# For training purposes only!

**INTERNATIONAL CONTEST-GAME**
**MATH KANGAROO**
**CANADA, 2019**

**INSTRUCTIONS**
**GRADE 11-12**

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.
2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.
3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.
4. The use of external material or aid of any kind is **not permitted**.
5. The figures *are not* drawn to scale. They should be used only for illustration purposes.
6. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.
7. At the end of the allotted time, please **give the response form to the contest supervisor**.
8. Do not forget to pick up your Certificate of Participation on your way out!

**Good luck!**

*Canadian Math Kangaroo Contest team*

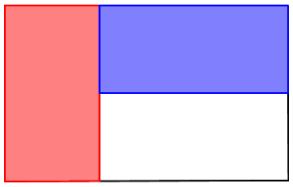
www.mathkangaroocanada.com

**Grade 11-12** **2019**

# Canadian Math Kangaroo Contest

## Part A: Each correct answer is worth 3 points

**1.** The flag of Kangaroland is a rectangle which is divided into three smaller equal rectangles as shown. What is the ratio of the side lengths of the white rectangle?

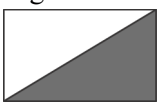
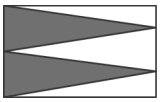
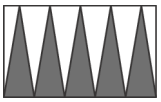
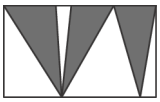


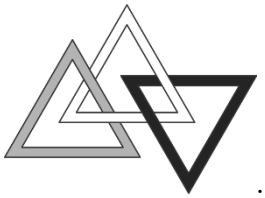
(A) 1:2 (B) 2:3 (C) 2:5 (D) 3:7 (E) 4:9

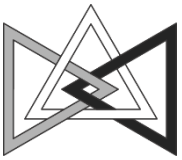
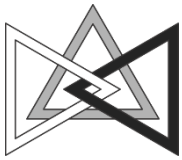
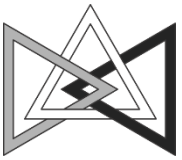
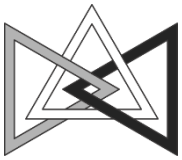
**2.** The numbers 1, 2, 3 and 4 are each written in different cells of the 2 × 2 table. After that, the sum of the numbers in each row and column is calculated. Two of these sums are 4 and 5. What are the other two sums?

(A) 6 and 6 (B) 3 and 5 (C) 4 and 5 (D) 4 and 6 (E) 5 and 6

**3.** A rectangle has been shaded in five different ways as shown. In which diagram does the shaded part have the largest area?

(A)  (B)  (C)  (D)  (E) 

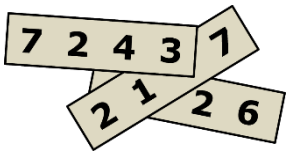
**4.** Three triangles are linked as shown.  . Which of the following pictures shows these three triangles linked in the same way?

(A)  (B)  (C)  (D)  (E) 

**5.** A pyramid has 23 triangular faces. How many edges does this pyramid have?

(A) 23 (B) 24 (C) 46 (D) 48 (E) 69

**6.** Three 4-digit numbers are written on three pieces of paper as shown.




The sum of the three numbers is 11126. Three of the digits are covered. What are the covered digits?

(A) 1, 4 and 7 (B) 1, 5 and 7 (C) 3, 3 and 3 (D) 4, 5 and 6 (E) 4, 5 and 7

**7.** What is the first (leftmost) digit of the smallest positive integer whose digits add up to 2019?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

**8.** Each of the faces of a die is marked with either 1, 2 or 3 dots so that the probability of rolling a 1 is $\frac{1}{2}$, the probability of rolling a 2 is $\frac{1}{3}$ and the probability of rolling a 3 is $\frac{1}{6}$. Which of the following cannot be a view of this die?

