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# Structural damage identification using co-evolution and frequency response functions

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## SUMMARY

A new damage identification strategy is presented in which damage scenarios and optimal tests are searched simultaneously. The proposed strategy, called Estimation-Exploration Algorithm (EEA), is based on co-evolutionary principles. Co-evolution is a biological process where populations of interacting individuals challenge each other in an ongoing cycle of adaptation. In EEA, a population of damage hypotheses evolves to predict physical tests that have been performed on a structure, while tests evolve to create discrepancy among current damage hypotheses that can explain the observed data. This co-evolutionary approach leads to physical experiments that carry optimal information and results in a fewer number of tests needed for the correct identification of the current damage state of a structure. EEA was introduced by the authors in the context of static testing and is extended in this work to steady-state dynamics. In this context, changes in frequency response functions are used to locate and quantify damage, while structural tests are defined by the location of excitation forces and sensors position. This work shows that EEA is a feasible methodology to alleviate the ill-posedness of inverse damage detection problems by providing an intelligent strategy for selecting tests that maximize information. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: genetic algorithms; damage identification; co-evolution; health monitoring; optimization; inverse problems

## 1. INTRODUCTION

Structural health monitoring is concerned with the identification and quantification of damage in aerospace, civil, and mechanical infrastructure [1], and has emerged as a modern area of research due to its key role in structural safety, maintenance, serviceability, and estimation of expected service life. Structural health monitoring systems use a combination of sensing, actuation, simulation, and reasoning components to infer the location and extent of damage in a

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structure. One of the main challenges in damage identification is to obtain sufficient information through sensing so that reasoning systems can uniquely and unambiguously characterize damage.

The identification of damaged elements is inherently an inverse problem in which the state of the structure is determined based on its response. In situations where sufficient data are not available or the noise-to-signal ratio is high, the inverse problem can be ill-posed in the sense that more than one solution can exist and/or the solution may be very sensitive to errors in the input data. To address the non-uniqueness of the solution, multiple tests can be performed to increase information about the damage state and constrain the number of possible solutions. Ghaboussi and Chou [2] and, Hjelmstad and Shin [3] used multiple simulated static tests in damage identification problems, recognizing that a simple test does not yield the necessary information for solution uniqueness. However, it is not trivial to know *a priori* which tests to perform on a structure so that its damage state can be unambiguously identified. Tests are expensive and carrying out random experiments on a structure can result in unacceptable costs.

A co-evolutionary strategy is presented in this work for intelligently generating tests needed for damage identification. Co-evolution is a biological process where populations of individuals interact with each other while trying to evolve in response to the evolution of the other populations. Co-evolutionary strategies have recently been proposed for nonlinear system identification and structural damage detection [4, 5–8]. The methodology proposed in this paper, called 'The Estimation-Exploration Algorithm (EEA)' hereafter, was introduced by the authors in the context of static analysis in [4] and provides an intelligent approach for selecting multiple tests in damage identification problems so that information contained in the measured response is maximized. In the co-evolution process, models evolve over generations to predict current tests, while current tests evolve to create discrepancy among models. The present work describes the theoretical foundations of EEA and its extension to structural dynamics applications.

The paper is organized as follows. The formulation of the problem is presented first, followed by a description of EEA. Then, the feasibility of the methodology is demonstrated through numerical examples. Finally, some preliminary results, discussion, and conclusions are presented.

## 2. FORMULATION OF THE PROBLEM

The dynamic behaviour of a linear system with viscous damping can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping ,and stiffness matrices of the structure, $\mathbf{x}$ is the displacement vector, and $\mathbf{f}$ is force vector acting on the structure.

Assuming harmonic excitation, the force acting on the structure can be expressed in terms of its angular frequency, $\omega$, and a complex forcing amplitude vector, $\bar{\mathbf{f}}$, as

$$\mathbf{f}(t) = \bar{\mathbf{f}}\, \mathrm{e}^{\mathrm{i}\omega t} \tag{2}$$

In addition, the steady-state response of the structure can be expressed as

$$\mathbf{x}(t) = \bar{\mathbf{x}}\, \mathrm{e}^{\mathrm{i}\omega t} \tag{3}$$



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where $\bar{\mathbf{x}}$ is the frequency component of the displacement. Substituting Equations (2) and (3) into Equation (1) yields

$$\bar{\mathbf{x}} = \mathbf{H}(\omega)\bar{\mathbf{f}} \tag{4}$$

where

$$\mathbf{H}(\omega) = (\mathbf{K} - \omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C})^{-1} \tag{5}$$

$\mathbf{H}(\omega)$ is called the frequency response function (FRF) matrix of the system or, more specifically, the receptance matrix. The *aj*th member of an FRF matrix represents the response (displacement, velocity or acceleration) of the *a*th degree of freedom (DOF) subjected to a harmonic force applied at the *j*th DOF.

The solution of Equation (4) is computationally expensive for large systems. A more efficient approach is to use the spectral decomposition of the receptance matrix $\mathbf{H}(\omega)$ to compute the frequency response at selected DOF. The spectral decomposition of the receptance matrix for a proportionally damped viscous dynamic system can be expressed as [9]

$$\mathbf{H}(\omega) = \mathbf{\Phi}\,\mathrm{diag}\left(\frac{1}{\omega_j^2 - \omega^2}\right)\mathbf{\Phi}^{\mathrm{T}} \tag{6}$$

It is usually assumed that damage is manifested as change in the stiffness of the structure. Considering one damage parameter per structural element, the updated local stiffness matrix of an element can be described as

$$\mathbf{K}_l^{ee} = (1 - \alpha_l)\mathbf{K}_{\mathbf{o}l}^{ee} \tag{7}$$

where $\mathbf{K}_{\mathbf{o}l}^{ee}$ is the undamaged stiffness matrix of element $l$, $\alpha_l$ is the damage ratio or index, and $\mathbf{K}_l^{ee}$ is the updated element stiffness matrix. The global stiffness matrix is assembled from individual element contributions as

$$\mathbf{K}(\boldsymbol{\alpha}) = \sum_{\text{elements}} \mathbf{K}^{ee} \tag{8}$$

Structural damage identification is usually cast as an optimization problem in which model parameters are updated in order to minimize the discrepancy between a mathematical model and the sensed behaviour of the actual structure [1–3, 9–13]. As a result of the above formulation, the receptance function will depend on the damage parameter vector, $\boldsymbol{\alpha}$, and on the frequency, $\omega$. Damage parameters are obtained by formulating an optimization problem in which the error between the computed and measured receptance functions is minimized. In essence, the structure is considered as a black box whose damage state is to be determined without detailed knowledge about its current condition. The error function for a given structural test, $q$, as used in this paper, is given as

$$E_q(\boldsymbol{\alpha}) = \sum_{a=1}^{R}\sum_{p=1}^{M}\frac{|H_{ak}(\omega_p,\boldsymbol{\alpha}) - \hat{H}_{ak}(\omega_p)|}{\max(\hat{H}_{ak}(\omega_p))} \tag{9}$$



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where $a$ indexes the measured DOF, $k$ is the excitation DOF, $p$ indexes the excitation frequencies, $R$ is the total number of sensors in the structure, $M$ is the total number of excitation frequencies, and $\hat{H}_{ak}$ is the measured receptance. The damage parameters are then obtained by solving

$$\text{Minimize } \frac{1}{N}\sum_{q=1}^{N} E_q(\boldsymbol{\alpha}) \quad \text{subject to } 0<\alpha_l<1 \tag{10}$$

where $N$ is the number of physical tests performed on the structure.

