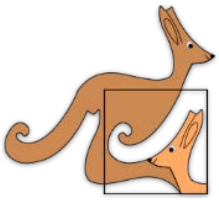
For training purposes only!



INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2019

INSTRUCTIONS
GRADE 9-10

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.
2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.
3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.
4. The use of external material or aid of any kind is **not permitted**.
5. The figures *are not* drawn to scale. They should be used only for illustration purposes.
6. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.
7. At the end of the allotted time, please **give the response form to the contest supervisor**.
8. Do not forget to pick up your Certificate of Participation on your way out!

Good luck!

Canadian Math Kangaroo Contest team

Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. What is the value of $20 \times 19 + 20 + 19$?

(A) 389 (B) 399 (C) 409 (D) 419 (E) 429

2. A model train takes exactly 1 minute and 11 seconds for each round on a course. How long does it take for six rounds?

(A) 6 minutes 56 seconds (B) 7 minutes 6 seconds (C) 7 minutes 16 seconds
(D) 7 minutes 26 seconds (E) 7 minutes 36 seconds

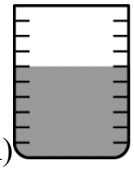
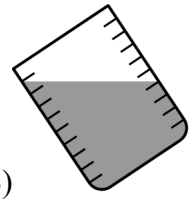
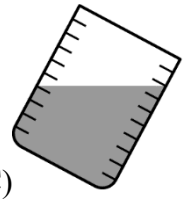
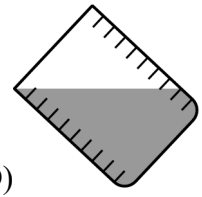
3. A barber wants to write the word SHAVE on a board in such a way that a client looking in to the mirror reads the word correctly. How should the barber write it on the board?

(A) SHAVE (B) SHAVƎ (C) ƎVAHS (D) EVAHS (E) ƎVAHS

4. How many different sums of dots can you get by rolling three standard dice simultaneously?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

5. Five identical glasses are partially filled with water. Four of them contain the same amount of water. Which one contains a different amount?

(A)  (B)  (C)  (D)  (E) 

6. A park has five gates. Monica wants to enter through one gate and to exit through a different one. In how many ways can she enter and exit the park?

(A) 25 (B) 20 (C) 16 (D) 15 (E) 10

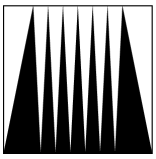
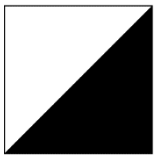
7. The weight of each of three kangaroos is a different whole number. The total weight of them is 97 kg. How much can the lightest of them weigh at most?

(A) 1 kg (B) 30 kg (C) 31 kg (D) 32 kg (E) 33 kg

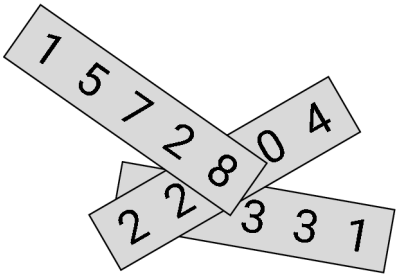
8. Tina's family consists of her mother, her father and her brother in addition to Tina herself. Tina added the age of everyone in the family and the sum was 88. How many years does it take until the sum of their ages is 100?

(A) 3 years (B) 4 years (C) 6 years (D) 10 years (E) 12 years

9. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

(A)  (B)  (C)  (D)  (E) 

10. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. Which are the covered digits?



(A) 0, 2 and 2 (B) 1, 2 and 9 (C) 2, 4 and 9 (D) 2, 7 and 8 (E) 5, 7 and 8

Part B: Each correct answer is worth 4 points

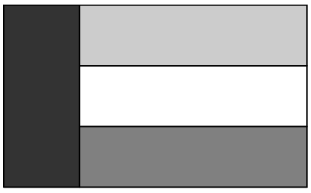
11. A square has vertices A, B, C, D labelled clockwise. An equilateral triangle is constructed with vertices A, E, C labelled clockwise. What is the measure of angle CBE in degrees?

(A) 30 (B) 45 (C) 135 (D) 145 (E) 150

12. The numbers a, b, c, d are distinct positive integers chosen from 1 to 10. What is the least possible value $\frac{a}{b}+\frac{c}{d}$ could have?

(A) $\frac{2}{10}$ (B) $\frac{3}{19}$ (C) $\frac{14}{45}$ (D) $\frac{29}{90}$ (E) $\frac{25}{72}$

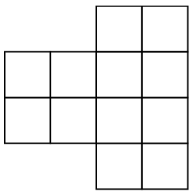
13. The flag of Kanguria is a rectangle with side lengths in the ratio 3:5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the side lengths of the white rectangle?



(A) 1:3 (B) 1:4 (C) 2:7 (D) 3:10 (E) 4:15

14. A 3×2 rectangle can be exactly covered by two of the L-shape figures in two different ways as shown below.

In how many different ways can the figure below be covered by the L-shape figures?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 48

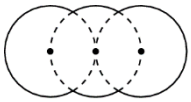
15. The numerator of the fraction was increased by 40%. How many percent should you then reduce the denominator to get a 2 times larger fraction than the original?

(A) 25% (B) 30% (C) 45% (D) 40% (E) 50%

16. Some diluted juice is to be made out of concentrate and water in the ratio 1:7 by volume. Juice concentrate is in a 1-litre flask, and the flask is half full. What fraction of this concentrate should be used to produce 2 litres of diluted juice?

