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Structural Damage Detection Using Modal Curvature and Fuzzy Logic

M. Chandrashekhar and Ranjan Ganguli*

Department of Aerospace Engineering, Indian Institute of Science Bangalore 560012, India

A fuzzy logic system (FLS) with a new sliding window defuzzifier is proposed for structural damage detection using modal curvatures. Changes in the modal curvatures due to damage are fuzzified using Gaussian fuzzy sets and mapped to damage location and size using the FLS. The first four modal vectors obtained from finite element simulations of a cantilever beam are used for identifying the location and size of damage. Parametric studies show that modal curvatures can be used to accurately locate the damage; however, quantifying the size of damage is difficult. Tests with noisy simulated data show that the method detects damage very accurately at different noise levels and when some modal data are missing.

Keywords damage detection · fuzzy logic · modal curvature · measurement noise · inverse problem

1 Introduction

Since structural failure can lead to severe economic loss and loss of life, structural damage detection is of great interest to many researchers [1,2]. While there are many techniques for structural health monitoring, modal-based methods have been popular due to their ease of practical implementation [3,4]. Modal methods are based on the fact that the modal parameters (natural frequency, mode shape, and modal damping) are functions of the physical parameters (mass, stiffness, and damping) and hence existence of damage leads to changes in the modal properties of the structure [3]. The modal characteristics used to detect structural damage include frequency response functions, natural frequencies, mode shapes, mode shape curvatures, modal flexibility, modal strain energy, etc. [5]. Among these data types, natural frequency is widely used as it can be measured most conveniently [6]. However, the feasibility of using frequency changes for the purpose of damage localization is limited for at least two reasons. First, significant damage may cause very small changes in natural frequencies, particularly for larger structures, and these changes may go undetected due to measurement or processing errors. Moreover, variations in the mass of the structure or measurement temperatures may introduce uncertainties in the measured frequency changes [7].

These difficulties with frequency damage indicators can be overcome to some extent by using changes in mode shapes due to damage [8–11]. The convincing feature is that changes in

*Author to whom correspondence should be addressed.
E-mail: ganguli@aero.iisc.ernet.in
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mode shapes are much more sensitive to local damage when compared to changes in natural frequencies. However, using mode shapes also has some drawbacks. First, damage is a local phenomenon and may not significantly influence mode shapes of the lower modes that are usually measured from vibration tests of large structures. Second, extracted mode shapes are affected by environmental noise from ambient loads or inconsistent sensor positions. Third, the number of sensors and the choice of sensor coordinates may have a crucial effect on the accuracy of the damage detection procedure [7].

Given the shortcomings of the frequency and mode shape damage indicators, some researchers used modal curvature which is the second spatial derivative of the mode shape. Pandey et al. [12] concluded that the absolute changes in the curvature mode shapes are localized in the region of damage and hence can be used to detect damage in a structure. The changes in the curvature mode shapes increase with increasing size of damage.

Wahab and Roeck [13] investigated the application of the change in modal curvatures to detect damage in a prestressed concrete bridge. Ratcliffe and Bagaria [14] proposed an experimental nondestructive vibration-based technique applying gapped smoothing damage detection method to the modal curvature yielding a damage index, which was used to locate the delamination in a composite beam. Hamey et al. [15] evaluated various damage detection techniques in carbon/epoxy composite beams with several possible damage configurations. Their study demonstrated that modal curvature-based method performed better than the other ones studied. Qiao et al. [16] carried out a 'study to evaluate dynamics-based damage detection techniques. They found that curvature-based methods performed very well for damage detection in composite plates.

Although the studies on damage detection clearly show the capability of modal curvatures, they largely ignored measurement uncertainty. Since noise in the mode shape gets amplified due to the two differentiation operators required to get the curvature, the solution of inverse problems using this method needs more research.

Some studies have addressed the issue of measurement noise in damage detection using vibration-based methods. Reddy and Ganguli [17] found that damage in fixed-fixed beams caused considerable change in the Fourier coefficients of the mode shapes which were found to be sensitive to both damage size and location. Pawar et al. [18] further investigated the effect of noise on damage detection in fixed-fixed beams using neural networks and Fourier analysis of mode shapes in the spatial domain. Liu [19] formulated an identification problem by minimizing the error-norm of eigenequation and addressed the effect of measurement noise on damage detection. Fuzzy logic and continuum damage mechanics were used by Sawyer and Rao [20] to process and analyze the uncertainties in damaged structure considering measurement noise. Ganguli [21] used a fuzzy logic system (FLS) and Pawar and Ganguli [22] proposed a genetic fuzzy system for damage detection using natural frequencies in helicopter rotor blades having uncertainties in measurement. However, these works involving fuzzy logic were restricted to frequencies and mode shapes and did not use modal curvature as the damage indicator.

A damage detection problem is an inverse problem, where damage is classified by mapping the changes in response of the damaged structure from undamaged state to the location and size of the structural damage. The neural network has been extensively researched as a tool for the solution of inverse problems in structural damage detection. However, neural networks have the reputation of being black boxes that are difficult to understand and can also require enormous computer time when the back-propagation algorithm is used. In contrast, fuzzy systems allow for easier understanding as they are expressed in terms of linguistic variables [23]. Fuzzy systems have a built-in fuzzification process at the front end that accounts for uncertainty. They do not need to be trained on several cycles of noisy data like neural networks to account for uncertainty [21].

In the present study, a finite element (FE) model of a cantilever beam is used to develop a FLS for damage detection in structures using modal curvature vectors. A new sliding window defuzzifier is proposed for fault isolation. The proposed FLS is tested for noisy data as well as for the case when some of the measurements are missing or faulty.

2 Modeling

2.1 Modeling of Beam

An Euler–Bernoulli beam structure is considered for damage detection in this study. The governing equation of motion for free vibration analysis is given by

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]+m(x)\frac{\partial^2 w(x,t)}{\partial t^2}=0 \qquad (1)$$

The beam equation is solved for natural frequencies using the FE method [24]. For an n degree of freedom system, the equation of motion in discrete form is obtained after assembly of the element matrices and application of the boundary conditions.