### 2.1. Genetic algorithms (GA)

Genetic algorithms (GA) are attractive for complex optimization problems because of their inherent advantages such as parallelism, convergence to global optima, adaptation, and the lack of need for the gradient of the objective function. Because of these advantages, GA have been successfully used in structural damage identification problems [1, 3, 9, 14–19]. Inspired by Darwin's theory of survival of the fittest, GA mimic the process of the evolution of an organism and can be used to solve a wide variety of problems in engineering and science [20, 21].

The information describing each solution (i.e. individual) is encoded in a string that is called a chromosome. The individual entries that form a chromosome are called genes. For instance, in damage identification problems, damage parameters are encoded in real-number genes, which form the chromosome of an individual solution. The algorithm starts with a set of solutions (initial population) selected randomly. Then, the fitness of each individual is calculated based on how well the response of the structure is predicted when the encoded set of damage parameters is used in a finite element analysis. For instance, the error norm defined in Equation (9) can be used as a measure of fitness.

In producing the next generation, pairs of individuals from the current population are selected based on their fitness to serve as parents. Operators such as crossover and mutation are crucial in GA and are used to produce new individuals. The crossover operation relates to the exchange of genetic information between parents, while the mutation operator relates to random changes in the individuals, promoting the exploration of diverse areas of the search space and preventing the evolutionary process from getting trapped in mediocre solutions (i.e. local minima). The schema theorem and minimal building blocks [20, 21] ensure that, on average, useful information is transmitted to the next generation through the genetic operators and better individuals (i.e. solutions closer to the desired goal) are produced. More detailed information on GA can be found in References [20, 21].

## 3. THE ESTIMATION-EXPLORATION ALGORITHM (EEA) IN STRUCTURAL DAMAGE IDENTIFICATION

As explained above, the damage identification problem can be ill-posed due to lack of information. The process of gathering additional information can prove to be complicated especially for large search domains (i.e. large number of possible tests). In these cases, blind selection of tests with the hope of gathering enough information can be futile. The goal of the co-evolutionary strategy presented herein is to search for the true damage state of the structure while minimizing the cost associated with additional tests.



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Two main stages are involved in EEA: the estimation phase and the exploration phase. A flowchart of this methodology is illustrated in Figure 1. In the estimation phase, the optimization problem defined through Equation (9) is addressed. Candidate solutions are sought based on information sensed from current physical tests. In the early stages of EEA, multiple solutions or candidate damage scenarios will be obtained due to the ill-posedness of the inverse problem.

In the exploration phase, the best candidate solutions found in the estimation phase are used to select the next physical test to be performed on the structure. The exploration phase is then cast as an optimization problem in which the objective is to maximize the discrepancy among candidate damage scenarios. Maximizing the discrepancy among candidate solutions can be interpreted as increasing information about the state of the structure since the selected damage scenarios can already explain all the existing test data. This helps the exploration phase to reduce the number of false hypotheses and direct the estimation phase towards the optimal solution.

Genetic algorithms are used in the implementation of the proposed co-evolutionary strategy. The most important aspects of the methodology are described in the next sections. For more details and additional applications of EEA the reader can consult the work of Kouchmeshky *et al.* [4] and Bongard and Lipson [5, 8, 22, 23].

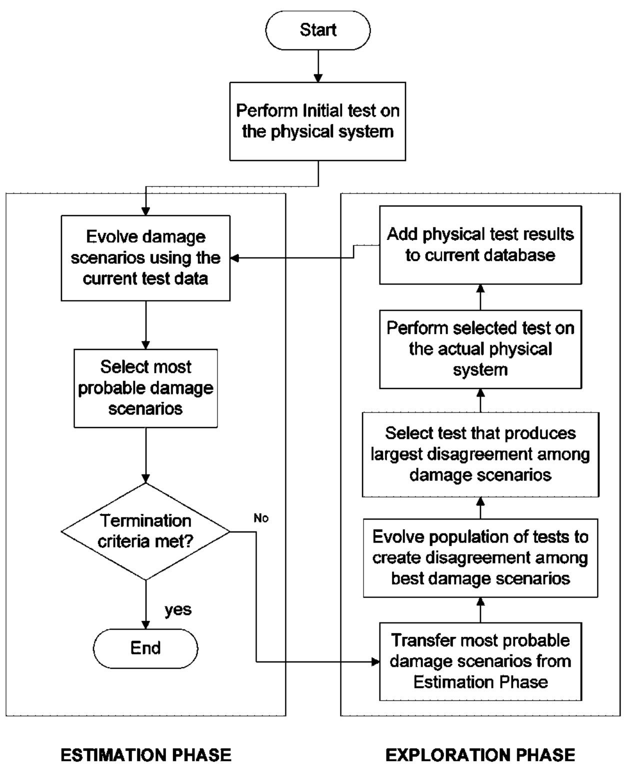


Figure 1. Flow chart for Estimation-Exploration Algorithm.



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### *3.1. Estimation phase*

The goal of the estimation phase of EEA is to find the best damage scenarios that can predict the data collected from current physical tests. Crucial components of the estimation phase are the encoding of test parameters in chromosomes, the selection of a suitable fitness measure, and maintaining diversity in the population of damage scenarios.

After the first estimation-exploration cycle is completed, the user has two options for the initial population of the estimation phase in subsequent cycles: (a) use the best individuals from the current population and randomly generate the rest of the population; and (b) carry the entire population of models throughout the entire co-evolution process and generate random individuals just for the first cycle. Approach (a) was used in this research. After the first estimation phase, the population at the end of a cycle was ranked by fitness (from best to worst) and the top 30% of the population was transferred to the next cycle. The other 70% of the population was then randomly generated. This is an elitism approach that guarantees that information learned from previous tests will be preserved, while the random generation of part of the population helps in exploring new areas of the search space and maintaining diversity. The elitist fraction of the population transferred to the next cycle (30% in the current case) is defined by the user. It was found in different simulation experiments that the algorithm is not sensitive to this parameter. The authors obtained satisfactory results when other elitist fractions (e.g. 50%) were used. Although the authors advocate approach (a) for structural damage identification, other researchers have used approach (b) successfully in other applications [5, 6, 8, 22].

*3.1.1. Solution encoding*. The encoding of solutions is one of the most important parts of GA and can have a great impact on the performance of this method. The simplest approach is to encode damage indexes for all elements in the structure. However, it has been shown by Chou and Ghaboussi [2] and Liszkai and Raich [9] that this is not a feasible approach. A more efficient method is to assume that the number of damaged elements in the structure is likely to be much smaller than the total number of elements and devise an encoding strategy accordingly. Chou and Ghaboussi used a solution representation based on binary strings in which only a small subset of the elements of the structure is assumed to be part of the solution. This approach, called the implicit redundant representation (IRR), produces much less variance in the solutions and has proven to be more reliable than searching for damage indexes in all elements at once.

As in IRR, the method used in this paper uses prior knowledge about the probability of having a small number of damaged elements. However, real-number chromosomes are used as opposed to the binary representation used by IRR. Each solution in the current approach is composed of two chromosomes. The first one, termed the 'identifier chromosome' hereafter, determines whether or not damage is present in an element, while the second one, called the 'damage chromosome' hereafter, encodes the damage ratios, $\alpha_l$.

