Stochastic blockmodels for relational events

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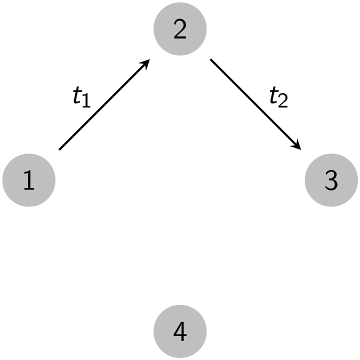
Dyadic event data



- Above example: 2 dyadic events occurring among 4 nodes
- Nodes and edges may have covariates
- Interested in inferences about the rate of particular events, conditioned on what we have observed

Relational event models

Multiplicative models for the hazard of a dyad (i, j):

$$\log \lambda_{ij}(t|\cdot) = \boldsymbol{\beta}' \mathbf{s}(t, i, j)$$

- $\lambda_{ij}(t|\cdot)$: "rate" of interaction from i to j at time t
- $\boldsymbol{\beta}$: vector of model parameters
- $\mathbf{s}(t, i, j)$: vector of sufficient statistics for (i, j) given the events before time t

See [2, 9, 1, 6, 7, 8, 5].

Relational event models: Likelihood

$$\mathcal{L}(\mathcal{A}|\theta) = \prod_{m=1}^{M} \lambda_{i_m,j_m}(t_m|\cdot) \prod_{(i,j)\in\mathcal{R}} \exp\{-(t_m - t_{m-1})\lambda_{ij}(t_m|.)\} \quad (1)$$

- M: number of events
- t_m: time of event m
- $\mathcal{R}$: set of dyadic events

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- Model block-wise interactions (mixing rates).

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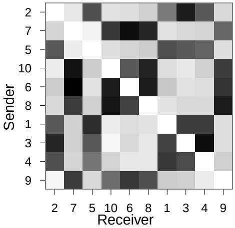

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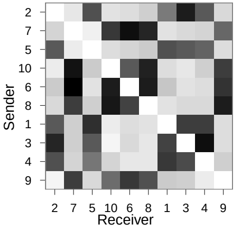

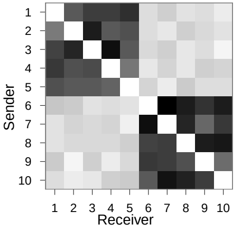

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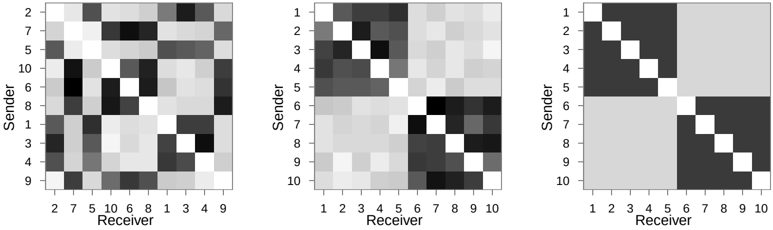

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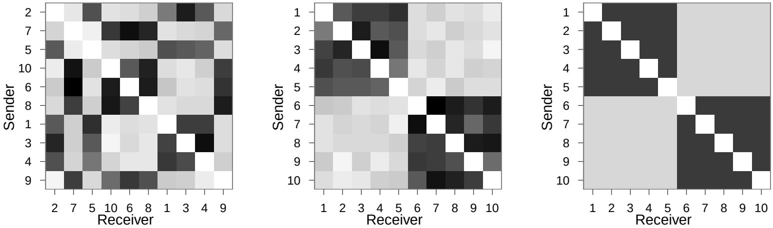

Mixing rates $$\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$$

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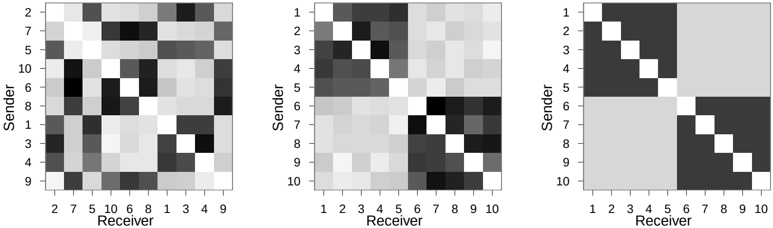