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{0} \qquad (2)$$

Here $\mathbf{M}$ is the $n \times n$ mass matrix of the system, $\mathbf{K}$ is the $n \times n$ stiffness matrix of the system, $\mathbf{q}$ is the $n \times 1$ vector of nodal degrees of freedom. We seek a solution of the form $\mathbf{q} = \Phi e^{(i\omega t)}$, which results in the eigenvalue problem:

$$\mathbf{K}\Phi = \omega^2 \mathbf{M}\Phi \qquad (3)$$

Solving this eigenvalue problem we get n eigenvalues and n eigenvectors which represent the natural frequencies and natural modes of the system, respectively.

2.2 Modeling of Damage

Damage in the structure is represented by reduction in the element stiffness and the damage parameter D is defined by [21]:

$$D = \frac{E^{(u)} - E^{(d)}}{E^{(u)}}100 \qquad (4)$$

where E is Young's modulus of the beam material and the superscripts u and d represent the undamaged and damaged states, respectively.

The beam is divided into 20 FEs of equal length. Each segment spanning 20% of the blade in Figure 1 is therefore divided into four FEs [21]. Relevant properties of the beam are shown in Table 1 [22]. First six natural frequencies for the undamaged beam are 142.4, 892.4, 2498.8, 4896.9, 8095.8, and 12096.27 rad/s, respectively. The structural damage in each segment is simulated by stiffness reduction (D) of 20, 40, and 60%. These damages are classified as 'slight damage', 'moderate damage,' and 'severe damage,' respectively. This type of representation of structural damage along various locations of the beam helps in development of a user-friendly decision system.

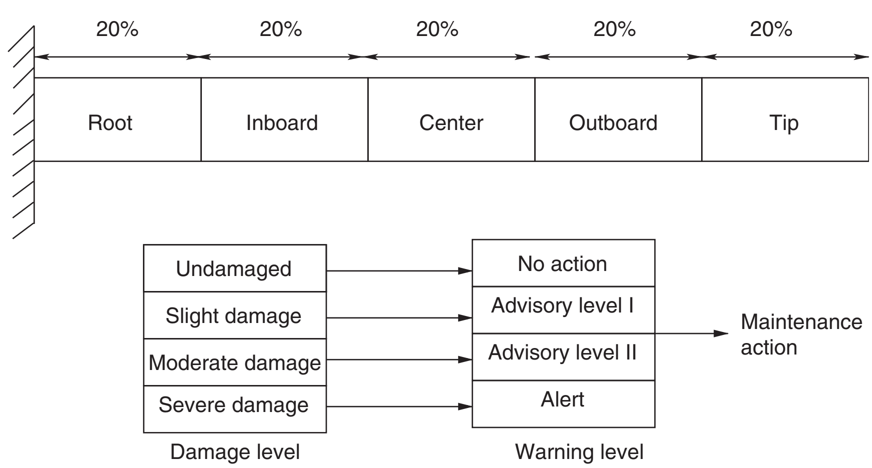

Figure 1 Schematic representation of beam structure and a decision system [21].

Table 1 Material and geometric properties of the beam.

Young's modulus (E)	2.0×10^5 N/mm^2
Mass density	7840×10^{-9} kg/mm^3
Cross-sectional area	240 mm^2
Moment of inertia	2000 mm^4
Length	600 mm

2.3 Modeling Damage Evaluation Parameter and Noise

The damage indicator referred as 'measurement delta (MD)' is the absolute difference between the modal curvatures of the damaged and undamaged beam. Reduction in the stiffness at some location for the damaged structure results in an increase of the local modal curvature [12].

Central difference approximation is used to estimate mode shape curvatures (MSC) from the mass normalized mode shapes obtained from the FE analysis. The MSC is obtained numerically as [12]:

$$\bar{\nu}_{i,j} = \frac{\phi_{(i+1),j} - 2\phi_{i,j} + \phi_{(i-1),j}}{h^2} \tag{5}$$

where $\bar{\nu}_{i,j}$ represents modal curvature, subscript i represents node number, and $\phi_{i,j}$ represents the mass normalized modal value for the i-th node in the j-th mode.

The change in mode shape curvature (CMSC) is obtained by subtracting the undamaged MSC vector from the damaged MSC:

$$\Delta\bar{\nu}_{i,j} = \bar{\nu}_{i,j}^{(d)} - \bar{\nu}_{i,j}^{(u)} \tag{6}$$

The CMSC is obtained using Equation (6) at each node of the beam FE model and is normalized to the same range using:

$$\Delta\nu_{i,j} = 1 + \frac{\Delta\bar{\nu}_{i,j}}{\max(\Delta\bar{\nu}_{i,j}) - \min(\Delta\bar{\nu}_{i,j})} \tag{7}$$

Therefore, different combinations of the three damage levels at five different locations of the beam give different sets of normalized measurement deltas ($\Delta\nu$), which are used to create the knowledge base composed of fuzzy rules. All results in the paper are for the normalized modal curvatures.

Though the use of modern instruments and signal processing can reduce measurement uncertainty, it can never be eliminated [25,26]. It can therefore be expected that uncertainty is present in measurement delta's ($\Delta\nu$). We shall assume Gaussian noise of 5% COV (Coefficient of Variation) to be present in the measurement delta [27].

The CMSC vectors obtained using Equation 7 for different damage conditions are shown in Figures 2–6. It can be seen from these figures that one can easily tell about the location of the damage just by the visual inspection of these plots but it is very hard to quantify the damage level. An automated reasoning system is therefore useful for processing this data and identifying the damage location and size.

3 Fuzzy Logic System

A FLS is a nonlinear mapping of an input feature vector into a scalar output. Fuzzy set theory and fuzzy logic provide the framework for the nonlinear mapping. The development of the FLS is briefly described in this section.

3.1 Input and Output

Inputs to the FLS are measurement delta's ($\mathbf{x}$) and outputs are structural damage size and location ($\mathbf{y}$). The objective is to find a functional mapping between $\mathbf{x}$ and $\mathbf{y}$. Mathematically, this can be represented as

$$\mathbf{y} = \mathbf{F}(\mathbf{x}) \tag{8}$$

where $\mathbf{y} = \{\text{Root, Inboard, Center, Outboard, Tip}\}^T$ and

$$\mathbf{x} = \left\{\Delta\nu_{i,1}, \Delta\nu_{i,2}, \Delta\nu_{i,3}, \Delta\nu_{i,4}\right\}^T.$$

Here, i varies from 1 to 21. Thus, $\Delta\nu_{i,j}$ represents the noisy CMSC vector corresponding to the j-th mode.