Solutions are parsed by evaluating entries in the identifier chromosome first, and then those in the damage chromosome string. The present encoding method is illustrated in Figure 2. String 1, the identifier chromosome, determines if a specific element of the structure is damaged. It contains real numbers in the range $[0, 1]$. If the value of any gene is less than a predefined damage probability parameter, $m$, the corresponding element is assumed to be damaged. Otherwise, a damage ratio of zero (i.e. element undamaged) is assigned to the corresponding gene in String 2. The parameter $m$ is defined by the user and represents an approximation to the



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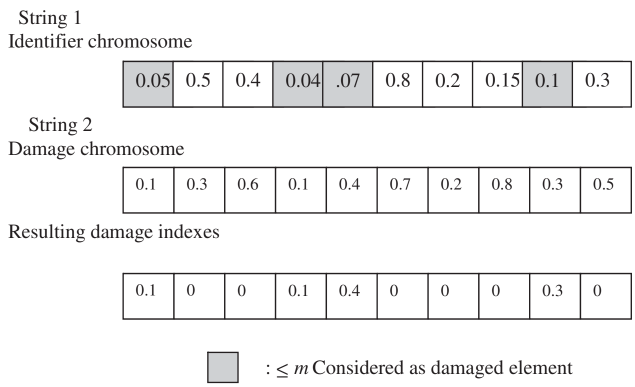


Figure 2. Example of solution encoding for damage indexes. For this example, there are 10 structural elements and $m = 0.3$.

expected number of damaged structural elements. Note that $m$ does not define the number of damaged elements; it just assigns a probability of having a certain number of damaged elements. A good value for $m$ would be an expected value of the number of damage elements in a structure given as a fraction of the total number of elements in the structure. String 2 contains the actual damage ratios, but only the genes for which damage has been identified in String 1 are active. It is important to realize that this method can still encode solutions ranging from no damaged element present to the extreme case of having all elements damaged. By using a damage probability parameter, $m$, the dimension of the search space is regulated such that there is a higher probability for the solution to lie in the range of the expected number of damaged elements.

*3.1.2. Maintaining diversity*. One of the key issues in EEA is to maintain diversity of the candidate models throughout the entire process [5]. Model diversity is critical in EEA since selection of good tests relies on their ability to create disagreement among different damage scenarios. Diversity also prevents loss of potentially good information during the estimation phase and helps the exploration stage by producing candidate solutions that carry different information about the damage state of the structure. Several methods have been proposed for maintaining diversity in GA. These methods are usually referred to as niching techniques and include crowding methods, neighbourhood methods, and fitness sharing, among others [24]. The deterministic crowding method was used in this work for maintaining diversity in the population of damage scenarios in the estimation phase. For details on this method the reader can consult Reference [24]. A simple two-point crossover and mutation were used in conjunction with the deterministic crowding method as genetic operators.

### *3.2. Exploration phase*

The goal of the exploration phase is to find a test that maximizes discrepancy among candidate solutions (i.e. damage scenarios) selected from the estimation phase. This task is cast as an optimization problem, which is also solved using a GA. The fitness of a test in the exploration phase is proportional to its ability to create disagreement among candidate solutions. The fitness



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function for tests used in this work is defined as

$$f_{\text{test}} = \sum_{i=1}^{r} \sum_{p=1}^{m} \sqrt{\frac{1}{z} \sum\nolimits_{a=1}^{z} [H_{ik}^{a}(\omega_p) - \bar{H}_{ik}(\omega_p)]^2} \tag{11}$$

where $z$ is the number of candidate models obtained from the estimation phase, $H_{ik}^{a}$ is the receptance function at DOF $i$ corresponding to model $a$, $\bar{H}_{ik}^{a}$ is the average receptance obtained from the candidate models at DOF $i$ for a given frequency. Equation (11) can be interpreted as the summation of the standard deviations of receptance at each DOF for all selected candidate solutions. As mentioned before, this fitness serves as a measure of how well the test can create discrepancy among the candidate solutions transferred from the estimation phase.

The population of tests is randomly generated at the beginning of the exploration phase. Then, selection, crossover, and mutation are applied over a number of generations. At the end of the evolution process, the test genome with the highest fitness is selected and is implemented in a physical experiment. The results from this physical experiment are added to the existing bank of data, and the estimation phase is invoked for the next cycle of the algorithm.

*3.2.1. Test encoding and genetic operators for the exploration phase.* Different strategies may be adopted for encoding tests during the exploration phase. These strategies depend on the type of excitation (e.g. static *vs* dynamic tests) and quantities being measured. Whatever test definition is used, an important issue to always consider is the conservation of building blocks in the encoding scheme in order to maximize the effectiveness of the evolution process. For instance, the encoding should assure that blocks of sensors and forces are transferred between individuals during crossover. The reason for this is that sensor locations that can detect localized damage depend on the forces acting on the structure.

For simplicity in demonstrating the feasibility of the method, it will be assumed in the numerical examples shown in this article that the force amplitude, the excitation frequency range, and the number of sensors are fixed. In this work, an individual test is defined by a chromosome composed of genes with integer values, which define the position of the excitation force and the location of sensors. For a more detailed description of test encoding in damage detection see Reference [4].

As in the estimation phase, individuals are paired randomly, without replacement, to reproduce. The genetic operators used in the exploration phase were a simple two-point crossover and a simple mutation that swaps the location of genes in a chromosome based on a probability parameter defined by the user. Elitism was used in the exploration phase by transferring the best individual to the next generation.

### *3.3. Stopping criteria*

The proposed algorithm terminates when one of the following conditions is met.

- Only one of the damage scenarios in the estimation phase can predict all the current test results obtained. This indicates that a potentially good solution has been found.
- Diversity is not maintained. At least two different individuals need to be transferred from the estimation phase to the exploration phase. Loss of diversity may be due to various factors such as inadequate parameters in the niching method, size of the population, number of generations, etc.



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- The exploration phase cannot find a test that causes disagreement among candidate solutions. In this case, the algorithm fails to find a unique solution. This situation may indicate that the problem is not fully observable (e.g. lack of sensitivity of the structural response to the damage parameters).

## 4. NUMERICAL EXAMPLES

The feasibility of the methodology is demonstrated in this section through several numerical examples. The structure used for the examples is a truss bridge subjected to harmonic excitation. EEA is compared to a control algorithm in which load position and sensor positions for each test are selected randomly. This comparison is used to show the advantages of EEA over the trivial approach of random test selection. In addition, the performance of EEA is compared to a simple structural damage identification strategy that consists of performing only one structural test and measuring the frequency response at all DOFs in the truss structure.

The four-span continuous truss used has 32 nodes and 75 elements. Six damaged elements were introduced and were located in one span of the structure, as shown in Figure 3. This element configuration was selected because it presents a more difficult damage identification problem than having elements scattered in the structure. Ghaboussi and Chou [2] demonstrated that damage identification in truss structures becomes more difficult when elements are connected at a node and only a limited number of DOFs are measured. The induced damage ratios in the simulated tests are shown in Table I. The Young's modulus and cross-sectional area used for the undamaged truss members were 200 GPa and 2500 mm$^2$, respectively.

