Applied Soft Computing 10 (2010) 919–925

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

The use of a fuzzy multi-objective linear programming for solving a multi-objective single-machine scheduling problem

Reza Tavakkoli-Moghaddam*, Babak Javadi, Fariborz Jolai, Ali Ghodratnama

Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

ARTICLE INFO

Article history:
Received 23 April 2007
Received in revised form 1 March 2009
Accepted 17 October 2009
Available online 25 October 2009

Keywords:
Single-machine scheduling
Multi-objective linear programming
Fuzzy multi-objective linear programming
Decision maker

ABSTRACT

This paper develops a fuzzy multi-objective linear programming (FMOLP) model for solving a multi-objective single-machine scheduling problem. The proposed model attempts to minimize the total weighted tardiness and makespan simultaneously. In this problem, a proposed FMOLP method is applied with respect to the overall acceptable degree of the decision maker (DM) satisfaction. A number of numerical examples are solved to show the effectiveness of the proposed approach. The related results are compared with the Wang and Liang's approach. These computational results show that the proposed FMOLP model achieves lower objective functions and higher satisfaction degrees.

1. Introduction

Scheduling consists of planning and arranging jobs in an orderly sequence of operations in order to meet customer's requirements [17]. The scheduling of jobs and the control of their flows through a production process are the most significant elements in any modern manufacturing systems. The single-machine environment is the basis for other types of scheduling problems. In single-machine scheduling, there is only one machine to process all jobs optimizing the system performance measures such as makespan, completion time, tardiness, number of tardy jobs, idle times, sum of the maximum earliness and tardiness [18,19].

In single-machine scheduling, most research is concerned with minimizing a single criterion. However, scheduling problems often involve more than one aspect and therefore may require multiple criteria analyses [14]. Ishi and Tada [11] considered a single-machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. A fuzzy precedence relation relaxes the crisp precedence relation and represents a satisfaction level with respect to the precedence between two jobs. Thus, the problem considers an additional objective in order to maximize the minimum satisfaction level that is obtained by the fuzzy precedence relations. An algorithm for determining non-dominated solutions is proposed based on a graph representation of the precedence relations.

Adamopoulos and Pappis [2] presented a fuzzy-linguistic approach to a multi-criteria sequencing problem. They considered a single machine, in which each job is characterized by fuzzy processing times. The objective was to determine the processing times of jobs and the common due as well as to sequence the jobs on the machine where penalty values are associated with due dates assigned, earliness, and tardiness. Another approach to solve a multi-criteria single-machine scheduling problem is presented by Lee et al. [12]. They used linguistic values to evaluate each criterion (e.g., very poor, poor, fair, good, and very good) and to represent its relative weights (e.g., very unimportant, unimportant, moderately important, important, and very important). Also, a tabu search method is used as a stochastic tool to find the near-optimal solution for an aggregated fuzzy objective function.

Chanas and Kasperski [6] considered two single-machine scheduling problems with fuzzy processing times and fuzzy due dates. They defined the fuzzy tardiness of a job in a given sequence as a fuzzy maximum of zero and the difference between the fuzzy completion time and the fuzzy due date of this job. In the first problem, they minimized the maximal expected value of a fuzzy tardiness. In the second one, they considered minimizing the expected value of a maximal fuzzy tardiness. Chanas and Kasperski [7] considered the single-machine scheduling problem with parameters given in the form of fuzzy numbers. It is assumed that the optimal schedule in such a problem cannot be determined precisely. In their paper, it is shown how to calculate the degrees of possible and necessary optimality of a given schedule in one of the special cases of single-machine scheduling problems.

Azizoglu et al. [4] studied the bi-criteria scheduling problem of minimizing the maximum earliness and the number of tardy

* Corresponding author. Tel.: +98 21 82084183; fax: +98 21 88013102.
E-mail address: tavakoli@ut.ac.ir (R. Tavakkoli-Moghaddam).

1568-4946/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.asoc.2009.10.010

jobs on a single machine. They assumed that idle time inserted is not allowed. First, they examined the problem of minimizing maximum earliness while keeping the number of tardy jobs to its minimum value. They also developed a general procedure to find an efficient schedule minimizing a composite function of the two criteria by evaluating only a small fraction of the efficient solutions. They adopted the general procedures for the bi-criteria problem of minimizing the maximum earliness and number of tardy jobs.

Eren and Güner [8] considered a bi-criteria single-machine scheduling problem with sequence-dependent setup times. The objective function is to minimize the weighted sum of total completion time and total tardiness. An integer programming model was developed for the problem, which belongs to the NP-hard class. For solving problems containing a large number of jobs, a special heuristic is also used for large-sized problems. To improve the performance of the tabu search (TS) method, the result of the proposed heuristic algorithm was taken as an initial solution of the TS method.