(A)  (B)  (C)  (D)  (E) 

**9.** Michael invented a new $\Diamond$ operation for real numbers, defined as $x \Diamond y = y - x$. If $a$, $b$, and $c$ satisfy $(a \Diamond b) \Diamond c = a \Diamond (b \Diamond c)$, which of the following statements is necessarily true?

(A) $a = b$ (B) $b = c$ (C) $a = c$ (D) $a = 0$ (E) $c = 0$

**10.** How many of the numbers from $2^{10}$ to $2^{13}$, inclusive, are divisible by $2^{10}$?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 16

## Part B: Each correct answer is worth 4 points

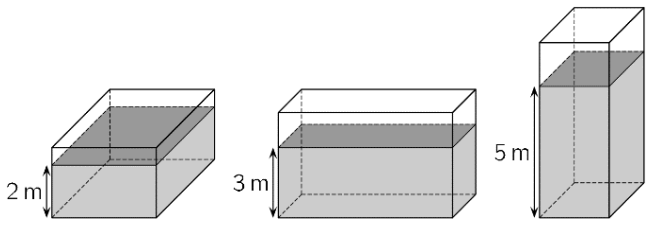
**11.** Which is the highest power of 3 dividing the number $7! + 8! + 9!$ ?

(A) $3^2$ (B) $3^4$ (C) $3^5$ (D) $3^6$ (E) a power of 3 higher than $3^6$

**12.** This year, the number of boys in my class has increased by 20% and the number of girls has decreased by 20%. We now have one student more than before. Which of the following could be the number of students in my class now?

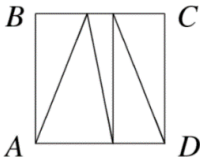
(A) 22 (B) 26 (C) 29 (D) 31 (E) 34

**13.** A container in the shape of a rectangular box is partially filled with 120 $m^3$ of water. The depth of the water is either 2 m or 3 m or 5 m, depending on which side of the box is on the ground, as shown (not to scale).
What is the volume of the container?




(A) 160 $m^3$ (B) 180 $m^3$ (C) 200 $m^3$ (D) 220 $m^3$ (E) 240 $m^3$

**14.** In the square $ABCD$ of side 1 we join $A$ and $D$ by a zigzag line touching the side $BC$ twice and $AD$ once. What is the shortest path length?

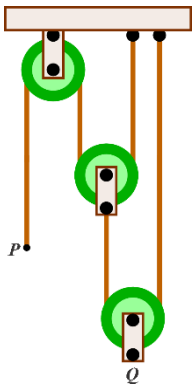



(A) $3\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3 + \sqrt{2}$ (D) $2 + 2\sqrt{3}$ (E) $\sqrt{17}$

**15.** A positive integer $n$ is called "good" if its largest divisor (excluding $n$) is equal to $n - 6$.
How many good positive integers are there?

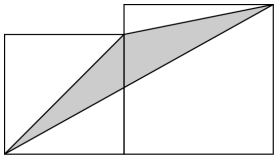
(A) 1 (B) 2 (C) 3 (D) 6 (E) infinitely many

**16.** The system shown consists of three pulleys with vertical sections of rope between them. The end P is moved down 24 centimeters. How many centimeters does point Q move up?




(A) 24 (B) 12 (C) 8 (D) 6 (E) 24/5

**17.** Two adjacent squares with side lengths $a$ and $b$ ($a < b$) are shown.



What is the area of the shaded triangle?

(A) $\sqrt{ab}$ (B) $\frac{1}{2}a^2$ (C) $\frac{1}{2}b^2$ (D) $\frac{1}{4}(a^2+b^2)$ (E) $\frac{1}{2}(a^2+b^2)$

**18.** What is the integer part of $\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20}}}}}$?

(A) 4 (B) 5 (C) 6 (D) 20 (E) 25

**19.** A box contains 5 treats (4 fruit chews and 1 chocolate) of the same shape and size. John and Mary take turns drawing a treat out of the box without replacement. Whoever draws the chocolate wins. John draws first. What is the probability that Mary wins?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) $\frac{1}{3}$

**20.** To calculate the result of $\frac{a+b}{c}$, Sara types $a + b \div c =$ on a calculator and the result is 11 ($a$, $b$, and $c$ are positive integers). She then types $b + a \div c =$ and she is surprised to see that the result is 14. She realizes that the calculator is designed to calculate divisions before additions.