In the examples presented herein, tests were defined by encoding the load and sensor locations in integer-valued chromosomes. That is, the range of excitation frequencies and the number of sensors remained fixed for all tests in each of the investigated cases. It was assumed that an

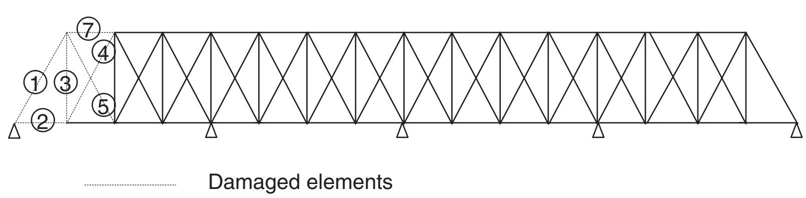


Figure 3. Truss bridge example.

Table I. Damage ratios used in truss example.

| Element label | Damage ratio, $\alpha$ |
|---|---|
| 1 | 0.35 |
| 2 | 0.30 |
| 3 | 0.40 |
| 4 | 0.25 |
| 5 | 0.30 |
| 7 | 0.20 |



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excitation force was applied in the vertical DOF at one node in the truss, while displacements in both DOFs were measured at different nodes. The GA parameters used in this problem are given in Table II. These parameters were selected based on common heuristics found in the GA literature (see Reference [21]). Because of the stochastic nature of GA, 10 runs were carried out for each example.

To study the sensitivity of the EEA to noise, uniformly distributed random noise was added to the receptance function in the simulated tests as

$$H_{ij} = H_{0ij}(1 + \beta e) \tag{12}$$

where $\beta$ is the noise-to-signal ratio, $H_{0ij}$ is the receptance function in the simulated tests without the effect of noise, and $e$ represents a uniformly distributed random variable in the range $[-1, 1]$.

### 4.1. Results and discussion

Four different cases were studied, each of which considered a different number of sensors in the test definition. In addition, two noise levels were used to study the sensitivity of the proposed algorithm to imperfect measurements and model errors. The cases studied included: (a) one load and one sensor; (b) one load and three sensors; (c) one load and five sensors; and (d) one load and all DOFs sensed (no test evolution in this case).

Figures 4(a) and (b) show the average damage indexes (averaged over 10 computer runs) found after 10 tests using the control algorithm and EEA, respectively. In this case only one load and one sensor with 10% noise were used for each test. It can be observed that both algorithms were able to find the target damaged elements after 10 tests. However, it can be observed from these plots that EEA outperformed the control algorithm in two main aspects. First, EEA produced fewer misidentifications than the control algorithm. Misidentifications are defined as non-damaged elements for which the algorithms produce damage indexes greater than zero. Second, EEA was more accurate in identifying the correct damage indexes for elements with damage ratios greater than zero.

When discussing co-evolutionary strategies it is useful to define metrics to monitor the performance of the algorithms. The error defined in Equation (10) will be called 'the subjective error' from hereon. This metric provides information about the ability of the individual solutions to predict the observed experimental data. However, subjective error may be deceiving, especially in the early stages of the algorithm, as different damage scenarios may predict the observed experimental data equally well due to the ill-posedness of the problem. For this reason, an additional metric, called the objective error, is used and it is defined as the

Table II. GA parameters used for truss example.

| | |
|---|---|
| *Estimation phase* | |
| Population size | 100 |
| Crossover probability (%) | 100 |
| Mutation probability (%) | 10 |
| Maximum number of tests (cycles) | 10 |
| Number of generations | 300 |
| *Exploration phase* | |
| Population size | 60 |
| Number of generations | 100 |



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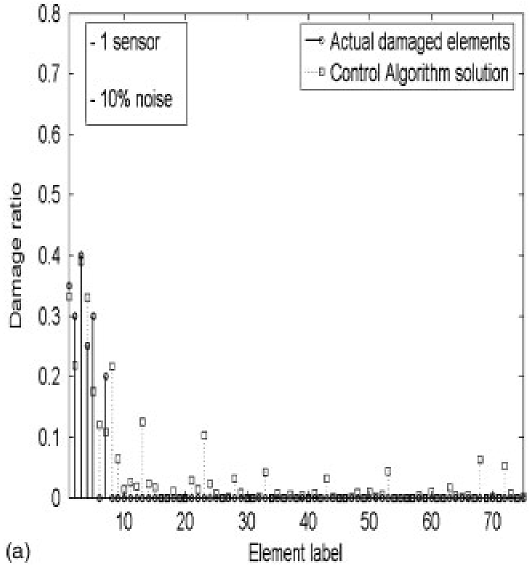


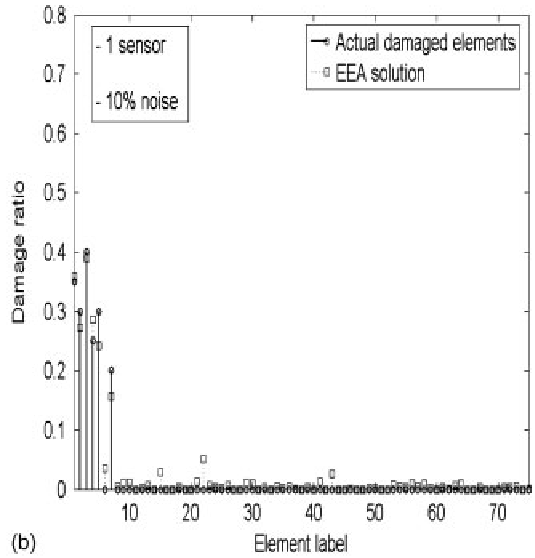


Figure 4. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to one sensor and 10% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.

Euclidian distance between the solution with lowest subjective error at the end of each estimation phase and the true damage state of the structure. Mathematically, this metric is defined as

$$E_{\mathrm{obj}} = \|\boldsymbol{\alpha}_{\mathrm{sol}} - \boldsymbol{\alpha}_{\mathrm{true}}\| \tag{13}$$

where $E_{\mathrm{obj}}$ is the objective error, $\boldsymbol{\alpha}_{\mathrm{sol}}$ is a vector whose entries are the damage indexes of the solution with lowest subjective error at the end of the estimation phase, and $\boldsymbol{\alpha}_{\mathrm{true}}$ is a vector whose entries are the true damage level induced in the simulated experiments. It is important to realize that this metric is not known in real-life scenarios and was used here for the sole purpose of monitoring the progress of the EEA and its convergence behaviour towards the true solution.

The subjective error *versus* number of tests for the control algorithm and EEA are shown in Figure 5(a), while the corresponding objective error *versus* tests is shown in Figure 5(b). These results correspond to the case of one sensor and 10% noise. The average of 10 computer runs is shown by the solid lines, while the error bars indicate standard deviation. An interesting behaviour is observed in Figure 5(a). For EEA, the subjective error increases with number of tests and then decreases, while for the control algorithm the subjective error decreases and then increases slightly. This behaviour can be attributed to the presence of noise in the data and the higher information content of tests selected by EEA. Note that due to the presence of noise in the data, the true solution will never yield a zero subjective error. Therefore, the decrease in subjective error displayed by the control algorithm indicates over fitting of the noisy results as the random tests generated by this strategy seem to fail to bring new information to distinguish noise from structural damage. On the other hand, the EEA solution moves towards a non-zero subjective error, which can be due to the effect of noise and the fact that the tests selected by this



