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Damage Detection at Element Level in Structures with Different Support Conditions

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Abstract: An iterative least square time domain system identification method is developed in this study to identify the stiffness of each member and detect damages in plane frames with different support conditions, i.e., fixed, hinged and semi-rigid. The frames are represented by two dimensional beam finite elements, while the semi-rigid supports are modeled by using rotational springs. The stiffness of the elements are identified using theoretical dynamic response measurements only. The time history of the dynamic excitation force is not required in the identification process and the developed method has no restriction on types of forces to be used to excite the plane frames. The procedure detects damages by tracking the changes in stiffness of each element. With the help of several numerical examples, it is shown that the method can identify the structures very accurately and can detect damages in members of plane steel frames with different support conditions. It is shown that the procedure has the potential to be used as a nondestructive health assessment technique. The accuracy of the method is much better than other currently available methods in the literature.

Key words: System identification, damage detection, finite element method, semi-rigid supports, unknown dynamic force

INTRODUCTION

Health monitoring of existing structures has recently become an important and challenging issue to the engineering profession. It generated a multi-disciplinary research interest. Structures deteriorate with time as they age. They can also suffer damage due to a natural event like a major earthquake or high wind. Man made events like explosions can also cause damage to them. In all cases, the performances of structures are expected to change reflecting the amount of degradation they have suffered. Objective health assessment is thus an integral part performance-based design concept now being promoted in the profession. Visual inspections on a regular basis or in an emergency basis just after a major natural or man-made event are generally suggested or recommended in the design guidelines. Damages invisible to the naked eyes cannot be detected. Visual inspections may be impractical in some cases and may be ineffective in some other cases. The outcomes of inspections of visually detectable damages may depend on many subjective factors including the experience of the inspectors, the type of structure being inspected and the location of the damages. An objective, efficient, economical and robust nondestructive inspection procedure is urgently needed to evaluate the health of the existing structures.

Global dynamic structural responses can be used for the health assessment purpose since they depend on the health of all structural elements. Using the information on the exciting force and the dynamic responses caused by the force, the state of the structural system can be identified using a System Identification (SI) technique. By tracking the changes in the signature, the current state or health of all structural elements can be established. It is a multi disciplinary research area. The available literature on the related topics is very extensive (Housner *et al*., 1997). However, only recently the potential of SI as a nondestructive structural health assessment technique has received attention from the research community.

Many currently available SI techniques are based on the frequency domain approach (Alampalli and Fu, 1997; Law *et al*., 1998; Vestroni and Capecchi, 2000; Shi *et al*., 2002; Lam *et al*., 2004; Beck and Yuen, 2004; Yuen and Beck, 2003; Yuen *et al*., 2004; Barroso and Rodriguez, 2004). There are many advantages of the frequency domain approaches. The modal information can be expressed in countable forms in terms of frequencies and mode shape vectors for comparison and changes in them can indicate the current health of a structure. Since, modal information represents global information, there may be an averaging effect of the presence of noise or error in the response measurements. It is also not necessary to manipulate a large amount of response data in this approach. However, the changes in the frequencies and

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mode shapes can help to establish the altered or the damage state of a structure, i.e., whether the structure is damaged in the global sense or not, but the locations of damages at the element level cannot be detected with this approach. Furthermore, the frequencies may not change significantly even in the presence of major damages, indicating low sensitivity for the damage assessment purpose. The modal information also loses it effectiveness as a damage indicator if only first few modes are used. Higher order modes are difficult to evaluate, particularly for large structural systems and there is no guidelines on how many modes are required for the effective damage identification purpose. The time domain approaches would be more appropriate for identifying damages at the local element level (Wang and Haldar, 1994; Koh *et al*., 2006; Ta *et al*., 2006).

