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COST-EFFECTIVE LIFETIME STRUCTURAL HEALTH MONITORING BASED ON AVAILABILITY

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Abstract

The state of a structural system subjected to deterioration processes is changing continuously. This state can not be reliably predicted without considering both aleatory and epistemic uncertainties. To reduce the epistemic uncertainty, inspection and structural health monitoring (SHM) should be performed, and the performance prediction model should be updated periodically. Continuous monitoring is needed to reliably assess and predict the performance of structures. However, due to limited financial resources continuous monitoring is not practical. Therefore, a cost-effective SHM strategy is necessary. In this paper, the probability that the performance prediction model based on monitoring data is usable in the future is computed by using the statistics of extremes and availability theory. This probability represents the availability of the monitoring data over non-monitoring periods. The monitoring cost and availability can be found by solving a bi-objective optimization problem. This problem consists in simultaneously minimizing the total monitoring cost and maximizing the availability of the monitoring data for performance prediction. Pareto solutions associated with monitoring duration and prediction duration are obtained. The proposed approach is applied to an existing bridge.

Keywords: Availability, Structural Health Monitoring, Optimization, Uncertainty, Monitoring Cost

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Introduction

Structural health monitoring (SHM) provides great potential to assess and predict the structural performance under uncertainty (Frangopol and Messervey, 2009). The general objectives of SHM include (a) assessing structural performance; (b) predicting the remaining service life of a structure; and (c) providing a decision tool for optimum maintenance planning (Frangopol *et al*., 2008b). These objectives are mainly related to the reduction of epistemic uncertainty. A life-cycle analysis is usually dependent on structural assessment and prediction models under uncertainty, and the accuracy associated with them can be considerably improved if the data from SHM are used efficiently (Peil, 2005). Therefore, recently, integration of SHM into maintenance management has been considered as a significant tool for rational maintenance planning.

In the past decade, there have been significant efforts on the life-cycle performance prediction based on structural performance indicators (e.g., reliability index, condition index, redundancy index) of bridges (Frangopol *et al.*, 2001; Kong and Frangopol, 2003; Frangopol and Liu, 2007; Okasha and Frangopol, 2010; Ghosn *et al.*, 2010). Tradeoff maintenance planning solutions with respect to condition index, reliability index, and cumulative maintenance cost profiles including uncertainties have been investigated (Liu and Frangopol, 2004; Neves *et al*., 2006a and 2006b; Frangopol, 2010). In general, the uncertainty associated with the life-cycle performance prediction increases as the structural performance is predicted further into the future. SHM can substantially reduce the expected failure cost and the expected maintenance cost of deteriorating structural systems by improving the accuracy of predicted structural performance (Frangopol and Messervey, 2007). In order to maximize this potential benefit of SHM, information from monitoring must be used appropriately. Ideally, continuous monitoring is needed to establish the optimal maintenance plan. However, this is not practical due to economical constraints and limited potential benefit of monitoring program. For this reason, the cost of monitoring and

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reliable performance prediction can be simultaneously considered in a bi-objective optimization formulation.

In this paper, the probability that the performance prediction model based on monitoring data is usable in future is computed by using the statistics of extremes. This probability represents the availability of the monitoring data over future non-monitoring period. The optimum availability of the prediction model and optimum monitoring cost can be formulated as an optimization problem with two conflicting criteria: minimization of the total monitoring cost and maximization of the availability of the monitoring data for performance prediction. This bi-objective optimization problem provides a Pareto solution set (i.e., optimum-balanced monitoring plan). Structural managers can choose the best monitoring plan among the Pareto set according to their preference and purpose. An alternative approach based on decision theory is also proposed. This approach is applied to the monitored data of the I-39 Northbound Bridge over the Wisconsin River in Wisconsin, USA, obtained by the ATLSS Engineering Research Center at Lehigh University.

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Exceedance Probability for Prediction Model

Efficient use of monitoring data based on prediction model

Sensors of monitoring systems can provide information at specific locations. Continuous combination of information provided by sensors in space and time can allow the assessment of the space- and time-dependent system performance (Frangopol and Messervey, 2009). Several approaches have been proposed for determining the optimal sensor placement to minimize the number of sensors (Shi *et al.*, 2000; Worden and Burrows, 2001; Meo and Zumpano, 2005). However, significant efforts related to the efficient inclusion of monitoring data in the

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assessment, prediction of structural performance, and optimized intervention planning of maintenance actions are still needed.

SHM to monitor the response of a structural system to external loadings (e.g., live load, temperature) requires a large storage system if all the data are recorded. The size of data depends on the monitoring frequency and the number of installed sensors on the structural system. The monitoring frequency and the number of sensors can be affected by the expected variation of the physical quantities in time and space, the planned monitoring period, and financial constraints (Frangopol and Messervey, 2009).

Generally, in order to reduce the data amount and manage it effectively, the information associated with the extreme physical quantities is recorded (Mahmoud *et al.*, 2005). This information can be mainly used to evaluate fatigue structural performance, but can be also used for serviceability performance. The prediction function based on monitored extreme data can provide helpful information to predict effective stress range and the number of cycles using the relation among the variation of monitored extreme data in time, the effective stress range and the number of cycles. Therefore, the prediction function based on monitored extreme data can be effective for the assessment and prediction of fatigue structural performance (Frangopol *et al.*, 2008b; Strauss *et al.*, 2008).

Statistical modeling of extreme values

The extreme values of random variables can be treated as random variables themselves and can have their own probability density function (PDF) that is related to the distribution of the initial variables (Ang and Tang, 1984). The PDF for the extreme values can be derived from the statistical data associated with the n initial sample values. The maximum value of initial variable X is defined as:

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$$Y_{max} = max\{X_1, X_2, \ldots, X_n\} \tag{1}$$

If the random variables $X_1, X_2, \ldots, X_n$ are assumed to be statistically independent and identically distributed, the cumulative distribution function (CDF) of Y_{max} can be obtained for all n initial values of $X_1, X_2, \ldots, X_n$ as:

$$F_{Y_{max}}(y) \equiv P(Y_{max} \leq y) = P(X_1 \leq y, X_2 \leq y, \ldots, X_n \leq y) = \left[F_X(y)\right]^n \tag{2}$$

Eq.(2) represents the exact CDF of the extremes of n samples with identical distribution. As $n \to \infty$, asymptotic (or limiting) forms of Eq. (2) may converge to a particular distribution type which depends on each end of tail's behavior of the initial distribution. Gumbel (1958) categorized the asymptotic distributions into three types: (a) Type I asymptotic form (i.e., the double exponential form that holds for initial distributions of the exponential type), (b) Type II asymptotic form (i.e., the exponential form), and (c) Type III asymptotic form (i.e., the exponential form with upper bound). For example, the largest values of the initial variables with normal and exponential distributions having exponential tails correspond to Type I asymptotic distribution, Type II asymptotic distribution is the largest value distribution of lognormal distribution with a polynomial tail in the direction of the largest one, and the distribution of the extreme values of a uniform and triangular distributions with an upper or a lower limit converge to the Type III asymptotic distribution (Ang and Tang, 1984). The three representative distributions mentioned previously are not exhaustive.

Linear regression function and its residuals

To establish the prediction model, the relation between time (predictor variable) and physical quantity (response variable) can be assumed as a function. The prediction model can be approximated to first, second or third order regression function based on the extreme value

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which can be relevant to the assurance of structural performance. The relationship between the real physical quantity and predictor variables is (Rosenkrantz, 1997)

$$O_t = f(t) + x_t \tag{3}$$

where O_t is observed data at time t, $f(t)$ and x_t are the regression function and the residual between values from the observed data and values from the regression at time t, respectively. The regression function can be expressed as:

$$f\left(t\right) = \sum_{i=0}^{k} a_i \cdot t^i \tag{4}$$

where a_i = coefficient, k = order of the regression function (i.e., k = 1, 2, or 3), and t = time. As shown in Fig. 1, where maximum values of the physical quantity for each day are plotted, if the regression model based on monitored data is defined as linear regression function for mathematical simplicity, the order of the function should be 1.0 and the coefficients, a_1 and a_0 can be obtained by minimizing the sum of the squared residuals (i.e., method of least squares). In general, the residuals between the values from the first order linear regression model and the actual data can be assumed normally distributed with mean value 0, if the data are mutually independent, and the number of data is large enough, and the regression model is obtained appropriately (Rosenkrantz, 1997).

Exceedance probability

The extreme values from the residuals as initial variate have their own probability distribution. If the residuals are normally distributed, their extreme values can be expressed as the double exponential form (i.e.,Type I asymptotic form) (Ang and Tang, 1984). The CDF of the double exponential form for the maximum positive value of the residual is

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$$F_{Ymax}(y) = P(Y_{max} \leq y) = exp[-e^{-\alpha_{max}(y - \beta_{max})}] \tag{5a}$$

and the CDF associated with the minimum negative value of the residual is

$$F_{Ymin}(y) = P(Y_{min} > y) = 1 - exp[-e^{\alpha_{min}(y - \beta_{min})}] \tag{5b}$$

where, α_{max} and α_{min} = the scale parameters for Y_{max} and Y_{min}, respectively; and β_{max} and β_{min} = the characteristic maximum and minimum values, respectively, of the initial variables which are the residuals between the predicted values and the observed values. If n samples associated with Eq. (5a) are chosen as n daily maximum positive residuals (i.e., $Y_{max(1)}$, $Y_{max(2)}$, ..., $Y_{max(n)}$), and if each sample is statistically independent and identically distributed, according to Eq. (2), the CDF of the largest value, $Y_{max,n}$, among n samples would be

$$F_{Ymax,n}(y) = [F_{Ymax}(y)]^n = exp[-e^{-\alpha_{max}(y - \beta_{max,n})}] \tag{6}$$

where $\beta_{max,n}$ = the characteristic maximum values of Y_{max}. Based on Eq. (6), the CDF of the maximum value, $Y_{max,N}$, among future N samples can be derived as follows:

$$F_{Ymax,N}(y) = [F_{Ymax}(y)]^N = \{[F_{Ymax}(y)]^n\}^{N/n} \tag{7}$$

If $\beta_{max,n}$ is assumed to be $Y_{max,n}$ which is the largest positive residual among n current samples in Eqs. (5a) and (6), the probability that the maximum positive residual, $Y_{max,N}$, in N future observations will be larger than the maximum positive residual, $Y_{max,n}$, among n current samples is given (Ang and Tang, 1984) as follows:

$$P(Y_{max,N} > Y_{max,n}) = 1 - \{[F_{Ymax}(Y_{max,n})]^n\}^{N/n} = 1 - exp[-e^{-\alpha_{max}(Y_{max,n} - Y_{max,n} - ln(N/n)/\alpha_{max})}] = 1 - e^{-N/n} \tag{8}$$