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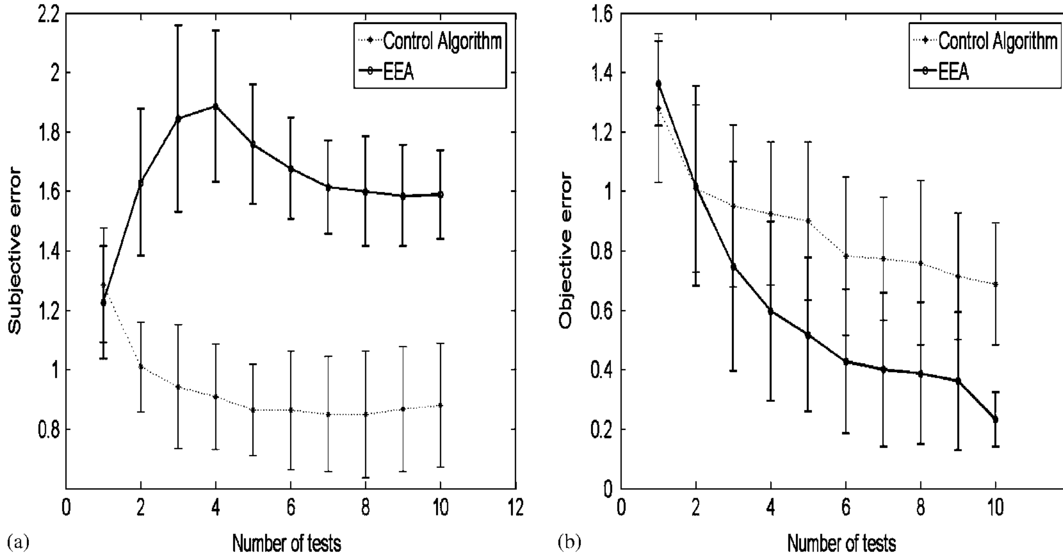


Figure 5. Comparison between performance of EEA and control algorithm using one sensor and 10% noise: (a) control algorithm and (b) Estimation-Exploration Algorithm.

algorithm contain more information. Note that a higher subjective error indicates that the best current solution could not produce a structural response that perfectly matches the experimental data. Then, after the fourth test the subjective error decreases and seems to asymptotically approach a non-zero value, which is a desired behaviour that prevents overfitting of the noise. This behaviour stems from the co-evolutionary basis of EEA, which strives to produce tests that challenge current damage scenarios, hence, elucidating new and useful information about the system [8].

The foregoing argument is further supported by the objective error behaviour shown in Figure 5(b). It can be observed that the solution found by EEA moves at a faster rate towards the true solution (i.e. zero axis) than the solution found by the control algorithm, indicating that tests suggested by EEA carry more information about the true damage state than those that are selected randomly.

In order to test the tolerance of EEA to high noise-to-signal ratios, the noise level was increased to 30% and the identification process was repeated using EEA and the control algorithm. The average damage ratios for all elements at the end of 10 tests found by EEA and the control algorithm are shown in Figures 6(a) and (b), respectively. It can be observed from these figures that both algorithms produce more misidentifications at this high level of noise, but again, EEA produces fewer misidentifications than the control algorithm. In addition, the accuracy in predicting the damage index degrades in both algorithms as the level of noise is increased, as expected. However, EEA produced more accurate damage indexes than the control algorithm as was found for the 10% noise case.

Common sense may dictate that using more sensors in damage identification problems would result in more accurate solutions since more information is gathered in each test. However, common sense would also dictate that placement of the sensors would also be crucial if the testing process is to be optimal. In order to test the effect of using multiple sensors on the



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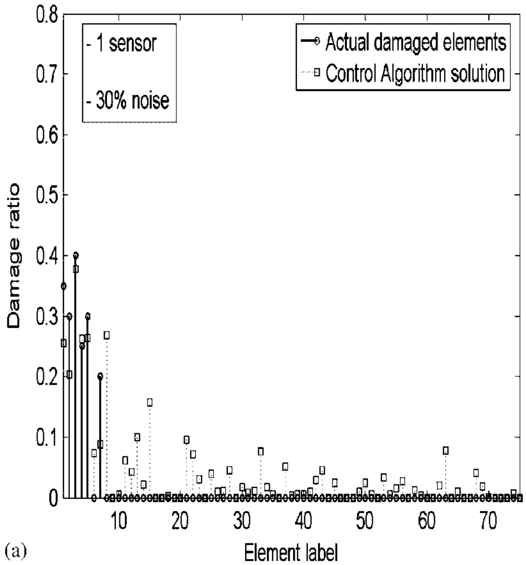
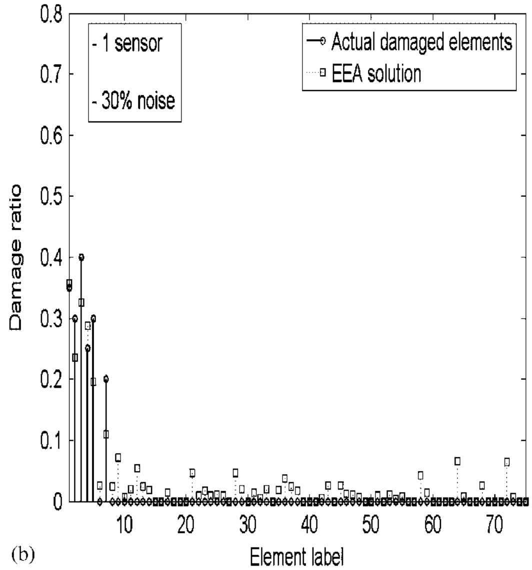



(a)




(b)

Figure 6. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to one sensor and 30% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.

performance of EEA, the identification process was repeated using three and five sensors. The reader should keep in mind that tests are defined in each case by the location of the excitation force and the locations of the sensors (the number of sensors was fixed for each case).

The average damage indexes found at the end of 10 tests for cases when three sensors and five sensors were used are shown in Figures 7 and 8, respectively. A 10% noise-to-signal ratio was introduced in the simulated tests. It can be observed from these plots that EEA again produced fewer misidentifications and more accurate damage indexes than the control algorithm when three and five sensors were used. An interesting trend is observed in these plots as compared to the case when one sensor was used. The accuracy of both EEA and the control algorithm degraded as more sensors were used. This trend in the results can be explained as follows. By introducing more sensors, the optimization problem becomes more difficult, which can be realized by noticing that the complexity of the error surface in the estimation phase defined by Equation (9) increases as more sensors are added. In addition, the search space of structural tests increases exponentially as the number of sensors increases. Therefore, it can be expected that the algorithms would have more difficulty in finding the global optimum as the complexity of the error surface increases. It is important to bear in mind that we cannot conclude in general that using fewer sensors would yield better solutions, and it is reasonable to expect that an optimum number of sensors must exist. More studies related to this issue are needed before any definite conclusion can be drawn.

