

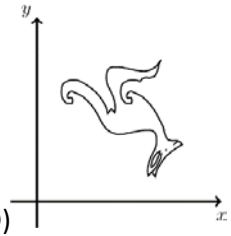
6. A set of points forms a picture of a kangaroo in the xy-plane as shown on the right. For each point the x and y coordinates are swapped. What is the result?

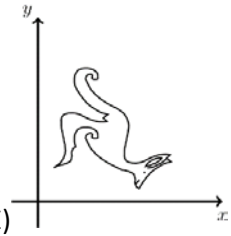


(A) 

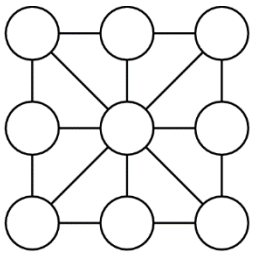
(B) 

(C) 

(D) 

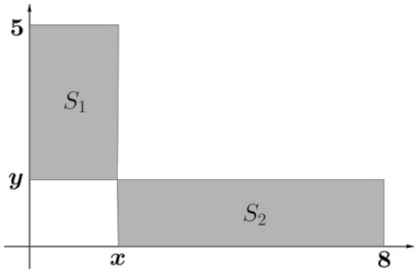
(E) 

7. Diana wants to write nine integers into the circles on the diagram so that, for the eight small triangles whose vertices are joined by segments the sums of the numbers in their vertices are identical. What is the greatest number of different integers she can use?



(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

8. The rectangles S_1 and S_2 in the picture have the same area. Determine the ratio $\frac{x}{y}$.



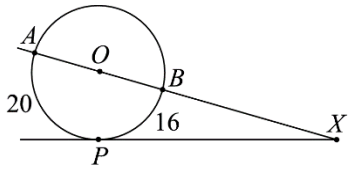
(A) 1 (B) $\frac{3}{2}$ (C) $\frac{4}{3}$

(D) $\frac{7}{4}$ (E) $\frac{8}{5}$

9. What is the value of $x + \frac{2}{x}$ if $x^2 - 4x + 2 = 0$?

(A) –4 (B) –2 (C) 0 (D) 2 (E) 4

10. The lengths of arc $\overset{\frown}{AP}$ and arc $\overset{\frown}{BP}$ are 20 and 16, respectively, as shown in the figure.



What is the size of the angle $\angle AXP$?

(A) 30^o (B) 24^o (C) 18^o (D) 15^o (E) 10^o

Part B: Each correct answer is worth 4 points

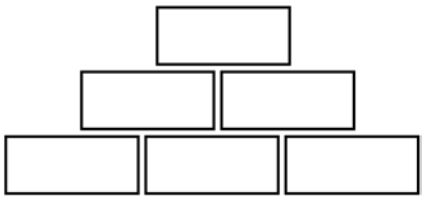
11. When a positive integer n is divided by 6, the remainder is 5. What is the remainder when n^2 is divided by 12?

(A) 1 (B) 4 (C) 6 (D) 13 (E) none of the previous

12. The four numbers a, b, c, d are positive integers satisfying: $a + 2 = b - 2 = c \cdot 2 = d \div 2$. Which of the four numbers is the greatest?

(A) a (B) b (C) c (D) d (E) This is not uniquely determined.

13. In this pyramid of numbers every block contains a number which is the product of the numbers on the two blocks directly underneath. Which of the following numbers cannot appear on the top block, if the three bottom blocks only contain integers greater than 1?



(A) 56 (B) 84 (C) 90 (D) 105 (E) 220

14. What is x_4, if $x_1 = 2$ and $x_{n+1} = {x_n}^{x_n}$ for $n \geq 1$?

(A) 2^{2^3} (B) 2^{2^4} (C) $2^{2^{11}}$ (D) $2^{2^{16}}$ (E) $2^{2^{768}}$

15. In rectangle $ABCD$ the length of the side $\overline{BC}$ is half the length of the diagonal $\overline{AC}$. Let M be a point on $\overline{CD}$ such that $|\overline{AM}| = |\overline{MC}|$. What is the size of the angle $\angle CAM$?