3.2 Fuzzification

The structural damages are considered as crisp numbers. To get a degree of resolution of

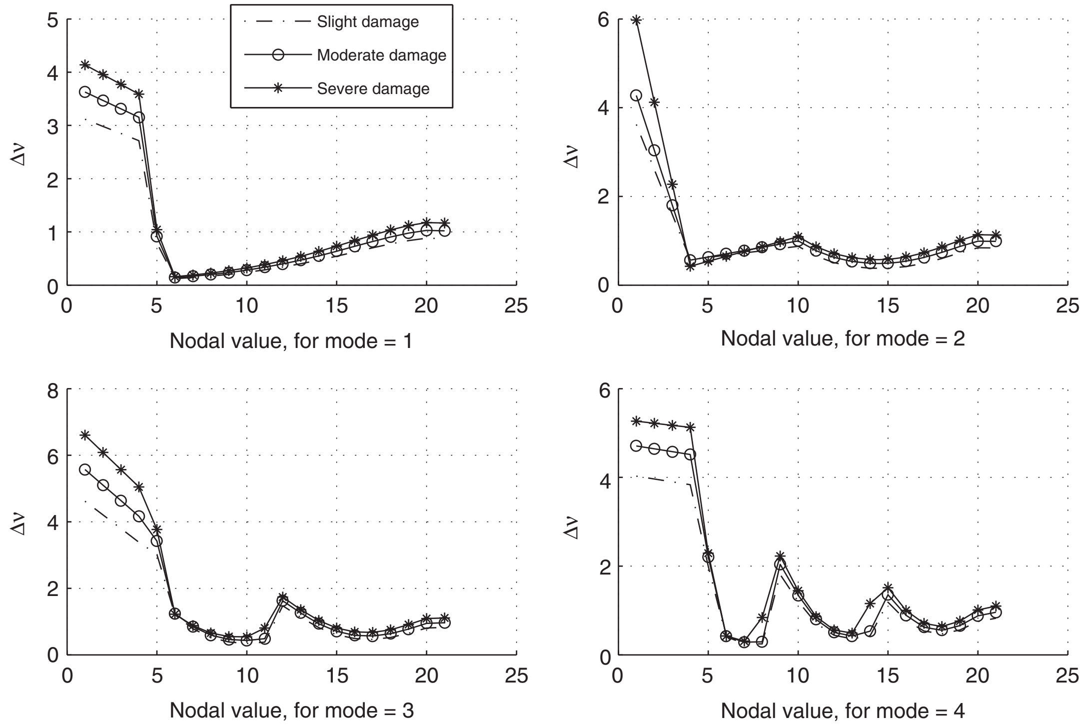

Figure 2 Change in normalized mode shape curvatures ($\Delta\nu$) for damage at 'root location' with different severity levels.

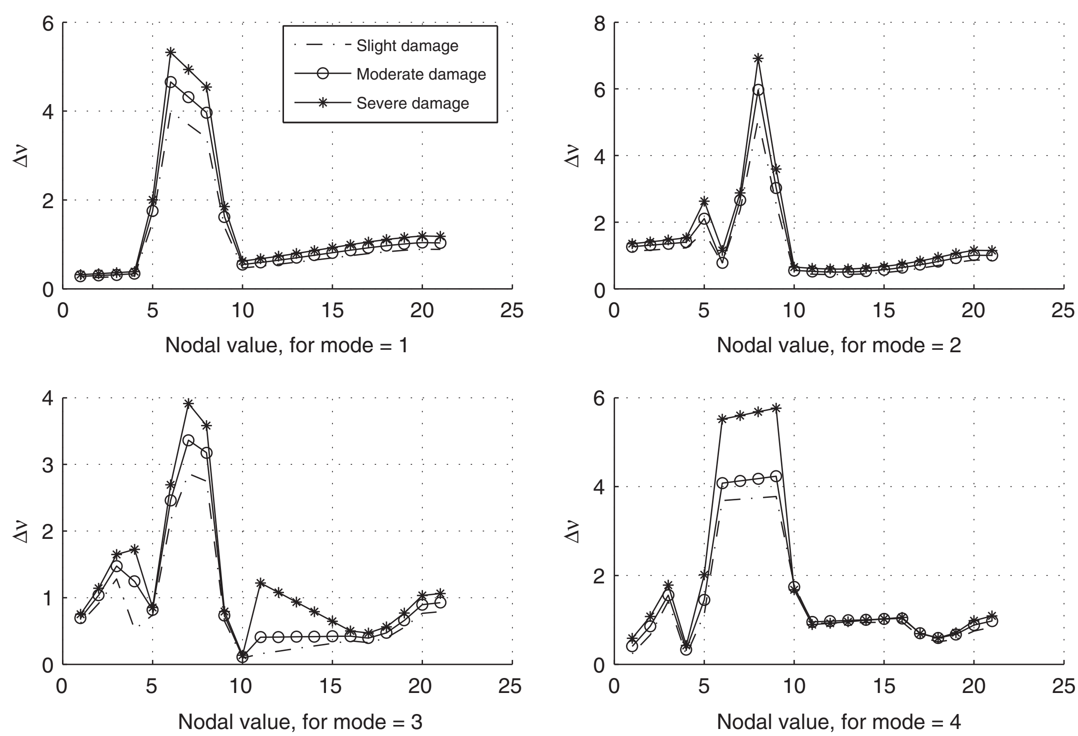

Figure 3 Change in normalized mode shape curvatures ($\Delta\nu$) for damage at 'inboard location' with different severity levels.