The runs with three and five sensors were repeated adding 30% noise to the simulated tests. The results are shown in Figures 9 and 10. The trends observed before held for these cases. First, EEA outperformed the control algorithm in producing fewer misidentifications and more



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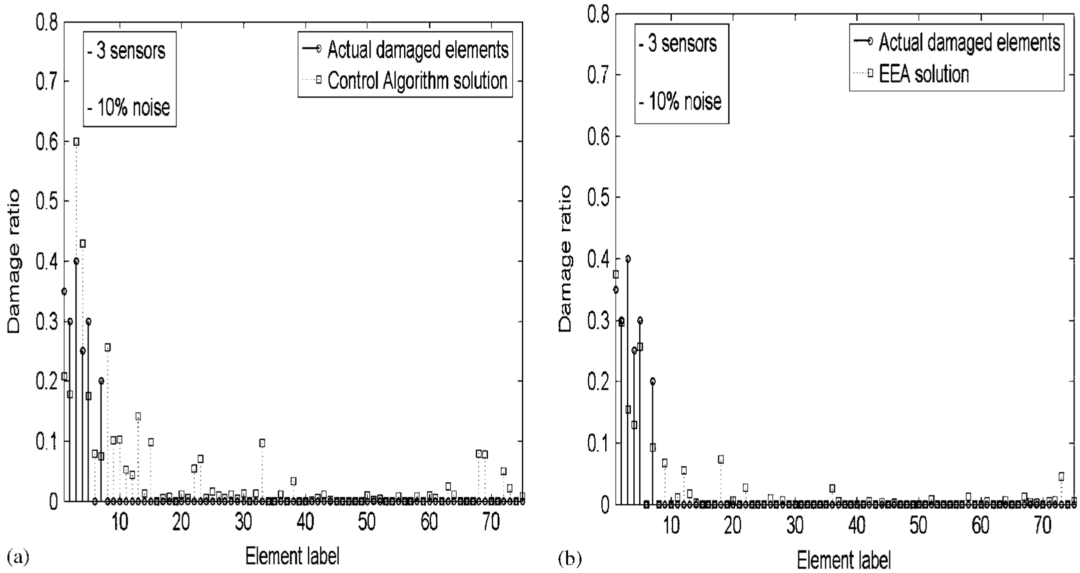


Figure 7. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to three sensors and 10% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.

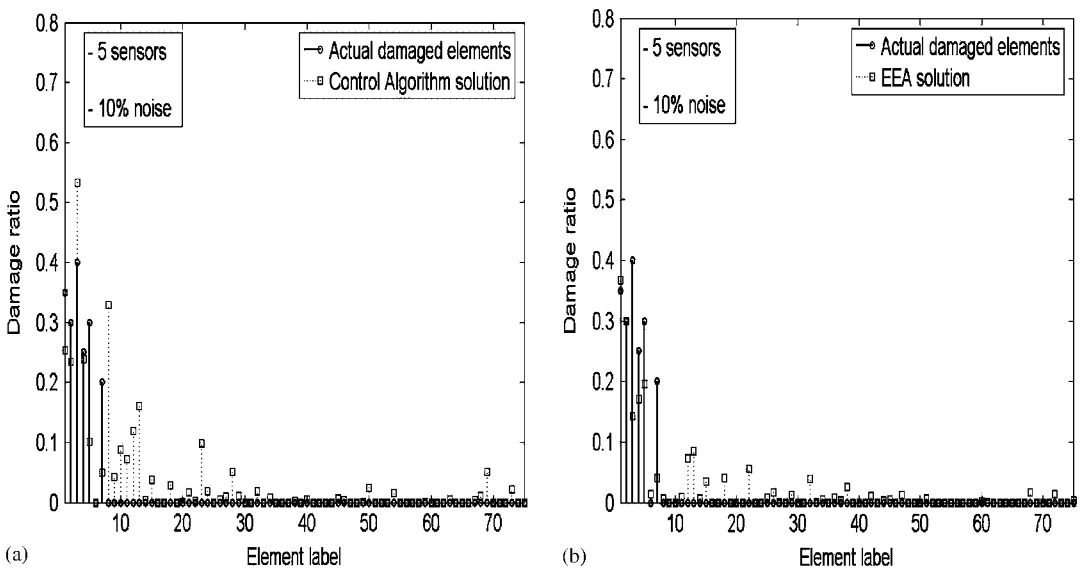


Figure 8. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to five sensors and 10% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.



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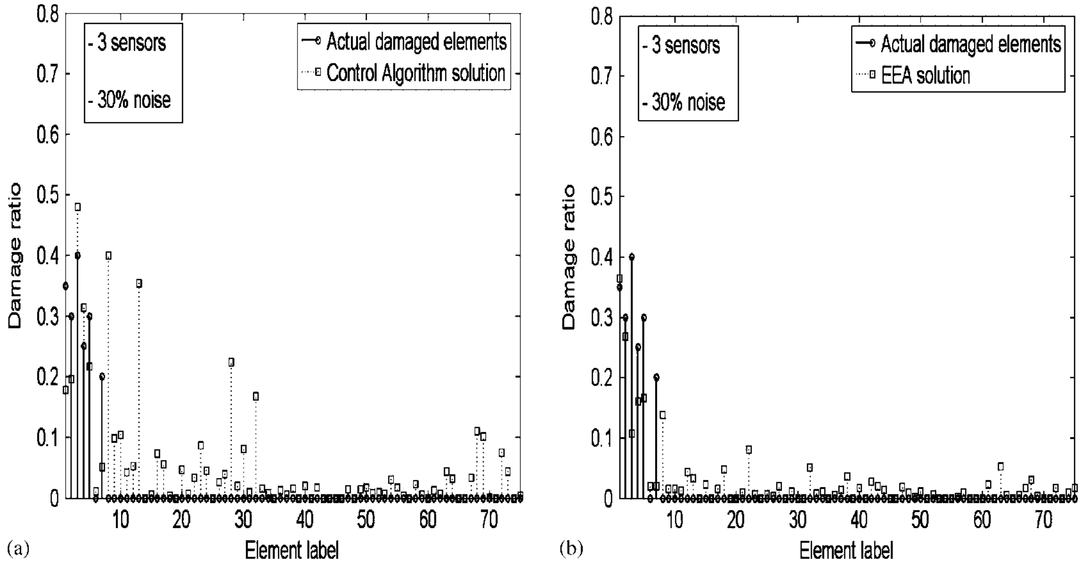


Figure 9. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to three sensors and 30% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.

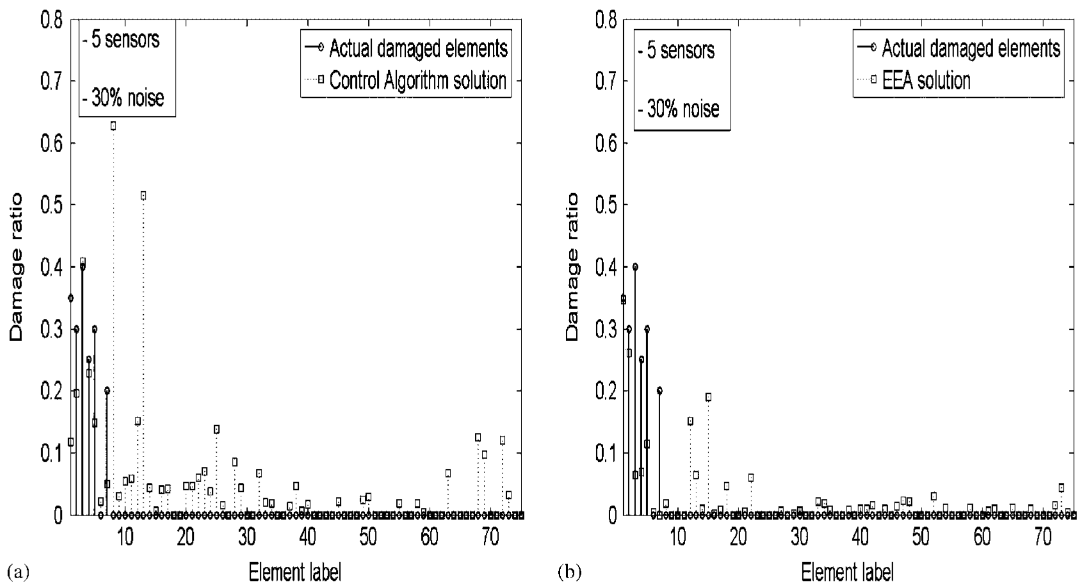


Figure 10. Average of estimated damage indexes using the EEA and control algorithm at the end of 10 tests. Case corresponding to five sensors and 30% noise used for the simulated physical tests: (a) control algorithm and (b) Estimation-Exploration Algorithm.



