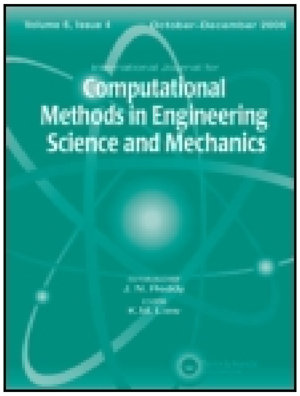

International Journal for Computational Methods in Engineering Science and Mechanics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/ucme20

Structural Optimization of Rotating Disk Using Response Surface Equation and Genetic Algorithm

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Accepted author version posted online: 23 Oct 2012.Version of record first published: 13 Feb 2013.

International Journal for Computational Methods in Engineering Science and Mechanics, 14:124–132, 2013

ISSN: 1550-2287 print / 1550-2295 online
DOI: 10.1080/15502287.2012.698712

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This study is focused on the structural optimization of an axial flow compressor rotating disk in a more simplistic way. Since optimization involves several runs, performing Finite Element Analysis (FEA) for a disk model in each run increases computational cost. Alternative response surface design models have been developed using the design of experiment (DOE) method to represent the FE disk model. These response equations are validated and used in optimization runs. A constraint nonlinear optimization procedure based on genetic algorithms has been used. FEA is completely avoided during the optimization runs with the help of response equations, resulting in less computational time and cost.

Keywords Rotating disk, Optimization, Genetic algorithm, Finite element analysis (FEA), Design of experiment (DOE), Response surface equation

1. INTRODUCTION

A rotating disk is a crucial component of any rotating machinery and has a wide variety of applications, such as in household appliances, medical equipment, machine tools, automobiles, airplanes, space vehicles, nano machines, etc. Success in a sophisticated structural analysis is measured by providing a positive, but near zero, structural margin of safety. For this purpose, an optimization technique is used. The optimization technique modifies and improves the design of rotating disks, if they have negative margins of safety.

Timoshenko [1] obtained a closed form solution for a homogeneous rotating disk. Leopold [2] calculated elastic stress distributions in rotating disks with variable thickness by using a semi-graphical method. Design of axially symmetric, rotating disks for uniform strength has been discussed by Ranta [3]. Chern et al. [4] showed an optimal design of a rotating disk for given radial displacement of edge. An interactive optimization procedure was applied to the design of a gas turbine disk by Luchi et al. [5]. Numerical solutions have been proposed taking into account centrifugal loads and frequency constraints by De Silva [6]. Shu-Yu et al. [7] used sequential linear programming methods for optimization of rotating disk profiles based on employing gradients of constraints and objective function. Also, in a different approach, Cheu [8] optimized a rotating disk profile by combining the finite element method with the feasible direction method and sequential linear programming for the optimization procedure. Lautenschlager et al. [9] used a design of experiments method to design a flywheel by employing the response surface shape optimization. Jahed et al. [10] optimized the inhomogeneous disk by subdividing into a number of finite ring elements, and taking ring thickness as design variable.

There is a growing interest in the use of genetic algorithms in structural optimization. Genta [11] has evaluated the possibility of application of genetic algorithm technique to the shape optimization of rotating disks. Rao et al. [12] used a constraint nonlinear optimization procedure based on genetic algorithms for designing rolling element bearings. Feng et al. [13] presented an optimization of rotor shafts by combining the genetic algorithm with simulated annealing. Despite using evolutionary GA for optimization, the reduction of computation involved in analyzing the disk is necessary to decrease the cost. Song et al. [14] coupled a real-coded genetic algorithm with a kriging surrogate model in order to reduce computational cost without sacrificing the ability of the GA in finding the global optimum. Gutzwiller et al. [15] focused on disk design with development of a one-dimensional plane stress model, disk parameterization methods, and implementation of a genetic algorithm for shape optimization. Huang et al. [16] presented a design optimization method based on kriging surrogate models, proposed and applied to the shape optimization of an aero engine turbine disk. Zeno [17] proposed to structurally optimize the cross-section of a spinning disk using a genetic algorithm and Latin Hypercube Sampling.

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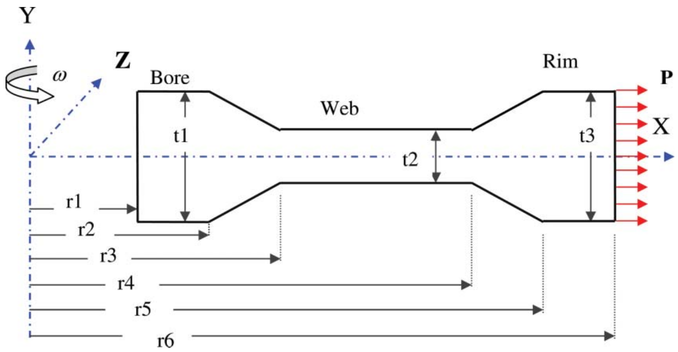

FIG. 1. Cross-section of rotating disk (axisymmetric model). (Color figure available online.)