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Since n is the number of daily maximum positive residuals and N is the number of daily maximum positive residuals in future, the probability that the largest positive residual in future t days will exceed the largest positive residual among τ_m monitoring days can be obtained by modification of Eq. (8) as:

$$p_e = 1 - exp\left(-t / \tau_m\right) \tag{9}$$

If the residuals between the values from the first order linear regression model and the actual data are not normally distributed, the procedure associated with Eqs (5) to (8) can be applied after determining the appropriate distribution of the initial variate (i.e., the residuals) through distribution fitting tests. For example, if the extreme value from an initial distribution decays with an exponential tail (i.e., Type I), the same exceedance probability as that in Eq. (9) is obtained. Furthermore, if the extreme value from an initial distribution decays with a polynomial tail (i.e., Type II), the final formulation of exceedance probability will be as that in Eq. (9). In this paper, Eq. (9) can be defined as exceedance probability for the monitoring data to predict structural performance. The exceedance probability for the monitoring data based on monitoring duration τ_m can have various values according to the number of future exceedances of the maximum positive residual from τ_m monitoring days. Furthermore, by taking into account the relation between Eq. (9) and the Poisson process, the probability associated with the number of future exceedances, N_e, can be expressed as:

$$P\left(N_e = n_e\right) = \frac{\left(t / \tau_m\right)^{n_e}}{n_e!} exp\left(-t / \tau_m\right) \tag{10}$$

From Eq. (10), the probability that the number of future exceedances, N_e, will be at least one is

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$$P(N_e \geq 1) = 1 - P(N_e = 0) = 1 - exp(-t/\tau_m) \tag{11}$$

which is identical with Eq. (9). In this manner, the exceedance probability associated with various numbers of future exceedances can be formulated. For instance, the probability that the number of future exceedances will be at least two is

$$P(N_e \geq 2) = 1 - \{P(N_e = 0) + P(N_e = 1)\} = 1 - \left(\frac{t + \tau_m}{\tau_m}\right) \cdot exp(-t/\tau_m) \tag{12}$$

Furthermore, the number of future exceedances of the minimum negative residuals as well as the maximum positive residuals from τ_m monitoring days can also be considered and formulated through Eq. (6) to Eq. (8), similarly with derivation of exceedance probability for the maximum positive residual. The formulation of the exceedance probability considering both the minimum negative residuals and the maximum positive residual can be developed if the probability that the maximum residuals in future t days will exceed the largest positive residual among τ_m monitoring days or the minimum residual in t days will exceed the minimum negative residual among τ_m days is considered. This exceedance probability can be formulated as:

$$p_e = 1 - exp\left(-\frac{2t}{\tau_m}\right) \tag{13}$$

Table 1 summarizes exceedance probabilities with various numbers of future exceedances.

Availability of Prediction Model for Monitoring at Regular Time Intervals

The availability of a system can be defined as the probability that the system is in operating state (Ang and Tang, 1984). A system in an operating state can become non-operating due to deterioration. Conversely, a system in a non-operating state can be returned to an operating state

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through appropriate repair. The availability of monitoring data for structural performance prediction is defined as the probability that the prediction model based on monitoring data can be usable in the future. Similarly with the availability of a system, the prediction model can become non-usable, and can be restored to a usable state by the introduction of an update prediction model based on new monitoring data.

The average availability of the prediction model during a time period τ characterized by two mutually exclusive and collectively exhaustive events (i.e., E_1 = prediction model is usable and E_2 = prediction model is not usable) is

$$\overline{A} = P_a(\tau) + \frac{T}{\tau} \cdot P_{ua}(\tau) \qquad 0 < T \leq \tau \tag{14}$$

where T is time to loose usability of prediction model, $P_a(\tau)$ and $P_{ua}(\tau)$ are availability and unavailability of the prediction model during τ, respectively (see Fig. 2), and $P_a(\tau) + P_{ua}(\tau) = 1.0$. In this paper, the criterion for using monitoring data for prediction is associated with the maximum residual between values from prediction model and monitoring. If this residual exceeds the maximum residual during monitoring duration τ_m, the prediction model cannot be used. According to this criterion, the unavailability of monitoring data $P_{ua}(\tau)$ can be replaced by the exceedance probability p_e for the six cases of exceedance probabilities (cases O1, O2, O3, B1, B2, and B3) indicated in Table 1. Cases O1, O2, and O3 correspond, respectively, to at least once, twice, and three-time exceedance considering the largest value. Cases B1, B2, and B3 correspond, respectively, to at least once, twice, and three-time exceedances considering both the largest value and the smallest value, respectively.

The expected average availability of the monitoring data for prediction can be derived from Eq. (14) (Ang and Tang, 1984) as:

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$$E\left(\bar{A}\right) = P_a\left(\tau\right) + \frac{E\left(T\right)}{\tau} \cdot P_{ua}\left(\tau\right) = P_a\left(\tau\right) + \frac{1}{\tau}\left(\frac{1}{P_{ua}\left(\tau\right)}\int_0^{\tau}\frac{\partial P_{ua}\left(t\right)}{\partial t} \cdot t\ dt\right) \cdot P_{ua}\left(\tau\right) = \frac{1}{\tau} \cdot \int_0^{\tau} P_a\left(t\right) dt \qquad (15)$$

For instance, using Eq. (15), the expected average availability within prediction duration τ of case O1 in Table 1 is computed as:

$$E\left(\bar{A}\right) = \frac{1}{\tau} \cdot \int_0^{\tau}\left(1 - p_e\right) dt = \frac{1}{\tau} \cdot \int_0^{\tau} exp\left[-t / \tau_m\right] dt = \frac{\tau_m}{\tau}\left(1 - exp\left[-\frac{\tau}{\tau_m}\right]\right) \qquad (16)$$

The expected average availability is formulated by using the variables τ_m and τ. Figs. 3(a) and 3(b) show the relations between the ratio r_m of monitoring duration τ_m to prediction duration τ and expected average availability $E\left(\bar{A}\right)$ for cases O1, O2, O3, and B1, B2, B3 in Table 1, respectively. It can be seen that higher the expected average availability of the monitoring data for prediction is, longer monitoring duration τ_m is required relatively to prediction duration τ. The expected average availability $E\left(\bar{A}\right)$ of case O3 has the largest value in Fig 3(a), since the prediction model associated with this case is less conservative than those associated with cases O1 and O2. Similarly, case B3 is associated with the largest expected average availability in Fig 3(b).

Cumulative Monitoring Cost

In general, monitoring cost is the result of the following actions: (a) general preparation and project coordination; (b) sensors, wiring, data acquisition system, and their maintenance; (c) analysis of data and preparation of reports; (d) continuous review of data (Frangopol *et al*. 2008a). Under the assumption that the total monitoring cost is proportional to the monitoring duration and all actions related to monitoring program are conducted only during the monitoring duration,

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the cumulative monitoring cost C_M over a prescribed duration is

$$C_M = \left(\frac{\tau_m}{\tau_{mo}} \cdot C_o \right) \cdot \sum_{i=1}^{n} \left(\frac{1}{(1+r)^{(i-1)(\tau+\tau_m)}} \right) \tag{17}$$

where C_o = reference monitoring cost during τ_{mo} days, r = daily discount rate of money, and n = total number of monitoring periods over a prescribed duration (days).

Optimum Balance of Monitoring Time Interval and Monitoring Cost

The potential benefit of SHM can be maximized by reducing the expected failure cost and maintenance cost of structural systems. Through appropriate SHM, structure managers can establish more rational maintenance strategies under uncertainty. A reliable performance prediction model will lead to cost-effective maintenance and repair actions. However, more reliable monitoring data and more frequent monitoring action require high cost, and, as a result, it may be difficult to obtain the monitoring benefit in financial terms. Therefore, in order to find the optimal balance between the two conflicting criteria, bi-objective optimization should be applied. This approach minimizes the total monitoring cost and maximizes the availability of the monitoring data for performance prediction. Alternatively, through decision analysis theory, these conflicting objectives can also be formulated into a decision analysis problem to find the minimum expected monetary loss.

In order to obtain well-balanced solutions, NSGA-II (Non-Dominated Sorting in Genetic Algorithms) program is used (Deb *et al*., 2002). The optimization problem requires (a) design variables, (b) objectives formulated by including the variables, and (c) constraints for the variables and for the objectives. In this paper, the two conflicting-objectives can be defined as: (a) maximize the expected average availability of the monitoring data for prediction $E\left(\bar{A}\right)$

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indicated in Eq. (15); and (b) minimize the cumulative total monitoring cost C_M indicated in Eq. (17).

As an alternative method, decision analysis can be used to find the optimal solution. In general, if the decision is expressed in terms of a monetary value, the decision associated with the maximum expected monetary value (EMV) (i.e., minimum monetary loss) is the solution. EMV of the *i*th alternative is (Ang and Tang, 1984)

$$E(a_i) = \sum_j p_{ij} C_{ij} \tag{18}$$

where p_{ij} = the probability of the *j*th consequence associated with alternative a_i, and C_{ij} = the expected monetary of the *j*th consequence associated with alternative a_i. According to the maximum monetary value criterion, the optimal alternative a_{opt} is determined as the alternative having maximum EMV among *n* alternatives as:

$$C(a_{opt}) = max\{E(a_1),\ E(a_2),\ ...,\ E(a_i),\ ...,\ E(a_n)\} \tag{19}$$

EMV for cost-effective SHM can be formulated by using the expected average availability of the model and monitoring cost associated with different monitoring durations τ_m and future non-monitoring durations τ. As shown in Fig.4, monitoring plan *i* has two events (j = 1, 2) that are mutually exclusive and collectively exhaustive: the monitoring data is usable and not usable during prediction duration τ_i. For the usable case over future non-monitoring period, the probability p_{ij} and the expected monetary C_{ij} of monitoring plan *i* in Eq. (18) are replaced by the expected average $E_i(\overline{A})$ and the monitoring cost $C_{i,u}$, respectively. On the other hand, for non-usable case over future non-monitoring period, the probability p_{ij} in Eq. (18) can be

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computed as $1-E_i\left(\bar{A}\right)$. The cost associated with the non-usable case $C_{i,nu}$ can include the potential loss occurred from the use of non-usable monitoring data for prediction and the monitoring cost $C_{i,u}$ as well. Therefore, EMV of monitoring plan i is

$$E\left(Plan\ i\right)=C_{i,u}\cdot E\left(\bar{A}\right)+C_{i,nu}\cdot\left(1-E\left(\bar{A}\right)\right) \tag{20}$$

If the monitoring cost is proportional to the duration of monitoring, the monitoring cost per day for the usable case can be calculated based on the reference monitoring cost C_o during τ_{mo} days as:

$$C_{i,u}=\left(\frac{C_o}{\tau_{mo}}\cdot\tau_{m,i}\right)/\left(\tau_{m,i}+\tau_i\right) \tag{21}$$

where $\tau_{m,i}$ = the monitoring duration, and τ_i = the prediction duration for monitoring plan i. Therefore, the monitoring cost for the non-usable case adding potential loss, C_L, is

$$C_{i,nu}=\left(\frac{C_o}{\tau_{mo}}\cdot\tau_{m,i}\right)/\left(\tau_{m,i}+\tau_i\right)+C_L \tag{22}$$

As a result, substituting Eqs. (21) and (22) into Eq. (20), EMV for plan i is

$$E(Plan\ i)=\left\{\left(\frac{C_o}{\tau_{mo}}\cdot r_{m,i}\right)/\left(r_{m,i}+1\right)\right\}\cdot E_i\left(\bar{A}\right)+\left\{\left(\frac{C_o}{\tau_{mo}}\cdot r_{m,i}\right)/\left(r_{m,i}+1\right)+C_L\right\}\cdot\left(1-E_i\left(\bar{A}\right)\right) \tag{23}$$

where $r_{m,i}$ is the ratio of the monitoring duration, $\tau_{m,i}$, to the prediction duration, τ_i.