Accepting the proposition that a time domain SI approach will be more appropriate, the implementation potential of the method to identify damages in real structures needs to be investigated. Outside the highly controlled laboratory environment, the dynamic excitation force is very difficult to measure. The measured dynamic excitation force could be so noisy that the overall SI concept might not be applicable. Furthermore, the information on the excitation force may not be available after natural or man made events. The application potential of a time domain SI approach in detecting damages at the element level could be significantly improved, if a structural system can be identified using only response information. The task is expected to be very challenging since two of the three elements of the SI concept will be unknown.

Wang and Haldar (1994) established a conceptual framework for such an approach. They called it an Iterative Least Square with Unknown Input (ILS-UI) method. They used viscous damping in the governing equation of motion. They identified shear-type buildings, the simplest mathematical representation of complicated structural systems. They demonstrated that the method is robust and accurate and can identify a system. Ling and Haldar (2004) improved the method particularly for large real structures by proposing Rayleigh-type damping instead of viscous damping. They called it a modified ILS-UI method.

In this study, an iterative least square method is developed to identify the stiffness of each element in 2 dimensional frames with different support conditions including semi-rigid. The main objective is to detect damages whether small or large in some members without knowing the dynamic excitation force. The responses of the structure to any dynamic loading are obtained by using finite element software (ADINA). The structure is modeled as a two dimensional frame, while the semi-rigid supports are modeled by using rotational springs.

The acceleration, velocity and displacement or rotation time histories for each node are expected to provide the necessary signature to identify the structure and detect damages in two dimensional framed structures with different support conditions. The proposed iterative technique can be used to identify members' stiffness as well as to detect any degradation in the stiffness which might occur due to any dynamic loading.

THEORY OF THE METHOD

The equation of motion of a linear system with Rayleigh damping can be expressed as:

$$K\,x(t) + (\alpha M + \beta K)\dot{x}(t) = f(t) - M\,\ddot{x}(t) \tag{1}$$

where, is the unknown stiffness matrix, M is the known mass matrix; $\ddot{x}(t)$ and x(t) are vectors containing the known dynamic responses in terms of acceleration, velocity and displacement respectively; factor α is the mass-proportional damping coefficient, factor β is the stiffness-proportional damping coefficient and f(t) is the unknown dynamic excitation force vector.

Eq. 1 can be written in a matrix form as:

$$A(t) * P = F(t) \tag{2}$$

where, A(t) is the matrix composed of the system responses vectors of velocity and displacement with stiffness matrix; P is vector composed of the unknown system parameters- damping and stiffness-that need to be known at the element level and F(t) is vector composed of the unknown dynamic excitation and inertia forces.

The A matrix for two dimensional frames can be expressed as:

$$A(t) = [\,R^1 x(t)\; R^2 x(t) \ldots R^{ne}\, x(t) R^1 \dot{x}(t) R^2 \dot{x}(t) \ldots R^{ne}\, \dot{x}(t)\; M\ddot{x}(t)\,] \tag{3}$$

where, R^i is 6×6 stiffness matrix of beam element to be defined later according to support conditions for the ith element; ne is the total number of elements or members of the structure and $\ddot{x}(t)$ and x(t) are vectors containing the known dynamic responses in terms of velocity and displacement, respectively, defined earlier.

The P vector is the unknown system parameters and can be written as:

$$P = [k_1, k_2, \ldots, k_{ne}, \beta k_1, \beta k_2, \ldots, \beta k_{ne}, \alpha]^T \tag{4}$$

where, $k_i = E_i I_i / L_i$ is the unknown stiffness parameter for the ith beam element that needs to be identified, Ei, Ii and

Ai are the Young's modulus, moment of inertia and area of the cross-section of the i^{th} element of the beam, respectively.

The F(t) is the force vector and can be defined as:

$$F(t) = f(t) - M\,\ddot{x}(t) \tag{5}$$

where, f(t) is the unknown dynamic excitation force vector, M is the known consistent mass matrix to be defined later according to support conditions and $\ddot{x}(t)$ is the vector containing the known dynamic responses in terms of acceleration as stated earlier.