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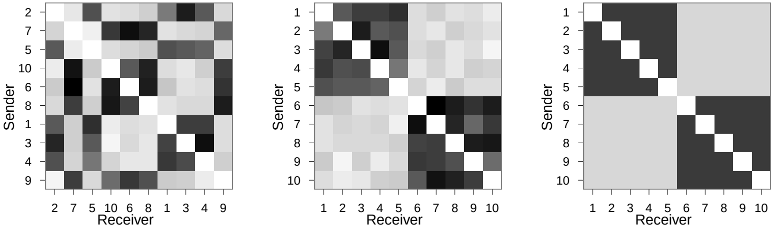

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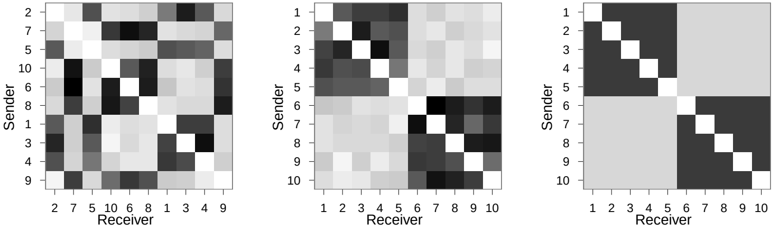

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See [4, 3].

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Subsets of interactions may have distinct dynamics

- Core-periphery structures
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- Core-periphery structures
  (e.g. periphery only sends, core reciprocates immediately)
- Cohesive subgroups with differing behavior
  (e.g. students and professors)

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$$\log \lambda_{ij}(t|\mathcal{A}_t, \mathbf{z}) = \boldsymbol{\beta}_{z_i,z_j} \mathbf{s}(t, i, j|\mathcal{A}_t). \tag{2}$$

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Notation:

- $\boldsymbol{\beta}_{k,l}$: parameter vector for relational events from an actor in block k to an actor in block l
- $\mathcal{A}_t$: history prior to time t
- $\mathbf{z}$: vector of class assignments

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Benefits:

- Adjust for unobserved heterogeneity.
- Obtain clusters of nodes who have similar interaction dynamic with other parts of the network.

Model specification for $\mathbf{s}(t, i, j)$

Participation shifts adjust for the next event

- AB-BA: Reciprocity
- AB-BY: Turn-taking
- AB-AY: Turn-continuing
- AB-XA: Turn-usurping
- AB-XB: Turn-usurping

Degree effects count the number of previous events

- sent by sender
- received by sender
- sent be receiver
- received by receiver
- involving this dyad

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Only use covariates for (i, j) that involve either i or j.

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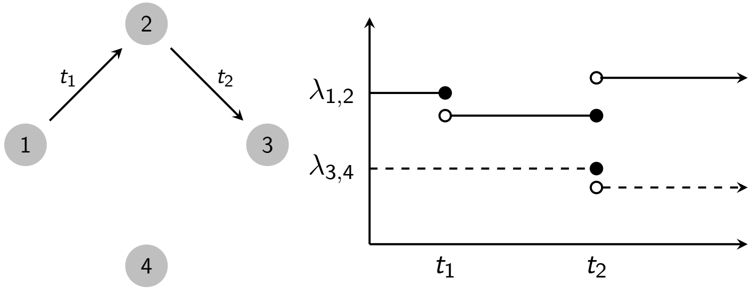

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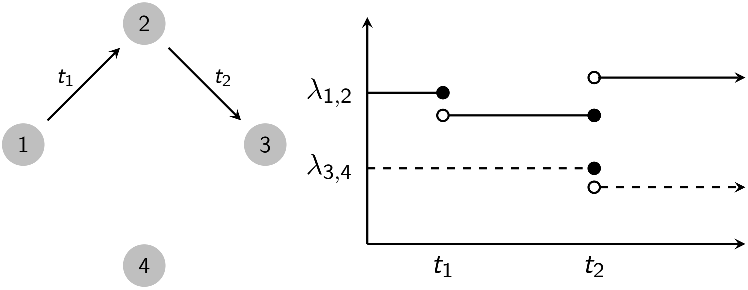
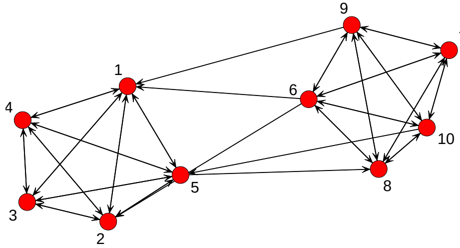
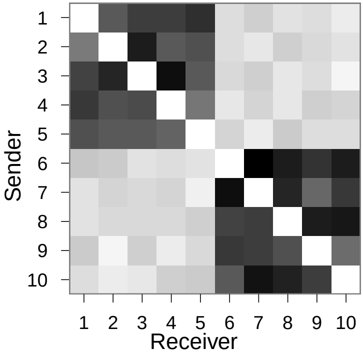