Application Example

The proposed approach is applied to the I-39 Northbound Bridge I-39 located in Wisconsin, USA. According to Mahmoud *et al.* (2005), this is a five span continuous steel plate girder bridge,

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which carries the northbound traffic of Interstate 39 and US51 over the Wisconsin River. As shown in Fig. 5(a), the bridge is symmetrical about the mid point of the third span and has a total length of 188.8m. The monitoring program was performed to estimate serviceability of the bridge focusing on a fatigue assessment, predict remaining fatigue life, and monitor the structural responses of the bridge under the actual traffic. Bondable resistance strain gages and displacement sensors were installed for the monitoring program. Controlled load tests using test trucks and long-term monitoring test with random traffic were performed (Mahmoud *et al.*, 2005). The methodology proposed in this study is applied to the long-term monitored data from the strain gage CH4 which was installed on the bottom flange of the third girder in the first span as shown in Figs. 5(b) and 5(c). This long-term monitoring program was conducted from July 29, 2004 to November 3, 2004, for a total of 95 days. The first 80 days monitored data are used in this example.

Expected average availability of monitoring extreme data for prediction

The linear regression model as a performance prediction model is based on the ten maximum daily stresses during 80 monitored days as shown in Fig. 6(a). The residuals between the monitored data and values from the performance prediction model have the mean value of 0.0 MPa and the standard deviation of 4.7 MPa. The probability paper is used as shown in Fig. 6(b) to check whether the appropriate distribution for these residuals is a normal distribution. For construction of the normal probability paper, 800 residuals are arranged in increasing order, and the ith residual value among the 800 data is plotted at the standard normal variate $s_i = \Phi^{-1}(i / (N+1))$, where $N = 800$ and Φ^{-1} is the inverse standard normal CDF. The linear regression line of these residuals on the normal probability paper can be obtained by the method of least square as shown in Fig. 6(b). The slope of the regression line and y-intercept represent the standard

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deviation of the residual (4.68 MPa) and the mean value of the residual (0 MPa) respectively. For evaluating how well the estimated regression line fits the data, the coefficient of determination denoted by R^2 is used (Rosenkrantz, 1997). This coefficient is defined as:

$$R^2 = \frac{\sum\left(y_i - \overline{y}\right)^2}{\sum\left(f_i - \overline{y}\right)^2} \tag{24}$$

where y_i = ith residual value, $\overline{y}$ = mean value of residual values, f_i = value on the regression line associated with s_i. As R^2 is close to 1.0, most of the data can be captured by the linear regression model (Rosenkrantz, 1997). R^2 associated with Fig. 6(b) is 0.9829. Additionally, the relative goodness of fit tests (i.e., the Chi-square test, Kolmogorov-Smirnov test, and Anderson-Darling test) were performed with several candidate distributions in order to select the most appropriate distribution which fits the residuals. As a result, normal distribution was selected as the best-fit distribution for the residuals. Therefore, the maximum and the minimum values have the Type I asymptotic distribution (*i.e.*, the double exponential form). The scale parameters α_{max} and α_{min} of the maximum and the minimum values are $\sqrt{2\,ln\,N}\,/\,\sigma = \sqrt{2\,ln\,800}\,/\,4.68 =$ 0.78 and the values of the characteristic maximum, β_{max}, and minimum, β_{min}, are assumed to be the maximum and minimum residuals of 14.97MPa and -11.49MPa, respectively, among 800 monitoring data. Therefore, the CDFs of the double exponential asymptotic form for the maximum and the minimum value of the residuals can be formulated (see. Eq. (5)), respectively,

$$F_{Ymax}(y) = P(Y_{max} \le y) = exp[-e^{-0.78\,(y\,-14.97)}] \tag{25a}$$

$$F_{Ymin}(y) = P(Y_{min} > y) = 1-\,exp[-e^{0.78\,(y\,+11.49)}] \tag{25b}$$

Fig. 6(c) shows the histogram from the residual, its appropriate distribution (i.e., normal distribution), and the PDFs of the extreme values of the initial variate (i.e., the residual values).

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The expected average availability $E(\bar{A})$ of the monitoring data for prediction can be obtained from Eq. (15), and, as a result, it can be formulated with monitoring duration τ_m and prediction duration τ. The relations between $E(\bar{A})$ for 80 days of monitoring duration τ_m and the prediction duration τ for the six cases in Table 1 are plotted in Figs 7(a) and 7(b). $E(\bar{A})$ decreases as the prediction duration τ increases.

Bi-objective optimum solutions

The design variables of the bi-objective problem are the monitoring duration τ_m and the prediction duration τ. The variables τ_m and τ are assumed to be between 50 days and 3000 days. The target life is assumed 7,300 days (i.e., about 20 years), and the reference monitoring cost C_o during τ_{mo} = 80 days is assumed \$10,000. For each case indicated in Table 1, 1,000 Pareto solutions are obtained using genetic algorithm after 100th generations as shown in Figs 8, 9, and 10. The values of objectives and design variables for some of these solutions are provided in Tables 2 and 3.

Fig. 8(a) represents the 1,000 Pareto solutions for cases O1, O2, and O3 without considering the discount rate of money (i.e., r = 0.0 %). In order to provide an expected average availability of the prediction model $E(\bar{A})$ = 0.2 for case O1 (i.e., solution A1 in Figs. 8(b) and 8(c)), the required monitoring duration and prediction duration have to be τ_m = 405 days and τ = 2,035 days, respectively (see design space in Fig. 8(b)), and the expected total monitoring cost has to be \$151,875 (see Fig. 8(c)). If $E(\bar{A})$ has to increase four times (i.e., $E(\bar{A})$ = 0.8 for case O1; see solution D1 in Figs. 8(b) and 8(c)), the required monitoring duration and prediction duration have to be 1,665 days and 770 days, respectively (see design space in Fig. 8(b)), and the expected total monitoring cost has to be \$624,375 (see Fig. 8(c)). The solutions B1 and C1

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associated with case O1, where the expected average availability is 0.4 and 0.6, are also indicated in Figs. 8(b) and 8(c), respectively. Since the number of exceedance probabilities is larger for case O2 than O1, and larger for case O3 than O2 (see Table 1), the total monitoring cost associated with the same expected average availability will be maximum for case O1 and minimum for case O3 (see Figs. 8 and 9). Figs. 8(d) and 8(e) indicate three solutions (D1, D2, and D3) associated with the same expected average availability (i.e., $E\left(\bar{A}\right)$ = 0.8) for cases O1, O2, and O3. If the discount rate of 0.016% per day (6% annual discount rate of money) is considered, the solutions in Figs. 9(a) to 9(e) are obtained for each case and the associated results are indicated in Table 3. A substantial reduction in total monitoring cost is observed by comparing results in Table 3 with those in Table 2. The solutions E1, F1, G1, and H1 (see Figs. 9(b) and 9(c)) are much less expensive than solutions A1, B1, C1, and D1 (see Figs. 8(b) and 8(c)), respectively. The same observation is valid for solutions H1, H2, and H3 (see Fig. 9(d) and 9(e)) as compared to solutions D1, D2, and D3 (see Fig. 8(d) and 8(e)). This is due to the fact that both monitoring duration and prediction duration are highly affected by the discount rate (compare results in Fig. 8(e) with those in Fig. 9(e)). Figs. 10(a) and 10(b) show the solutions for cases B1, B2, and B3, without and with discount rate, respectively. It is worth noting that the total monitoring costs associated with cases B1, B2, and B3 are higher than those associated with cases O1, O2, and O3, respectively.

Optimum solutions from decision analysis

The expected monetary value (EMV) associated with various monitoring plans can be obtained by using Eq. (23) with C_o = \$10,000 and τ_{mo} = 80 days. Figs. 11(a) and 11(b) show the relation between the EMV per day and the ratio of the monitoring duration τ_m to the prediction duration τ . From these figures, it is clear that as the potential monitoring loss C_L from unavailability of

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the monitoring for prediction increases, the plan having larger ratio of monitoring duration to prediction duration becomes the optimum monitoring plan according to minimum monetary loss criterion. Therefore, structures with very high potential loss need long-term monitoring. In this case, the continuous monitoring program is the optimal plan. If structures have a moderate potential loss (e.g., C_L = \$100/day), the optimal monitoring plan will correspond to the ratio of monitoring duration to prediction duration with the maximum EMV (or minimum monetary loss). For example, \$100/day of the potential loss value yields the optimum design value of r_m = 0.63 and the maximum EMV (or minimum monetary loss) = -\$98.2 /day as shown in Fig. 11(b).

Conclusions

The main objective of SHM is to provide reliable information to structure managers in order to implement cost-effective lifetime maintenance planning. To obtain the maximum benefit from SHM, an optimal monitoring plan is needed by balancing the availability of monitoring and monitoring cost over the lifetime of structures. In this paper, this optimization problem under uncertainty has been formulated as bi-objective problem related to the availability of monitoring data for structural performance prediction and the cumulative monitoring cost. Additionally, as an alternative approach, decision analysis theory has been used based on the minimum monetary loss criterion. The following conclusions can be drawn from this paper.

1. The optimum monitoring plan is affected by the discount rate of money and the criterion for using monitoring data for prediction. A higher discount rate of money leads to an optimal monitoring plan with shorter monitoring duration and shorter time interval between monitorings. The criterion for using monitoring data for prediction is dependent on the number of exceedances allowed for the largest positive and/or negative residual in a prescribed time interval.

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2. In order to apply the proposed approach, managers have to assign the threshold for the expected average availability of monitoring data according to the importance and state of structural deterioration. As indicated in Kim and Frangopol (2010), structural members with high contribution to structural system performance need a larger threshold of the expected average availability. The value of this threshold can be changed over time, and the monitoring plan can be updated.
3. The potential loss from unavailability of the monitoring for prediction has a significant effect on EMV.

In conclusion, it is emphasized that although the proposed methodology for cost-effective lifetime SHM under uncertainty appears to be adequate at this time for structural components, much further effort is needed to establish a general methodology to deal with optimized performance monitoring of structural systems under conflicting criteria.

