GENETIC ALGORITHM FOR FATIGUE CRACK DETECTION IN TIMOSHENKO BEAM

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Abstract: Presented paper deals with a method of detecting fatigue cracks in their early state of growth. The method uses optimisation tool consisting of genetic algorithm and gradient method. The applied fitness function is based on the changes in propagating waves.

Keywords: genetic algorithm, crack detection, spectral element method, Timoshenko beam

1. INTRODUCTION

In order to improve the safety, reliability and operational life, it is urgent to monitor the integrity of structural systems. Techniques of non-destructive damage detection in mechanical engineering structures are essential [1], [2], [3]. Previous approaches to non-destructive evaluation of structures to assess their integrity typically involved some form of human interaction. Recent advances in smart materials and structures technology has resulted in a renewed interest in developing advanced self-diagnostic capability for assessing the state of structure without any human interaction. The goal is to reduce human interaction while at the same time monitor the integrity of the structure. With this goal in mind, many researchers have made significant

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strides in developing damage detection methods for structures based on traditional modal analysis techniques. These techniques are often well suited for structures which can be modeled by discrete lumped-parameter elements where the presence of damage leads to some low frequency change in the global behavior of the system [4], [5], [6], [7]. On the other hand small defects such as cracks are obscured by modal approaches since such phenomena are high frequency effects not easily discovered by examining changes in modal mass, stiffness or damping parameters. This is because at high frequency modal structural models are subject to uncertainty. This uncertainty can be reduced by increasing the order of the discrete model, however, this increases the computational effort of modal-based damage detection schemes. There is also a group of methods which utilize thermodynamic damping for assessment of the structural integrity of vibrating structures [8], [9], [10].

Changes in propagating waves are very sensitive to any discontinuities in the structures. Analysis of the process of wave propagation are possible with utilization the spectral element method. This method allows an exact assessment of the inertia of the system [11]. Mathematical site of this method gives only simple set of equations to solve. An important thing is that only one spectral element allows to calculate precisely an infinite number of frequencies and mode shapes of the examined structure. With the spectral element method it is also possible to compare the impact signal and the system response directly in the time domain. Differences in those signals give information about the state of structure.

In the paper a new finite spectral Timoshenko beam element with a transverse open and not propagating crack is introduced. Up till now there are models of a cracked beam in the literature available [11, 12], however there is no spectral model of a cracked Timoshenko one. The crack was modelled with consideration of the influence of the plasticity zone around the crack tip. This approach allows to model the changes of stiffness according to crack appearance in examined structure in more precise way.

For the searching process of parameters of the crack there was used special optimization method, which consisted of simple genetic algorithm and a gradient method. Numerical tests done show that evaluated approach is very sensitive for damage introduction in the structure and the same allows to detect it in very early stage. This fact is very promising for future work in the field of structural health monitoring.

2. MATHEMATICAL MODEL OF THE BEAM

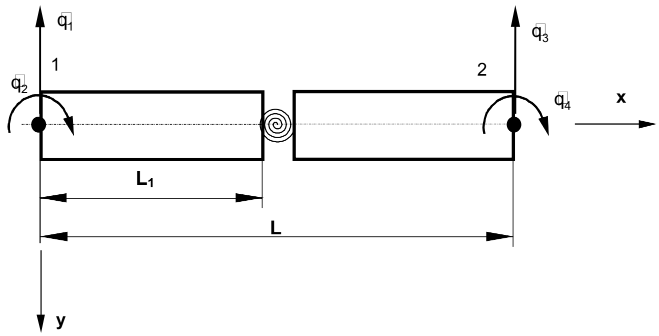

Figure 1. The physical model of the beam.

A spectral Timoshenko beam finite element with a transverse open and non-propagating crack is presented in Fig. 1. The length of the element is L, and its area of cross-section is A. The crack is substituted by a dimensionless and massless spring, whose bending θ_b and shear θ_s flexibilities are calculated using Castigliano's theorem and laws of the fracture mechanics.

Nodal spectral displacements $\hat{w}$ and rotations $\hat{\phi}$ are assumed in the following forms, for the left and right part of the Timoshenko beam:

