Canadian Math Kangaroo Contest

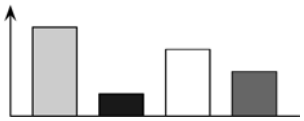
Part A: Each correct answer is worth 3 points

1. Andrea was born in 1997, her younger sister Charlotte in 2001.
The age difference of the two sisters is therefore in any case
(A) less than 4 years (B) at least 4 years (C) exactly 4 years
(D) more than 4 years (E) not less than 3 years

2. How many perfect square numbers greater than 0 and less than 2015 are also perfect cubes?
(A) 2 (B) 3 (C) 4 (D) 6 (E) 12

3. How many solutions does the equation $2^{2x} = 4^{x+1}$ have?
(A) 0 (B) Infinitely many (C) 2 (D) 1 (E) 3

4. Diana drew a bar chart representing the quantity of the four tree species registered during a biology excursion. Jasper thinks that a pie chart would better represent the ratios of the different tree species. What does the corresponding pie chart look like?

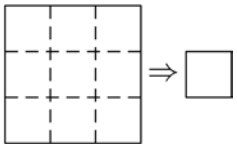


(A)  (B)  (C)  (D)  (E) 

5. The opposite vertices of a cube are at $A(1,1,1)$ and $G(3,4,5)$ in the Cartesian plane.
Find the edge length of the cube.
(A) 3 (B) $\sqrt{\frac{29}{3}}$ (C) 4 (D) $\sqrt{\frac{50}{3}}$ (E) 5

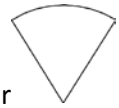
6. We add the 31 integers from 2001 to 2031 and divide the sum by 31. What result do we get?
(A) 2012 (B) 2013 (C) 2015 (D) 2016 (E) 2496

7. A square piece of paper is folded along the dashed lines one after the other in any order or direction. One corner is cut off of the obtained small square. The resulting piece of paper is then unfolded. How many holes are there in the interior of the unfolded piece of paper?



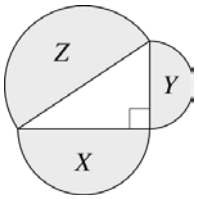
(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

8. A drinking glass has the shape of a truncated cone (see figure). The outside of the glass (without the base) should now be covered with coloured paper. What shape does the paper need to be in order to cover the whole glass completely without overlaps?

(A) rectangle  (B) trapezoid  (C) circle sector 

(D) parallel strip  (E) part of a circle sector 

9. Three semicircles have diameters which are the sides of a right-angled triangle. Their areas are X cm², Y cm² and Z cm², as shown. Which of the following is necessarily true?



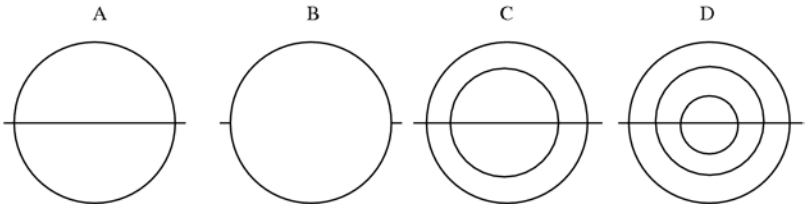
(A) $X + Y < Z$ (B) $\sqrt{X} + \sqrt{Y} = \sqrt{Z}$ (C) $X + Y = Z$
(D) $X^2 + Y^2 = Z^2$ (E) $X^2 + Y^2 = Z$

10. Which of the following is the complete list of the number of acute angles a convex quadrilateral can have?
(A) 0, 1, 2 (B) 0, 1, 2, 3 (C) 0, 1, 2, 3, 4 (D) 0, 1, 3 (E) 1, 2, 3

Part B: Each correct answer is worth 4 points

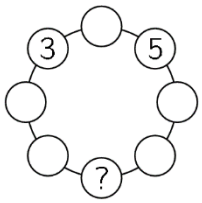
11. Into how many regions do the x-axis and the graphs of the functions $f(x) = 2 - x^2$ and $g(x) = x^2 - 1$ divide the Cartesian plane?
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

12. How many of the following figures can be drawn with one continuous line without drawing a segment twice?



(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

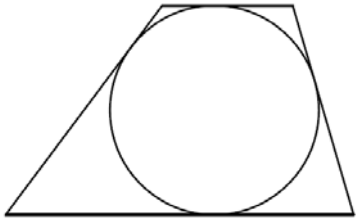
13. Ella wants to write a number in each circle in the picture such that each number is the sum of its two neighbours. Which number must Ella write in the circle with the question mark?



(A) -5 (B) -16 (C) -8 (D) -3 (E) This is impossible.

14. A circle has radius 7cm? Of all squares with integer side lengths that fit entirely inside this circle, what is the side length of the largest one?
(A) 6cm (B) 7cm (C) 8cm (D) 9cm (E) 10cm

15. The trapezoid in the figure has a circle inscribed into it (so that the trapezoid's sides are tangent to the circle). The sum of the lengths of the two bases (parallel sides) is 29cm, and the difference of the remaining two sides' lengths is 5cm. How many such trapezoids exist that have integer side lengths? (A trapezoid that is a symmetrical image of another one is considered the same.)

