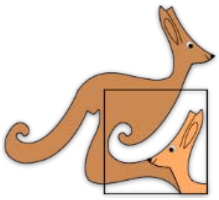
For training purposes only!



INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2018

INSTRUCTIONS
GRADE 5-12

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.
2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.
3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11 -20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.
4. The use of external material or aid of any kind is **not permitted**.
5. The figures *are not* drawn to scale. They should be used only for illustration purposes.
6. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.
7. At the end of the allotted time, please **give the response form to the contest supervisor**.
8. Do not forget to pick up your Certificate of Participation on your way out!

Good luck!

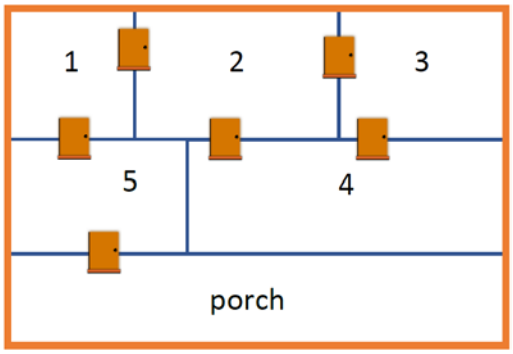
Canadian Math Kangaroo Contest team

Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

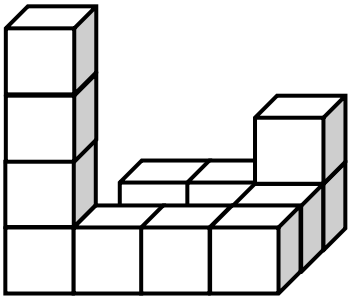
1. Which of the following numerical expressions has the highest value?
(A) $2-0\times1+8$ (B) $2+0\times1\times8$ (C) $2\times0+1\times8$
(D) $2\times(0+1+8)$ (E) $2\times0+1+8$

2. The figure shows the floor plan of Renate's house. Renate enters her house from the porch and walks through each door exactly once. In which room does she end up?
(A) 1 (B) 2 (C) 3
(D) 4 (E) 5



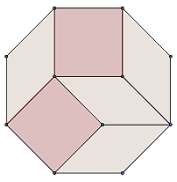
3. Thor has seven stones and a hammer. Every time he hits a stone with the hammer it breaks into exactly five smaller stones. He does this several times. Which of the following numbers could be the number of stones he may end with?
(A) 17 (B) 20 (C) 21
(D) 23 (E) 25

4. The shape shown is made of 12 cubes glued together.
The shape is dipped into a bucket of paint covering the surface entirely.
How many of the cubes will be painted on exactly four of their faces?
(A) 11 (B) 7 (C) 8 (D) 9 (E) 10



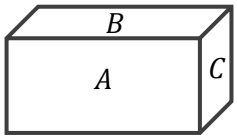
5. The following two statements are true: Some aliens are green, the others are purple. Green aliens live only on Mars. Therefore, it logically follows that:
(A) all aliens live on Mars (B) only green aliens live on Mars.
(C) some purple aliens live on Venus. (D) all purple aliens live on Venus.
(E) no green aliens live on Venus.

6. Four identical rhombuses and two squares are put together to make a regular octagon.
What is the measure of the larger angle of each rhombus?
(A) 135° (B) 140° (C) 144° (D) 145° (E) 150°



7. There are 65 balls in a box. 8 are white and the rest of the balls are black. In one move, at most 5 balls can be taken out of the box. It is not allowed to put any balls back in the box. What is the smallest number of moves needed to ensure that at least one white ball is taken out?
(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

8. The faces of a rectangular brick have areas A, B and C as shown. What is the volume of the brick?

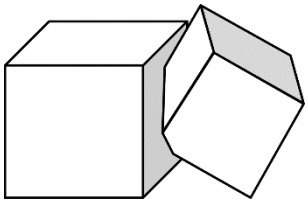


(A) ABC (B) $\sqrt{ABC}$ (C) $\sqrt{AB+BC+CA}$
(D) $\sqrt[3]{ABC}$ (E) $2(A+B+C)$

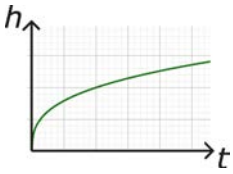
9. In how many ways can the number 1001 be written as the sum of two primes?
(A) none (B) one (C) two (D) three (E) more than three

10. If a and b are digits and $\frac{a}{b} = b.a$ ($b.a$ is a decimal), find $a+b$.
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