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Figure Captions

Fig. 1 Residuals between values from prediction model and monitoring data

Fig. 2 Timeline of monitoring and prediction at regular time intervals

Fig. 3 The relation between the ratio of monitoring duration τ_m to prediction duration τ and the expected average availability of monitoring data; (a) cases O1, O2, and O3; and (b) cases B1, B2, and B3

Fig. 4 Decision tree for monitoring plan

Fig. 5 I-39 Northbound Bridge: (a) plan view; (b) detail A; and (c) detail B (adapted from Mahmoud *et al*., 2005)

Fig. 6 (a) Linear regression model based using 800 monitored data from the sensor CH 4; (b) normal probability paper for the residuals; and (c) PDF of the residuals and its extremal asymptotic distributions

Fig. 7 Prediction duration versus expected average availability of monitoring data for 80 monitoring days; (a) cases O1, O2, and O3 and (b) cases B1, B2, and B3

Fig. 8 (a) Pareto solution sets of the bi-objective problem without discount rate for cases O1, O2, and O3; (b) solutions and design space for A1, B1, C1, and D1; (c) monitoring plans of solutions A1, B1, C1, and D1; (d) solutions and design space for D1, D2, and D3; and (e) monitoring plans for solutions D1, D2, and D3

Fig. 9 (a) Pareto solution sets of the bi-objective problem with discount rate r = 0.016% / day for cases O1, O2, and O3; (b) solutions and design space for E1, F1, G1, and H1; (c) monitoring plans for solutions E1, F1, G1, and H1; (d) solutions and design space for H1, H2, and H3; and (e) monitoring plans for solutions H1, H2, and H3

Fig. 10 Pareto solution sets of multi-objective problem for case B1, B2, and B3; (a) without discount rate of money; and (b) with discount rate of money r = 0.016% / day

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Fig. 11 Expected monetary value per day versus ratio of the monitoring duration to the prediction duration; (a) for C_L = \$50 / day, and C_L = \$500 / day; and (b) for C_L = \$100 / day

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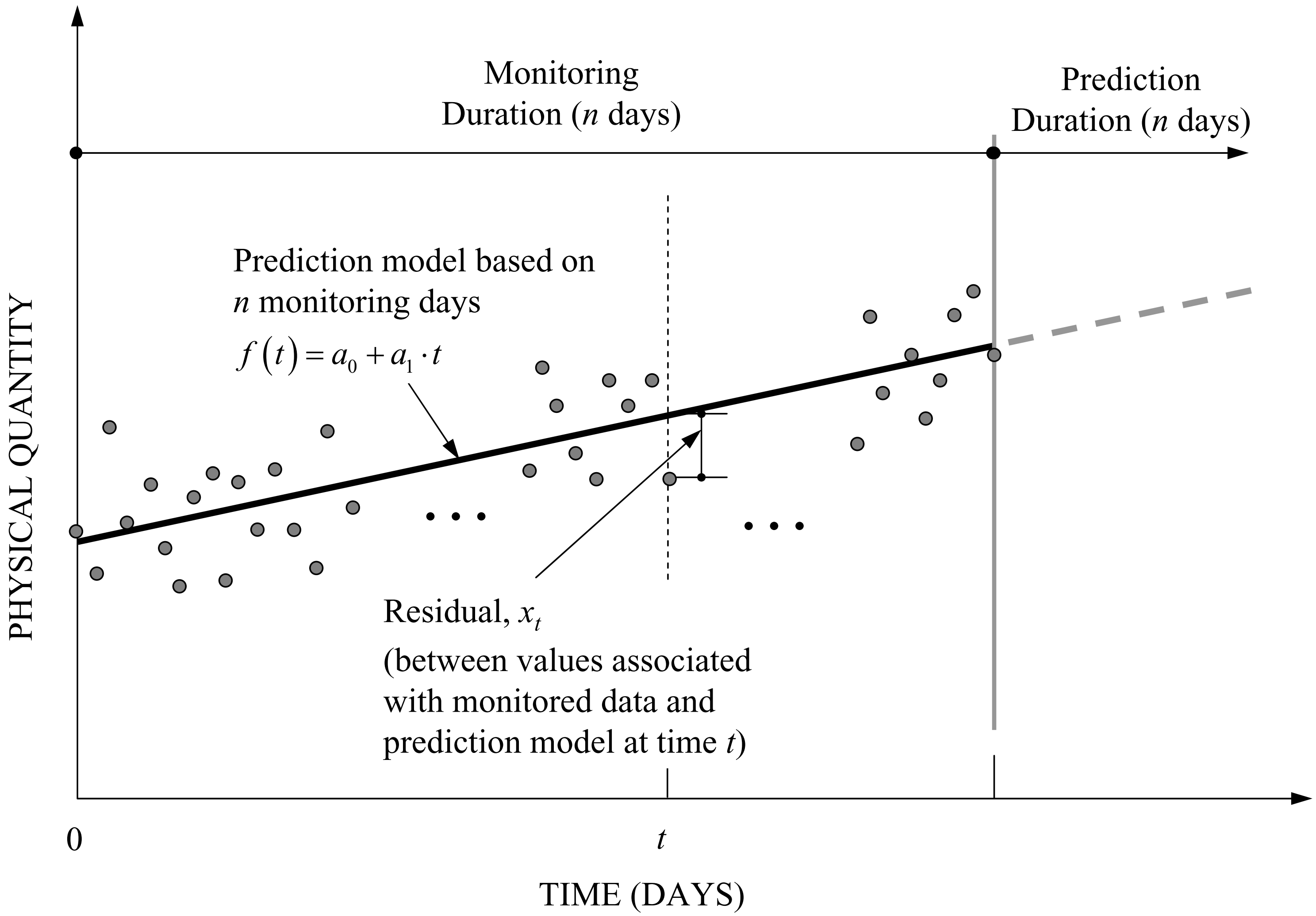

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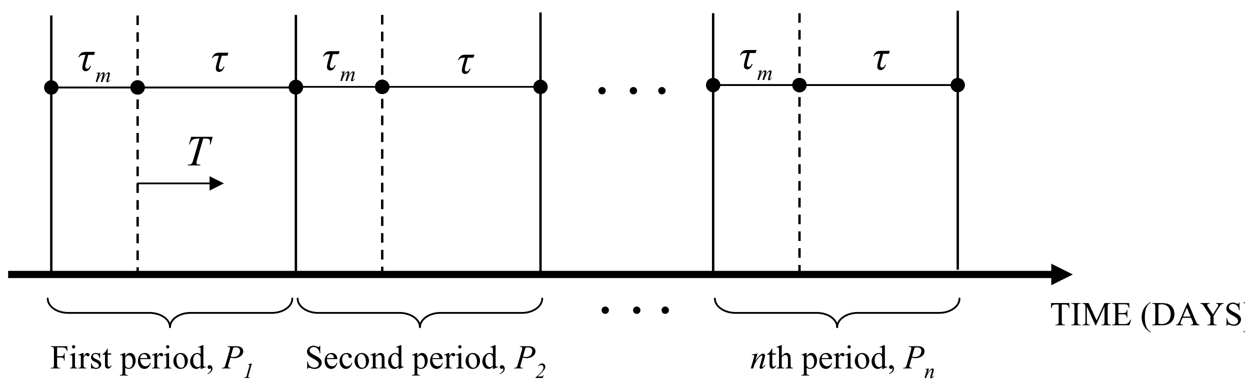

τ_m: Monitoring duration (days)

τ : Prediction duration (days)

T : Time to loose usability of prediction model ($0 < T \leq \tau$)

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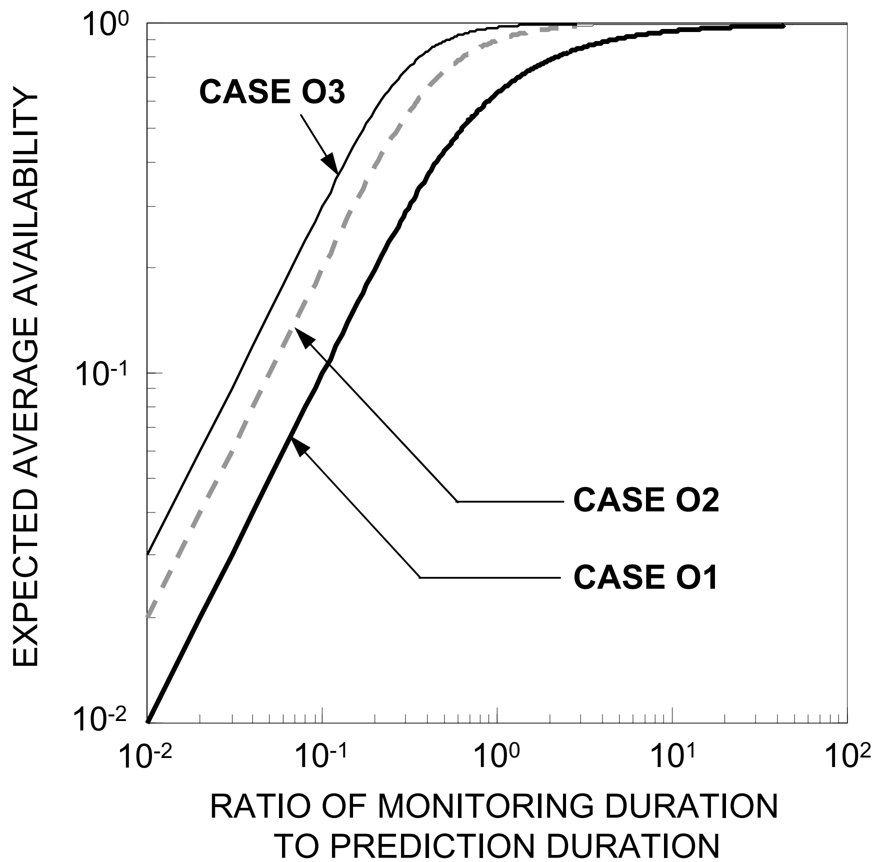

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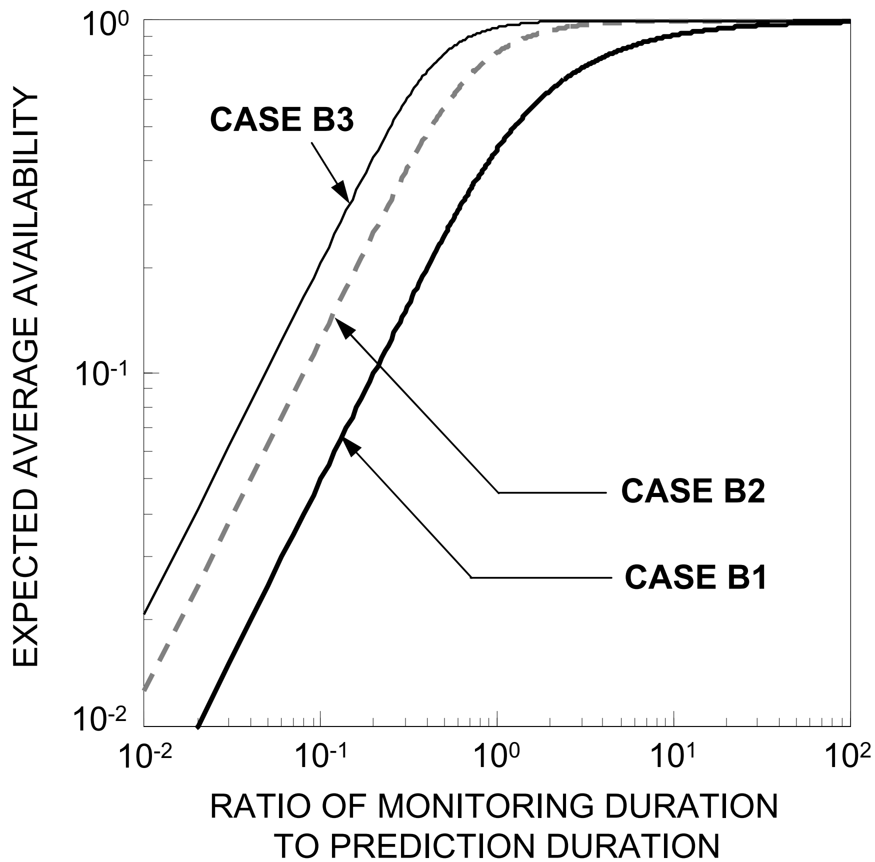