What is the correct result of $\frac{a+b}{c}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

## Part C: Each correct answer is worth 5 points

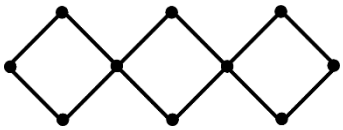
**21.** The number $2a$ has exactly 10 divisors, and the number $3a$ has exactly 6 divisors. Find the sum of the first and last digits of the number $2019a$.

(A) 8 (B) 10 (C) 12 (D) 13 (E) Impossible to determine

**22.** Find all values of the parameter $a$ for which the equation $2 - |x| = ax$ has exactly two distinct roots.

(A) $-\infty < a \leq -1$ (B) $-1 < a < 1$ (C) $1 \leq a < +\infty$ (D) $a = 0$ (E) $a = \pm 1$

**23.** The vertices of the network shown are labelled with the numbers from 1 to 10. The sum $S$ of the four labels on each square is the same. What is the least possible value of $S$?



(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

**24.** Let ABC be an equilateral triangle and $K$, $L$ be interior points of the side $BC$ with $|BK| = |KL| = |LC|$. What is the ratio of diameters of circumcircles of the triangle $AKL$ and $ABC$ equal to?

(A) 2:3 (B) 7:9 (C) 5:6 (D) 8:9 (E) Another answer

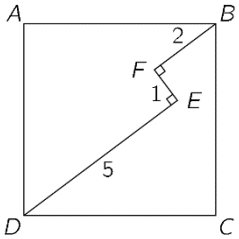
**25.** Four distinct straight lines pass through the origin of the coordinate system. They intersect the parabola $y = x^2 - 2$ at eight points. What can be the product of the $x$-coordinates of these eight points?

(A) 16 (B) –16 (C) 8 (D) –8 (E) There are several possible products.

**26.** If $a < b < c$ are consecutive primes and the equation $ax^2 + cx + b = 0$ has an integer root, how many are the possible values of $b$?

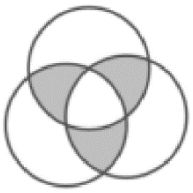
(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

**27.** A path $DEFB$ with $DE \perp EF$ and $EF \perp FB$ lies inside the square $ABCD$ as shown. Given that $|DE| = 5$, $|EF| = 1$ and $|FB| = 2$, what is the length of the side of the square?




(A) $3\sqrt{2}$ (B) $\frac{7\sqrt{2}}{2}$ (C) $\frac{11}{2}$ (D) $5\sqrt{2}$ (E) none of the previous

**28.** Three congruent circles of radius 2 cm intersect with their centres at intersection points as shown.



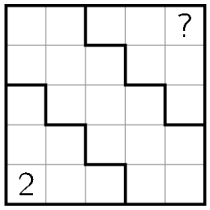
What is the area of the shaded region? Express your answer in terms of π.

(A) π (B) 3π (C) π/2 (D) 2π (E) 4π

**29.** Three different numbers are chosen at random from the set {1, 2, 3,…,10}. What is the probability that one of them is the average of the other two?

(A) $\frac{1}{10}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

**30.** The square shown is filled with numbers in such a way that each row and each column contains the numbers 1, 2, 3, 4 and 5 exactly once. Moreover, the sum of the numbers in each of the three bold-bordered areas is equal. What number is in the upper right corner?




(A) 1 (B) 2 (C) 3 (D) 4 (E) 5