In this paper, a study of optimization of a simple, axial flow compressor rotating disk is presented. So far, the references mentioned above do not account for a simplified model, except Song et al. [14] and Hang et al. [16]. Even though they used kriging surrogate models, FE analysis was not completely avoided in the algorithm. In the present work, FE analysis is completely avoided in optimization runs by the use of a robust alternative simplified model. A simple response surface design model, representing a rotating disk, is developed as an alternative to FEA model. A genetic algorithm is coupled with a response surface design model of a rotating disk in order to reduce computational cost. The schematic diagram of the rotating disk design problem (Figure 1) has been formulated and a procedure is presented to solve the constrained nonlinear optimization problem.

A total of six design variables have been considered, out of which two design parameters are the bore and web thicknesses. The other four design variables are inner and outer radius of bore, and inner and outer radius of web. The area of disk, maximum hoop stress, and radial stress have been derived in terms of design variables with the help of FEM analysis (using ANSYS). The output responses and the design variables are used to derive the response surface equations using MINITAB trial software. In the present study, second-order functions are used to obtain the nonlinear and flexible response surface equations for better approximation. These response equations are used in the optimization process.

Simple binary-coded GA has been developed in the MATLAB environment for optimization. To adapt suitable GA parameters, a study of GA parameters has been done. New adapted parameters resulted in significant improvement in the result. Constraints have been handled by penalizing the violated solution. Penalization is achieved here using a simple penalty approach. This GA approach in optimizing the disk has resulted in significant reduction in the weight of the disk.

The main advantage of the methodology presented in this paper (Finite Element Analysis) is completely avoided during optimization runs with the help of response surface equations, which are derived before starting the optimization run. This leads to less computational time and cost.

2. THEORETICAL BACKGROUND

2.1 Mathematical Formulation of Structural Analysis

2.1.1 The Stress Field in Rotating Disk

A simple analytical formula can be used for the analysis of a rotating disk of uniform thickness (Figure 2). The stresses generated through the thickness are neglected, since they are assumed to be small in comparison with the in-plane radial and tangential stresses.

Using Airy's stress function and applying boundary conditions$(\sigma_r)_{r=a} = (\sigma_r)_{r=b} = 0$, radial and hoop stresses can be expressed as [18]:

$$\sigma_r = \frac{3+\nu}{8}\rho\omega^2\left(a^2 + b^2 - \frac{a^2b^2}{r^2} - r^2\right) \tag{1}$$

$$\sigma_\theta = \frac{3+\nu}{8}\rho\omega^2\left(a^2 + b^2 + \frac{a^2b^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right) \tag{2}$$

Where ρ and ν are the respective material density and Poisson's ratio, r is the disk radius, and ω is the disk rotational speed.

The above expressions do not account for thermal loadings. In such cases, the boundary conditions should be modified to reflect the thermal stresses. The main structural design requirements for the rotating disk are that the factor of safety based on yield strength and ultimate strength are 1.1 and 1.5, respectively.

2.1.2 Finite Element Formulation

With the availability of many powerful finite element packages, it was felt unnecessary to develop solution programs for the current study. Commercial software ANSYS, a finite element package, is used to model and analyze the disk.

PLANE42 element is used for 2-D modeling of axisymmetric solid rotating disk structure. The element is defined by four nodes having two degrees of freedom at each node, and they are translations in the r and z directions, respectively. A two-dimensional axisymmetric analysis of disk is based on a full 360-degree rotation. A 4-node quadrilateral element is as shown in Figure 3 and its FE formulations are given below [19]:

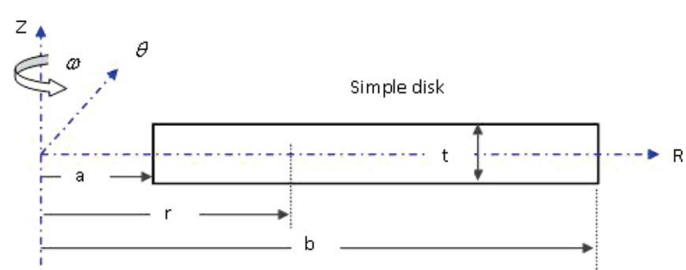

FIG. 2. The simple disk geometry. (Color figure available online.)

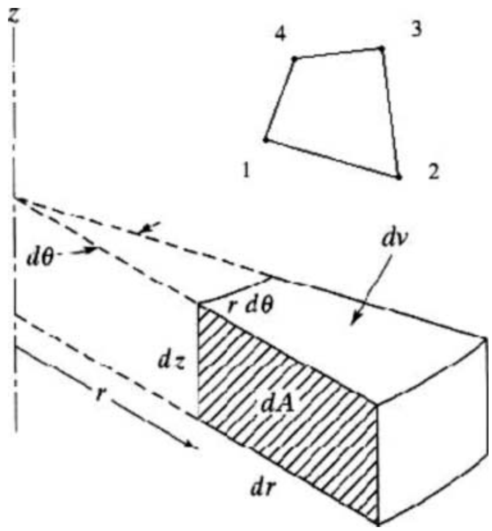

FIG. 3. Axisymmetric element.