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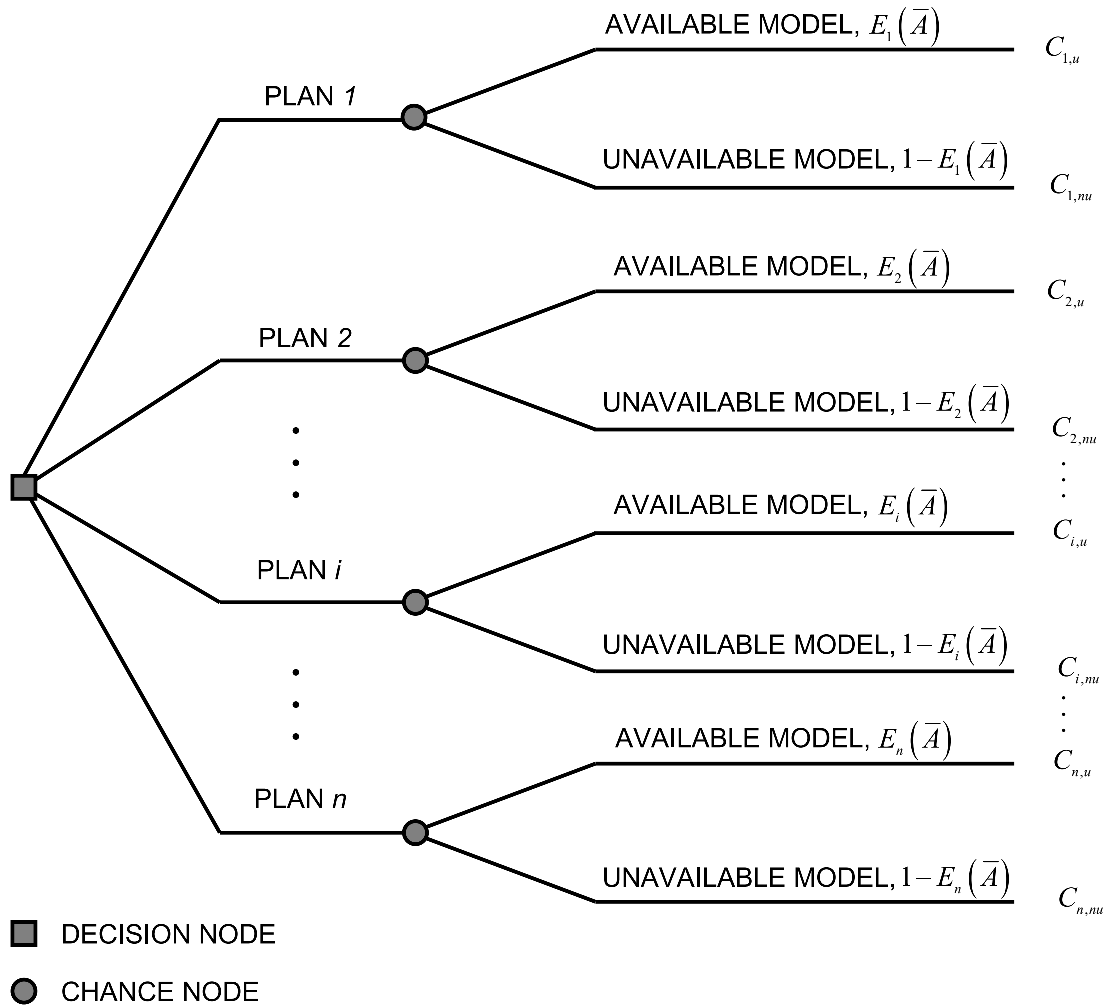

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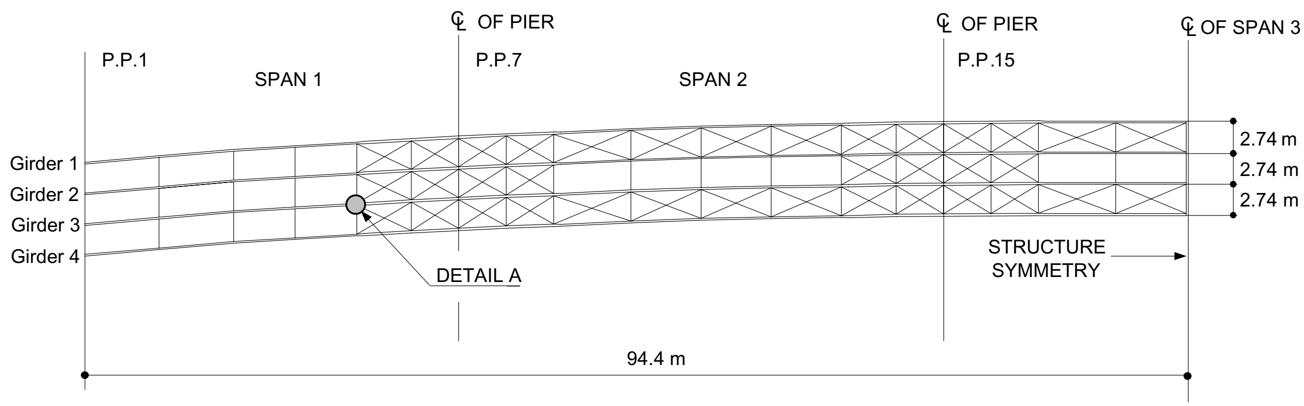

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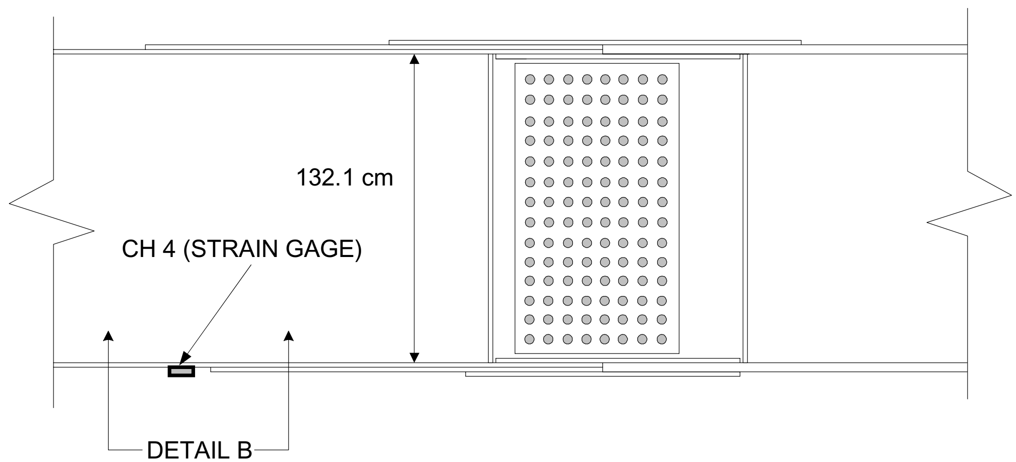

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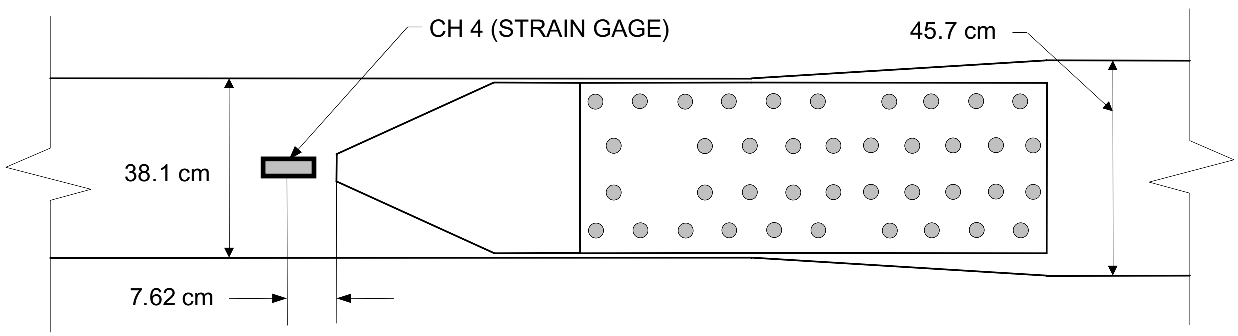