Tavakkoli-Moghaddam et al. [20] presented a fuzzy goal programming approach for solving a mixed-integer model of a single-machine scheduling problem minimizing the total weighted flow time and total weighted tardiness. Because of the conflict between these two objectives, they proposed a fuzzy goal programming approach to solve the extended mathematical model of a single-machine scheduling problem. This approach was constructed based on desirability of the decision maker (DM) and tolerances considered on goal values. Huo et al. [10] considered bi-criteria single-machine scheduling problems involving the maximum weighted tardiness and number of tardy jobs. They gave NP-hardness proofs for the scheduling problems when one of these two criteria is the primary criterion and the other one is the secondary criterion. They considered complexity relationships between the various problems and proposed polynomial algorithms for some special cases as well as fast heuristics for the general case.

It is well known that the optimal solution of single-objective models can be quite different if the objective is different (e.g., for the simplest model of one machine without any additional constraint, the shortest processing time (SPT) rule is optimal to minimize $\bar{F}$ (i.e., mean flow time) but the earliest due date (EDD) rule is optimal to minimize the maximal tardiness ($T_{\max}$)). In fact, each particular decision maker often wants to minimize the given criterion. For example, the commercial manager of a company is interested in satisfying customers and then minimizing tardiness. On the other hand, the production manager wishes to optimize the use of machines by minimizing the makespan or the work in process by minimizing the maximum flow time. In addition, each of these objectives is valid from a general point of view. Since these objectives are conflicting, a solution may perform well for one objective, but giving bad results for others. For this reason, scheduling problems often have a multi-objective nature [14].

Zimmermann [22] first extended his FLP approach to a conventional multi-objective linear programming (MOLP) problem. For each of the objective functions of this problem, it was assumed that the DM has a fuzzy goal such as 'the objective functions should be essentially less than or equal to some value'. Then, the corresponding linear membership function is defined and the minimum operator proposed by Bellman and Zadeh [5] is applied in order to combine all objective functions. By introducing an auxiliary variable, this problem can be transformed into equivalent, conventional LP problem and can be easily solved by the simplex method. Subsequent work on fuzzy goal programming (FGP) is given in [9,13,15,16].

The aim of this paper is to develop a fuzzy multi-objective linear programming (FMOLP) model for solving the multi-objective single-machine scheduling problem in the fuzzy environment. First, a MOLP model of a multi-objective single-machine scheduling problem is constructed. The model attempts to minimize the makespan and total weighted tardiness. Furthermore, this model is converted into an FMOLP model by integrating fuzzy sets and objective programming approaches.

2. The multi-objective single-machine scheduling model

2.1. Problem formulation

The following notation and definitions are used to describe the multi-objective single-machine scheduling problem.

2.1.1. Indices and parameters

- N = number of jobs,
- P_i = processing time of job i (i = 1, 2,. . ., N),
- R_i = ready time of job i (i = 1, 2,. . ., N),
- D_i = due date of job i (i = 1, 2,. . ., N),
- W_i = importance factor (or weight) related to job i (i = 1, 2,. . ., N), and
- M = a large positive integer value.

2.1.2. Decision variables

$$X_{ij} = \begin{cases} 1 & \text{if job } j \text{ is scheduled after job } i, \\ 0 & \text{otherwise.} \end{cases}$$

2.1.3. Mathematical model

In this model, the objective is to find the best (or optimal) schedule minimizing the total weighted tardiness (i.e., Z_1) and makespan (i.e., Z_2) of a manufacturing system. It is worthy noting that these two objectives conflict each other [14]:

$$\text{Min}\, Z_1 = \sum_{i=1}^{N} W_i T_i \tag{1}$$

$$\text{Min}\, Z_2 = C_{\max} = \max(C_1, C_2, \ldots, C_N) \tag{2}$$

s.t.

$$C_i \geq R_i + P_i \quad \forall i \tag{3}$$

$$X_{ij} + X_{ji} = 1 \quad \forall i, j; i \neq j \tag{4}$$

$$C_i - C_j + MX_{ij} \geq P_i \quad \forall i, j; i \neq j \tag{5}$$

$$T_i = \max\{0, C_i - D_i\} \quad \forall i, j; i \neq j \tag{6}$$

$$X_{ij} \in \{0, 1\} \quad \forall i, j; i \neq j \tag{7}$$

Constraint (3) ensures that the completion time of a job is greater than its release time plus processing time. Constraint (4) specifies the order relation between two jobs scheduled. Constraint (5) stipulates relative completion times of any two jobs. M should be large enough for constraint (5). Constraint (6) specifies the tardiness of each job.

3. Fuzzy multi-objective linear programming (FMOLP) model

The original MOLP model can be converted to the FMOLP model using the piecewise linear membership function given in [9] in order to represent the fuzzy goals of the DM in the MOLP model given in [5]. In general, a piecewise linear membership function given in [5] can be adopted in order to convert the problem to be solved into an ordinary LP problem. The algorithm includes the following steps.

Table 1
Membership function $f_i(Z_i)$.