To solve for vector P in Eq. 2; the Least Square technique is used. It is based on minimizing the total error. It is simple to solve for vector P provided that the force vector F(t) and A(t) are known. However, as mentioned earlier, the input excitation f(t) is not known. The proposed method solves for the vector P by using an iteration process. The iteration process cannot be initiated without using information on the dynamic excitation force. Since, the force is not available, to start the iteration the input excitation force f(t) is assumed to be zero for all time points.

With this assumption, the F(t) vector can be obtained and a first estimation of the unknown system parameters P can be evaluated. Using Eq. 2 and the estimated system parameters P, the information on the dynamic excitation force f(t) can be generated at all time points. Using the information on the generated excitation force, the estimation on the system parameters can be updated. The algorithm will iterate until the system parameters are evaluated with a pre-determined accuracy. The convergence criterion is set with respect to the evaluated dynamic excitation force. The procedure will continue until there is a convergence in the input excitation with a predetermined tolerance (ϵ). The tolerance (ϵ) is set to be 10^{-6}. The convergence requires $|f^{i+1}-f^{i}| \le 10^{-6}$ applied for all time points. It is interesting to note that the algorithm not only identifies unknown stiffness parameters of all the elements, it also identifies the time history of the unknown dynamic excitation force.

Consistent mass matrix (M) and stiffness matrix (K) for fixed and hinged supports: The two dimensional plane steel frames are represented by two dimensional beam elements. There are three Degrees of Freedoms (DOFs) at each node. Two are translational DOFs; one is along the length of the element (x-axis) and the other is perpendicular to the x-axis, i.e., along the y-axis and the third DOF represents the rotation of the node.

The consistent local mass matrix, M^i for can be represented as:

$$M^i = \frac{\bar{m}_i L_i}{420}\begin{bmatrix} 140 & & & & & \\ 0 & 156 & & & \text{Sym.} & \\ 0 & 22L_i & 4L_i^2 & & & \\ 70 & 0 & 0 & 140 & & \\ 0 & 54 & 13L_i & 0 & 156 & \\ 0 & -13L_i & -3L_i^2 & 0 & -22L_i & 4L_i^2 \end{bmatrix} \tag{6}$$

where, L_i is the element length and $\bar{m}_i$ is the mass per unit length.

The local stiffness matrix, K^i can be represented by:

$$K^i = \frac{E_i I_i}{L_i}\begin{bmatrix} A_i/I_i & 0 & 0 & -A_i/I_i & 0 & 0 \\ 0 & 12/L_i^2 & 6/L_i & 0 & -12/L_i^2 & 6/L_i \\ 0 & 6/L_i & 4 & 0 & -6/L_i & 2 \\ -A_i/I_i & 0 & 0 & A_i/I_i & 0 & 0 \\ 0 & -12/L_i^2 & -6/L_i & 0 & 12/L_i^2 & -6/L_i \\ 0 & 6/L_i & 2 & 0 & -6/L_i & 4 \end{bmatrix} \tag{7}$$

This equation can be rewritten as:

$$K^i = k_i R^i \tag{8}$$

where, $k_i = E_i I_i / L_i$ is the unknown stiffness parameter for the ith beam element that needs to be identified and R^i is the 6×6 matrix shown in Eq. 7 in the square bracket for the ith element.

The global mass M and stiffness K matrices for a frame can be assembled from the local mass and stiffness matrices of all the elements by considering their connectivity and the direct stiffness method, respectively.