Stress strain relationship is given by

$$\{\sigma\} = \begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \times \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{zr} \end{Bmatrix} = [D]\{\varepsilon\} \qquad (3)$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{zr} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} \frac{\partial N_i}{\partial r} & 0 \\ 0 & \frac{\partial N_i}{\partial z} \\ \frac{N_i}{r} & 0 \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} \end{bmatrix} \begin{Bmatrix} u_i \\ w_i \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{\partial N_1}{\partial r} & 0 & \frac{\partial N_2}{\partial r} & 0 & \frac{\partial N_3}{\partial r} & 0 & \frac{\partial N_4}{\partial r} & 0 \\ 0 & \frac{\partial N_1}{\partial z} & 0 & \frac{\partial N_2}{\partial z} & 0 & \frac{\partial N_3}{\partial z} & 0 & \frac{\partial N_4}{\partial z} \\ \frac{N_1}{r} & 0 & \frac{N_2}{r} & 0 & \frac{N_3}{r} & 0 & \frac{N_4}{r} & 0 \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial z} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial z} & \frac{\partial N_3}{\partial r} & \frac{\partial N_4}{\partial z} & \frac{\partial N_4}{\partial r} \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \\ u_4 \\ w_4 \end{Bmatrix} = [B]\{d\} \qquad (4)$$

Where N_i are shape functions of the element.

The stiffness of the element is given by

$$K^e = \iiint_{z\,r\,\theta} B^T D B d\theta dr dz = 2\pi \iint_{r\,z} r B^T D B dr dz \qquad (5)$$

$$f_c = 2\pi \iint_{r\,z} r N^T F_c dr dz; where,\ F_c = \begin{Bmatrix} f_c^r \\ f_c^z \end{Bmatrix} = \begin{Bmatrix} \rho\omega^2 r \\ 0 \end{Bmatrix},$$

is centrifugal force. (6)

$$f_t = 2\pi \oint_\Gamma r N^T F_t ds;$$ where Ft is traction force or surface loading. (7)

FE solution is obtained using the well-known global equation:

$$\{\mathrm{d}\} = [\mathrm{K}]^{-1}\{\mathrm{f}\}. \qquad (8)$$

Where {d} is displacement vector, [K] is global stiffness matrix, and {f} is global force vector.

2.2 Response Surface Design

Response surface methods are used to examine the relationship between one or more response variables and a set of quantitative experimental variables or factors. This method requires an identification of a few vital controllable factors, called design variables. The relationship between the response variable y and independent variables is usually unknown. Depending on the approximation of unknown function, either first-order or second-order models are employed. When there is a curvature in the response surface, the first-order model is insufficient. A second-order model is useful in approximating a portion of the true response surface with parabolic curvature. Using a second-order model with q number of design variables, a single response y can be expressed as follows [20]:

$$y = \beta_0 + \sum_{j=1}^{q} \beta_j x_j + \sum_{i=1}^{q} \sum_{j=1}^{q} \beta_{ij} x_i x_j + \varepsilon \qquad (9)$$

The response y is a function of the design variables x_1, $x_2, \ldots, x_q$, the experimental error ε, and estimated coefficients $\beta_0, \beta_j, \beta_{ij}$. The second-order model is flexible, because it can take a variety of functional forms and approximates the response surface locally. Therefore, this model is usually a good estimation of the true response surface.

2.3 GA Implementation

Genetic algorithm (GA) is a population-based probabilistic search and optimization technique, which works based on Darwin's principle of natural selection. GA is implemented here in solving the constrained nonlinear optimization problem. In GA, the term chromosome typically refers to a candidate solution to a defined problem, and fitness is the objective function value of the candidate solution. The method of handling constraints in optimization problems is a critical issue in the domain of GAs. If constraints are violated, the solution is infeasible and the fitness is penalized as follows:

Fitness function of ith solution is $F_i(x) = f_i(x) \pm P_i$, (+ for minimization problem, − for maximization problem), where P_i indicates penalty used to penalize infeasible solution. P_i is set equal to 0.0 for feasible solution, whereas for the infeasible solution it is set to a positive real number approximately equal to average fitness. In this study, reproduction is carried out by

