Stochastic blockmodels for relational events

Christopher DuBois[1] Carter Butts[2] Padhraic Smyth[3]

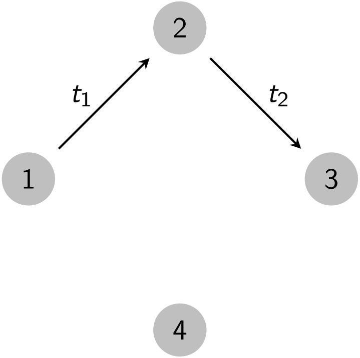
[1]Department of Statistics
University of California, Irvine

[2]Department of Sociology
University of California, Irvine

[2]Department of Computer Science
University of California, Irvine

Sunbelt 2012

Dyadic event data



- Above example: 2 dyadic events occurring among 4 nodes
- Nodes and edges may have covariates
- Interested in inferences about the rate of particular events, conditioned on what we have observed

Relational event models

Multiplicative models for the hazard of a dyad (i, j):

$$\log \lambda_{ij}(t|\cdot) = \boldsymbol{\beta}'\mathbf{s}(t, i, j)$$

- $\lambda_{ij}(t|\cdot)$: "rate" of interaction from i to j at time t
- $\boldsymbol{\beta}$: vector of model parameters
- $\mathbf{s}(t, i, j)$: vector of sufficient statistics for (i, j) given the events before time t

See [2, 9, 1, 6, 7, 8, 5].

Relational event models: Likelihood

$$\mathcal{L}(\mathcal{A}|\theta) = \prod_{m=1}^{M} \lambda_{i_m,j_m}(t_m|\cdot) \prod_{(i,j)\in\mathcal{R}} \exp\{-(t_m - t_{m-1})\lambda_{ij}(t_m|.)\} \quad (1)$$

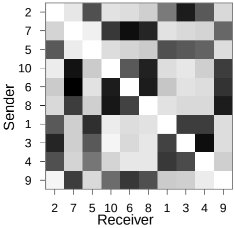
- M: number of events
- t_m: time of event m
- $\mathcal{R}$: set of dyadic events

Stochastic blockmodels

- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).

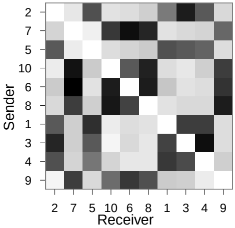
Stochastic blockmodels

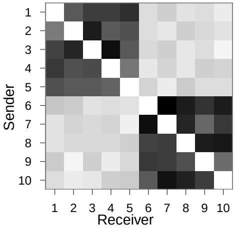
- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).



Stochastic blockmodels

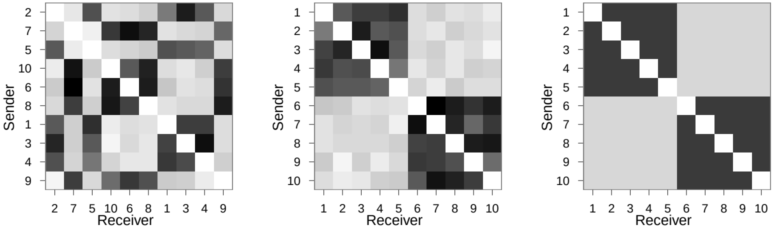
- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).





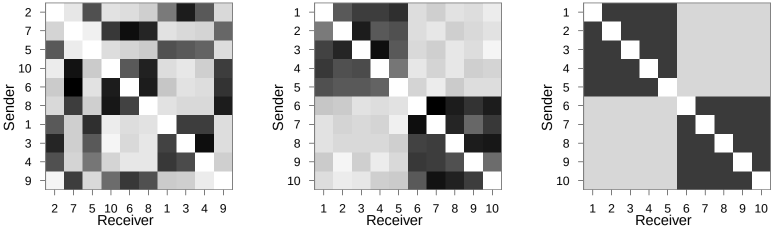
Stochastic blockmodels

- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).



Stochastic blockmodels

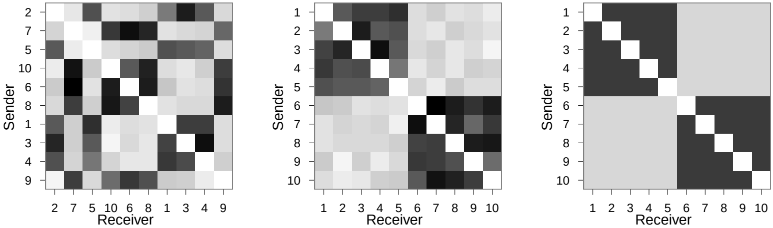
- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).