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accurate damage indexes. Second, the accuracy of the algorithms degraded as more sensors were used.

A common approach to structural health monitoring is to perform one test in which a large amount of data is collected and then to carry out damage identification using these data. In order to compare the performance of EEA to this common approach, a test was simulated in which all the DOFs of the truss were measured, as shown in Figure 11. The location of the excitation force for this test is shown in the same figure. A GA with the same encoding and parameters as those used for the estimation phase of EEA were used for the single-test damage identification case (see Table II). The algorithm was run for 3000 generations to obtain as many function evaluations as those used by EEA in 10 tests. A 10% noise-to-signal ratio was used in this case. The average results from 10 runs are shown in Figure 12. The results show that the single-test strategy was able to identify all damaged elements. However, EEA in all the previous cases with 10% noise produced more accurate damage indexes in general than the single-test approach as can be noticed by the large misidentified damage indexes produced by the latter.

*4.1.1. Evolution of tests.* Figures 13 and 14 illustrate an example of the sequence of tests and best damage scenarios found by EEA when three sensors were used with 10% noise. It can be observed by the third test that only three of the six damaged elements could be found, which is a

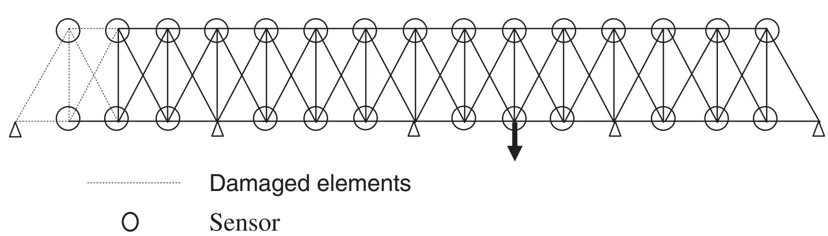


Figure 11. Force and sensor layout for the single-test case.

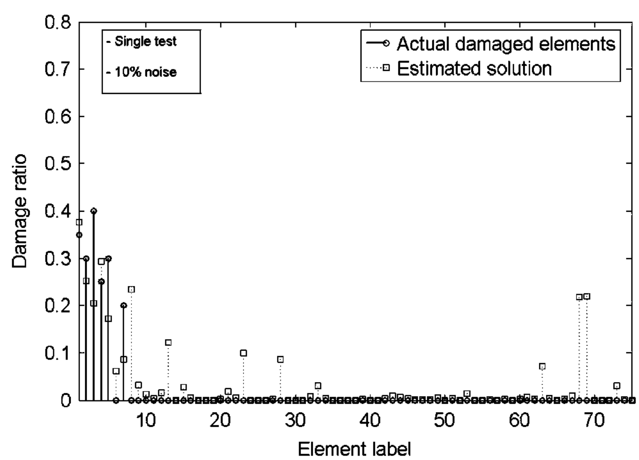


Figure 12. Average damage ratios found using a single-test approach.



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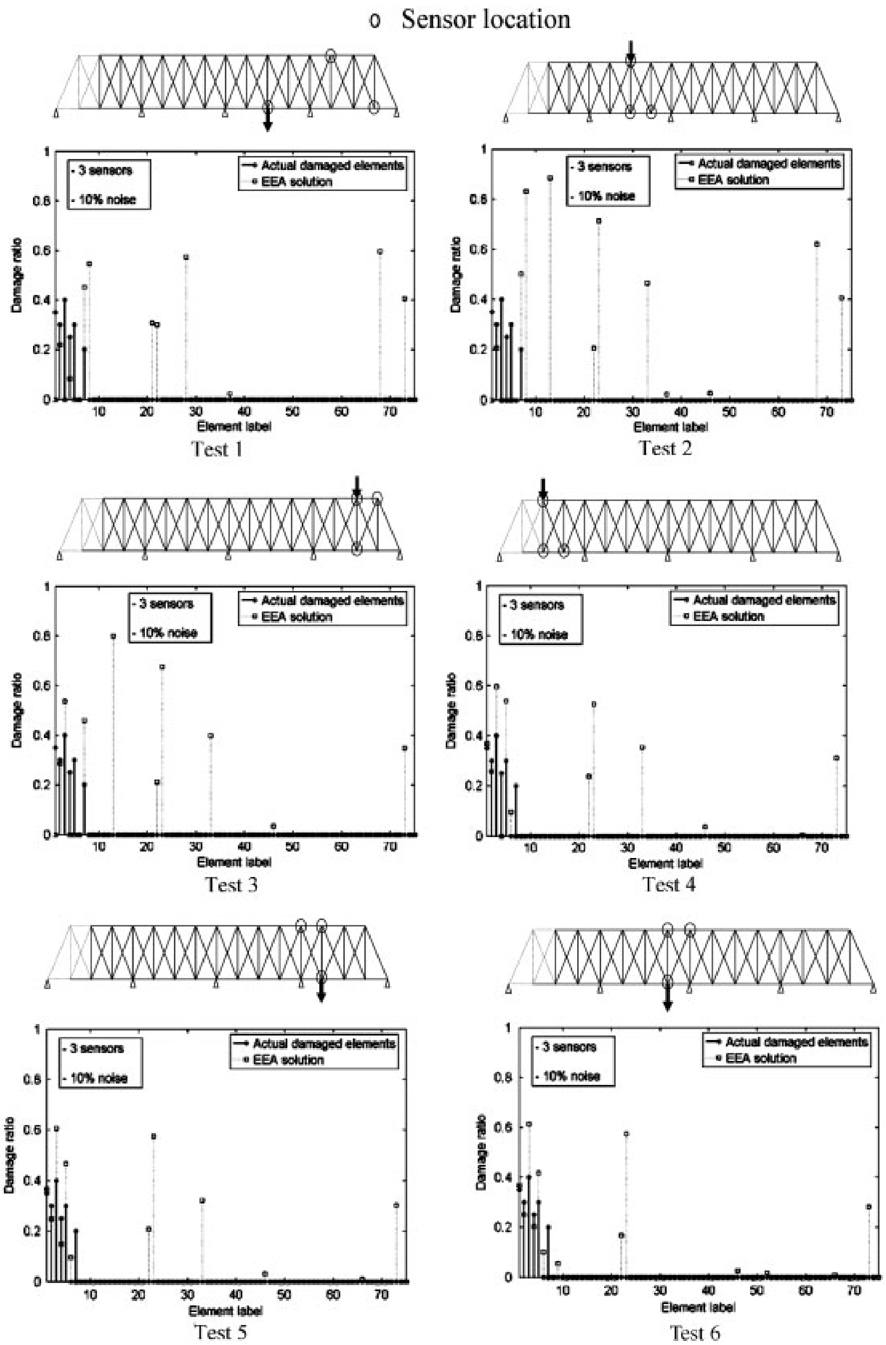


Figure 13. Sequence of tests and damage scenarios selected by EEA for a case with three sensors and 10% noise. Tests 1–6.