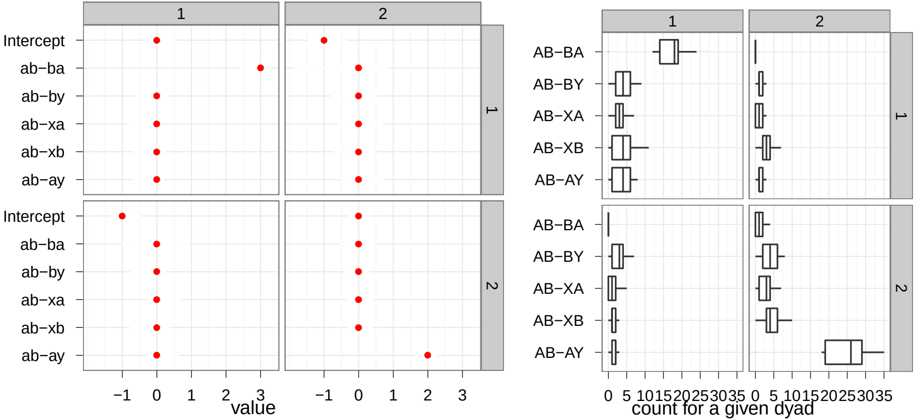
- Restricts the class of effects one can use
- Complexity of likelihood computation reduced $(N^2 \to N)$

Synthetic example: 2000 events among 10 actors

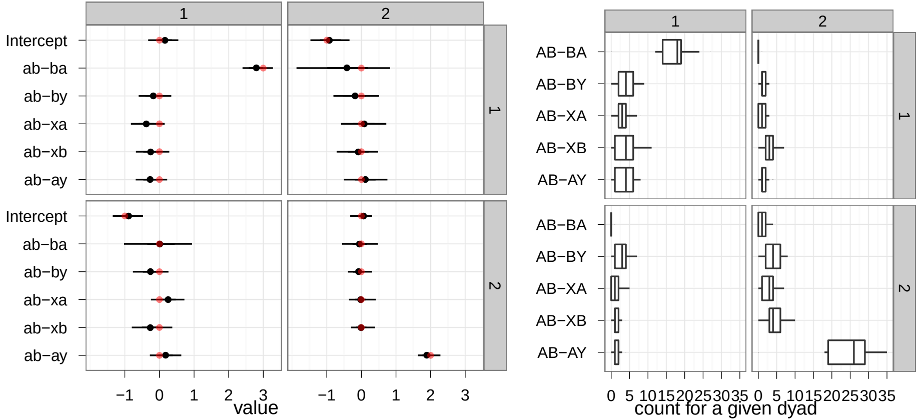




Synthetic example: β and observed counts



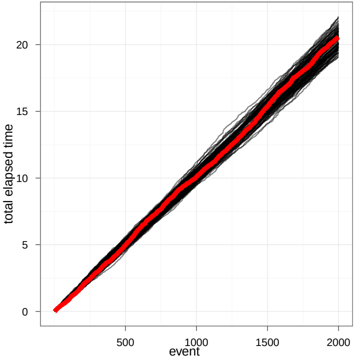
Synthetic example: slice sampling for β and Gibbs on $\mathbf{z}$