Mixing rates $$\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$$

Stochastic blockmodels

- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).

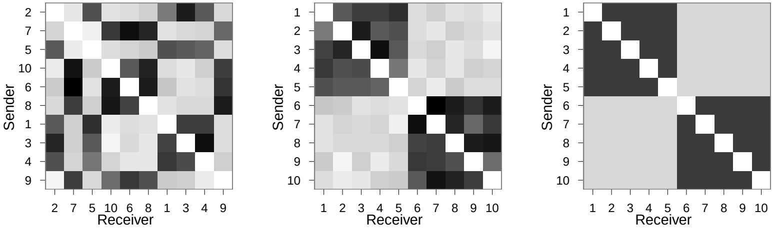


Mixing rates $\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$

Actor i's class $z_i \sim \text{Categorical}(\pi)$

Stochastic blockmodels

- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).



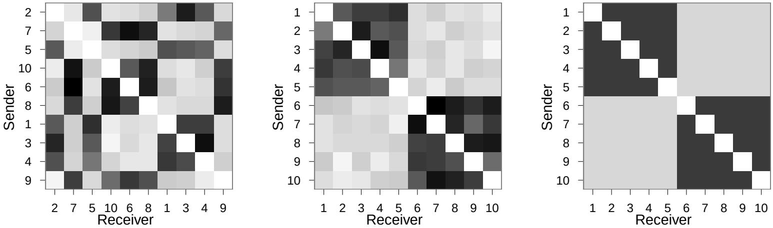
Mixing rates $$\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$$

Actor i's class $z_i \sim \text{Categorical}(\pi)$

Dyad (i,j) $y_{ij} \sim F(\eta_{z_i,z_j})$

Stochastic blockmodels

- Each actor belongs to a latent class.
- Model block-wise interactions (mixing rates).



Mixing rates $\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$

Actor i's class $z_i \sim \text{Categorical}(\pi)$

Dyad (i,j) $y_{ij} \sim F(\eta_{z_i, z_j})$

See [4, 3].

Stochastic blockmodels for relational event data

Subsets of interactions may have distinct dynamics

- Core-periphery structures
  (e.g. periphery only sends, core reciprocates immediately)

Stochastic blockmodels for relational event data

Subsets of interactions may have distinct dynamics

- Core-periphery structures
  (e.g. periphery only sends, core reciprocates immediately)
- Cohesive subgroups with differing behavior
  (e.g. students and professors)

Stochastic blockmodels for relational event data

Idea:

- Assume each actor i belongs to a latent class z_i.
- Model event dynamics between blocks.

$$\log \lambda_{ij}(t|\mathcal{A}_t, \mathbf{z}) = \boldsymbol{\beta}_{z_i, z_j} \mathbf{s}(t, i, j|\mathcal{A}_t, \mathbf{z}). \tag{2}$$

Stochastic blockmodels for relational event data

Idea:

- Assume each actor i belongs to a latent class z_i.
- Model event dynamics between blocks.

$$\log \lambda_{ij}(t|\mathcal{A}_t, \mathbf{z}) = \boldsymbol{\beta}_{z_i,z_j}\mathbf{s}(t, i, j|\mathcal{A}_t, \mathbf{z}). \tag{2}$$

Notation:

- $\boldsymbol{\beta}_{k,l}$: parameter vector for relational events from an actor in block k to an actor in block l
- $\mathcal{A}_t$: history prior to time t
- $\mathbf{z}$: vector of class assignments

Stochastic blockmodels for relational event data

Idea:

- Assume each actor i belongs to a latent class z_i.
- Model event dynamics between blocks.

$$\log \lambda_{ij}(t|\mathcal{A}_t, \mathbf{z}) = \boldsymbol{\beta}_{z_i,z_j}\mathbf{s}(t, i, j|\mathcal{A}_t, \mathbf{z}). \tag{2}$$

Benefits:

- Adjust for unobserved heterogeneity.
- Obtain clusters of nodes who have similar interaction dynamic with other parts of the network.