Z_1	$>Y_{10}$	Y_{10}	Y_{11}	Y_{12}	...	Y_{1v1}	Y_{1,V_1+1}	$< Y_{1,V_1+1}$
$f_1(Z_1)$	0	0	q_{11}	q_{12}	...	q_{1v1}	1	1
Z_2	$>Y_{20}$	Y_{20}	Y_{21}	Y_{22}	...	Y_{2v2}	Y_{2,V_2+1}	$< Y_{2,V_2+1}$
$f_2(Z_2)$	0	0	q_{21}	q_{22}	...	q_{2v2}	1	1

Note: $(0 \leq q_{ib} \leq 1, q_{ib} \leq q_{ib+1}, i = 1,2, b = 1, 2, \ldots, v_i)$.

Table 2
Generated data for 6-jobs.

Jobs	P_i	d_i	R_i	w_i
1	24	63	7	12
2	21	56	5	11
3	26	126	4	15
4	29	92	4	15
5	29	67	9	13
6	30	67	8	15

3.1. Algorithm

Step 1 Specify a degree of a membership function for several values of each objective function Z_i $(i = 1, 2)$ (see Table 1).
Step 2 Draw the piecewise linear membership function.
Step 3 Formulate the linear equations for each of the piecewise linear membership functions $f_i(Z_i)$ $(i = 1, 2)$.

The intervals for possible values of each objective function Z_i was specified by the user as $[Y_{i,V_i+1}, Y_{i_0}]$ implicating a piecewise membership function (see Table 1). In generally, piecewise membership functions can be divided into two main intervals. The first interval, $[0, Y_{i,V_i+1}]$, represents "*risk free*" values in the sense that a solution should almost be implementable and realistic. On the other hand, the second interval, $[Y_{i,V_i+1}, Y_{i_0}]$, represents "*full risk*" values that are most certainly unrealistic, impossible, and the solution obtained by these values is not implemental. While moving from "*risk free*" toward "*full risk*" values, it is moved from solutions with a high degree to those with a low degree [3]. In general, Y_{i,V_i+1} and Y_{i_0} indicate the optimistic and pessimistic viewpoints of the DM, respectively.

Step 3.1 Convert the membership functions $f_i(Z_i)$ into the form:

$$f_i(Z_i) = \sum_{b=1}^{P_i} \alpha_{ib}|Z_i - Y_{ib}| + \beta_i Z_i + \theta_i, \quad i = 1, 2, \tag{8}$$

where,

$$\alpha_{ib} = -\frac{\gamma_{i,b+1} - \gamma_i}{2}, \qquad \beta_i = \frac{\gamma_{i,V_i+1} + \gamma_{i1}}{2},$$
$$\theta_i = \frac{S_{i,V_i+1} + S_{i_1}}{2}. \tag{9}$$

Assume that $f_i(Z_i) = \gamma_{ir} Z_i + S_{ir}$ for each segment $Y_{i,r-1} \leq Z_i \leq Y_{ir}$, where γ_{ir} denotes the slope and S_{ir} is the y-intercept of the line segment on $[Y_{i,r-1}, Y_{ir}]$ in the piecewise linear membership function. Hence, we have:

$$\begin{aligned} f_i(Z_i) = &- \left(\frac{\gamma_{i2} - \gamma_{i1}}{2}\right)|Z_i - Y_{i1}| - \left(\frac{\gamma_{i3} - \gamma_{i2}}{2}\right)|Z_i - Y_{i2}| \\ &- \ldots - \left(\frac{\gamma_{i,V_i+1} - \gamma_{iV_i}}{2}\right)|Z_i - Y_{ip_i}| + \left(\frac{\gamma_{i,V_i+1} + \gamma_{i1}}{2}\right) \\ &Z_i + \frac{S_{i,V_i+1} + S_{i1}}{2}\left(\frac{\gamma_{i,b+1} - \gamma_{ib}}{2}\right) \neq 0, \\ &i = 1, 2, \quad b = 1, 2, \ldots, V_i, \end{aligned} \tag{10}$$

where,

$$\gamma_{i_1} = \left(\frac{q_{i_1} - 0}{Y_{i_1} - Y_{i_0}}\right), \qquad \gamma_{i_2} = \left(\frac{q_{i_2} - q_{i_1}}{Y_{i_2} - Y_{i_1}}\right),$$
$$\ldots, \gamma_{i,V_i+1} = \left(\frac{1 - q_{iV_i}}{Y_{i,V_i+1} - Y_{iV_i}}\right), \tag{11}$$