(A) $12.5°$ (B) $15°$ (C) $27.5°$ (D) $42.5°$ (E) some other angle

16. Diana cut up a rectangle of area 2016 into 56 equal squares. The lengths of the sides of the rectangle and of the squares are integers. For how many different rectangles could she do this cutting?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 0

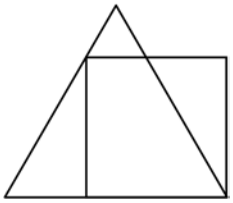
17. On the Island of Knights and Knaves every citizen is either a Knight (who always speaks the truth) or a Knave (who always lies). During your travels on the island you meet 7 people sitting around a bonfire. They all tell you "I'm sitting between two Knaves!" How many Knaves are there?

(A) 3 (B) 4 (C) 5 (D) 6
(E) You need more information to determine this.

18. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real roots. It is known that the sum of squares of the roots of the first equation is equal to the sum of squares of the roots of the second equation, and $a \neq b$. What is the value of $a + b$?

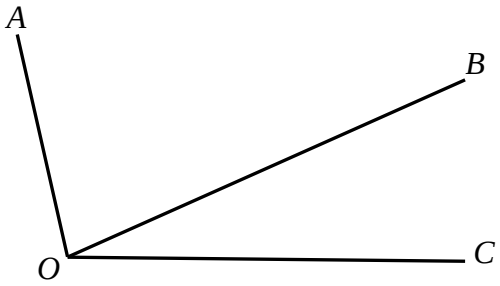
(A) 0 (B) – 2 (C) 4 (D) – 4 (E) It is impossible to determine.

19. The perimeter of the square in the figure equals 4. What is the perimeter of the equilateral triangle?



(A) 4 (B) $3 + \sqrt{3}$ (C) 3 (D) $3 + \sqrt{2}$ (E) $4 + \sqrt{3}$

20. If the difference between $\angle BOA$ and $\angle COB$ angles is 30°, what is the value of the angle between the bisectrix of $\angle COA$ and the $\overline{OB}$ segment?



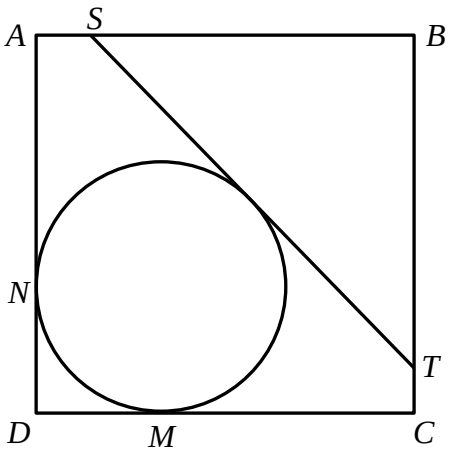
(A) 30° (B) 25° (C) 20°
(D) 15° (E) 10°

Part C: Each correct answer is worth 5 points

21. How many different solutions are there to the equation $(x^2 - 4x + 5)^{x^2+x-30} = 1$?

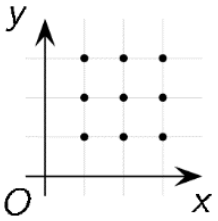
(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

22. In the picture, the circle touches two sides of square $ABCD$ at points M and N. Points S and T lie on the sides of the square so that $|\overline{AS}| = |\overline{CT}|$ and $\overline{ST}$ is tangent to the circle. If the diameter of the circle is 2 and so is $\overline{MC}$, what is the length of $\overline{ST}$?



(A) $\sqrt{8}$ (B) $4\sqrt{2} - 2$ (C) $2\sqrt{3}$
(D) 3 (E) $\sqrt{6} + 1$

23. How many quadratic functions of x have a graph passing through at least three of the marked points?



(A) 6 (B) 18 (C) 20 (D) 22 (E) 27

24. In a right-angled triangle ABC (right angle at A) the bisectors of the acute angles intersect at point P. If the distance from P to the hypotenuse is $\sqrt{8}$, what is the distance from P to A?