Modeling semi-rigid supports: The semi-rigid supports at the base are modeled by adding rotational springs to hinged supports. The local consistent mass (M^i) and stiffness matrices (K^i) in Eq. 6 and 7 for the beam elements that are connected to the semi-rigid supports are to be modified as follows:

$$M^i_{SR} = \frac{\bar{m}_i L_i}{420}\begin{bmatrix} 140 & & & & & \\ 0 & 156 & & & \text{Sym.} & \\ 0 & 22L_i & 4L_i^2 & & & \\ 70 & 0 & 0 & 140 & & \\ 0 & 54 & 13L_i & 0 & 156 & \\ 0 & -13L_i & -3L_i^2 & 0 & -22L_i & 4L_i^2 + M_f \end{bmatrix} \tag{9}$$

where, M^i_{SR} is the local consistent matrix for beam element that are connected to semi-rigid supports and M_f is the mass of the rotational spring that is added at the base to model the semi-rigid support.

$$K_{SR}^{i} = \frac{E_i I_i}{L_i}\begin{bmatrix} A_i/I_i & 0 & 0 & -A_i/I_i & 0 & 0 \\ 0 & 12/L_i^2 & 6/L_i & 0 & -12/L_i^2 & 6/L_i \\ 0 & 6/L_i & 4 & 0 & -6/L_i & 2 \\ -A_i/I_i & 0 & 0 & A_i/I_i & 0 & 0 \\ 0 & -12/L_i^2 & -6/L_i & 0 & 12/L_i^2 & -6/L_i \\ 0 & 6/L_i & 2 & 0 & -6/L_i & 4+K_f \end{bmatrix} \quad (10)$$

where, K^i_{SR} is the local stiffness matrix for beam element that are connected to semi-rigid supports and K_f is the rotational stiffness of the added spring at the base to model the semi-rigid support.

NUMERICAL EXAMPLES

In order to verify the method in term of its effectiveness in identifying the stiffness for undamaged and damaged elements in two dimensional plane frames; three different numerical examples are adopted. First for a three story one bay steel frame with fixed support, the second for the same frame with hinged support and finally for the same frame with semi-rigid supports.

Example 1: Steel frame with fixed supports: A three story one bay plane steel frame, as shown in Fig. 1, is considered. The frame consists of 6 columns and 3 beams. W18×71 steel section is used for all the members. The supports in this frame are assumed fixed; i.e., nodes 7 and 8. The structure is represented by 18 DOFs. The masses of all beams and columns are known. The theoretical stiffness of beams and columns (k = EI/L) are calculated to be 10650 and 26625 kN m, respectively.

Two cases were considered, namely, damage-free and damaged. The damaged one was resembled in the frame by a 5% reduction in the stiffness of a column; element 3; and a beam; element 7. The frame is excited by two blast forces applied horizontally at the third floor at node 1 and at the second floor at node 3, as shown in Fig. 1. The blast force at node 1 is assumed to be rectangular pulse with force magnitude of 44.48 kN acting for duration of 0.05 sec and the other blast force at node 3 is assumed to be rectangular pulse with force magnitude of 22.24 kN acting for duration of 0.05 sec. The responses of the frame are calculated in terms of displacements, velocities and accelerations at all nodes using finite element software (ADINA).

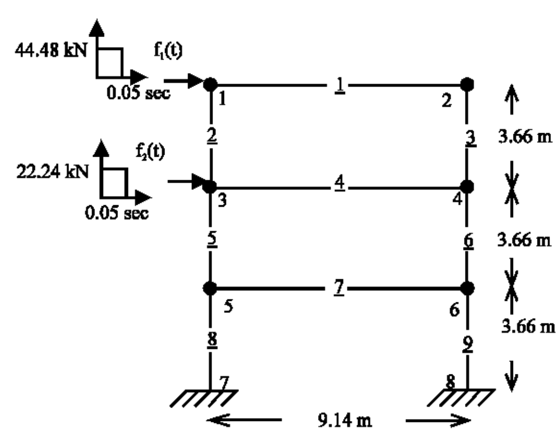

Fig. 1: Three story steel frame used in example 1

The responses are recorded at 0.01 sec time intervals from 0.03 to 1.20 sec providing 118 time points. After the theoretical responses are evaluated, the information on the blast forces is completely ignored. Using the response measurements only, the elements of the steel frame are identified by the method. The theoretical and identified stiffness parameters (EI/L) of all elements for the damage-free and damaged cases are shown in Table 1.