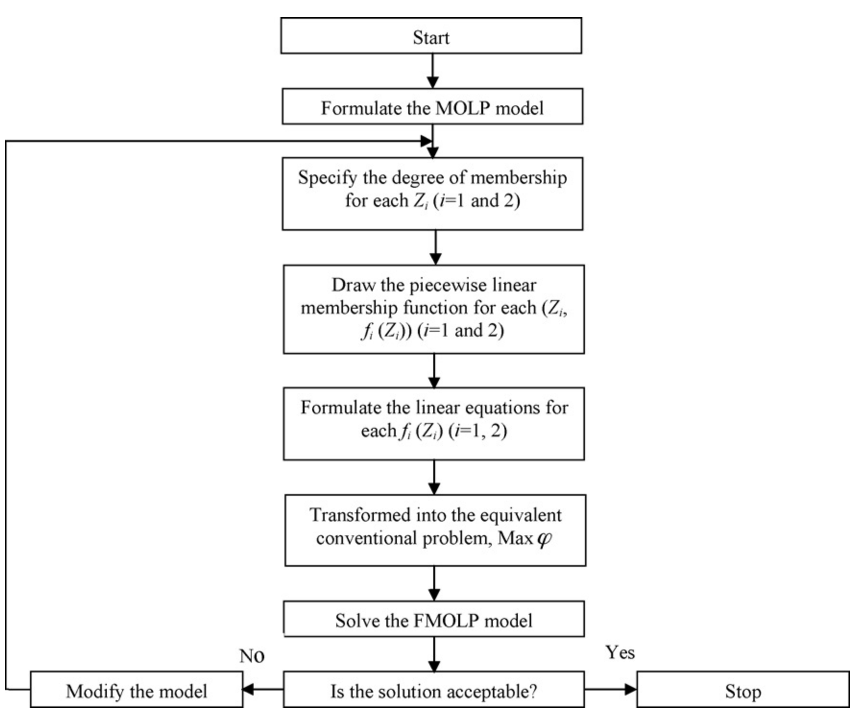

Fig. 1. The block diagram of the interactive FMOLP model development.

Table 3
Generated data for 7-jobs.

Jobs	P_i	d_i	R_i	w_i
1	24	94	8	16
2	21	180	3	13
3	26	80	5	12
4	29	86	3	16
5	29	93	9	19
6	30	176	6	12
7	20	96	10	17

V_i is the number of broken points of the ith objective function and S_{i,V_i+1} is the y-intercept for the section of the line segment on $[Y_{i,V_i}, Y_{i,V_i+1}]$.

Step 3.2 Introduce the following non-negative variables.

$$Z_i + d_{ib}^- - d_{ib}^+ = Y_{ib}, \quad i = 1, 2, \quad b = 1, 2, \ldots, V_i \tag{12}$$

where, d_{ib}^+ and d_{ib}^- denote the deviational variables in positive and negative directions at the ith point and Y_{ib} represents the values of the ith objective function at the ith point.

Step 3.3 Substituting Eq. (12) into Eq. (10) yields the following equation:

$$\begin{aligned} f_i(Z_i) = &-\left(\frac{\gamma_{i_2}-\gamma_{i_1}}{2}\right)(d_{i_1}^- - d_{i_1}^+) - \left(\frac{\gamma_{i_3}-\gamma_{i_2}}{2}\right)(d_{i_2}^- - d_{i_2}^+) \\ &- \ldots - \left(\frac{\gamma_{i,V_i+1}-\gamma_{iV_i}}{2}\right)(d_{iV_i}^- - d_{iV_i}^+) \\ &+ \left(\frac{\gamma_{i,V_i+1}-\gamma_{i1}}{2}\right) Z_i + \frac{S_{i,V_i+1}+S_{i1}}{2}, \quad i = 1, 2. \end{aligned} \tag{13}$$

Step 4 Introducing a two-phase approach for the auxiliary variable φ and then the problem can be transformed into the equivalent ordinary LP problem. The variable φ can be interpreted to represent an overall degree of the satisfaction with the DM's multiple fuzzy goals.

Phase 1 Using the "*max–min*" operator proposed by Bellman and Zadeh [5] and φ_0 satisfaction degree, the FMOLP problem can be solved as a single-objective problem:

$$\text{Max}\,\varphi_0 \tag{14}$$

s.t.

$$\begin{aligned} \varphi_0 \le &-\left(\frac{\gamma_{i_2}-\gamma_{i_1}}{2}\right)(d_{i_1}^- - d_{i_1}^+) - \left(\frac{\gamma_{i_3}-\gamma_{i_2}}{2}\right)(d_{i_2}^- - d_{i_2}^+) \\ &- \ldots - \left(\frac{\gamma_{i,V_i+1}-\gamma_{iV_i}}{2}\right)(d_{iV_i}^- - d_{iV_i}^+) \\ &+ \left(\frac{\gamma_{i,V_i+1}-\gamma_{i_1}}{2}\right) Z_i + \frac{S_{i,V_i+1}+S_{i_1}}{2}, \quad i = 1, 2. \end{aligned} \tag{15}$$

$$Z_i + d_{ib}^- - d_{ib}^+ = Y_{ib}, \quad i = 1, 2,\ b = 1, 2, \ldots, V_i \tag{16}$$

Constraints (3)–(7).