(A) 8 (B) 3 (C) $\sqrt{10}$ (D) $\sqrt{12}$ (E) 4

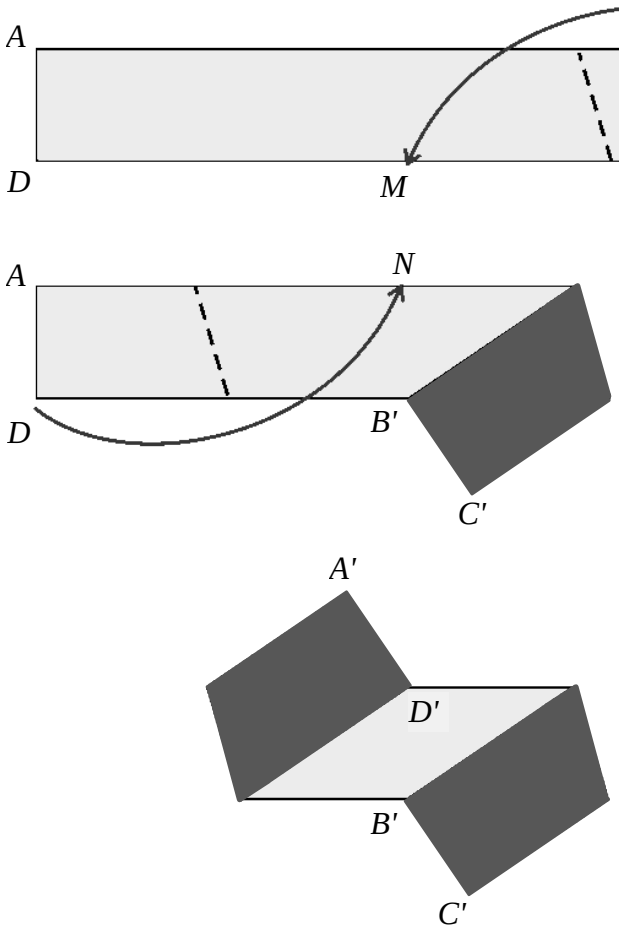
25. Three three-digit numbers are formed from the digits from 1 to 9 (each digit is used exactly once). Which of the following numbers couldn't be equal to the sum of these three numbers?

(A) 1500 (B) 1503 (C) 1512 (D) 1521 (E) 1575

26. A cube is dissected into six pyramids by connecting a given point in the interior of the cube with each vertex of the cube. The volumes of five of these pyramids are 2, 5, 10, 11 and 14. What is the volume of the sixth pyramid?

(A) 1 (B) 4 (C) 6 (D) 9 (E) 12

27. A rectangular strip $ABCD$ of paper, 5 cm wide and 50 cm long, is light grey on one side and dark grey on the other side. Folding the strip, Cristina makes the vertex B coincide with the midpoint M of the side $\overline{CD}$. Folding again, she makes the vertex D coincide with the midpoint N of the side $\overline{AB}$.



What is the area, in cm^2, of the visible light grey part of the folded strip in the picture?

(A) 50 (B) 60 (C) 62.5 (D) 100 (E) 125

28. Ann chose a positive integer n and wrote down the sum of all positive integers from 1 to n. A prime number p divides the sum, but not any of the summands. Which of the following could be $n + p$?

(A) 217 (B) 221 (C) 229 (D) 245 (E) 269

29. We have boxes numbered as 1, 2, 3, … We put a ball with the number 1 into the box number 1. We put two balls numbered 2 and 3 into the box number 2. We put three balls numbered 4, 5 and 6 into the box number 3. And so on. What is the box number containing ball 2016?

(A) 50 (B) 53 (C) 60 (D) 63 (E) 70

30. The positive integer N has exactly six distinct (positive) divisors including 1 and N. The product of five of these divisors is 648. Which of the following numbers is the sixth divisor of N?

(A) 4 (B) 8 (C) 9 (D) 12 (E) 24