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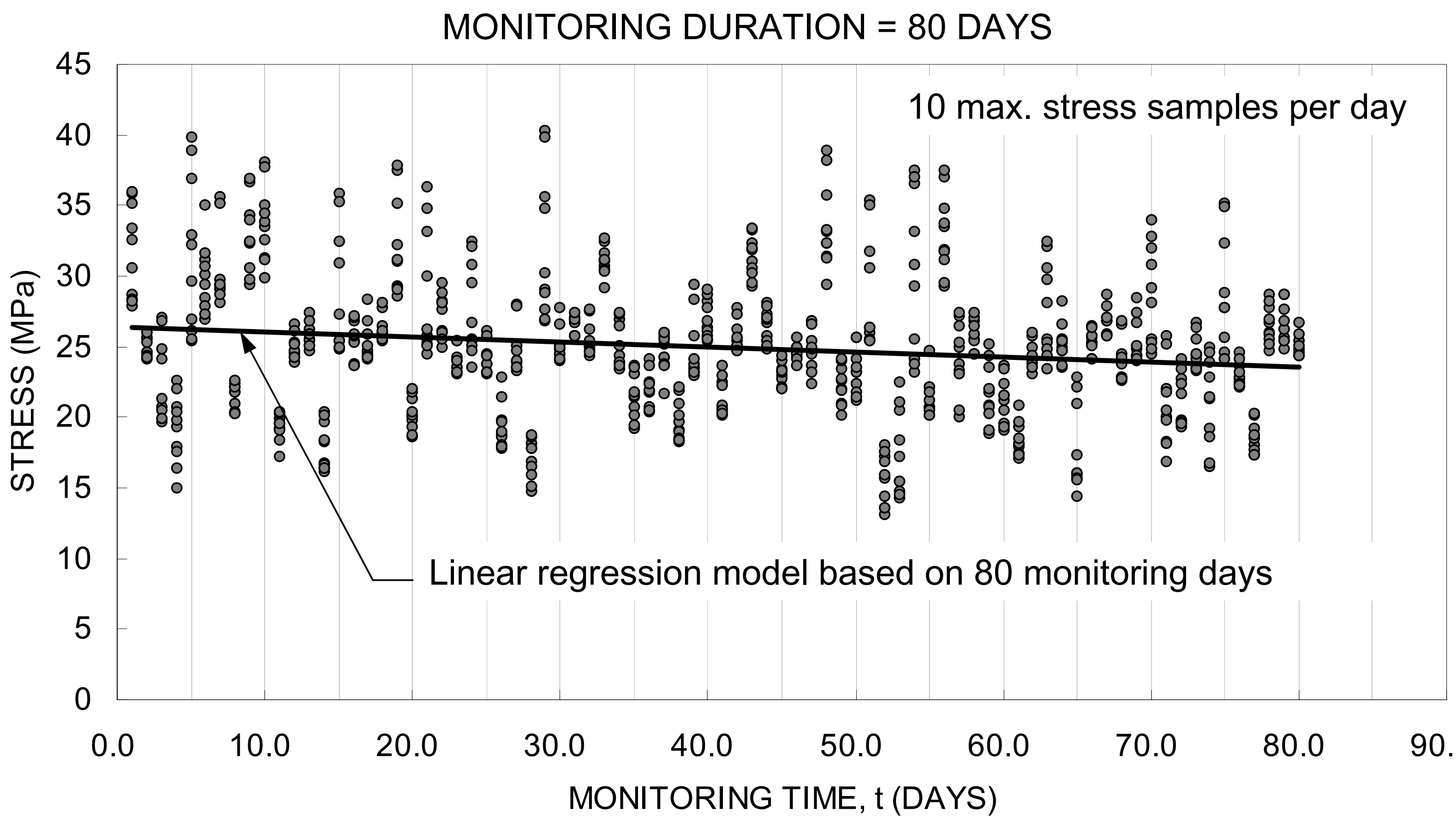

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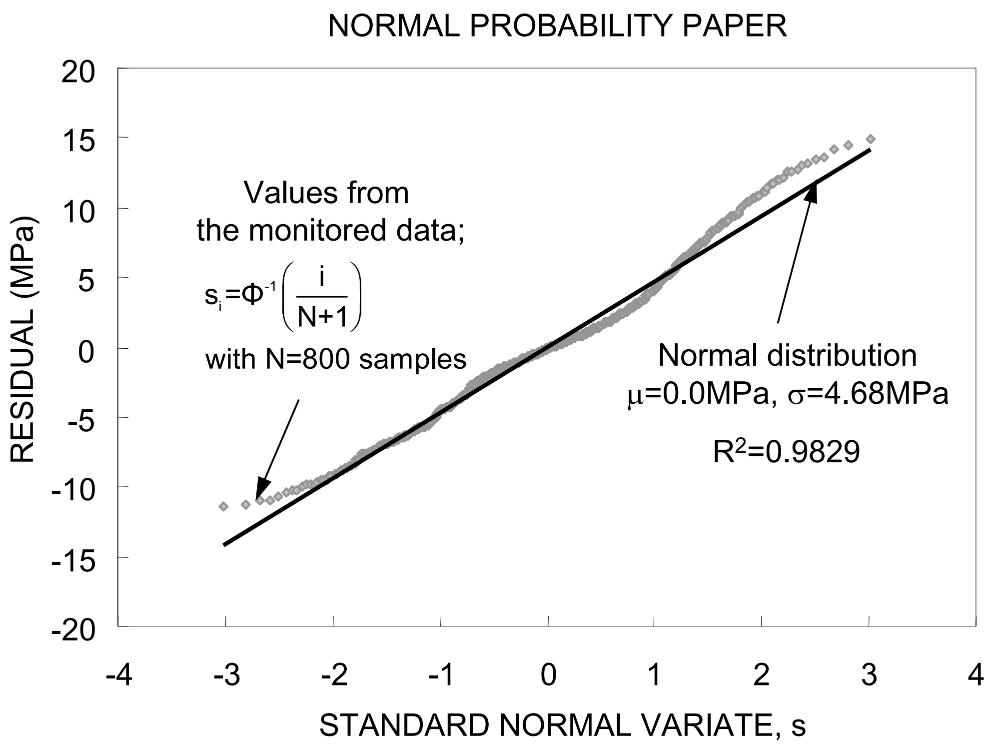

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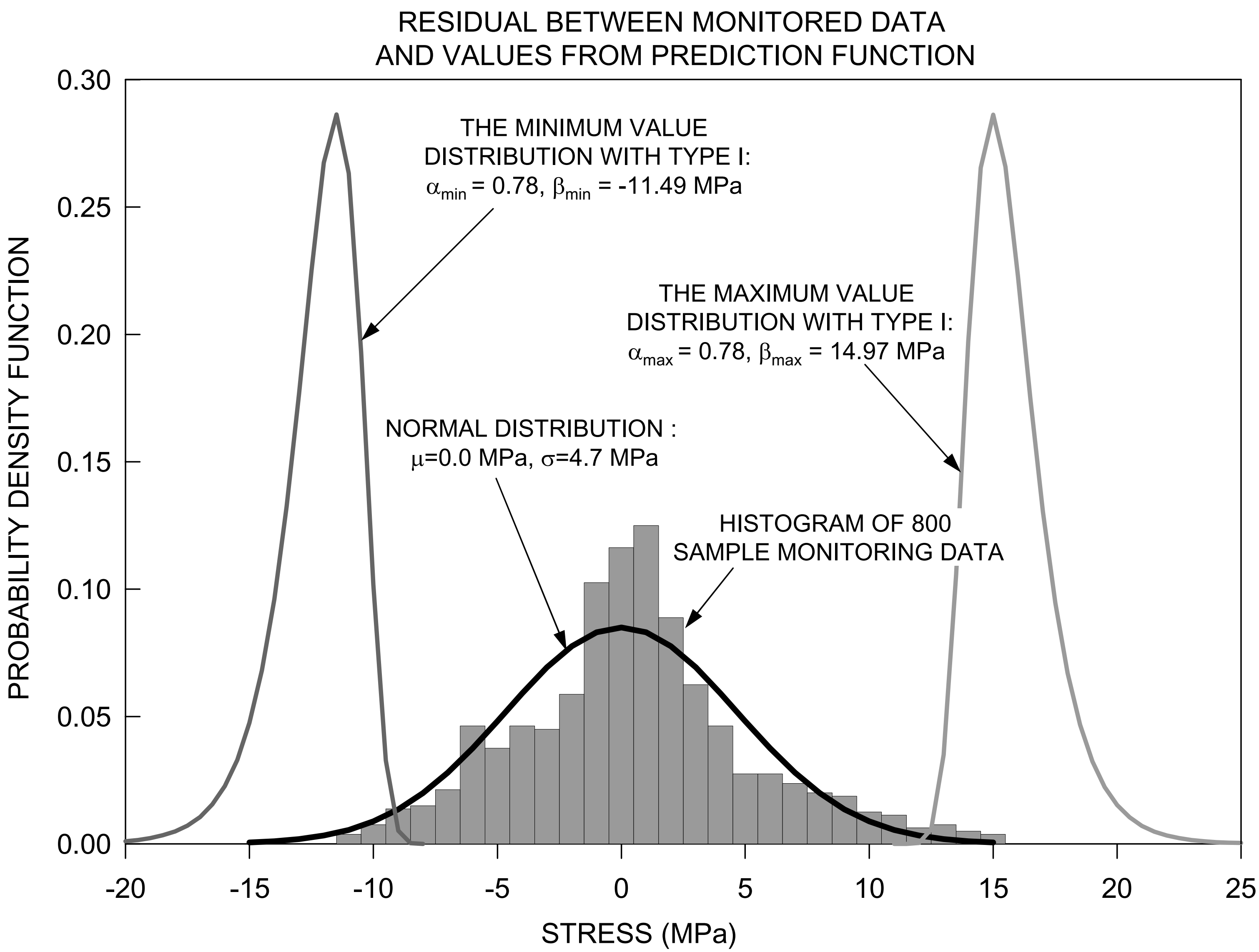

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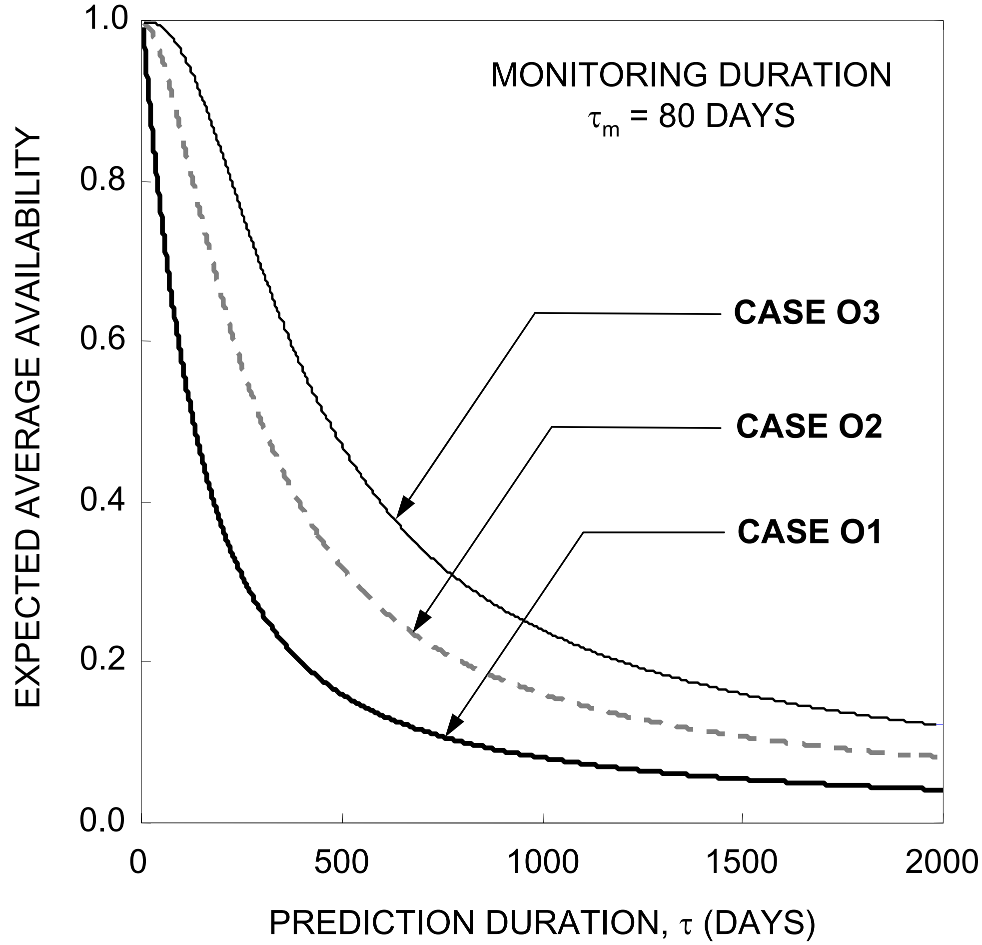