Phase 2 We use the result of the presented model to overcome disadvantages of Phase (1). In this phase, the solution is forced to improve, modify, and dominate the one obtained by the "*max–min*" operator. Also, we add constraints and a new auxiliary objective function to Phase (2) in order to achieve at least the satisfaction degree obtained in Phase (1). Thus, the proposed problem is as follows:

$$\text{Max}\,\varphi = \varphi_0 + \frac{1}{2}\sum_{i=1}^{2}(\varphi_i - \varphi_0) \tag{17}$$

s.t.

$$\begin{aligned} \varphi_0 \le \varphi_i \le &-\left(\frac{\gamma_{i_2}-\gamma_{i_1}}{2}\right)(d_{i_1}^- - d_{i_1}^+) \\ &-\left(\frac{\gamma_{i_3}-\gamma_{i_2}}{2}\right)(d_{i_2}^- - d_{i_2}^+) - \ldots \\ &-\left(\frac{\gamma_{i,V_i+1}-\gamma_{iV_i}}{2}\right)(d_{iV_i}^- - d_{iV_i}^+) + \left(\frac{\gamma_{i,V_i+1}-\gamma_{i_1}}{2}\right) Z_i \\ &+\frac{S_{i,V_i+1}+S_{i_1}}{2}, \quad i = 1, 2. \end{aligned} \tag{18}$$

$$Z_i + d_{ib}^- - d_{ib}^+ = Y_{ib}, \quad i = 1, 2,\ b = 1, 2, \ldots, V_i \tag{19}$$

Constraints (3)–(7).

Step 5 Execute and modify the interactive decision process. If the DM is not satisfied with the initial solution, the model must be changed until a satisfactory solution is found.

Fig. 1 illustrates the block diagram of the interactive FMOLP model development.

Table 4
Membership functions for the 6-jobs example.

Z_1	>2600	2400	2200	2000	1800	<1800
$f_1(Z_1)$	0	0	0.5	0.8	1	1
Z_2	>240	220	200	180	160	<160
$f_2(Z_2)$	0	0	0.4	0.7	1	1

4. Numerical examples and performance analysis of FMOLP

4.1. Basic data for numerical examples

Tables 2 and 3 summarize the data used for two numerical examples of 6- and 7-jobs. We consider the following assumptions:

1. The processing times (P_i) are integers and are generated from a uniform distribution on [20,30].
2. The due dates (d_i) are discrete uniform distribution on $[P(1-T-R/2),\ P(1-T+R/2)]$ where $P = \sum_{i=1}^{N} P_i$ given in [14]. The values of T and R are set to 0.4 and 0.6, respectively; when T increases the due dates are more restrictive and when R increases the due dates are more diversified.
3. The ready times (R_i) are generated from a uniform distribution on [1,10].
4. The jobs' weights (w_i) are uniformly generated from discrete uniform distribution on [10,20].

4.2. Formulate the FMOLP model

First, we determine the initial solutions for each objective function by using the conventional LP model. The results are obtained

Table 5
Membership functions for the 7-jobs example.

Z_1	>1500	1300	1100	900	700	<700
$f_1(Z_1)$	0	0	0.5	0.8	1	1
Z_2	>260	240	220	200	180	<180
$f_2(Z_2)$	0	0	0.4	0.7	1	1

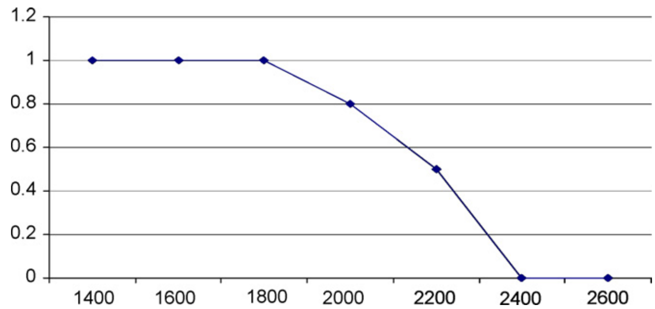

Fig. 2. Shape of membership function $(Z_1, f(Z_1))$.

by $Z_1 = 1846$ and $Z_2 = 163$. Then, we formulate the FMOLP model by using the initial solutions and the MOLP model was presented in Section 3. Tables 4 and 5 give the piecewise linear membership functions of the proposed model for the 6- and 7-jobs examples, respectively. Figs. 2 and 3 illustrate the corresponding shapes of the piecewise linear membership functions for the 6-jobs example.

The complete FMOLP model of the 6-jobs numerical example is given below.

$$\text{Max}\,\varphi = \varphi_0 + \left(\frac{1}{2}\right)((\lambda_1 - \varphi_0) + (\lambda_2 - \varphi_0)) \quad (20)$$

s.t.