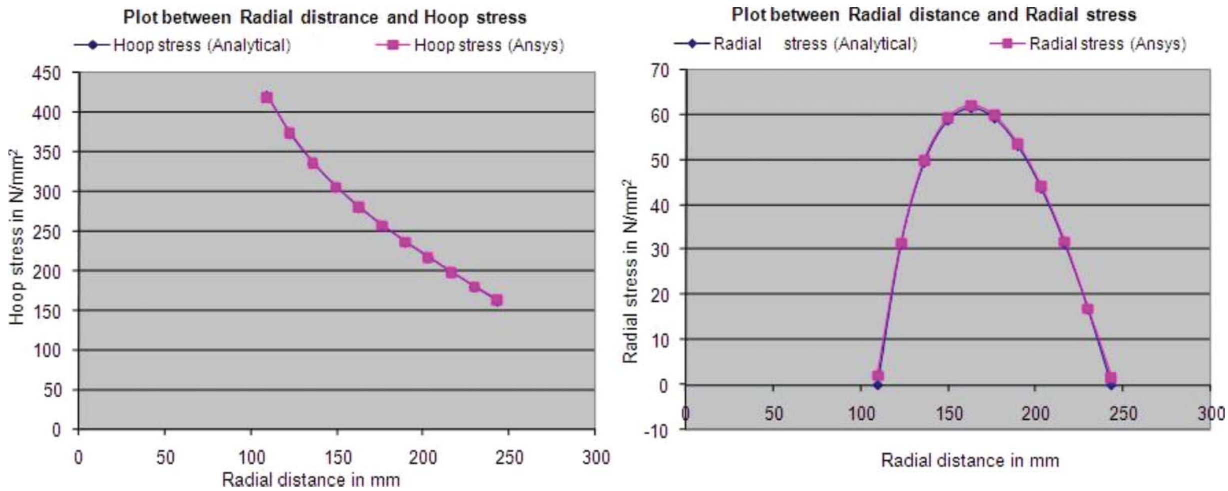

FIG. 4. Comparisons of stresses for a simple annular disk. (Color figure available online.)

choosing a first elite string (in terms of best fitness) and then tournament selection has been implemented to select the remaining population. A uniform crossover operator is used here with the probability of crossover P_c. Then the bitwise mutation is performed with a probability of mutation P_m. Following the rule of reproduction, crossover and mutation, new generations are evaluated. After many such generations, an optimal solution is obtained after satisfying the convergence criteria.

3. RESULTS AND DISCUSSION

3.1 Analytical Validation

In order to validate the finite element analysis procedure for rotating disk, a simple annular rotating disk with constant thickness has been analyzed. To find the magnitude of radial and tangential stress over an annular ring under rotation is calculated using Eq. (1) and Eq. (2). This approximation only holds if the outside radius of the ring, or disk, is large compared to the thickness, and also with assumption that the thickness of the ring is constant and the stresses are constant over the thickness.

The analysis model for comparison purpose is as shown in Fig. 2. Inner radius is 109.22 mm, outer radius is 243.22 mm, and disk thickness is 37 mm. The material used is Ti-6Al-4V (titanium-6 wt% aluminum-4 wt% vanadium), which has E = 114GPa, Poisson ratio = 0.342, Density = 4.43 g/cc, Yield strength (Sy) = 880 MPa. The disk is operated at 13000 RPM and analyzed for static load condition.

Figure 4 shows the results of both the finite element method using ANSYS and the analytical method. The FE results compare well with those of the analytical results. This gives confidencefor the procedure used in ANSYS for rotating disk analysis.

3.2 Analysis of Rotating Disk

The rotating disk considered for this study is from the energy-efficient engine's high-pressure second-stage compressor [21]. ANSYS is used for finite element analysis of the disk. Simple response surface equations are developed using the design of experiment (DOE) method, representing the FE disk model. These response equations are validated and used in optimization runs. GA has been done to minimize the weight without exceeding the stress limits. Upon violation of any of these constraints, the disk geometry will be modified until the conditions for all constraints are met and the weight is minimal.

3.2.1 Problem Description

Our goal is to determine the optimum geometrical distribution of the rotating disk (Fig. 1) such that the structural weight is minimized. Constraints are placed on maximum allowable stresses at bore and web. The rotating disk dimensions used [21] in this design are given in Table 1.

The material used is Ti-6Al-4V (titanium-6 wt% aluminum-4 wt% vanadium), which has E = 114 GPa, Poisson ratio = 0.342, Density = 4.43 g/cc, Yield strength (S_y) = 880 MPa. The disk is operated at 13000 rpm and analyzed for static load

TABLE 1
Initial disk dimensions

Dimensions	r1	r2	r3	r4	r5	r6	t1	t2	t3
mm	109.22	123.19	154.94	198.12	229.87	243.84	31.75	6.35	31.75