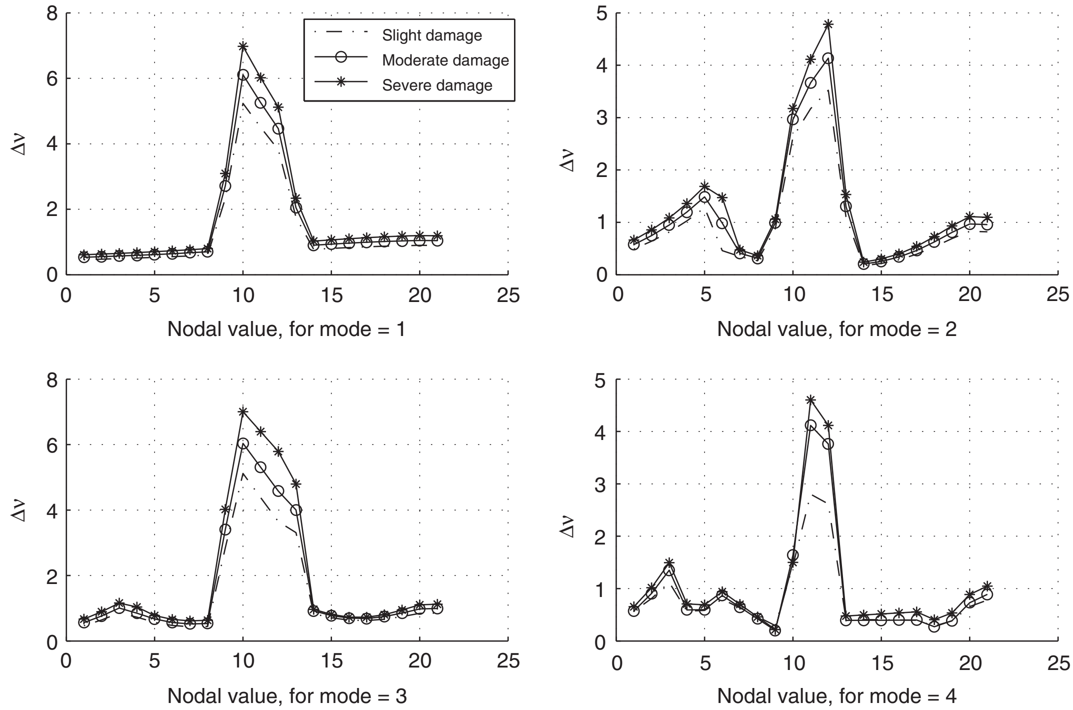

Figure 4 Change in normalized mode shape curvatures ($\Delta\nu$) for damage at 'center location' with different severity levels.

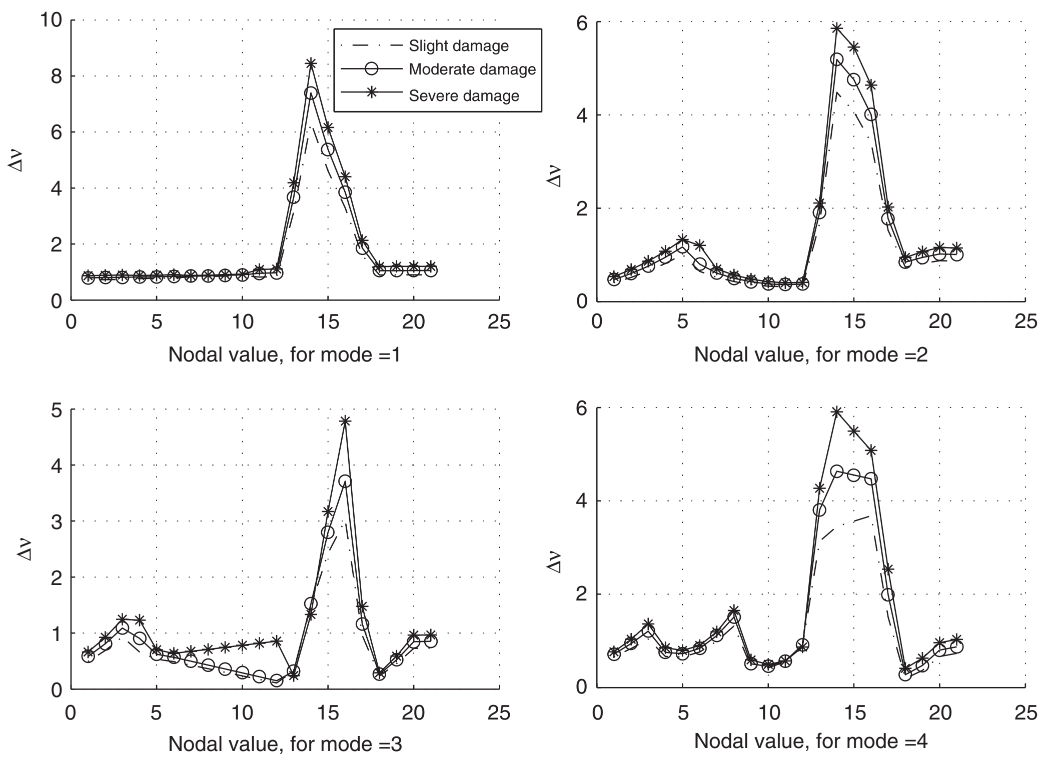

Figure 5 Change in normalized mode shape curvatures ($\Delta\nu$) for damage at 'outboard location' with different severity levels.