$$
\begin{aligned}
&\hat{w}_1(x \in (0,L_1)) = R_1 A_1 e^{-ik_1 x} + R_2 B_1 e^{-ik_2 x} - R_1 C_1 e^{-ik_1(L_1-x)} + \\
&\qquad - R_2 D_1 e^{-ik_2(L_1-x)} \\
&\hat{\phi}_1(x \in (0,L_1)) = A_1 e^{-ik_1 x} + B_1 e^{-ik_2 x} + C_1 e^{-ik_1(L_1-x)} + D_1 e^{-ik_2(L_1-x)} \\
&\hat{w}_2(x \in (0,L-L_1)) = R_1 A_2 e^{-ik_1(x+L_1)} + R_2 B_2 e^{-ik_2(x+L_1)} + \\
&\qquad - R_1 C_2 e^{-ik_1[L-(L_1+x)]} - R_2 D_2 e^{-ik_1[L-(L_1+x)]} \\
&\hat{\phi}_2(x \in (0,L-L_1)) = A_2 e^{-ik_1(x+L_1)} + B_2 e^{-ik_2(x+L_1)} + \\
&\qquad + C_2 e^{-ik_1[L-(L_1+x)]} + D_2 e^{-ik_1[L-(L_1+x)]}
\end{aligned}
\tag{1}
$$

where: L_1 denotes the location of the crack, L is the total length of the beam, R_n is the amplitude ratios given by [12]:

$$
R_n = \frac{ik_n GAS_1}{GAS_1 k_n^2 - \rho A \omega^2} \qquad \text{for } (n=1,2) \tag{2}
$$

whereas: $S_1 = \left(\frac{0.87 + 1.12\nu}{1+\nu}\right)^2$ is shear coefficient for displacement [12], ν is Poisson ratio, G is shear modulus, ρ denotes density of the material, ω is a frequency and i is imaginary unit given as $i = \sqrt{-1}$.

The wave numbers k_1 and k_2 are the roots of the characteristic equation in the general form:

$$\begin{aligned}&(GAS_1EJ)k^4 - (GAS_1\rho JK_2\omega^2 + EJ\rho A\omega^2)k^2 + \\ &\quad + (\rho JS_2\omega^2 - GAS_1)\rho A\omega^2 = 0\end{aligned} \qquad (3)$$

where: $S_2=12K_1/\pi^2$ is shear coefficient for rotation [12], E denotes Young's modulus and J is second moment of area. The coefficients A_1, B_1, C_1, D_1 A_2, B_2, C_2 and D_2 can be calculated as a function of the nodal spectral displacements using the boundary conditions with additional assumption at the crack place: at the left and right end of the beam displacements and rotations are known, at the crack location transverse displacements, bending moments and shear forces for the left and right part of the beam are the same, whereas the drop in rotations is proportional to the bending moment multiplied by the flexibility of the crack calculated with the fracture mechanics laws [13, 14, 15].

3. FLEXIBILITY AT THE CRACK LOCATION

A coefficients of a beam flexibility matrix at the crack location (in general form) can be calculated using Castigliano theorem [13]:

$$c_{ij} = \frac{\partial^2 U}{\partial S_i \partial S_j} \qquad (\text{for } i = 1..6, \quad j = 1..6) \qquad (4)$$

where: U denotes the elastic strain energy of the element caused by the presence of the crack and S are the independent nodal forces acting on the element.

For the analyzed beam, the following relation can express the elastic strain energy due to the crack appearance [14]:

$$U = \frac{1}{E}\int_A (K_I^2 + K_{II}^2)dA, \qquad (5)$$

where: A denotes the area of the crack, K_I and K_{II} are a stress intensity factors corresponding to the first and second mode of the crack growth [15].

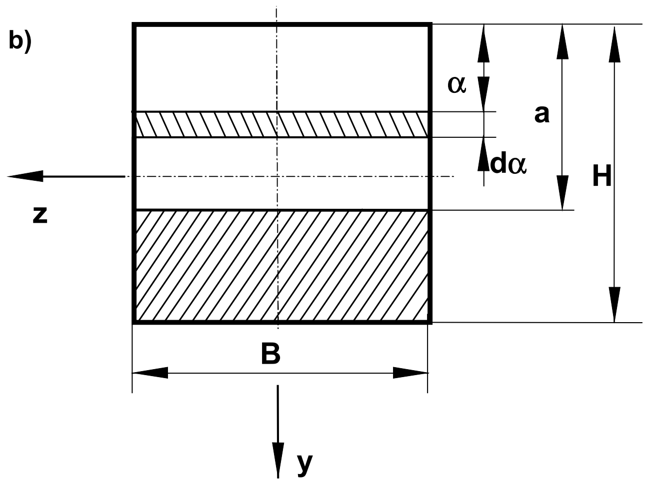

Figure 2. The cross section at the crack location.