Model specification for $\mathbf{s}(t, i, j)$

Participation shifts adjust for the next event

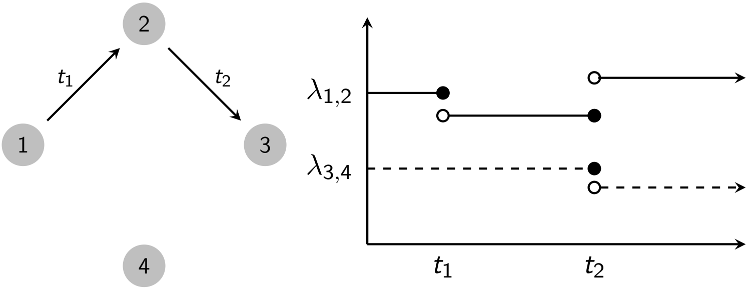
- AB-BA
- AB-BY
- AB-AY
- AB-XA
- AB-XB

Degree effects count the number of previous events

- sent by sender
- received by sender
- sent be receiver
- received by receiver
- previous occurrences of this dyad

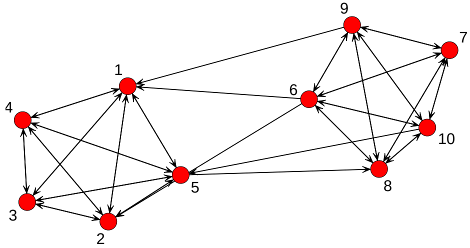
Scalability

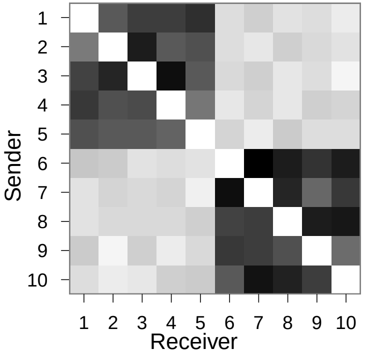
Only use covariates for (i, j) that involve either i or j.



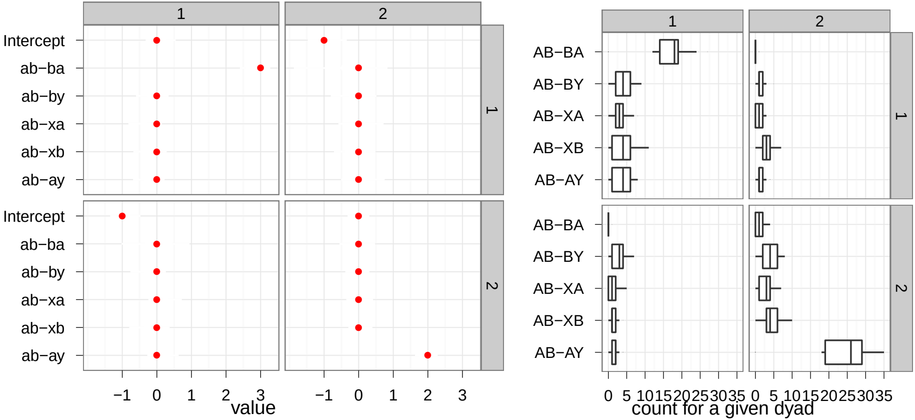
- Restricts the class of effects one can use
- Complexity of likelihood computation reduced ($N^2 \to N$)

Synthetic example: 2000 events among 10 actors

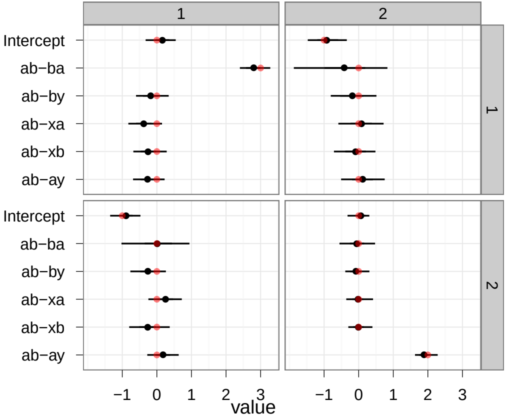


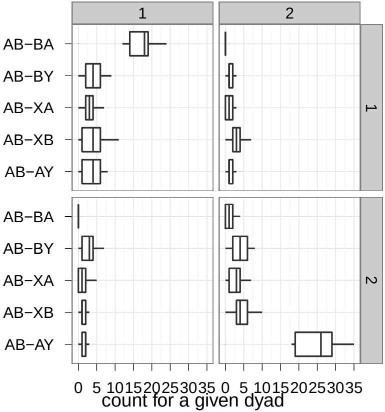


Synthetic example: β and observed counts



Synthetic example: slice sampling for β and Gibbs on $\mathbf{z}$



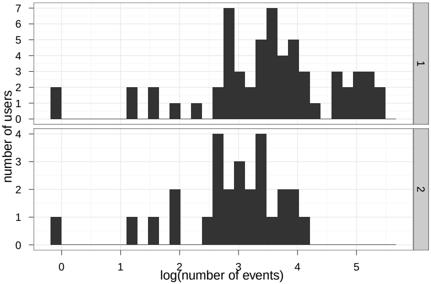


Prediction experiment: Synthetic data

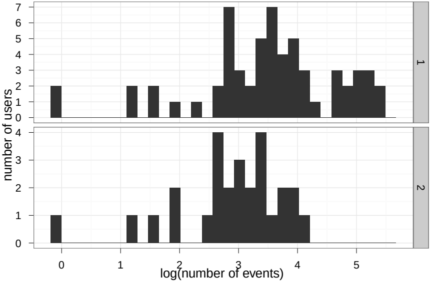
Mean log likelihood on observed events and held out events.