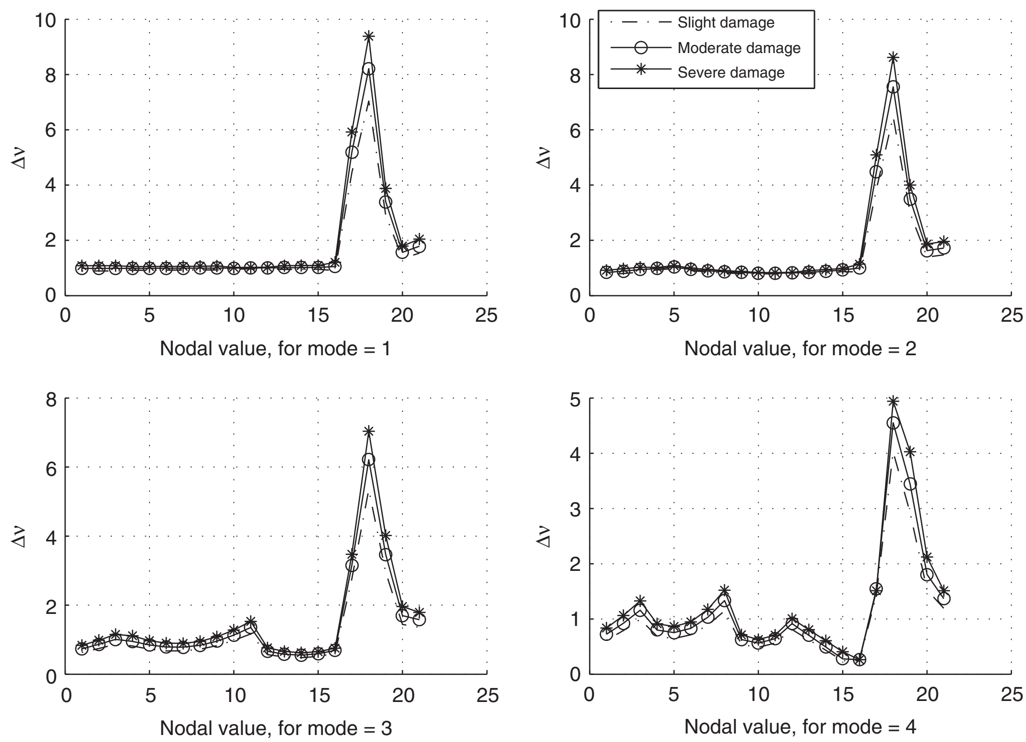

Figure 6 Change in normalized mode shape curvatures ($\Delta\nu$) for damage at 'tip location' with different severity levels.

the extent of damage, each of these damage locations is allowed several levels of damage and split into linguistic variables (i.e., Slight Damage, Moderate Damage, and Severe Damage).

The measurement delta's $\Delta\nu_{i,1}, \Delta\nu_{i,2}, \Delta\nu_{i,3}$, and $\Delta\nu_{i,4}$ are treated as fuzzy variables. To get a high degree of resolution, they are further split into linguistic variables. For example, consider $\Delta\nu_{i,1}$ as a linguistic variable. It can be decomposed into a set of terms:

$$T(\Delta\nu_{i,1}) = (\text{Negligible, VeryLow, Low, Low Medium, Medium, MediumHigh, High, VeryHigh, VeryVeryHigh})$$

where each term in $T(\Delta\nu_{i,1})$ is characterized by a fuzzy set in the universe of discourse $U(\Delta\nu_{i,1}) = \{-1, 9\}$. The other three measurement delta vectors are defined using the same set of terms as that for $\Delta\nu_{i,1}$. Measurement deltas larger than covered by the universe of discourse will represent an extensive structural damage indicative of a catastrophic failure and are not considered.

Fuzzy sets with Gaussian membership fun–ctions are used for the input variables. These fuzzy sets can be defined using the following equation

$$\mu(x) = e^{-0.5((x-m)/\sigma)^2} \tag{9}$$

where m is the midpoint of the fuzzy set and σ is the spread (standard deviation) associated with the variable. Gaussian fuzzy membership functions are quite popular in the fuzzy logic literature, as they are the basis for the connection between fuzzy system and radial basis function neural networks [28]. Table 2 gives the linguistic measure associated with each fuzzy set and the midpoint of the set for each measurement delta. The midpoints are selected to span the region ranging from an undamaged beam (all measurement deltas are zero) to one with significant damage.

Figure 7 shows the membership functions for each of the nine input fuzzy sets. The standard deviations for the membership functions are selected as 0.3 to provide enough intersection between the fuzzy sets so as to give good accuracy of detection [21].

3.3 Rule Generation

Rules for the fuzzy system are obtained by fuzzification of the numerical values obtained from the FE analysis using the following procedure [21]:

(1) A set of four measurement delta vectors corresponding to a given structural fault is input to the FLS and the degree of membership of the elements of $\Delta v_{i,1}$, $\Delta v_{i,2}$, $\Delta v_{i,3}$, and $\Delta v_{i,4}$ are obtained. Therefore, each measurement delta has nine degree of memberships based on the linguistic measures in Table 2.

Table 2 Gaussian fuzzy sets.

Linguistic measure	*Symbol*	*Midpoint $\Delta\omega$*
Negligible	N	0
Very low	VL	1
Low	L	2
Low–medium	LM	3
Medium	M	4
Medium–high	MH	5
High	H	6
Very high	VH	7
Very very high	VVH	8

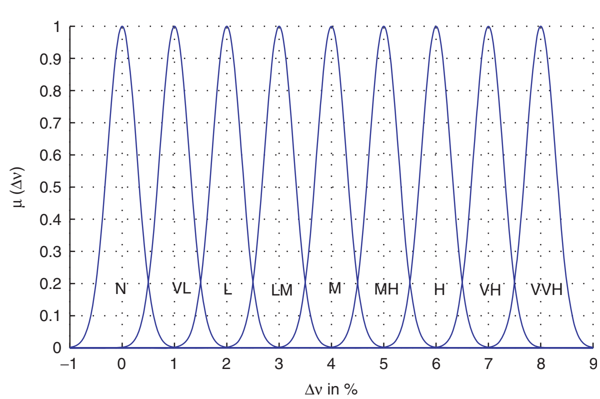

Figure 7 Fuzzy sets representing measurement deltas over universe of discourse (−1 to 9).

(2) Each measurement delta is then assigned to the fuzzy set with maximum degree of membership.

(3) One rule is obtained for each fault by relating the measurement deltas with maximum degree of membership to a fault.

The fuzzy rules are given in Tables 3–8. The linguistic symbols used in these tables are defined in Table 2. There is a separate fuzzy rule set for each fault location which can be interpreted as a pattern classifier.

The rule for the ‘undamaged case’ is given in Table 3 and shows ‘very low’ level of change in all the measurements. Rules for the fault ‘Damage at root’ at the three different damage severity levels are given in Table 4 for a matrix consisting of the four CMSC vectors. Table 4 clearly shows that as damage becomes more severe, the indicators move from ‘low’ to ‘medium’ and ‘high’ levels. The rules for the other faults in Tables 5–8 can be similarly interpreted.

Table 3 Rules for fuzzy system at ‘Undamaged condition’.

	Measurement deltas			
Node no.	*Mode 1 (Δv)*	*Mode 2 (Δv)*	*Mode 3 (Δv)*	*Mode 4 (Δv)*
1	VL	VL	VL	VL
2	VL	VL	VL	VL
3	VL	VL	VL	VL
4	VL	VL	VL	VL
5	VL	VL	VL	VL
6	VL	VL	VL	VL
7	VL	VL	VL	VL
8	VL	VL	VL	VL
9	VL	VL	VL	VL
10	VL	VL	VL	VL
11	VL	VL	VL	VL
12	VL	VL	VL	VL
13	VL	VL	VL	VL
14	VL	VL	VL	VL
15	VL	VL	VL	VL
16	VL	VL	VL	VL
17	VL	VL	VL	VL
18	VL	VL	VL	VL
19	VL	VL	VL	VL
20	VL	VL	VL	VL
21	VL	VL	VL	VL

A close observation of the results in Tables 3–8 shows that each rule represents a unique signature and is different from all the other rules. Therefore, the fuzzy system is a good pattern classifier. These rules provide a knowledge base and represent how a human engineer would interpret data to isolate structural damage using changes in modal curvatures. It is difficult for a human to memorize and process such a large number of patterns. However, the task is straight forward for computer interpretation.