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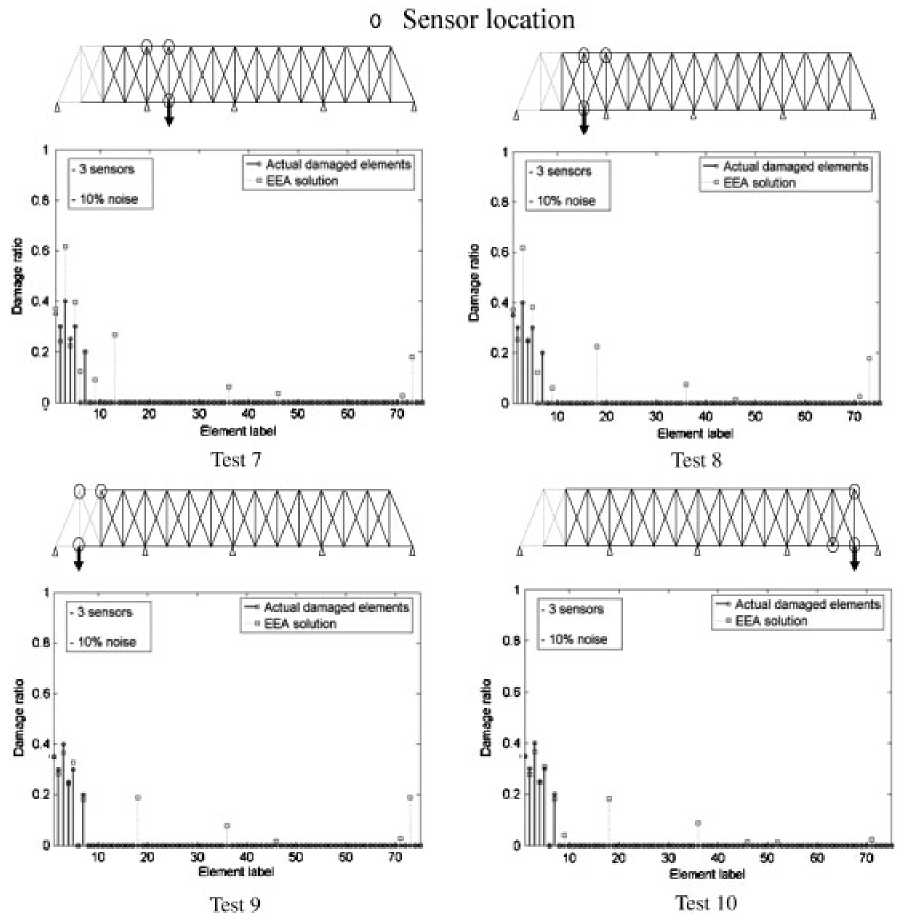


Figure 14. Sequence of tests and damage scenarios selected by EEA for a case with three sensors and 10% noise. Tests 7–10.

clear indication of the ill-posedness of the problem. The information gathered in the fourth test significantly improved the identification process as four out of the six damaged elements could be identified by the algorithm at this point, but the accuracy of the damage indexes was still low and there were several misidentified elements. As the testing continued, it can be noticed that the accuracy of the damage indexes improved significantly and the number of misidentified elements decreased.

By the 10th test, the six damaged elements and their damage indexes were accurately identified, and only three misidentifications with low damage indexes persisted. Some misidentifications are always expected due to the noise in the data. It is important to keep in mind that the sequence of tests shown in Figures 13 and 14 is non-unique as it depends on the best damage scenarios found by EEA in each cycle. If the algorithm were executed again, holding the initial test population constant, a different sequence of optimal tests would be expected, but the goal defined in the exploration phase would still be satisfied.



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An interesting trend can be noticed in the sequence of tests shown in Figures 13 and 14. It can be observed that sensors were always grouped around the load location. This result is a consequence of the goal defined in the exploration phase of maximizing the standard deviations of the frequency response produced by candidate damage scenarios at measured DOFs. For the structure used in this example, maximum response usually occurred near the point of loading and the nodes around it. Therefore, the absolute maximum value of the function shown in Equation (11) was consistently produced by the configuration of sensors selected by the algorithm. Other possible sensor configurations could be obtained by using other fitness functions in the exploration phase.

The evolution of tests in this work was limited to a fixed number of sensors, loads, and frequencies. In general, the proposed algorithm can be used to evolve more general tests with variable excitation frequency range, variable number of sensors, and multiple load components. This is a very desirable capability that is currently being explored by the authors. It is important to realize that this next step is not trivial since, as the number of parameters for the test increases, the complexity of the optimization problem increases and more computational effort would be required to find good tests in the exploration phase. In addition, close attention should be paid to how tests are encoded in the exploration phase.

## 5. GENERAL REMARKS

The implementation of EEA presented in this paper used GA as the main optimization tool. However, it is important to recognize that EEA is general and can be implemented through other optimization techniques. In addition, it is recognized that because of the use of GA, a very large number of function evaluations were required in the examples presented herein. This large number of function evaluations was possible due to the simple nature of the structures considered. For more complex cases such as continuum structures modelled with finite or boundary elements, it is still possible to apply the EEA algorithm. This may be achieved, for instance, by using reduced-order modelling approaches such as proper orthogonal decomposition [25] and surrogate model strategies [26].

Another important point to bear in mind is that random Gaussian noise may not be the best way to test the robustness of an algorithm to the presence of data imperfection. Therefore, only real experimental data will dictate how robust the proposed strategy is. The authors intend to develop an experimental program to validate the co-evolutionary algorithm presented herein. Furthermore, the influence of environmental effects such as temperature and humidity changes on the effectiveness of the proposed identification algorithm will be studied.

## 6. CONCLUSIONS

A new methodology for active structural damage identification called the Estimation-Exploration Algorithm (EEA) was presented in this paper. The proposed methodology was described in the context of steady-state dynamics and using truss structures. EEA was compared to a control algorithm, in which tests were selected randomly, and to a typical structural health monitoring strategy, in which a single test was performed. In all the studied cases, it was found from numerical simulations that EEA was more accurate in identifying damage indexes and



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produced fewer misidentifications than the control algorithm and the single-test strategy for all cases considered. In addition, through the use of the objective error measure, it was shown that EEA approaches the true solution at a faster rate (with fewer tests) than the control algorithm. The effect of high noise to signal ratios on the performance of EEA was also studied. It was found that EEA was more resilient to high levels of signal to noise ratio than the control algorithm by producing fewer misidentifications and more accurate damage indexes.

As more sensors were used in the structural tests, it was found that the accuracy of the solution degraded for both EEA and the control algorithm. However, EEA still outperformed the control algorithm by producing fewer misidentifications and more accurate damage indexes. The loss of accuracy in the results as the number of sensors increased was explained by noticing that the optimization problem becomes more difficult. That is, the search space for tests becomes larger and the error surface in the estimation phase becomes more complex. Further studies are needed to investigate this effect and draw more definite conclusions about optimality of the number of sensors used.

The results presented in this paper demonstrate the feasibility of EEA, but are still at the proof of concept level. Future research by the authors will expand the capabilities of EEA to evolve a variable number of sensors, excitation frequencies, and loads. Furthermore, the authors will validate the proposed methodology through laboratory and field studies.

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