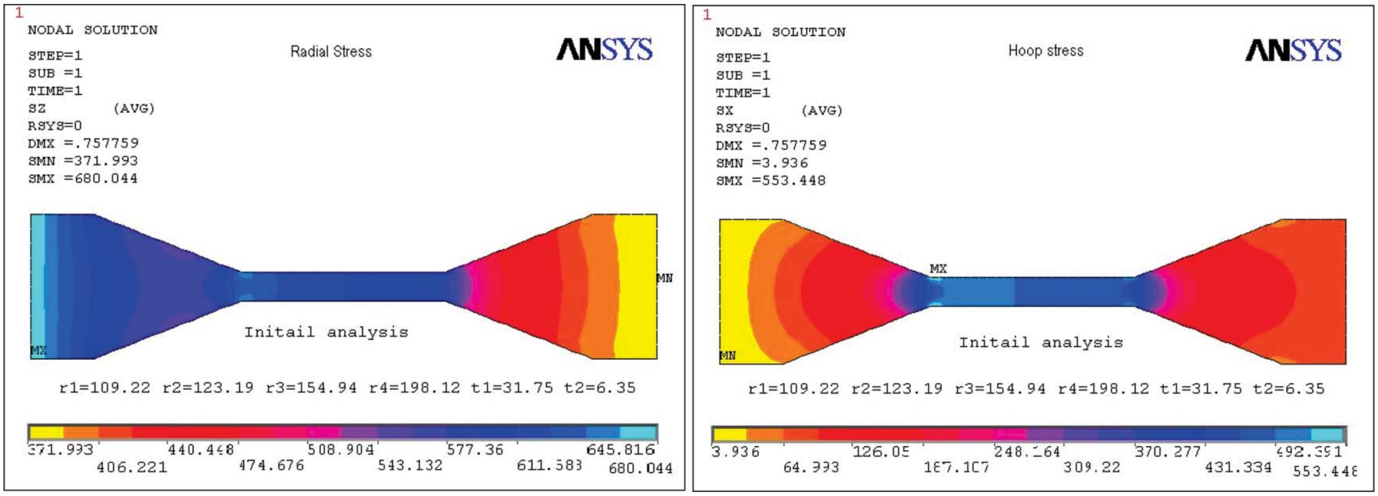

FIG. 5. Radial and hoop stress plot for initial disk geometry. (Color figure available online.)

condition, including both centrifugal load due to its own mass and uniform pressure of 77 MPa at rim due to blade load.

3.2.2 FEM Analysis

Axisymmetric model of the initial disk has been modeled and analyzed using ANSYS. Figure 5 reveals that the maximum stresses have not yet reached the maximum allowable stress. Therefore there is a scope to reduce the disk area.

In order to go for response surface design, first we need to develop various combinations of geometry of the disk by varying the variables within the design space using the design of experiments. Here r_1, r_2, r_3, r_4 and t_1 are varied with ± 5 mm, and t_2 by ± 2 mm. When performing an experiment, varying the levels of the factors simultaneously rather than one at a time is efficient in terms of time and cost, and also allows for the study of interactions between the factors. Interactions are often the driving force in the process. Without the use of factorial experiments, important interactions may remain undetected.

With the use of full factorial design and one center point, there will be 65 sets of design of experiments. Once the DOE are developed, analysis of disk for all sets of designs has been done using ANSYS and responses are extracted for each case. Responses are obviously the maximum radial stress at the web and maximum hoop stress at the bore, because these are the critical stresses that can affect the design of rotating disk, as is shown in Fig. 5. From DOE it is observed that hoop stress in the web region is also critical. Therefore this stress is also extracted from the FE analysis and taken as response.

3.2.3 Response Surface Design

Once the maximum stresses (responses) are extracted from the FEM analysis, we need to develop the response surface, which will represent the responses corresponding to the variables. In this rotating disk problem, as mentioned earlier, hoop stress is critical at the bore and web regions, and radial stress is critical at the web. These stresses are the major factors that affect the design. The objective in optimization of the disk is to minimize its area. Hence response surface analysis has been done for these three stresses, plus the area of disk. Transfer functions for these responses are developed using MINITAB trial version software.

For smooth curvature, approximating response functions are defined by using a second-order model. For fitting a second-order model, the most popular central composite design is used. The Minitab computes the linear, quadratic, and interaction terms in the model. Analysis of variance indicates that there are significant interactions between the factors.

The response design will give the coefficients corresponding to the linear and square terms. By allowing those coefficients whose probability of becoming zero is less than 5%, we can remove the unnecessary coefficients. Also, possible residual error should be small and sequential sum of squares and adjusted sum of squares should be closer for transfer function to be acceptable. Removing the unnecessary terms, the new coefficients are generated and are displayed in Table 2. The response surface equations for stresses can be represented as follows:

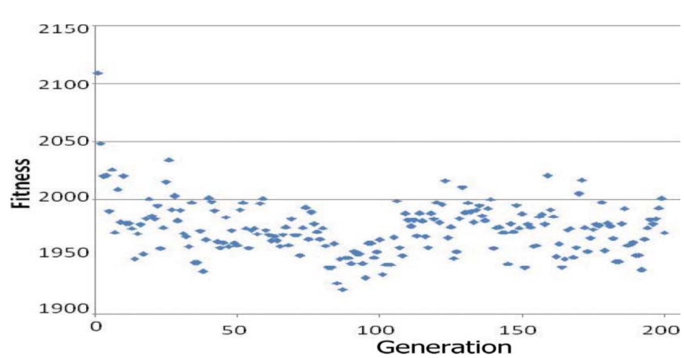

FIG. 6. Results of the GA optimization showing the fitness values vs generation. (Color figure available online.)