4 Damage Detection

Once the fuzzy rules are applied to a given measurement, we have a set of degree of memberships for each fault. For fault isolation, we are interested in the most likely fault. Therefore, the so called maximum matching method has been widely used for defuzzification in damage detection work [29,30]. In this paper, we present a new defuzzification approach which gives better results than the maximum matching method.

4.1 Sliding Window Method

Algorithmic development of the sliding window technique is explained in this section. Here, ΔN represents the noisy measurement delta for a given damage condition. Hence,

$$\Delta N = \left\{ (\Delta\nu_{i,1})_{noisy}, (\Delta\nu_{i,2})_{noisy}, (\Delta\nu_{i,3})_{noisy}, (\Delta\nu_{i,4})_{noisy} \right\}^T.$$

Take a window of maximum limit equal to the maximum(ΔN) and the minimum limit equal to the minimum(ΔN) for any given arbitrary encountered fault. Slide this window over the fuzzy rule base sets with a maximum move limit (M) defined by the difference between the midpoints of two successive fuzzy sets to scan for the possible variability of the measurement delta's

Table 4 Rules for fuzzy system for the fault at 'Root'.

	Measurement deltas ($\Delta\nu$)											
	Slight damage				Moderate damage				Severe damage			
	Mode no.				Mode no.				Mode no.			
Node No.	1	2	3	4	1	2	3	4	1	2	3	4
1	LM	M	MH	M	M	M	H	MH	M	H	VH	MH
2	LM	LM	M	M	M	LM	MH	MH	M	M	H	MH
3	LM	L	M	M	LM	L	MH	MH	M	L	H	MH
4	LM	VL	LM	M	LM	VL	M	MH	M	N	MH	MH
5	VL	VL	LM	L	VL	VL	M	L	VL	VL	M	L
6	N	VL	VL	N	N	VL	VL	N	N	VL	VL	N
7	N	VL	VL	N	N	VL	VL	N	N	VL	VL	N
8	N	VL	VL	N	N	VL	VL	N	N	VL	VL	VL
9	N	VL	N	L	N	VL	N	L	N	VL	VL	L
10	N	VL	N	VL	N	VL	N	VL	N	VL	VL	VL
11	N	VL	N	VL	N	VL	N	VL	N	VL	VL	VL
12	N	VL	VL	N	N	VL	L	VL	N	VL	L	VL
13	N	N	VL	N	N	VL	VL	N	VL	VL	VL	N
14	N	N	VL	N	VL	VL	VL	VL	VL	VL	VL	VL
15	VL	N	VL	VL	VL	VL	VL	VL	VL	VL	VL	L
16	VL	N	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
17	VL	N	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
18	VL	VL	VL	N	VL	VL	VL	VL	VL	VL	VL	VL
19	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
20	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL

Table 5 Rules for fuzzy system for the fault at 'inboard'.

	Measurement deltas ($\Delta\nu$)											
	Slight damage				*Moderate damage*				*Severe damage*			
	Mode no.				*Mode no.*				*Mode no.*			
Node No.	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*
1	N	VL	VL	N	N	VL	VL	N	N	VL	VL	VL
2	N	VL	VL	VL	N	VL	VL	VL	N	VL	VL	VL
3	N	VL	VL	VL	N	VL	VL	L	N	L	L	L
4	N	VL	VL	N	N	VL	VL	N	N	L	L	N
5	L	L	VL	VL	L	L	VL	VL	L	LM	VL	L
6	M	VL	L	M	MH	VL	L	M	H	VL	LM	H
7	M	L	LM	M	M	LM	LM	M	MH	LM	M	H
8	LM	MH	LM	M	M	H	LM	M	MH	VH	M	H
9	VL	LM	VL	M	L	LM	VL	M	L	M	VL	H
10	N	N	N	L	VL	VL	N	L	VL	VL	N	L
11	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL	VL
12	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL	VL
13	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL	VL
14	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL	VL
15	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL	VL
16	VL	VL	N	VL	VL	VL	N	VL	VL	VL	VL	VL
17	VL	VL	N	VL	VL	VL	N	VL	VL	VL	N	VL
18	VL	VL	N	VL	VL	VL	N	VL	VL	VL	VL	VL
19	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
20	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL

over the fuzzy sets. In this study, $M = 2 - 1 = 1$ (Figure 7), as 1 and 2 are the midpoints for any two successive fuzzy sets. Hence we scan by varying the given ΔN symmetrically by M by giving step increments of γ to ΔN. Thus we scan from $\Delta N - M/2$ to $\Delta N + M/2$. The step size (γ) is equal to M/n, where n is the total number of scans for the damage classification over the fuzzy sets. The number n is selected to provide a finer scanning while not being computationally too expensive. The following steps are followed for damage identification:

(a) Start with $\Delta N - M/2$ representing modified damage features as $\Delta N^0 = \Delta N - M/2$. This is counted as the zeroth step.

(b) Assign fuzzy sets to the modified modal measures for their maximum degree of memberships as explained in Section 3.3. Call this 'modified fuzzy sets for the given damage,' represented symbolically as 'MFS_{GD}'.

(c) Compare MFS_{GD} with the available fuzzy knowledge base (Tables 3–8) for a match. The matching fuzzy rule with this MFS_{GD} is selected as a damage condition for this modified damage features and it is counted. In case the MFS_{GD} does not match with any of the rules available with the fuzzy knowledge base, there is no hit.

(d) Give a step increment of γ to ΔN^0, representing the next modified damage features as $\Delta N^1 = \Delta N^0 + \gamma$. This is counted as the first step.

(e) Follow steps (b) and (c). Every hit for a different identified fault is counted.

(f) Modify ΔN^1 by adding γ to it, representing the modified damage indicators in second step as $\Delta N^2 = \Delta N^1 + \gamma$. In general the modified damage indicators set in i-th step is given by $\Delta N^i = \Delta N^{i-1} + \gamma$.