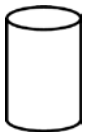
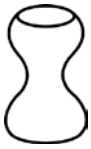
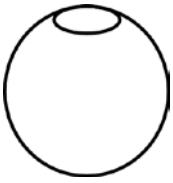
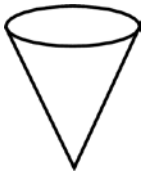
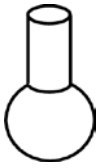
Part B: Each correct answer is worth 4 points



11. Two cubes of volumes V and W intersect. The part of the cube of volume V which is not common to the two cubes is 90% of its volume. The part of the cube of volume W which is not common to the two cubes is 85% of its volume. What is the relationship between V and W?
(A) $V = \frac{2}{3}W$ (B) $V = \frac{3}{2}W$ (C) $V = \frac{85}{90}W$ (D) $V = \frac{90}{85}W$ (E) $V = W$



12. A vase is filled up to the top with water, at a constant rate. The graph shows the height h of the water as a function of time t.
Which of the following could be the shape of the vase?

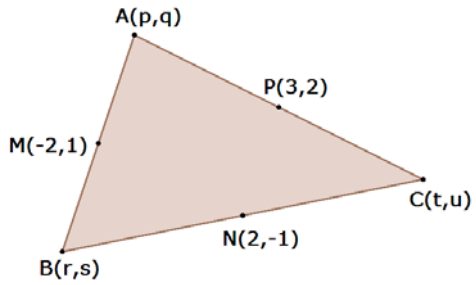
(A) 
(B) 
(C) 
(D) 
(E) 

13. Three of the five cards shown are given to Nadia and the rest to Riny. Nadia multiplies the 3 values of her cards and Riny multiplies the 2 values of his cards. It turns out that the sum of the two resulting products is prime. What is the sum of the values of Nadia's cards?
(A) 12 (B) 13 (C) 15 (D) 17 (E) 18

14. What is the value of $|\sqrt{17}-5| + |\sqrt{17}+5|$?
(A) 10 (B) $2\sqrt{17}$ (C) $\sqrt{34}-10$ (D) $10-\sqrt{34}$ (E) 0

15. An arc is one-sixth of the circumference of a circle whose area is 144π. The length of this arc is
(A) 3π (B) 4π (C) 2π (D) 6π (E) 12π

16. The vertices of a triangle are $A(p,q)$, $B(r,s)$ and $C(t,u)$ as shown. The midpoints of the sides of the triangle are the points $M(-2,1)$, $N(2,-1)$ and $P(3,2)$. What is the value of $p+q+r+s+t+u$?



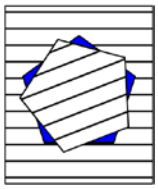
(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 5 (E) none of these

17. Five predictions were made before the football match between Real Madrid and Manchester United:
    1. The game will not end in a draw;
    2. Real Madrid will score;
    3. Real Madrid will win;
    4. Real Madrid will not lose;
    5. Three goals will be scored.

What was the final score of the match Real Madrid - Manchester United if exactly three of the predictions came true?

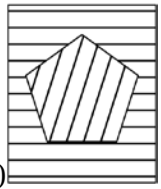
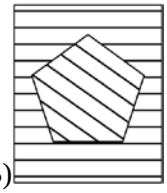
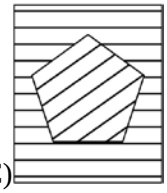
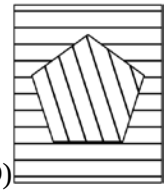
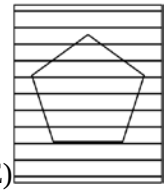
(A) 3-0 (B) 2-1 (C) 0-3 (D) 1-2 (E) this situation is not possible

18. We cut out a regular pentagon from a lined piece of paper. At each step we rotate the pentagon counter clockwise about its centre through 21°.



The situation after the first step is shown.

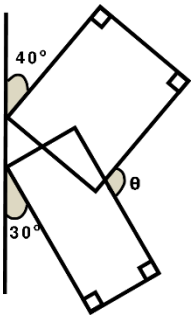
What will we see when the pentagon first fits back in the hole?

(A)  (B)  (C)  (D)  (E) 

19. Which of these five numbers does not divide $18^{2017}+18^{2018}$?

(A) 8 (B) 18 (C) 28 (D) 38 (E) 48

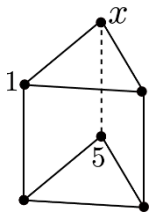
20. Two rectangles are inclined to the vertical line at angles 40° and 30° as shown. What is the measure of the angle θ?