$$\varphi_0 \le \lambda_1 \le -0.0005(d_{11}^- - d_{11}^+) - 0.00025(d_{12}^- - d_{12}^+)$$
$$-0.00175 \times \left\{\sum_{i=1}^{N} w_i T_i\right\} + 4.4 \quad (21)$$

$$\varphi_0 \le \lambda_2 \le -0.0025(d_{21}^- - d_{21}^+) - 0.0175 \times \{C_{\max}\} + 3.9 \quad (22)$$

$$\left\{\sum_{i=1}^{N} w_i T_i\right\} + d_{11}^- - d_{11}^+ = 2200 \quad (23)$$

$$\left\{\sum_{i=1}^{N} w_i T_i\right\} + d_{12}^- - d_{12}^+ = 2000 \quad (24)$$

$$\{C_{\max}\} + d_{21}^- - d_{21}^+ = 200 \quad (25)$$

$$C_i \ge R_i + P_i, \quad \forall i \quad (26)$$

$$X_{ij} + X_{ji} = 1, \quad \forall i, j; i \ne j \quad (27)$$

$$C_i - C_j + MX_{ij} \ge P_i, \quad \forall i, j; i \ne j \quad (28)$$

$$T_i = \max\{0, C_i - D_i\}, \quad \forall i, j; i \ne j \quad (29)$$

$$X_{ij} \in \{0, 1\}, \quad \forall i, j; i \ne j \quad (30)$$

$$\varphi, C_i, T_i, d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+, d_{21}^-, d_{21}^+ \ge 0 \quad (31)$$

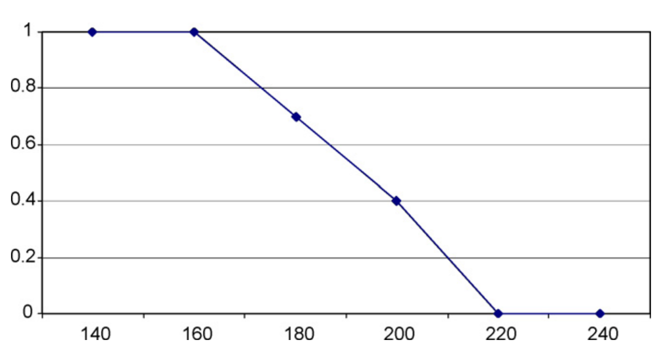

Fig. 3. Shape of membership function $(Z_2, f(Z_2))$.

Table 6
FMOLP model solutions for 6-jobs.

Jobs	C_i	T_i
1	50	0
2	26	0
3	135	9
4	109	17
5	164	97
6	80	13

Table 7
FMOLP model solutions for 7-jobs.

Jobs	C_i	T_i
1	56	0
2	182	2
3	131	51
4	32	0
5	85	0
6	161	0
7	105	9

Table 8
Results obtained from Scenario 1.

Z_1	$f_1(Z_1)$	Vary $(Z_2, f_2(Z_2))$	
		Z_2	$f_2(Z_2)$
>2600	0	>225	0
2400	0	225	0
2200	0.5	205	0.4
2000	0.8	185	0.7
1800	1	165	1
<1800	1	<165	1
Original levels			

Objective function: 0.9845 > 0.947. Status: the overall degree of satisfaction is accepted.

In constraints (18)–(21), d_{11}^+, d_{12}^-, and d_{21}^+ denote the deviational variables at the first and second points. In addition, 2200(Y_{11}), 2000(Y_{12}), and 200(Y_{21}) represent the values of the first and second objective function at the first and second point.

4.3. Output solutions

The Lingo computer package is used to run the proposed FMOLP model on an Intel® 1.61 GHz Processor with 512 Mb RAM. The results for the 6-jobs example are as follows: $Z_1 = 1846$ and $Z_2 = 164$. Also, the overall degree of satisfaction with the DM's multiple fuzzy goals is 0.947. Table 6 presents computational solutions of this numerical example for each decision variable within 2 s of CPU times. The best sequence of jobs is J_2–J_1–J_6–J_4–J_3–J_5.

Furthermore, we solve the 7-jobs example yielding the results as follows: $Z_1 = 791$, $Z_2 = 182$, and 93.95% for the overall degree of satisfaction. Table 7 illustrates the computational solutions of

Table 9
Results obtained from Scenario 2.

Z_2	$f_2(Z_2)$	Vary $(Z_1, f_1(Z_1))$	
		Z_1	$f_1(Z_1)$
>220	0	>2500	0
220	0	2500	0
200	0.4	2300	0.5
180	0.7	2100	0.8
160	1	1900	1
<160	1	<1900	1
Original levels			

Objective function: 0.997 > 0.947. Status: the overall degree of satisfaction is accepted.

Table 10
Comparison results for the 6-jobs example.

	LP-1	LP-2	Wang and Liang's method [20]	Proposed method (FMOLP)
Objective function	Min Z_1	Min Z_2	Max φ	Max φ
φ	100	100	0.94	0.947
Z_1	1846	2731	1860	1846
Z_2	164	163	164	164
Optimal sequence	J2–J1–J6–J4–J3–J5	J3–J2–J5–J6–J1–J4	J2–J6–J1–J4–J3–J5	J2–J1–J6–J4–J3–J5

Table 11
Comparison results for the 7-jobs example.

	LP-1	LP-2	Wang and Liang's method [20]	Proposed method (FMOLP)
Objective function	Min Z_1	Min Z_2	Max φ	Max φ
φ	100	100	90.9	93.95
Z_1	765	2462	791	791
Z_2	176	182	186.06	182
Optimal sequence	J4–J1–J5–J7–J3–J2–J6	J4–J3–J5–J6–J2–J7–J1	J4–J5–J1–J7–J3–J6–J2	J4–J1–J5–J7–J3–J6–J2

this numerical example for the decision variable. Also, it takes about 3 s of CPU times and the best sequence of this example is J4–J1–J5–J7–J3–J6–J2.