TABLE 2
Coefficients for response surface equations

B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_{11}	B_{12}	B_{13}
21886.51	−414.74	2.37947	13.8295	−1.4596	19.4561	33.5756	2.11654	−0.15495	−0.13604
B_{14}	B_{15}	B_{16}	B_{23}	B_{25}	B_{26}	B_{34}	B_{35}	B_{36}	B_{45}
0.043610	−0.17706	−0.48215	0.02602	0.19384	0.22149	−0.01626	−0.0794	0.143815	−0.02884
B_{56}	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_{11}	C_{13}
−1.19321	53864.3	−1128.2	49.7742	66.4587	−8.2834	79.3716	−63.723	4.89963	0.20720
C_{14}	C_{15}	C_{16}	C_{23}	C_{25}	C_{26}	C_{35}	C_{45}	C_{46}	C_{56}
0.089180	−0.1172	0.65938	−0.4815	1.24053	−0.7967	−1.0540	−0.1339	0.249804	−1.24657
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_{13}	D_{14}	D_{15}
−347.16	20.2343	−10.100	16.3592	−5.9762	7.191	−19.228	−0.1334	0.0497405	−0.0897
D_{25}	D_{35}	D_{56}	A_0	A_4	A_6	A_{15}	$A_{25}, A_{26}, A_{35}, A_{36}, A_{46}$		
0.324032	−0.2159	−0.4094	4092.73	−15.88	114.94	−1	0.5		

Maximum hoop stress at bore:

$$\begin{aligned}\sigma_{\theta-bore,\max} = {} & B_0 + B_1x_1 + B_2x_2 + B_3x_3 + B_4r_4 + B_5x_5 \\ & + B_6x_6 + B_{11}x_1x_1 + B_{12}x_1x_2 + B_{13}x_1x_3 \\ & + B_{14}x_1x_4 + B_{15}x_1x_5 + B_{16}x_1x_6 + B_{23}x_2x_3 \\ & + B_{25}x_2x_5 + B_{26}x_2x_6 + B_{34}x_3x_4xB_{35}x_3x_5 \\ & + B_{36}x_3x_6 + B_{45}x_4x_5 + B_{56}x_5x_6 \end{aligned} \tag{10}$$

Maximum radial stress at web:

$$\begin{aligned}\sigma_{r-web,\max} = {} & C_0 + C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 + C_5x_5 \\ & + C_6x_6 + C_{11}x_1x_1 + C_{13}x_1x_3 + C_{14}x_1x_4 \\ & + C_{15}x_1x_5 + C_{16}x_1x_6 + C_{23}x_2x_3 + C_{25}x_2x_5 \\ & + C_{26}x_2x_6 + C_{35}x_3x_5 + C_{45}x_4x_5 + C_{46}x_4x_6 \\ & + C_{56}x_5x_6 \end{aligned} \tag{11}$$

Maximum hoop stress at bore:

$$\begin{aligned}\sigma_{\theta-web,\max} = {} & D_0 + D_1x_1 + D_2x_2 + D_3x_3 + D_4x_4 \\ & + D_5x_5 + D_6x_6 + D_{13}x_1x_3 + D_{14}x_1x_4 \\ & + D_{15}x_1x_5 + D_{25}x_2x_5 + D_{35}x_3x_5 + D_{56}x_5x_6 \end{aligned} \tag{12}$$

Area of disk:

$$\begin{aligned}\text{Area} = {} & A_0 + A_4x_4 + A_6x_6 + A_{15}x_1x_5 + A_{25}x_2x_5 \\ & - A_{26}x_2x_6 + A_{35}x_3x_5 - A_{36}x_3x_6 \\ & + A_{46}x_4x_6 \end{aligned} \tag{13}$$

Where x_1,x_2,x_3,x_4,x_5,x_6 are r_1,r_2,r_3,r_4,t_1,t_2, respectively, and coefficients are as shown in Table 2.

To validate the response surface equations, two design models are arbitrarily chosen within the design space, and they are shown in Table 3. The responses for these models are obtained from the ANSYS, as well as response surface equations, Eqs. (10)-(13). This shows that the responses from the response surface equations are matching well with those of the ANSYS. Percentage error in the prediction of area of disk, maximum hoop stress at bore, maximum radial stress at web, and maximum hoop stress at web are calculated and they are marginal, as shown in Table 3. These errors are acceptable, as there is not much variation compared to those of ANSYS. Thus the response surface equations obtained are validated.