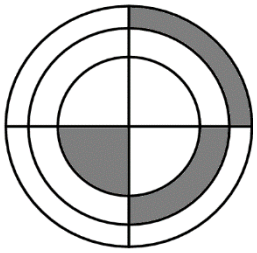


(A) 1 (B) 2 (C) 4 (D) a number greater than 5 (E) infinitely many

16. The geometric mean of a set of n positive numbers is defined as the n-th root of the product of those numbers. The geometric mean of a set of three numbers is 3 and the geometric mean of another set of three numbers is 12. What is the geometric mean of the combined set of six numbers?

(A) 4 (B) 6 (C) $\frac{15}{2}$ (D) $\frac{15}{6}$ (E) 36

17. In the figure shown there are three concentric circles and two perpendicular diameters. If the three shaded parts of the figure have equal area and the radius of the small circle is equal to one, what is the product of the three radii?



(A) $\sqrt{6}$ (B) 3 (C) $\frac{3\sqrt{3}}{2}$ (D) $2\sqrt{2}$ (E) 6

18. An automobile dealer bought two cars. He sold the first one for 40% more than he paid for it and the second one for 60% more than he paid for it. The money he received for the two cars was 54% more than what he paid for both. The ratio of the prices the dealer paid for the first and the second car was:

(A) 10:13 (B) 20:27 (C) 3:7 (D) 7:12 (E) 2:3

19. Billy has a die with the numbers 1, 2, 3, 4, 5 and 6 on its six faces. Tina has a die which is special: it has the numbers 2, 2, 2, 5, 5 and 5 on its six faces. When Billy and Tina roll their dice the one with the larger number wins. If the two numbers are equal it is a draw. What is the probability that Tina wins?

(A) $\frac{1}{3}$ (B) $\frac{7}{18}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$ (E) $\frac{11}{18}$

20. There are 2015 marbles in a box. The marbles are numbered from 1 to 2015. Marbles with equal digit sums have the same colour and marbles with different digit sums have different colours. How many different colours of marbles are there in the box?

(A) 10 (B) 27 (C) 28 (D) 29 (E) 2015

Part C: Each correct answer is worth 5 points

21. The following is the multiplication table of the numbers 1 to 10.

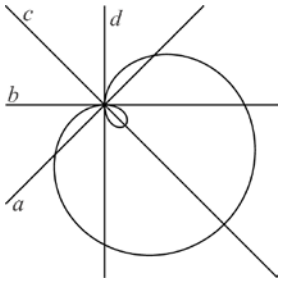
x	1	2	3	...	10
1	1	2	3	...	10
2	2	4	6	...	20
3	3	6	9	...	30
...	...	...	...	...	...
10	10	20	30	...	100

What is the sum of all 100 products in the complete table?

(A) 1000 (B) 2025 (C) 2500 (D) 3025 (E) 5500

22. The curve in the figure is described by the equation

$$(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2)$$



Which of the lines a, b, c, d represents the y-axis?

(A) a (B) b (C) c (D) d

(E) none of these

23. In each of the cells of a square $n \times n$ ($n > 3$) table a number is written so that the sums along all rows and columns of the table are the same. Furthermore, not all numbers are equal. Find the largest possible number of equal values in the table's cells.

(A) $(n-1)^2$ (B) $n(n-1)$ (C) $n^2 - 4$ (D) $n^2 - 3$ (E) $n^2 - 1$

24. How many regular polygons exist such that their angles (in degrees) are integers?

(A) 17 (B) 18 (C) 22 (D) 25 (E) 60

25. How many 3-digit positive integers can be represented as the sum of exactly nine different powers of 2?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. How many triangles ABC with $\angle ABC = 90°$ and $AB = 20$ exist such that all sides have integer lengths?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

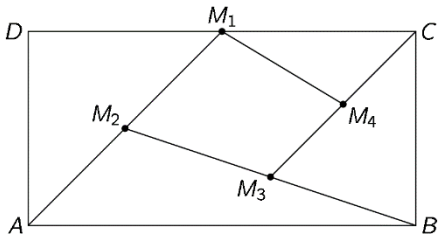
27. In the word KANGAROO Bill and Bob replace the letters by digits, so that the resulting numbers are multiples of 11. They each replace different letters by different digits and the same letters by the same digits ($K \neq 0$). Bill obtains the largest possible such number and Bob the smallest.
In both cases one of the letters is replaced by the same digit. Which digit is this?

(A) 0 (B) 3 (C) 4 (D) 5 (E) 6

28. A piece of paper has the shape of an equilateral triangle. What is the minimum number of straight lines parallel to its sides that divide the triangle into at least 100 regions?

(A) 16 (B) 25 (C) 36 (D) 81 (E) 100

29. In the rectangle $ABCD$ shown in the figure, M_1 is the midpoint of DC, M_2 is the midpoint of AM_1, M_3 is the midpoint of BM_2 and M_4 is the midpoint of CM_3. Find the ratio between the areas of the quadrilateral $M_1M_2M_3M_4$ and of the rectangle $ABCD$.



(A) $\frac{7}{16}$ (B) $\frac{3}{16}$ (C) $\frac{7}{32}$ (D) $\frac{9}{32}$ (E) $\frac{1}{5}$

30. Blue and red rectangles are drawn on a blackboard. Exactly 7 of the rectangles are squares. There are 3 red rectangles more than blue squares. There are 2 red squares more than blue rectangles. How many blue rectangles are there on the blackboard?

(A) 1 (B) 3 (C) 5 (D) 6 (E) 10