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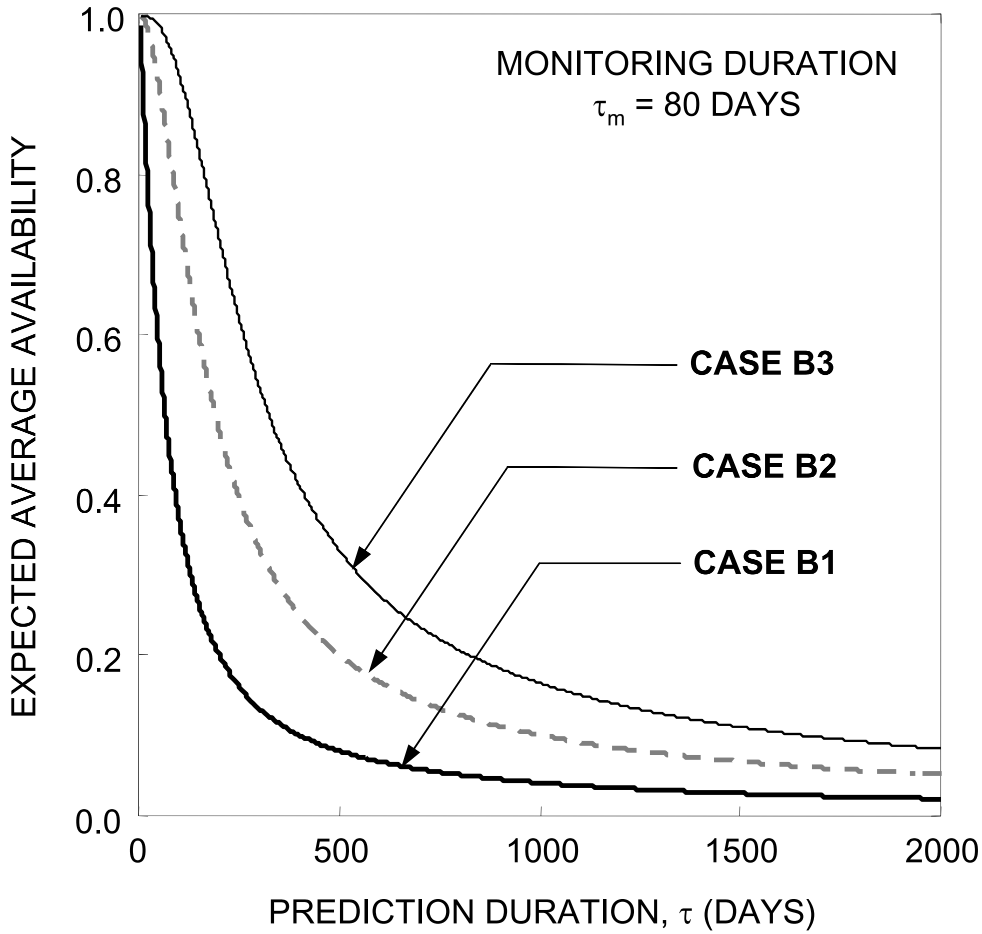

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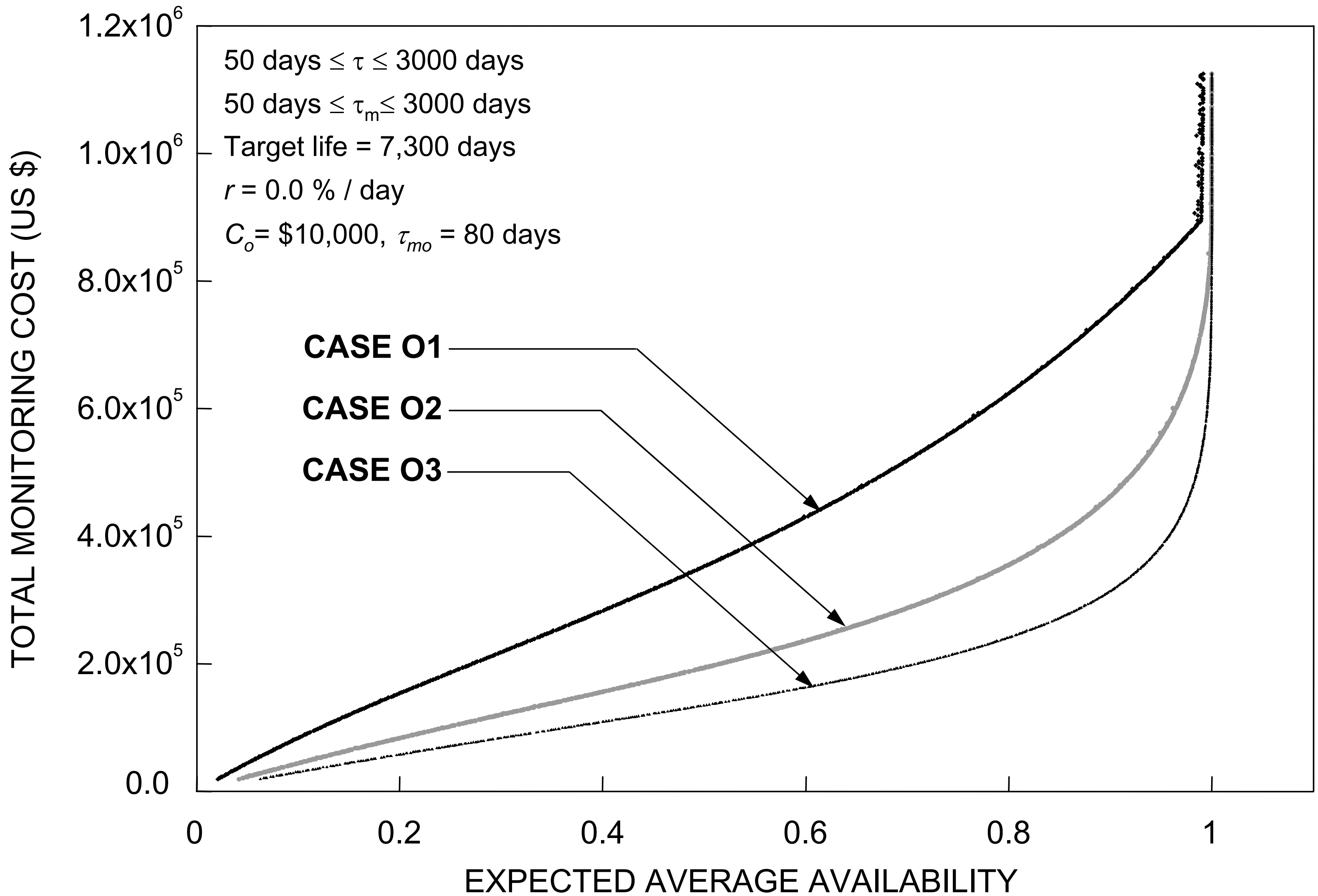

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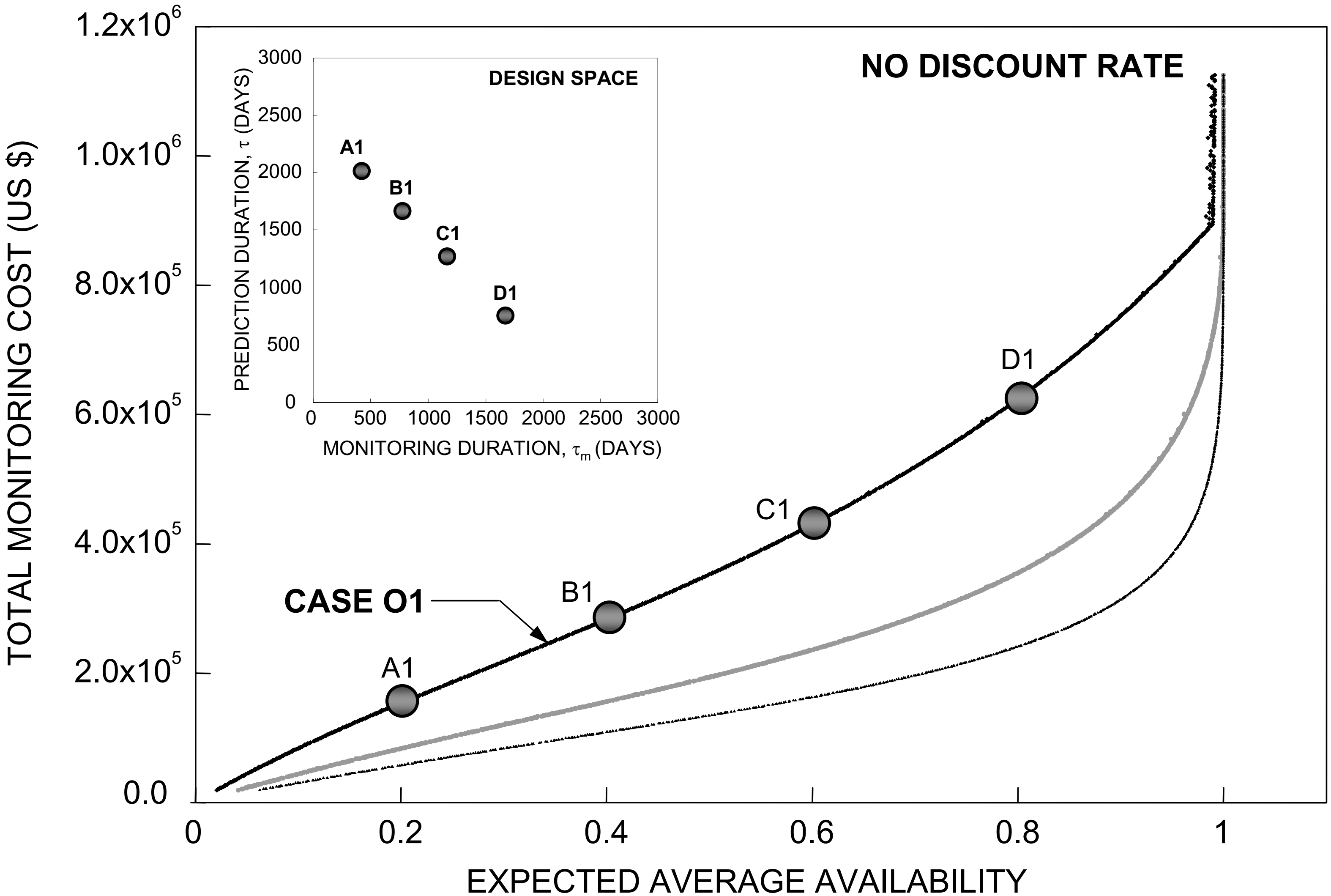