TABLE 3
Compression of response results obtained by response function and ANSYS

S. No	r_1	r_2	r_3	r_4	r_5	r_6	t_1	t_2	t_3
1	114.22	128.19	159.94	193.12	229.87	243.84	36.75	4.35	31.75
2	114.22	128.19	159.94	203.12	229.87	243.84	26.75	4.35	31.75

	Area in mm^2 - Eq. (13)			Hoop stress at bore in N/mm^2 - Eq. (10)			Radial stress at web in N/mm^2 - Eq. (11)			Hoop stress at web in N/mm^2 - Eq. (12)		
S. No	ANSYS	RS Eq.	% Error	ANSYS	RS Eq.	%Error	ANSYS	RS Eq.	%Error	ANSYS	RS Eq.	%Error
1	2417.08	2417.08	0.00008	753.8	754.25	0.06	951.77	952.87	0.116	784.64	786.7	0.269
2	1981.63	1981.63	0.0001	749.16	749.58	0.056	698.05	695.68	0.339	760.86	762.3	0.188

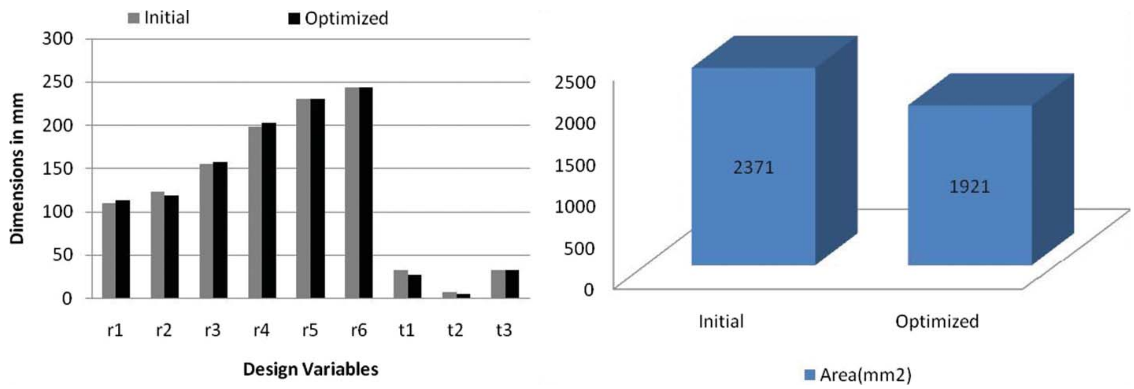

FIG. 7. Comparison of disk dimensions and disk area before and after optimization. (Color figure available online.)

3.3 Optimization Using Genetic Algorithm

The objective function is to minimize the disk cross-sectional area given in Eq. (13). Minimization of the disk cross-sectional area ensures the weight reduction of the rotating disk. Design variables are r_1, r_2, r_3, r_4, t_1 and t_2, as shown in Fig. 3. In the practical rotating disk r_5, r_6, and t_3 are fixed according to blade dimensions, hence they cannot be varied.

Inequality constraints are maximum hoop stress at bore given in Eq. (10), maximum radial stress at web given in Eq. (11), and maximum hoop stress at web given in Eq. (12). These stresses should be less than or equal to the yield stress divided by a factor of safety 1.1; i.e., 800 MPa.

In order to do optimization we need to develop design space by varying the variables. Here x_1, x_2, x_3, x_4 and x_5 are varied with ±5 mm, and x_6 by ±2 mm. This variation in dimensions will allow us to examine the stress variation for different combinations of variables. Hence side constraints will be:

$$
\begin{array}{lclcl}
104.22 & \leq & x_1 & \leq & 114.22 \\
118.19 & \leq & x_2 & \leq & 128.19 \\
149.94 & \leq & x_3 & \leq & 159.94 \\
193.12 & \leq & x_4 & \leq & 203.12 \\
26.75 & \leq & x_5 & \leq & 36.75 \\
4.35 & \leq & x_6 & \leq & 8.35
\end{array}
$$

Ensuring a proper balance between population diversity (exploration) and selection pressure (exploitation) is very important in order to have a very good performance of the genetic algorithm. The interactions among GA parameters were studied to select their optimal sense.

At first population size N, crossover probability P_c, mutation probability P_m and maximum allowable generation number G_{max} are set in the ranges of (20 to 60), (0.6 to 0.9), (0.05 to 0.2), and (50 to 500), respectively. Optimal GA-parameters are found by evaluating in four stages to be $P_c = .9$, $P_m = .05$, $N = 50$, $G_{max} = 200$.