For damage-free case; the results indicate that the method identified the stiffness very well with a maximum error in identification of 0.001%. On the other hand, for damaged case, the results indicate that the identified stiffness values of elements 3 and 7 decreased by 3.39 and 3.46%, respectively compared with the initial theoretical values. At the same time, the stiffness of the other elements increased indicating redistribution of stiffness. By comparing the initial theoretical stiffness with the identified values, it can be inferred that the damage are in elements 3 and 7.

Example 2: Steel frame with hinged supports: The same three story one bay plane steel frame shown in Fig. 1 is considered here again. In this example the supports are assumed to be hinged; i.e., nodes 7 and 8. The structure is represented by 20 DDOF. The frame is excited by the same two blast forces applied horizontally at the third floor at node 1 and at the second floor at node 3. Two cases were considered, namely, damage-free and damaged. The damaged one was the same as in example 1; i.e., 5% reduction in the stiffness of element 3 and 7.

Table 1: Stiffness (EI/L) identification for example 1

Member	Initial theoretical value (kN m)	Identified damage-free (kN m)	Error (%)	Identified damaged case (kN m)	Effect (%)
k_1	10650	10650	-0.001	10812	+1.52
k_2	26625	26625	-0.001	27014	+1.46
k_3	26625	26625	-0.001	25722	-3.39
k_4	10650	10650	-0.001	10825	+1.64
k_5	26625	26625	-0.001	27073	+1.68
k_6	26625	26625	-0.001	27026	+1.50
k_7	10650	10650	-0.001	10282	-3.46
k_8	26625	26625	-0.001	27043	+1.57
k_9	26625	26625	-0.001	27095	+1.76

Table 2: Stiffness (EI/L) identification for example 2

Member	Initial theoretical value (kN m)	Identified damage-free (kN m)	Error (%)	Identified damaged case (kN m)	Effect (%)
k_1	10650	10649	-0.013	10857	+1.944
k_2	26625	26622	-0.012	27122	+1.865
k_3	26625	26622	-0.013	25821	-3.020
k_4	10650	10649	-0.010	10847	+1.853
k_5	26625	26623	-0.010	27121	+1.861
k_6	26625	26623	-0.010	27086	+1.729
k_7	10650	10649	-0.009	10308	-3.214
k_8	26625	26623	-0.009	27127	+1.885
k_9	26625	26623	-0.009	27131	+1.901

The responses are recorded at all nodes using ADINA in the same way mentioned above at 0.01 sec time intervals from 0.02 to 1.30 sec providing 129 time points. The theoretical and identified stiffness parameters (EI/L) of all elements are shown in Table 2 for both cases. For damage-free case, the results indicate that the method identified the stiffness very well with a maximum error in identification of 0.013%. For the damaged one, the results indicate that the stiffness values of elements 3 and 7 decreased by 3.02 and 3.214%, respectively, indicating that the damage are in elements 3 and 7.

Example 3: Steel frame with semi-rigid supports: The same three story one bay plane steel frame shown in Fig. 1 is considered here again. In this example the supports are assumed to be semi-rigid supports. Rotational springs are added at nodes 7 and 8 to model the semi-rigid supports as stated earlier. The frame is excited by the same two blast forces mentioned in example 1 above.

- **Case 1:** Stiffness of the rotational springs is assumed to be 10% of the rotational stiffness of elements 8 and 9

Two cases were considered, namely, damage-free and damaged. The damaged one was the same as in example 1; i.e., 5% reduction in the stiffness of element 3 and 7. The responses are recorded at 0.01 sec time intervals from 0.01 to 1.0 sec providing 100 time points. Using those response measurements only, the elements of the steel frame are identified.