(g) Follow step e.

(h) Repeat the procedure by following (f) and (g). until ΔN^i equals $\Delta N + M/2$.

For fault isolation, the fault with the maximum hit count identified by the FLS is selected

Table 6 Rules for fuzzy system for the fault at 'center'.

	Measurement deltas ($\Delta\nu$)											
	Slight damage				*Moderate damage*				*Severe damage*			
	Mode no.				*Mode no.*				*Mode no.*			
Node No.	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*
1	N	VL	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
2	N	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
3	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	L
4	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
5	VL	VL	VL	VL	VL	VL	VL	VL	VL	L	VL	VL
6	VL	N	N	VL	VL	VL	VL	VL	VL	L	VL	VL
7	VL	N	N	VL	VL	N	VL	VL	VL	N	VL	VL
8	VL	N	N	N	VL	N	VL	N	VL	N	VL	N
9	L	VL	LM	N	LM	VL	LM	N	LM	VL	M	N
10	MH	LM	MH	VL	H	LM	H	L	VH	LM	VH	L
11	MH	LM	MH	LM	MH	M	MH	M	H	M	VH	MH
12	M	M	M	LM	MH	M	MH	M	MH	MH	H	M
13	L	VL	LM	N	L	VL	M	N	L	L	MH	N
14	VL	N	VL	N	VL	N	VL	N	VL	N	VL	VL
15	VL	N	VL	N	VL	N	VL	N	VL	N	VL	VL
16	VL	N	VL	N	VL	N	VL	N	VL	N	VL	VL
17	VL	N	VL	N	VL	N	VL	N	VL	VL	VL	VL
18	VL	VL	VL	N	VL	VL	VL	N	VL	VL	VL	N
19	VL	VL	VL	N	VL	VL	VL	N	VL	VL	VL	VL
20	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL

as the most likely fault. The new proposed method for fault isolation is logical as uncertainty associated with the measurement deltas causes the $\Delta\nu$'s to vary.

The FLS is tested using measurement deltas obtained from the FE model. Gaussian noise with 5% COV is added to the simulated data. Five thousand noisy data points are generated for each seeded fault and the percentage success rate for the fuzzy system in isolating a fault is calculated. The success rate S_R is defined as follows

$$S_R = \frac{N_c}{N}100 \tag{10}$$

where N_c is the number of correct classifications and N is the total number of data sets. We have used $N = 5000$ in this paper.

Table 9 compares the fault isolation results obtained using the highest degree of membership method [21] and the sliding window technique. The sliding window method gives a higher success rate for fault isolation than the previously proposed FLS [21], which is based on maximum degree of membership for damage classification. Note that the percentage success rate of 98.74% for the highest degree of membership method may not appear to be very different from 99.81% for the sliding window method. However, for 5000 samples of noisy data, this implies a decrease in mis-detection from 63 to only 10. This dramatic improvement is brought about by the new algorithm.

4.2 Effect of Different Noise Levels on Damage Detection

It is instructive to evaluate the damage detection algorithm at different noise levels. The FLS defined by the rules of four CMSC ($\Delta\nu$) vector inputs is tested for the measurement deltas with noise levels of 3% COV and 5% COV. The new FLS gives an average success rate of 99.98% for the measurement variability

Table 7 Rules for fuzzy system for the fault at 'outboard'.

	Measurement deltas (Δν)											
	Slight damage				*Moderate damage*				*Severe damage*			
	Mode no.				*Mode no.*				*Mode no.*			
Node No.	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*
1	VL	N	N	VL	VL	N	VL	VL	VL	VL	VL	VL
2	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
3	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
4	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
5	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
6	VL	VL	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
7	VL	VL	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
8	VL	N	N	VL	VL	VL	N	L	VL	VL	VL	L
9	VL	N	N	N	VL	N	N	VL	VL	N	VL	VL
10	VL	N	N	N	VL	N	N	N	VL	N	VL	VL
11	VL	N	N	VL	VL	N	N	VL	VL	N	VL	VL
12	VL	N	N	VL	VL	N	N	VL	VL	N	VL	VL
13	LM	L	N	LM	M	L	N	M	M	L	N	M
14	H	MH	VL	LM	VH	MH	L	MH	VVH	H	VL	H
15	MH	M	L	M	MH	MH	LM	MH	H	H	LM	H
16	LM	LM	LM	M	M	M	M	MH	MH	MH	MH	MH
17	L	L	VL	L	L	L	VL	L	L	L	L	LM
18	VL	VL	N	N	VL	VL	N	N	VL	VL	N	N
19	VL	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL
20	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL

of 3% COV and 99.81 for 5% COV, as shown in Table 10. It also classifies the undamaged structure with an accuracy of 100 and 99.65% for COV of 3 and 5%, respectively, minimizing the possibility of false alarms. The proposed FLS gives very good accuracy in classifying 'Slight Damage at Tip,' which is the most difficult damage to detect in a cantilever beam structure. Thus the FLS is robust to noisy data.

4.3 Damage Detection with Missing or Faulty Measurements

A damage detection algorithm should be evaluated for the case when some of the measurements are either missing or are faulty. Such situations can occur in practical implementation. We consider the cases when some of the modal vectors are wrongly measured or have noise levels much higher than the threshold value. A statistical threshold can be used to flag the measurement on such occasion [25,26]. The faulty measurement is replaced by a large number (say 100) and then the fuzzy system estimates the fault while ignoring that measurement [22].

The FLS is tested for several cases where a modal vector is missing. Table 11 shows the results obtained when the third or fourth modal vector is missing. Average success rates with different possible combinations of missing measurements are given in Table 12. In general, using fewer modes results in some loss in accuracy of damage detection.

Results obtained from the FLS by testing with missing measurements indicate that the methodology is quite accurate in damage classification. For the case when all the measurements are available within the threshold noise limit of 5% COV, the success rate of the FLS is 99.81%. Even for the case when two of the measurement vectors are missing, the FLS degrades to 98.70%, which is still a very good success rate for

Table 8 Rules for fuzzy system for the fault at 'tip'.