(A) 1/4 (B) 1/2 (C) 2/7 (D) 4/7 (E) All of the concentrate.

17. The given shape is made of parts of three identical circles of radius R that have their centres on a straight line. The middle circle passes through the centres of the other two, as shown. What is the perimeter of the shape?



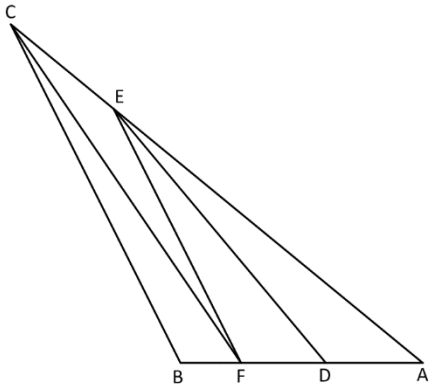
(A) 10/3 πR (B) 5/3 πR (C) (2√3)/3 πR (D) 2√3 πR (E) 4πR

18. The seven digits of the telephone number $aaabbbb$ add up to the two digit number $\overline{ab}$. What is the sum $a + b$?

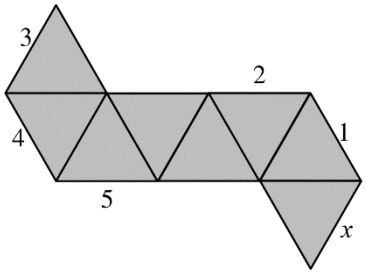
(A) 8 (B) 9 (C) 10
(D) 11 (E) 12

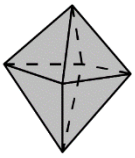
19. The triangle *ABC* is divided by segments *DE*, *EF* and *CF* into four triangles whose areas are equal.
Determine the ratio of *AF*: *BD*.

(A) 1 (B) 9:8 (C) 8:7
(D) 7:6 (E) 6:5



20. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment marked with the x?





(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Part C: Each correct answer is worth 5 points

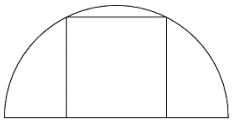
21. How many planes pass through exactly three vertices of a given cube?

(A) 1 (B) 2 (C) 4 (D) 8 (E) 12

22. What is the least number of elements we have to delete from the set {10,20,30,40,50,60,70,80,90} so that the product of the elements remaining in the set is a perfect square?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

23. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown.



The radius of the circle is 1 cm. What is the area of the square?

(A) $\frac{4}{5}$ cm^2 (B) $\frac{\pi}{4}$ cm^2 (C) 1 cm^2 (D) $\frac{4}{3}$ cm^2 (E) $\frac{2}{\sqrt{3}}$ cm^2

24. Two dots are marked in a disc that is rotating around its centre. One of them is 3 cm further than the other to the centre of the disc and moves at a constant speed that is 2.5 times as fast as the other. What is the distance from the centre of the disc to this far point?

(A) 10 cm (B) 9 cm (C) 8 cm (D) 6 cm (E) 5 cm

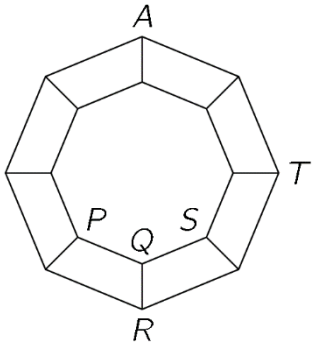
25. The integers from 1 to 99 are written in ascending order without gaps. The sequence of digits is then divided into triplets of digits:

123456789101112…979899 ⟶ (123)(456)(789)(101)(112)…(979)(899).

Which of the following is not one of the triplets?

(A) (222) (B) (444) (C) (464) (D) (646) (E) (888)

26. A graph consists of 16 vertices and some edges that connect them, as in the picture. An ant is now at the vertex labelled A. At each move, it can walk from one vertex to any neighbouring vertex crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 moves?



(A) only P, R or S, not Q and T (B) only P, R, S or T, not Q (C) only Q
(D) only T (E) all of these are possible

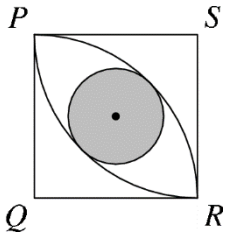
27. The number $3a$ has exactly 4 divisors, and the number $5a$ has exactly 6 divisors. Find the first digit of the number $2019a$.

(A) 1 (B) 3 (C) 8 (D) 9 (E) impossible to determine

28. On each vertex of a square, one positive integer is placed. For any two numbers joined by an edge of the square, one is a multiple of the other. However, for any two diagonally opposite numbers, neither is a multiple of the other. What is the smallest possible sum of the four numbers?

(A) 12 (B) 24 (C) 30 (D) 35 (E) 60

29. Two quarter circles, whose centres are at S and Q, fit exactly inside the square $PQRS$ of side length 2.



The largest possible circle is drawn inside the region where the quarter circles overlap.
What is the area of the circle?

(A) $(6-2\sqrt{2})\pi$ (B) $4(\sqrt{2}-2)\pi$ (C) $(2\sqrt{2}+2)\pi$ (D) $(3\sqrt{2}-2)\pi$ (E) $(6-4\sqrt{2})\pi$

30. If any digit of a given 4-digit number is deleted, the resulting 3-digit number is a divisor of the original number. How many 4-digit numbers have this property?

(A) 5 (B) 9 (C) 14 (D) 19 (E) 23