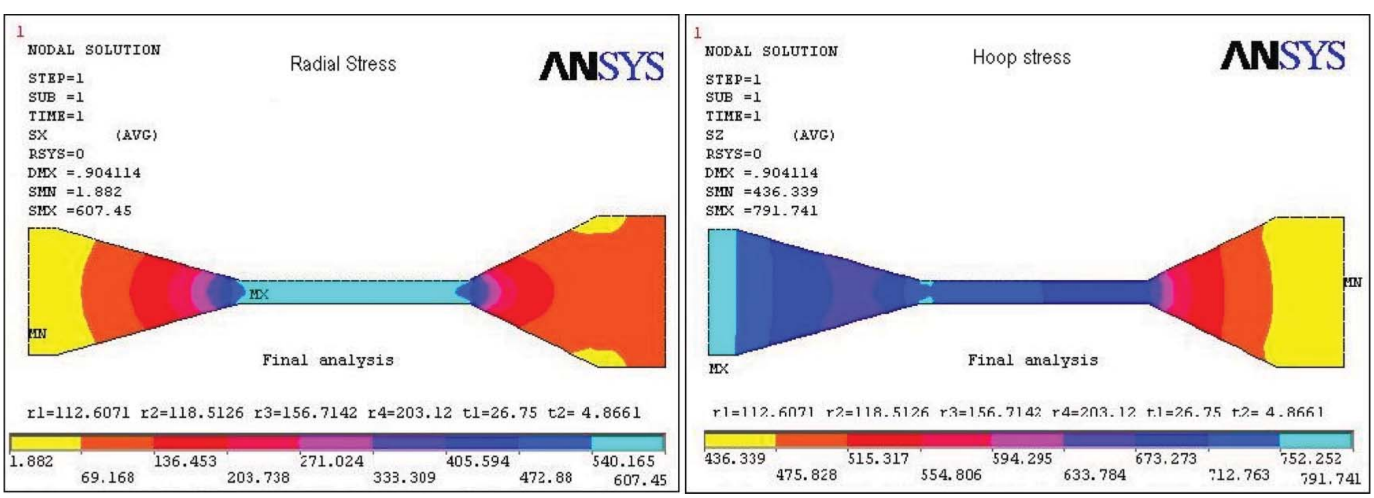

FIG. 8. Radial and hoop stresses plot for optimized disk. (Color figure available online.)

TABLE 4
Initial and optimized disk dimensions

Dimensions	r_1	r_2	r_3	r_4	r_5	r_6	t_1	t_2	t_3	Area (mm^2)
Initial (mm)	109.22	123.19	154.94	198.12	229.87	243.84	31.75	6.35	31.75	2371
Optimized (mm)	112.61	118.51	156.71	203.12	229.87	243.84	26.75	4.86	31.75	1921

Implementation of initial population, reproduction, crossover, and mutation fallowed by evaluation of fitness value for each generation has been executed through developed MATLAB codes. The corresponding fitness vs. generation graph is as shown in Figure 6. This graph shows best fitness values of each generation.

From the results obtained, it is clear that GA has been able to arrive at an optimal solution. The best solution is found at best fitness and hence the corresponding variables are declared to be the optimal design. A comparison of initial and final dimensions and area is shown in Table 4. Comparisons are also plotted in a bar chart for dimensions and area and are shown in Figure 7.

In Fig. 7, the bar chart for dimensions shows the increase in radius r1 and r4, but this increment will actually contribute to reduce the area. In the same figure, the bar chart for area clearly shows a considerable reduction in area from 2371 mm^2 to 1921 mm^2. The weight reduction is 24% with respect to the initial geometry. To validate the results, the optimized disk geometry has been analyzed using ANSYS. Figure 8 shows maximum radial stress of 607.45 MPa and maximum hoop stress of 791MPa, which are well within the allowable yield stress limit 800MPa with factor of safety.

4. CONCLUSIONS

A study of a simple axial flow compressor rotating disk is presented for its weight optimal design. The method is economical as well as practical for optimum design of disks used in a wide variety of applications. An alternative to the FEA model, a simple response surface design model of a rotating disk, is derived. Many researchers have tried to simplify the disk model by various techniques. The response surface model, although approximate, is showing better performance in optimizing disks. Accuracy of the design in this approach mainly depends on the precision of the response surface equations generated. This method is used in many fields, but has not yet been implemented in the design of rotating disks. First, maximum stresses are obtained at critical locations by analyzing different configurations with the help of FEA and DOE. It is observed that the maximum radial stress occurs at the web region and the maximum hoop stress occurs at the bore region of the rotating disk. DOE shows that hoop stress in the web region is also critical. Hence these three stresses are the driving factors for structural optimization of the rotating disk. The response equations obtained using a response surface design are validated and utilized to optimize the disk with the help of efficient evolutionary GA. Constraints handling plays a major role in arriving at an optimal solution for constrained optimization problems using evolutionary techniques. In the final run, the optimization has resulted in significant reduction in the disk area. Finally, the stresses calculated with optimized disk dimensions are well within the allowable limit. Use of response surface equations, instead of finite element analysis in optimization runs, will result in less computational time and cost.

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