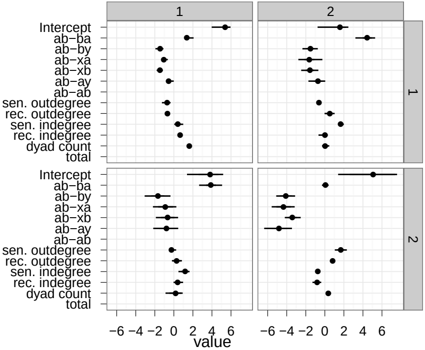
method	brem.train	brem.test	multin.train	multin.test
truth	-0.11	-0.12	-3.69	-3.72
full.1	-0.58	-0.59	-4.16	-4.18
full.2	-0.11	-0.13	-3.69	-3.73
marg	-0.92	-0.92	-4.59	-4.61
online	-0.69	-0.64	-4.28	-4.23
uniform	-0.93	-0.92	-4.61	-4.61

Application: Modeling email (88 people, 2000 events)



Application: Modeling email (88 people, 2000 events)





Prediction experiment: Eckmann subset

Mean log likelihood on observed events and held out events.

method	brem.train	brem.test	multin.train	multin.test
full.1	-6.55	-6.15	-6.51	-6.05
full.2	-6.35	-6.11	-6.44	-6.24
full.3	-6.43	-6.45	-6.46	-6.34
marg	-7.72	-7.73	-7.89	-7.93
online	-7.31	-6.66	-7.46	-6.85
uniform	-8.80	-8.76	-8.95	-8.95

Discussion

Interpretation of parameter estimates

- The proposed method clusters individuals with respect to shared relational event dynamics with other clusters

Discussion

Interpretation of parameter estimates

- The proposed method clusters individuals with respect to shared relational event dynamics with other clusters

Main takeaways

- Detailed, highly dependent model for local structure (using relational event models)
- Latent variable model to capture meso-level structure (using stochastic block model)

Discussion

Interpretation of parameter estimates

- The proposed method clusters individuals with respect to shared relational event dynamics with other clusters

Main takeaways

- Detailed, highly dependent model for local structure (using relational event models)
- Latent variable model to capture meso-level structure (using stochastic block model)

Future directions

- Consider a mixed-membership approach
- Dirichlet process instead of fixed K

References I

Ulrik Brandes, Jürgen Lerner, and Tom a.B. Snijders.
Networks Evolving Step by Step: Statistical Analysis of Dyadic Event Data.
2009 International Conference on Advances in Social Network Analysis and Mining, pages 200–205, July 2009.

Carter T. Butts.
A Relational Event Framework for Social Action.
Sociological Methodology, 38(1):155–200, July 2008.

Charles Kemp, Joshua B Tenenbaum, and Thomas L Griffiths.
Learning Systems of Concepts with an Infinite Relational Model.
pages 381–388.

Krzysztof Nowicki and Tom A B Snijders.
Estimation and Prediction for Stochastic Blockstructures.
Journal of the American Statistical Association, 96(455):1077–1087, 2001.

Tore Opsahl and Bernie Hogan.
Modeling the evolution of continuously-observed networks : Communication in a Facebook-like community 1 Introduction.
Communication, pages 1–22, 2011.

Patrick O Perry and Patrick J Wolfe.
Point process modeling for directed interaction networks.
New York, 2011.

Christoph Stadtfeld.
Who Communicates with Whom? Measuring Communication Choices on Social Media Sites.
2010 IEEE Second International Conference on Social Computing, pages 564–569, August 2010.

References II

Christoph Stadtfeld and Andreas Geyer-Schulz.
Analyzing event stream dynamics in two-mode networks: An exploratory analysis of private communication in a question and answer community.
Social Networks, pages 1–15, October 2011.

Duy Q Vu, Arthur U. Asuncion, David R. Hunter, and Padhraic Smyth.
Continuous-Time Regression Models for Longitudinal Networks.
NIPS, pages 1–9, 2011.

Thanks

Work supported by National Defense Science and Engineering Graduate Fellowship.

Special thanks to Carter Butts and Padhraic Smyth.