Q. A small grocery store has only one checkout counter. Customers arrive at the checkout counter at random from 1–8 minutes apart. Each possible value of inter arrival time has the same probability. The service time varies from 1–6 min with probability given in table. The problem is to analyse the system by simulating the arrival & service of 20 customers

Service Time	Probability	Cumulative probability	Random digit assignment
1	0.10	0.10	01 – 10
2	0.20	0.30	11 – 30
3	0.30	0.60	31 – 60
4	0.25	0.85	61 – 85
5	0.10	0.95	86 – 95
6	0.05	1.00	96 – 00

Arrival time	Probability	cumulative probability	Random digit
1	0.125 (1/8)	0.125	001 – 125
2	0.125	0.250	126 – 250
3	0.125	0.375	251 – 375
4	0.125	0.500	376 – 500
5	0.125	0.625	501 – 625
6	0.125	0.750	626 – 750
7	0.125	0.875	751 – 875
8	0.125	1.000	876 – 000

random numbers for service time

20

84
10
74
53
17
79
91
67
89
38

32
94
79
05
79
84
52
55
30
50

For arrival

913
727
015
948
309
922
753
235
302
109

093
607
738
359
888
106
212
493
535

Arrival of first ⇒ start of clock
No need random digit

Cust. no.	Time since last Arrival	Arrival time	Service time	Time service begins	customer waits in queue	time services ends	Time spent in system	id
1.	0	0	4	0	0	4	4	
2.	8	8	1	8	0	9	1	
3.	6	14	4	14	0	18	4	
4.	1	15	3	18	3	21	6	0
5.	8	23	2	23	0	25	2	0
6.	3	26	4	26	0	30	4	1
7.	8	34	5	34	0	39	5	4
8.	7	41	4	41	0	45	4	0
9.	2	43	5	45	2	50	7	0
10.	3	46	3	50	4	53	7	0
11.	1	47	3	53	6	56	9	0
12.	1	48	5	56	8	61	13	0
13.	5	53	4	61	8	65	12	0
14.	6	59	1	65	6	66	7	0
15.	3	62	4	66	4	70	8	0
16.	8	70	4	70	0	74	4	0
17.	1	71	3	74	3	77	6	0
18.	2	73	3	77	4	80	7	0
19.	4	77	2	80	3	82	5	0
20.	5	82	3	82	0	85	3	0
			67		51		118	18

Average waiting time of each customer = 2.55 (51/20)

Probability that a customer has to wait in queue = no. of customers

=0.55

Probability of idle time of server = $\frac{\text{total idle time}}{\text{total run time of simulation}}$

$= \frac{18}{85} = 0.211$

Probability of server busy time $= 1 - \frac{18}{85} = 0.788$

Average service time (observed service time) = $\frac{\text{total service time}}{\text{total customer}}$

$= \frac{67}{20} = 3.35$

This result can be compared with the expected service time by finding the mean of service time distribution using the equation

$$E(S) = \sum_{s=0}^{\infty} S \times p(S)$$

$= (4 \times 0.25) + (1 \times 0.1) + (2 \times 0.2) +$
$(3 \times 0.3) + (5 \times 0.1) + (6 \times 0.05) = 3.2$

Average time btw arrivals = $\frac{\text{sum of all time btw arrivals}}{\text{no. of arrivals} - 1}$

$= \frac{82}{19} = 4.32$

This result can be compared to the expected time btw arrivals by finding mean of discrete uniform distribution whose end points are

$a = 1 \qquad b = 8$

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5$$

Average waiting time of those customers who wait in queue = $\frac{\text{Total time customers wait in queue}}{\text{Total no of customers waiting}}$

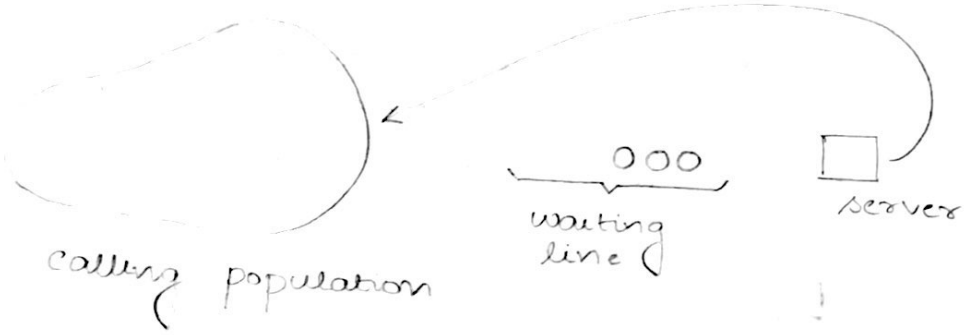
$= \frac{51}{11} = 4.63$

Average time customer spends in the system = $\frac{\text{total time customer spent}}{\text{total customer}} = \frac{118}{20} = 5.9$

Second way of computing the same result is

= average time customer spends waiting + average time customer spends in service

= 2.55 + 3.35 = 5.9 min



Queueing system

A queueing system is described by its calling population, the nature of arrivals, the service mechanism, system capacity & the queueing discipline.

		Queue Status	
		Empty	Not empty
Server Status	Idle	Enter service	Impossible
	busy	Enter queue	Enter queue

		Queue status	
		Empty	Not empty
Server Status	Idle	✓	✗
	busy	✓	✓

Potential (customer) unit

Action upon arrival

Server outcomes after Scanning Completion