The stress intensity factors can be calculated as follows:

$$K_I = \frac{6M}{BH^2}\sqrt{\pi\alpha}F_I\left(\frac{\alpha}{H}\right)$$
$$K_{II} = \frac{\beta T}{BH}\sqrt{\pi\alpha}F_{II}\left(\frac{\alpha}{H}\right) \tag{6}$$

where: M is a bending moment, β denotes shear factor [16] T is a shear force, B,H α are dimensions – see Fig.2, F_I and F_{II} are a correction function in the form [15]:

$$F_I\left(\frac{\alpha}{H}\right) = \sqrt{\frac{\tan(\pi\alpha/2H)}{\pi\alpha/2H}} \cdot \frac{0.752 + 2.02(\alpha/H) + 0.37[1-\sin(\pi\alpha/2H)]^3}{\cos(\pi\alpha/2H)}$$
$$F_{II}\left(\frac{\alpha}{H}\right) = \frac{1.30 - 0.65(\alpha/H) + 0.37(\alpha/H)^2 + 0.28(\alpha/H)^3}{\sqrt{1-(\alpha/H)}} \tag{7}$$

After simple transformations, the flexibilities of the elastic elements modeling of the cracked cross section of the Timoshenko beam spectral finite element, can be rewritten as:

$$c_b = \frac{72\pi}{BH^2}\int_0^{\bar{a}} \bar{\alpha} F_I^2(\bar{\alpha}) d\bar{a}$$
$$c_s = \frac{2\beta\pi}{B}\int_0^{\bar{a}} \bar{\alpha} F_{II}^2(\bar{\alpha}) d\bar{a} \tag{8}$$

where: $\bar{a} = \frac{a}{H}, \bar{\alpha} = \frac{\alpha}{H}$, (see Figure.2)

In the non-dimensional form the flexibilities can be expressed as:

$$\theta_b = \frac{EJc_b}{L}$$
$$\theta_s = \frac{GAc_s}{L} \tag{9}$$

4. OPTIMIZATION METHOD USED

As the genetic algorithm gives only approximate and fully stochastic solution [17] to improve the accuracy of the solution gradient method is implemented into optimization method used. The gradient method utilized was the Newton algorithm for a function with several variables. Gradient method started in two cases: when the mean value of the fitness function reached 95% of the assumed value or when the fitness function value for one "superindividual" reached 97% of the assumed value.

The parameters of the genetic algorithm used were as follows: 30 individuals per population, each individual consisted of 20 bites - 10 bites per variable, fitness function was scaled with linear scaling. Individuals were chosen for recombination with stochastic universal sampling. One-point crossover probability was 90%, mutation probability was 10%.

The fitness function was assumed as:

$$F_f = 1 - \left(\frac{|Q_m - Q_c|}{|Q_c|}\right)^2 \tag{10}$$

where: Q_m is the "measured" displacement of the beam, Q_c is the displacement calculated for every parameter generated with the genetic algorithm. When the values were the same fitness function was equal to one.

5. EXEMPLARY RESULTS

All numerical calculations were carried out for a steel cantilever beam with geometrical dimensions as follow: length 2 [m], height 0,02 [m], weight 0,02 [m]. The Young's modulus 210 [GPa], mass density 7860 [kg/m^3] and Poisson ratio was 0,3.

Next three figures present results obtained from numerical calculation of the inverse problem. We assumed that we know the size and location of the crack, that means that we calculated the system response for the known values of the crack parameters. Then the optimization method started. To show the advantages of the method there are several results presented.

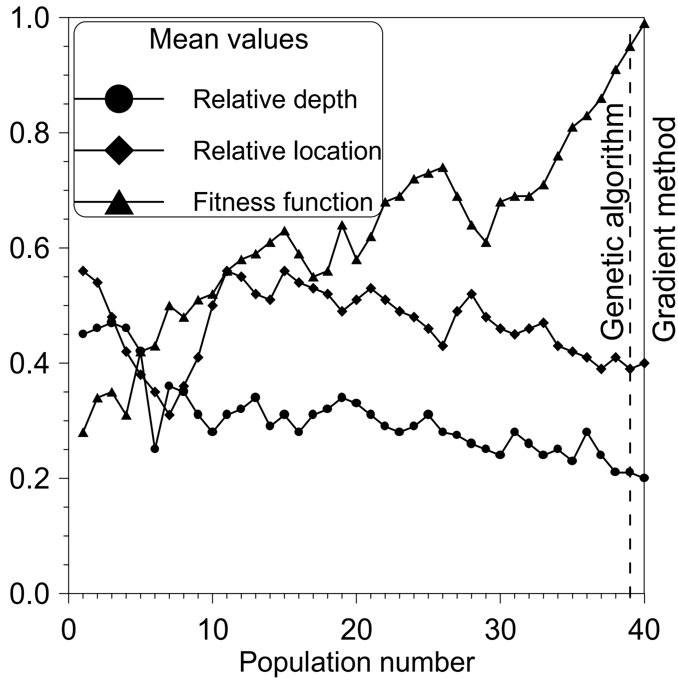

Figure 3. Change of mean values of fitness function, relative crack depth and location in populations.