	Measurement deltas (Δv)											
	Slight damage				*Moderate damage*				*Severe damage*			
	Mode no.				*Mode no.*				*Mode no.*			
Node No.	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*
1	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
2	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
3	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
4	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
5	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
6	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
7	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
8	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	L
9	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
10	VL	VL	VL	N	VL	VL	VL	VL	VL	VL	VL	VL
11	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	L	VL
12	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
13	VL	VL	N	VL	VL	VL	VL	VL	VL	VL	VL	VL
14	VL	VL	N	N	VL	VL	VL	N	VL	VL	VL	VL
15	VL	VL	VL	N	VL	VL	VL	N	VL	VL	VL	N
16	VL	VL	VL	N	VL	VL	VL	N	VL	VL	VL	N
17	M	M	LM	VL	MH	MH	LM	L	H	MH	M	L
18	VH	H	MH	M	VVH	VH	H	MH	VVH	VVH	VH	MH
19	LM	LM	LM	LM	LM	M	M	M	M	M	M	M
20	VL	VL	VL	L	L	L	L	L	L	L	L	L
21	L	VL	VL	VL	L	L	L	VL	L	L	L	L

practical applications. At 3% COV, the effect of missing measurement is less with the FLS success rate degrading from a best value of 99.98% to a value of 99.31%.

4.4 Closing Remarks

Since the primary aim of the paper is to demonstrate a new damage detection algorithm, the results in this work were illustrated for a cantilever beam using modal curvature-based damage indicator. There are some limitations of the damage indicator and structure selected which should be pointed out.

The success of the FLS depends on the knowledge base available for damage identification. Thus, the FLS can not detect a damage which does not have a rule representation available with the FLS. It can possibly detect a damage having closest features (damage indicators) to that of the existing damage condition. It is possible that we loose uniqueness of the rules for two different damage states for a complex structure (or possibly for damages occurring at more refined levels). Then we would need more powerful damage indicators based on wavelet or HHT transforms [31,32], which are more sensitive and localized with respect to damage to maintain the uniqueness of the fuzzy rules. These issues are subjects for future research. Though using numerical simulations with added noise is an established approach for testing damage detection algorithms [12,21], experimental validation is the true test. Again, that is a subject for future research.

5 Conclusions

A FLS with a new fault isolation (sliding window) technique is proposed for damage detection in structures using modal curvature vectors. Test results are obtained to simulate the uncertainties in the measurement delta's.

Table 9 Success rate (S_R) in percent for different defuzzification techniques using first four CMSC vectors considering variability of 5% COV in modal data due to measurement noise.

Fault	*Highest degree of membership [21]*	*New sliding window technique*
Undamaged	98.75	99.65
Slight damage at root	98.61	100
Moderate damage at root	99.02	100
Severe damage at root	99.48	100
Slight damage at inboard	98.54	100
Moderate damage at inboard	98.37	99.27
Severe damage at inboard	98.95	100
Slight damage at center	98.82	100
Moderate damage at center	98.93	100
Severe damage at center	99.44	100
Slight damage at outboard	97.71	98.71
Moderate damage at outboard	98.67	100
Severe damage at outboard	99.06	100
Slight damage at tip	98.26	99.47
Moderate damage at tip	98.43	100
Severe damage at tip	98.84	100
Average S_R	98.74	99.81

Table 10 Success rate in percent of the FLS with sliding window defuzzifier using first four CMSC vectors for different noise levels.

Rule no.	*Variability of 3% COV*	*Variability of 5% COV*
Undamaged	100	99.65
Slight damage at root	100	100
Moderate damage at root	100	100
Severe damage at root	100	100
Slight damage at inboard	100	100
Moderate damage at inboard	99.82	99.27
Severe damage at inboard	100	100
Slight damage at center	100	100
Moderate damage at center	100	100
Severe damage at center	100	100
Slight damage at outboard	100	98.71
Moderate damage at outboard	100	100
Severe damage at outboard	100	100
Slight damage at tip	100	99.47
Moderate damage at tip	100	100
Severe damage at tip	100	100
Average S_R	99.98	99.81

The following conclusions are made from this study:

(1) The proposed FLS with new sliding window method gives higher success rate of 99.81% in comparison to the FLS using highest degree of membership giving success rate of 98.74%, when measurements have variability of 5% COV due to noise.

(2) The fuzzy system performs as a good classifier even in the case when there are missing measurements. Considering the worst case scenario when two of the measurement vectors are unavailable, the success rate of the FLS remains over 98.70%.

(3) Even a slight damage at the tip location is accurately classified by the FLS with a success rate of 99.47% for measurements having variability of 5% COV due to noise.

Based on the numerical results in this work, the fuzzy system based on modal curvature is proposed as a robust tool for structural damage detection.

Table 11 Success rate in percent of the FLS with sliding window defuzzifier using different number of noisy modal data.

Rule no.	*With first three CMSC vectors*		*With first, second, and fourth CMSC vectors*	
	COV = 3%	*COV = 5%*	*COV = 3%*	*COV = 5%*
Slight damage at root	99.79	99.53	99.73	99.15
Moderate damage at root	100	100	100	100
Severe damage at root	100	100	100	100
Slight damage at inboard	100	100	100	100
Moderate damage at inboard	99.72	98.86	99.54	98.83
Severe damage at inboard	100	100	98.81	97.91
Slight damage at center	100	100	100	100
Moderate damage at center	100	100	100	100
Severe damage at center	100	100	100	100
Slight damage at outboard	100	98.23	100	98.69
Moderate damage at outboard	100	100	100	100
Severe damage at outboard	100	100	100	100
Slight damage at tip	99.68	98.06	99.76	98.99
Moderate damage at tip	100	99.78	100	100
Severe damage at tip	100	100	100	100
Average S_R	99.94	99.63	99.85	99.57

Table 12 Average success rate in percent for the FLS with sliding window defuzzifier tested with faulty or missing modal data at different noise levels.

Sr. no.	*Modes used*	*Average success rate*	
		3% COV	*5% COV*
1	First four modes	99.98	99.81
2	First three modes	99.94	99. 63
3	First, second, and fourth modes	99.85	99.57
4	First, third, and fourth modes	99.71	99.49
5	Second, third, and fourth modes	99.89	99.69
6	First and second modes	99.43	98.96
7	First and third modes	99.37	98.91
8	First and fourth modes	99.31	98.70
9	Second and third modes	99.47	99.08
10	Second and fourth modes	99.43	99.00
11	Third and fourth modes	99.40	98.93

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