The proposed model provides the most flexible decision-making and adjustment processes. For instance, if the DM does not accept the initial overall degree of satisfaction of 0.947 as given in the 6-jobs example, then the DM may try to adjust this φ value by taking account of relevant information in order to obtain a set of output solutions for making the decision. Two scenarios for the 6-job example are carried out in order to implement the FMOLP model by manipulating different alternatives and analyzing the sensitivity of decision parameters based on the preceding numerical example. It shows the process of modifying the initial overall degree of satisfaction.

Scenario 1: Set $(Z_1, f_1(Z_1))$ to their original values in the numerical example and vary $(Z_2, f_2(Z_2))$. Table 8 presents the data and results of implementation the Scenario 1.

Scenario 2: Set $(Z_2, f_2(Z_2))$ to their original value in the numerical example and vary $(Z_1, f_1(Z_1))$. Table 9 presents the data and results of implementation the Scenario 2.

4.4. Performance analysis

We consider the solution of the illustrative examples using a different method to evaluate the performance of the proposed approach, Tables 10 and 11 compare the results obtained by using the single-objective LP model and Wang and Liang's approach [21] with the proposed FMOLP method for two given examples. Minimizing the total weighted tardiness (Z_1) yields an optimal value of 1846 and 765 for the 6- and 7-jobs examples, respectively. Minimizing the makespan (Z_2) yields an optimal value of 163 and 182 for the 6- and 7-jobs examples, respectively. Alternatively, we use the Wang and Liang's approach [21] that minimizes the total weighted tardiness and the makespan simultaneously. The associated results are $Z_1 = 1860$ and 791 for the 6- and 7-job examples, respectively, and $Z_2 = 164$ and 186.06 for the 6- and 7-jobs examples, respectively. In addition, the overall degrees of the DM satisfaction are 0.94 and 0.909 for the 6- and 7-job examples, respectively. In contrast with the proposed FMOLP method, the results are $Z_1 = 1846$ and 791 for the 6- and 7-jobs examples, respectively, and $Z_2 = 164$ and 182 for the 6- and 7-jobs examples, respectively. Also, the overall degrees of the DM satisfaction are 0.947 and 0.9395 for the 6- and 7-job examples, respectively. These tables indicate that the results by using the proposed FMOLP method under an acceptable degree of the DM satisfaction in a fuzzy environment.

We define the following family of distance functions [1] in order to determine the degree of closeness of the FMOLP solutions to the ideal solution:

$$D_p(w, K) = \left[\sum_{k=1}^{K} w_k^p (1 - d_k)^p\right]^{1/p} \tag{32}$$

where d_k represents the degree of closeness of the preferred solution vector to the optimal solution vector with respect to the kth objective function. $w = (w_1, w_2, \ldots, w_k)$ is the vector of the objectives satisfaction degrees. Power p represents a distance parameter, $1 \le p \le \infty$. Assuming $\sum_{k=1}^{K} w_k = 1$, we can write $D_p(w, K)$ with $p = 1$, 2 and ∞ as follows:

$$D_1(w, K) = 1 - \sum_{k=1}^{K} w_k d_k \quad \text{(the Manhattan distance)} \tag{33}$$

$$D_2(w, K) = \left[\sum_{k=1}^{K} w_k^2 (1 - d_k)^2\right]^{1/2} \quad \text{(the Euclidean distance)} \tag{34}$$

$$D_\infty(w, K) = \max_k \{w_k(1 - d_k)\} \quad \text{(the Tchebycheff distance)} \tag{35}$$

where in a minimization problem, d_k takes the form: d_k = (the optimal solution of Z_k)/(the preferred compromise solution Z_k). We consider $w_1 = w_2 = 0.5$ in the above equations.

Thus, we conclude that the approach derived a preferred solution is better than the other approaches if: Min $D_p(\lambda, K)$ is achieved by its solution with respect to some p as discussed in [1]. We compare the degree of closeness of both Wang and Liang's approach and our proposed FMOLP approach with the ideal solutions summarized in Table 12. In this table, it is clear that our approach to find the preferred solutions outperforms the approaches given in [21] for all the distance functions D_1, D_2 and D_∞. This comparison shows our proposed FMOLP approach is better than the Wang and Liang's approach.

Table 12
Comparison of the degree of closeness.

	Wang and Liang's approach [21]		Proposed FMOLP approach	
	6-Jobs	7-Jobs	6-Jobs	7-Jobs
D_1	0.0068	0.0273	0.0030	0.0164
D_2	0.0048	0.0197	0.0030	0.0164
D_∞	0.0038	0.0164	0.0030	0.0164

5. Conclusion

In most real-world single-machine scheduling problems, the decision maker (DM) must handle conflicting objectives simultaneously. This paper developed an FMOLP method for solving single-machine scheduling problems with multiple fuzzy objectives and piecewise linear membership functions. The proposed method aims at minimizing the total weighted tardiness and makespan simultaneously. Moreover, the proposed FMOLP method provides a systematic framework that facilitates the fuzzy decision-making process to obtain a satisfactory solution.