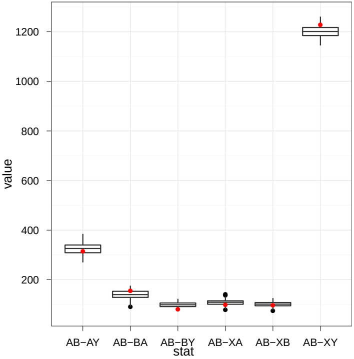


Posterior predictive checks

- Investigate whether observed data is reasonable under model.
- Interested in a particular statistic of a sequence $T(Y)$ (e.g. total time, degree distribution, etc.)
- Simulate $Y^{(i)} \sim \mathrm{REM}(\beta^{(i)}, \mathcal{A})$
- Compare $T(Y)$ to distribution of $T(Y^{(i)})$

Posterior predictive checks



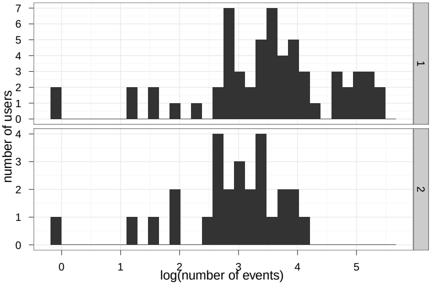


Prediction experiment: Synthetic data

Mean log likelihood for observed events and held out events.

method	brem.train	brem.test	multin.train	multin.test
truth	-0.11	-0.12	-3.69	-3.72
full.1	-0.58	-0.59	-4.16	-4.18
full.2	-0.11	-0.13	-3.69	-3.73
marg	-0.92	-0.92	-4.59	-4.61
online	-0.69	-0.64	-4.28	-4.23
uniform	-0.93	-0.92	-4.61	-4.61

Application: Modeling email (88 people, 2000 events)



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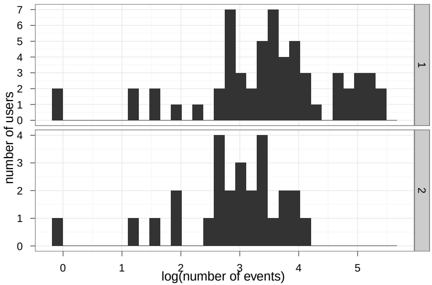
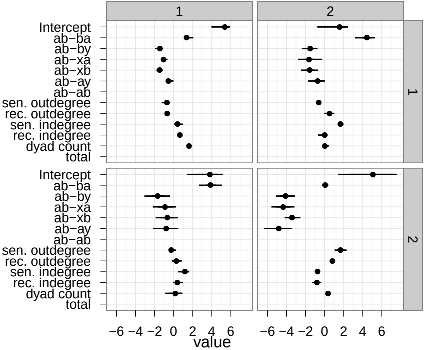


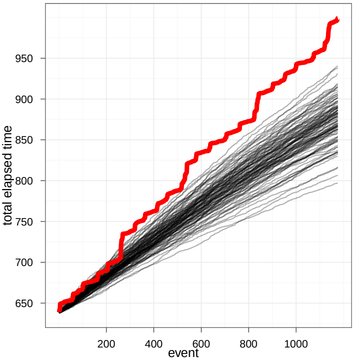


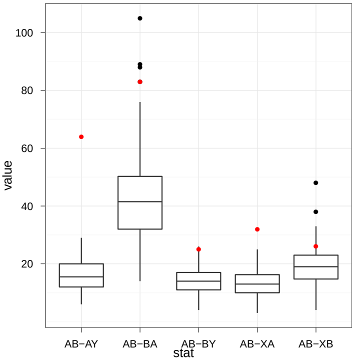
Prediction experiment: Eckmann subset

Mean log likelihood for observed events and held out events.

method	brem.train	brem.test	multin.train	multin.test
full.1	-6.55	-6.15	-6.51	-6.05
full.2	-6.35	-6.11	-6.44	-6.24
full.3	-6.43	-6.45	-6.46	-6.34
marg	-7.72	-7.73	-7.89	-7.93
online	-7.31	-6.66	-7.46	-6.85
uniform	-8.80	-8.76	-8.95	-8.95

Posterior predictive checks for $K = 2$ model





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Interpretation of parameter estimates

- The proposed method clusters individuals with respect to shared relational event dynamics with other clusters

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Main takeaways

- Detailed, highly dependent model for local structure (using relational event models)
- Latent variable model to capture meso-level structure (using stochastic block model)

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Future directions

- Consider a mixed-membership approach
- Intelligent way to share parameters as K grows

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