In the first example (Fig.3) it was assumed that the crack with the depth equal to 20% of the beam height is located in the relative distance from the fixed end equal to 0,4. As the figure shows in the 39th generation there was jump into the gradient method, because the mean value of the fitness function reached 97% of the assumed value. Crack parameters calculated with the genetic algorithm, being the starting point for the gradient method were as follows: the crack depth 0,21 and location 0,39. Values calculated with the gradient method were equal to 0,2 and 0,4, what is the exact value of the searched crack parameters.

In the second numerical example the "searched" relative crack parameters were: the depth 0,15, the location 0,7. According to the figure the gradient method started in the 67th generation, when the mean value of the fitness function reached 97% of the maximum. The gradient method started with values 0,17 and 0,743. The finally calculated values were 0,15 and 0,75. They are exactly the searched crack parameters.

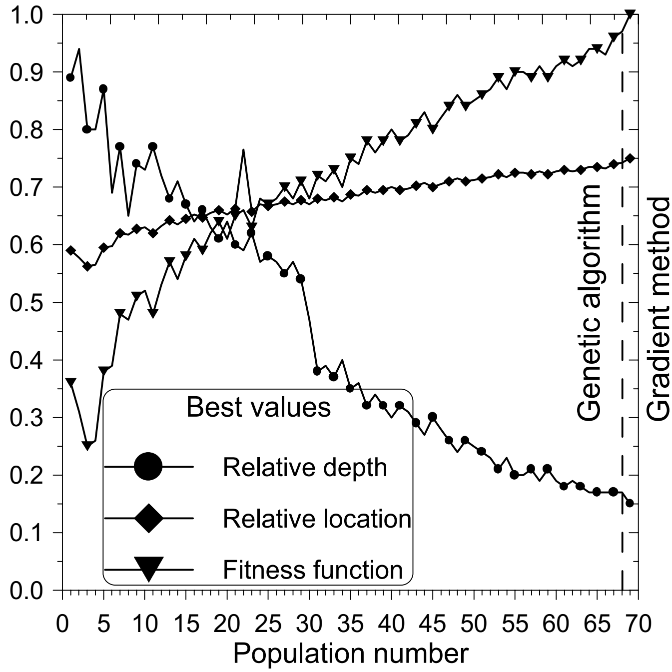

Figure 4. Change of mean values of fitness function, relative crack depth and location in populations.

Last example presented (Fig.5) shows the change of the fitness function and the crack parameters calculated with 53 generations of the genetic algorithm. For this case it was assumed that the crack is located in the relative distance equal to 0,8 and its relative depth is 0,05. The genetic algorithm ended in 53rd generation. Last iteration with values 0,07 and 0,76 was the gradient method. The final calculation values were equal to 0,05 and 0,8, what is the exact match with assumed crack parameters.

As presented above the proposed optimization method allows to find parameters of the crack in a very precise way.

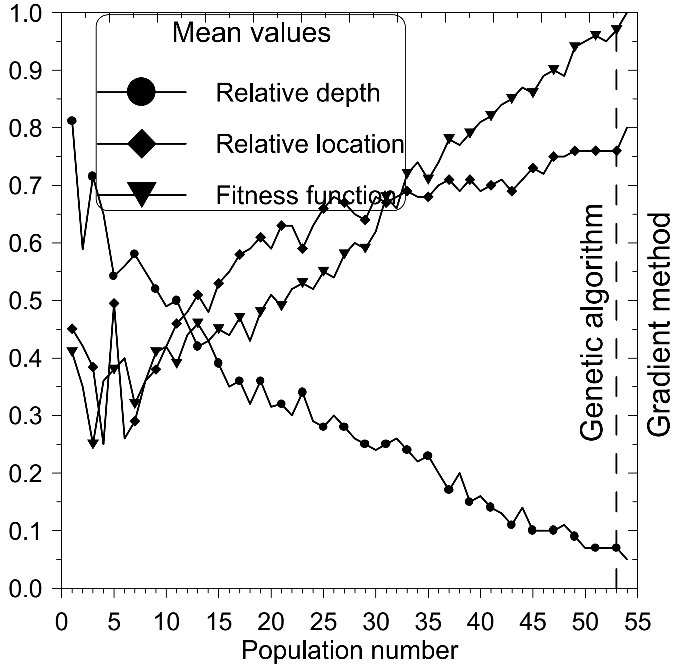

Figure 5. Change of mean values of fitness function, relative crack depth and location in populations.

6. CONCLUSION

The proposed spectral model of a Timoshenko beam with a crack provides analysis of wave propagation in the structure. This fact makes possible utilization of propagating waves for damage detection. Implementing optimization method into the process of searching parameters of the crack saves calculation time and gives precise solution. Proposed method will be developed into elements with more complicated geometry.

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