The theoretical and identified stiffness parameters (EI/L) of all elements are shown in Table 3. The results indicate that the method identified the stiffness very well with a maximum error in identification for case one of 0.812% for the damage-free case. For the damaged case, the results indicate that the (EI/L) values of elements 3 and 7 decreased by 3.899% and 3.977%, respectively indicating that the damage are in elements 3 and 7.

Table 3: Stiffness (EI/L) identification for case 1 for example 3

Member	Initial theoretical value (kN m)	Identified damage-free (kN m)	Error (%)	Identified damaged case (kN m)	Effect (%)
k_1	10650	10564	-0.812	10754	+0.972
k_2	26625	26420	-0.772	26874	+0.935
k_3	26625	26418	-0.779	25587	-3.899
k_4	10650	10572	-0.733	10760	+1.032
k_5	26625	26427	-0.743	26902	+1.039
k_6	26625	26433	-0.724	26866	+0.903
k_7	10650	10575	-0.702	10227	-3.977
k_8	26625	26476	-0.562	26950	+1.220
k_9	26625	26473	-0.572	26958	+1.250

Table 4: Stiffness (EI/L) identification for case 2 for example 3

Member	Initial theoretical value (kN m)	Identified damage-free (kN m)	Error (%)	Identified damaged case (kN m)	Effect (%)
k_1	10650	10446	-1.91	10617	-0.31
k_2	26625	26142	-1.82	26550	-0.28
k_3	26625	26139	-1.83	25280	-5.05
k_4	10650	10458	-1.81	10634	-0.15
k_5	26625	26136	-1.84	26585	-0.15
k_6	26625	26144	-1.81	26547	-0.30
k_7	10650	10452	-1.86	10098	-5.19
k_8	26625	26157	-1.76	26599	-0.10
k_9	26625	26152	-1.78	26614	-0.04

- **Case 2:** Stiffness of the rotational springs is assumed to be 30% of the rotational stiffness of elements 8 and 9

Two cases were considered, namely, damage-free and damaged. The damaged one was the same as in example 1; i.e., 5% reduction in the stiffness of element 3 and 7. The theoretical and identified stiffness parameters (EI/L) of all elements are shown in Table 4. The results indicate that the method identified the stiffness very well with a maximum error in identification for case one of 1.91% for the damage-free case. For the damaged case, the results indicate that the (EI/L) values of elements 3 and 7 decreased by 5.05 and 5.19%, respectively indicating that the damage are in elements 3 and 7.

RESULTS AND DISCUSSION

For all the numerical examples showed in this study, the stiffness of all the elements are identified accurately with a maximum error of 0.001, 0.013, 0.812 and 1.91% for one bay three floors frame with fixed, hinged and semi-rigid supports, respectively. The maximum error in identification is very small and can be considered acceptable. The proposed iterative least square method identified the stiffness of the local elements very well. On the other hand, for the damaged cases, the method was capable of identifying the pre-imposed damages in elements 3 and 7. The results showed that the stiffness of the damaged elements decreased indicating that the damage is in those elements, while the stiffness of the other elements of the frame increases indicating redistribution of stiffness.

Small time points of responses were used in the proposed method to identify stiffness and detect damages, i.e., 100, 118 and 129. The results showed that the number of time points is not an issue for the accuracy of the results. This can be considered an advantage for the method. Also, as a by-product, the method identified the unknown dynamic forces subjected to the structures very accurately since it is the convergence criteria used in the algorithm.

This can be considered another advantage of the method since most of the methods available in the literature need the time-history of the dynamic force.

CONCLUSION

An Iterative least square time domain system identification method is developed in this study to identify the stiffness of each member in plane frames with different support conditions, including semi-rigid supports. The stiffness are identified using dynamic response measurements only. The time history of the dynamic excitation force is not required in the identification process. The results show that the method identified stiffness of all elements very well and detect the damages at local element level for different support conditions. The number of time points of the response measurements was considered not an issue in the identification process. The procedure has the potential to be used as a structural health assessment and monitoring technique.

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