(A) 105° (B) 120° (C) 130°
(D) 135° (E) None of these

Part C: Each correct answer is worth 5 points

21. The prism in the picture is formed of two triangles and three squares. The six vertices are numbered from 1 to 6 in such a way that the sum of the four vertices of each square is the same for all three squares. Numbers 1 and 5 are already shown. What number is at the vertex labelled x?



(A) 2 (B) 3 (C) 4 (D) 6 (E) the situation is impossible

22. m and n are the roots of the equation $x^2 - x - 2018 = 0$. What is the value of $n^2 + m$?
(A) 2016 (B) 2017 (C) 2018 (D) 2019 (E) 2020

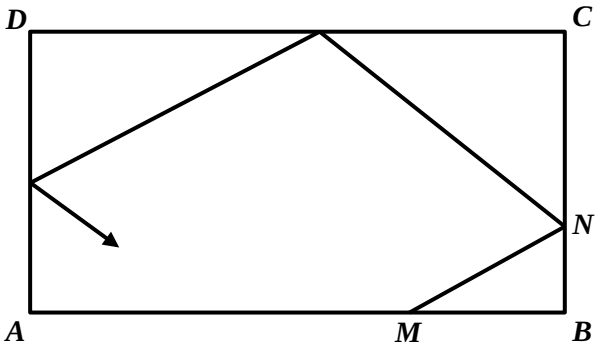
23. Four brothers named A, B, C and D have different heights. They state the following:
- A: I am neither the tallest nor the shortest.
- B: I am not the shortest.
- C: I am the tallest.
- D: I am the shortest.

Exactly one of them is lying. Who is the tallest?
(A) A (B) B (C) C (D) D (E) We do not have enough information

24. Let f be a function such that $f(x + y) = f(x)f(y)$ for all integers x and y. If $f(1) = 1/2$, find the value of $f(0) + f(1) + f(2) + f(3)$.
(A) $\frac{1}{8}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{15}{8}$ (E) 6

25. A quadratic function $f(x) = x^2 + px + q$ is such that its graph intersects the x-axis and the y-axis in three different points. The circle through these three points intersects the graph of f in a fourth point. What are the coordinates of this fourth point?
(A) $(0, -q)$ (B) (p, q) (C) $(-p, q)$ (D) $(-\frac{q}{p}, \frac{q^2}{p^2})$ (E) $(1, p + q + 1)$

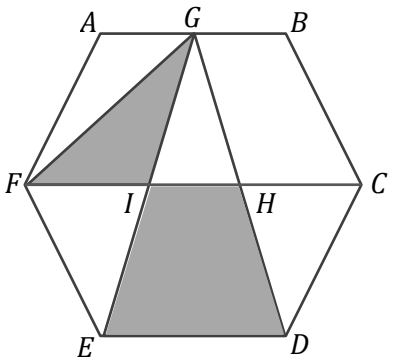
26. We are given a rectangular billiard table with sides of length 3 m and 2 m. A ball is shot from the point M on one of the longer sides. It reflects once on every other side as shown. At what distance from point A will it hit the initial side if $BM = 1.2$ m and $BN = 0.8$ m?
(A) 1.2 m (B) 1.5 m (C) 2 m
(D) 2.8 m (E) 1.8 m



27. How many real solutions does the equation $||4^x - 3| - 2| = 1$ have?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

28. $ABCDEF$ is a regular hexagon. G is the midpoint of AB. H and I are the points of intersection of the segments GD and GE with FC respectively. What is the ratio between the area of the triangle GIF and the area of the trapezoid $IHDE$?



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{3}}{4}$

29. There are 40% more girls than boys in a class. How many pupils are in this class if the probability that a two-person delegation selected at random consists of a girl and a boy equals $\frac{1}{2}$?

(A) 20 (B) 24 (C) 36 (D) 38 (E) This situation is not possible.

30. Archimedes calculated 15!. The result is written on the board. Unfortunately, two of the digits, the second and the tenth, are not visible. Which are these two digits?

$$1\blacksquare 0\,7\,6\,7\,4\,3\,6\blacksquare 0\,0\,0$$

(A) 2 and 0 (B) 4 and 8 (C) 7 and 4 (D) 9 and 2 (E) 3 and 8