A numerical example is used to demonstrate the feasibility of applying the proposed FMOLP method to the single-machine scheduling problem. The proposed method yields an efficient solution and acceptable degree of DM satisfaction. The proposed FMOLP method is based on Hannan's method, which implicitly assumes that the minimum operator is the appropriate representation of the human DM who combines fuzzy sets by logical 'and' operations.

Acknowledgement

The authors would like to thank the anonymous reviewers for their helpful comments and suggestions, which greatly improved the presentation of this paper. In addition, this study was partially supported by the University of Tehran under the research Grant No. 8106043/1/12. The first author is grateful for this financial support.

References

[1] W.F. Abd El-Wahed, S.M. Lee, Interactive fuzzy goal programming for multi objective transportation problems, Omega: International Journal of Management Science 34 (2006) 158–166.
[2] G.I. Adamopoulos, C.P. Pappis, A fuzzy linguistic approach to a multi-criteria sequencing problem, European Journal of Operational Research 92 (1996) 628–636.
[3] F. Askin, Z. Gungor, A parametric model for cell formation and exceptional elements problems with fuzzy parameters, Journal of Intelligent Manufacturing 16 (2005) 103–114.
[4] M. Azizoglu, S. Kondakci, M. Köksalan, Single machine scheduling with maximum earliness and number tardy, Computers and Industrial Engineering 45 (2) (2003) 257–268.
[5] R.E. Bellman, L.A. Zadeh, Decision-making in a fuzzy environment, Management Science 17 (1970) 141–164.
[6] S. Chanas, A. Kasperski, On two single machine scheduling problems with fuzzy processing times and fuzzy due dates, European Journal of Operational Research 147 (2003) 281–296.
[7] S. Chanas, A. Kasperski, Possible and necessary optimality of solutions in the single machine scheduling problem with fuzzy parameters, Fuzzy Sets and Systems 142 (2004) 359–371.
[8] T. Eren, E. Güner, A bicriteria scheduling with sequence-dependent setup times, Applied Mathematics and Computation 179 (1) (2006) 378–385.
[9] E.L. Hannan, Linear programming with multiple fuzzy goals, Fuzzy Sets and Systems 6 (1981) 235–248.
[10] Y. Huo, J.Y.-T. Leung, H. Zhao, Bi-criteria scheduling problems: number of tardy jobs and maximum weighted tardiness, European Journal of Operational Research 177 (1) (2007) 116–134.
[11] H. Ishi, M. Tada, Single machine scheduling problem with fuzzy precedence relation, European Journal of Operational Research 87 (1995) 284–288.
[12] H.T. Lee, S.H. Chen, H.Y. Kang, Multi-criteria scheduling using fuzzy theory and tabu search, International Journal of Production Research 40 (5) (2002) 1221–1234.
[13] H. Leberling, On finding compromise solutions in multi-criteria problems using the fuzzy min-operator, Fuzzy Sets and Systems 6 (1981) 105–118.
[14] T. Loukil, J. Teghem, D. Tuyttens, Solving multi-objective production scheduling problems using meta-heuristics, European Journal of Operational Research 161 (2005) 42–61.
[15] M.K. Luhandjula, Compensatory operators in fuzzy programming with multiple objectives, Fuzzy Sets and Systems 8 (1982) 245–252.
[16] M. Sakawa, An interactive fuzzy satisfying method for multi-objective linear fractional programming problems, Fuzzy Sets and Systems 28 (1988) 129–144.
[17] D.R. Sule, Industrial Scheduling, PWC Publishing Company, 1997.
[18] R. Tavakkoli-Moghaddam, G. Moslehi, M. Vasei, A. Azaron, Optimal scheduling for a single machine to minimize the sum of maximum earliness and tardiness considering idle insert, Applied Mathematics and Computation 167 (2) (2005) 1430–1450.
[19] R. Tavakkoli-Moghaddam, G. Moslehi, M. Vasei, A. Azaron, A branch-and-bound algorithm for a single machine to minimize the sum of maximum earliness and tardiness with idle insert, Applied Mathematics and Computation 174 (1) (2006) 388–408.
[20] R. Tavakkoli-Moghaddam, B. Javadi, N. Safaei, Solving a mixed-integer model of a single machine scheduling problem by a fuzzy goal programming approach, Wseas Transactions on Business and Economics 3 (2) (2006) 45–52.
[21] R.C. Wang, T.F. Liang, Application of fuzzy multi-objective linear programming to aggregate production planning, Computer and Industrial Engineering 46 (2004) 17–41.
[22] H.J. Zimmermann, Description and optimization of fuzzy systems, International Journal of General Systems 2 (1976) 209–215.