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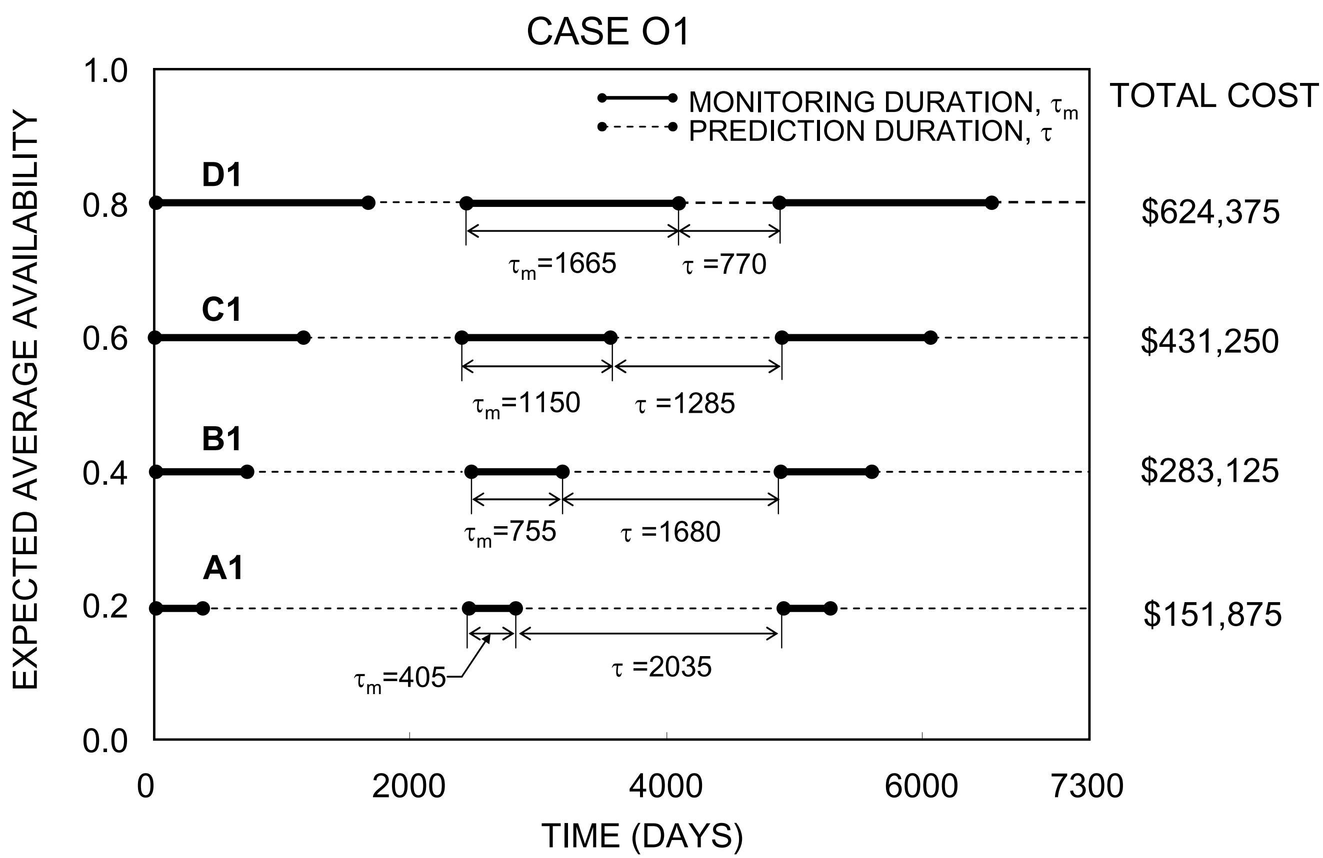

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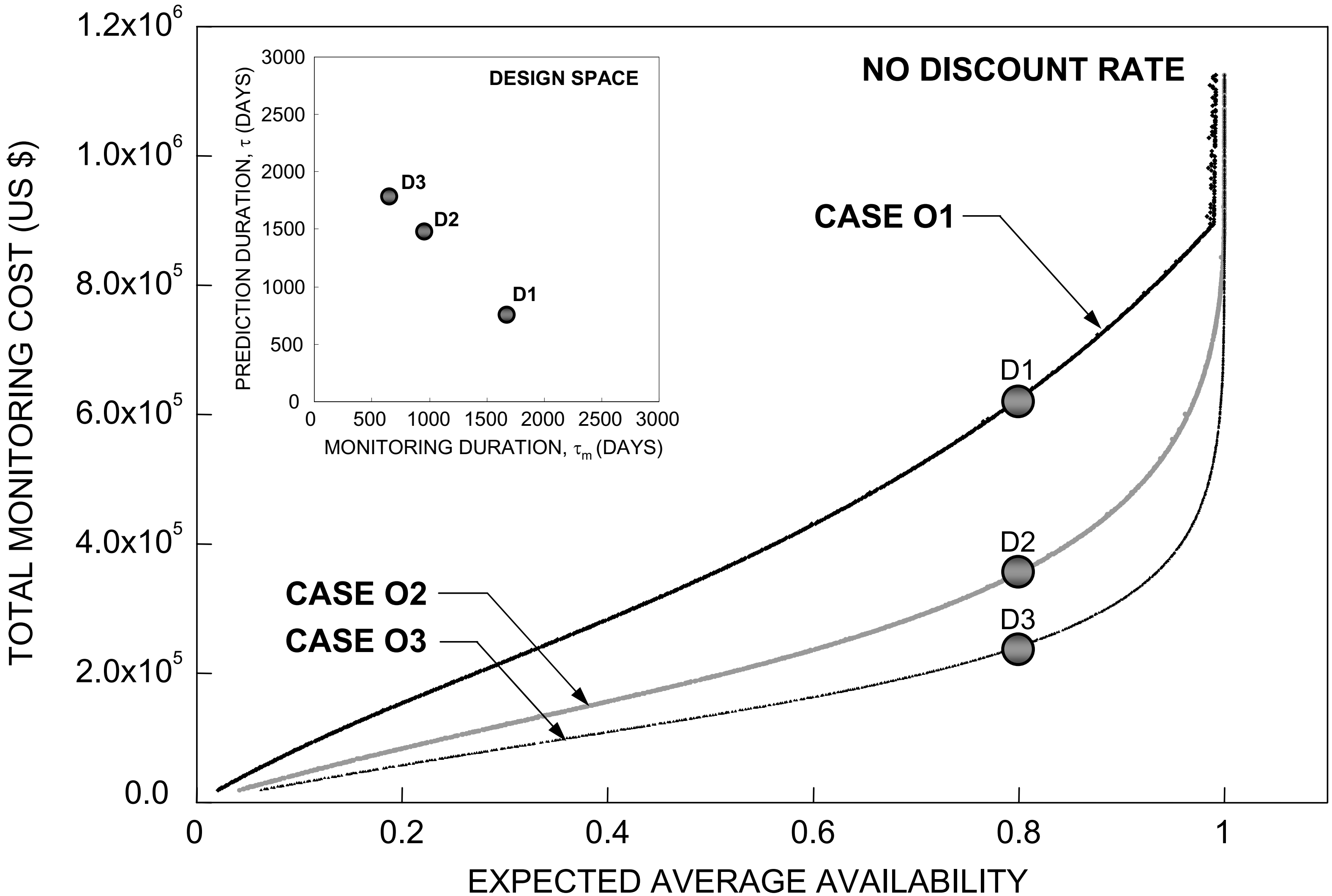

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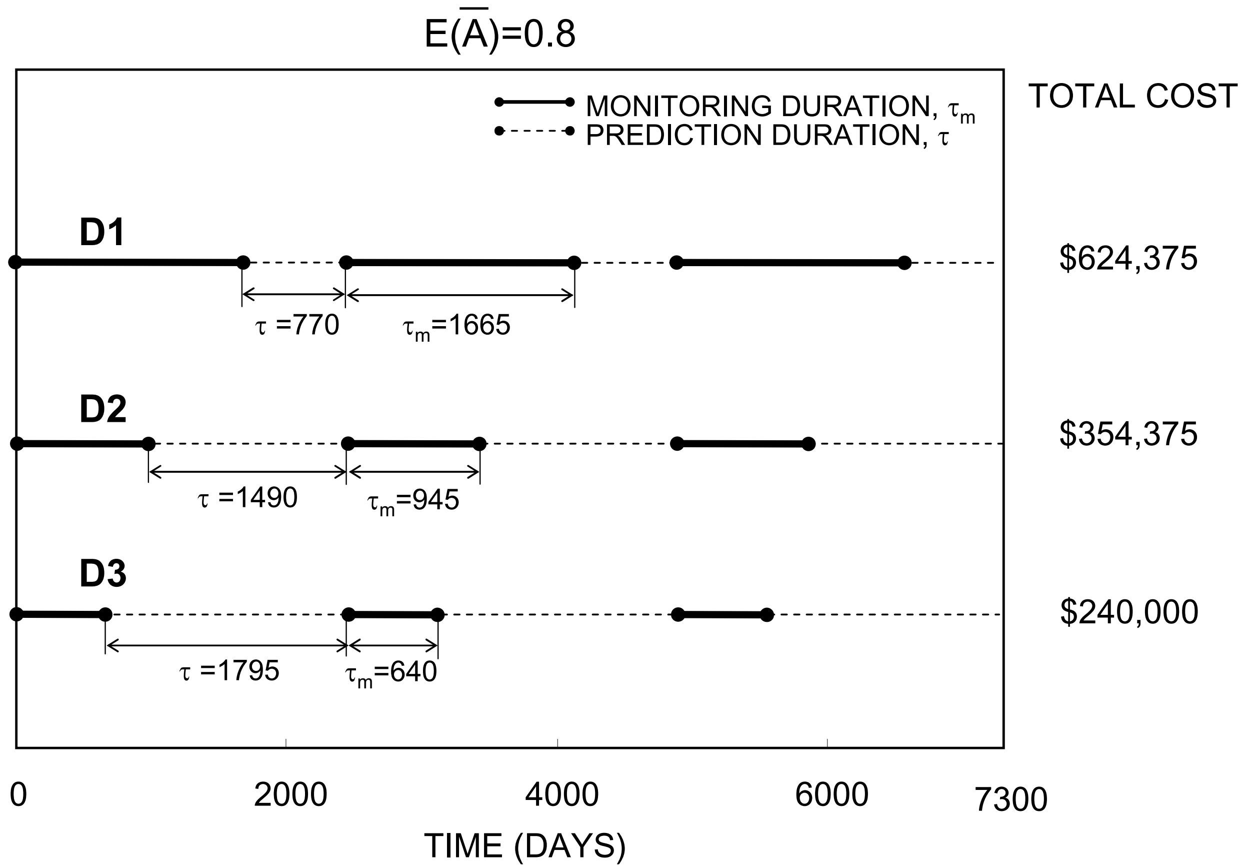

Figure. 8(e)

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Figure09a

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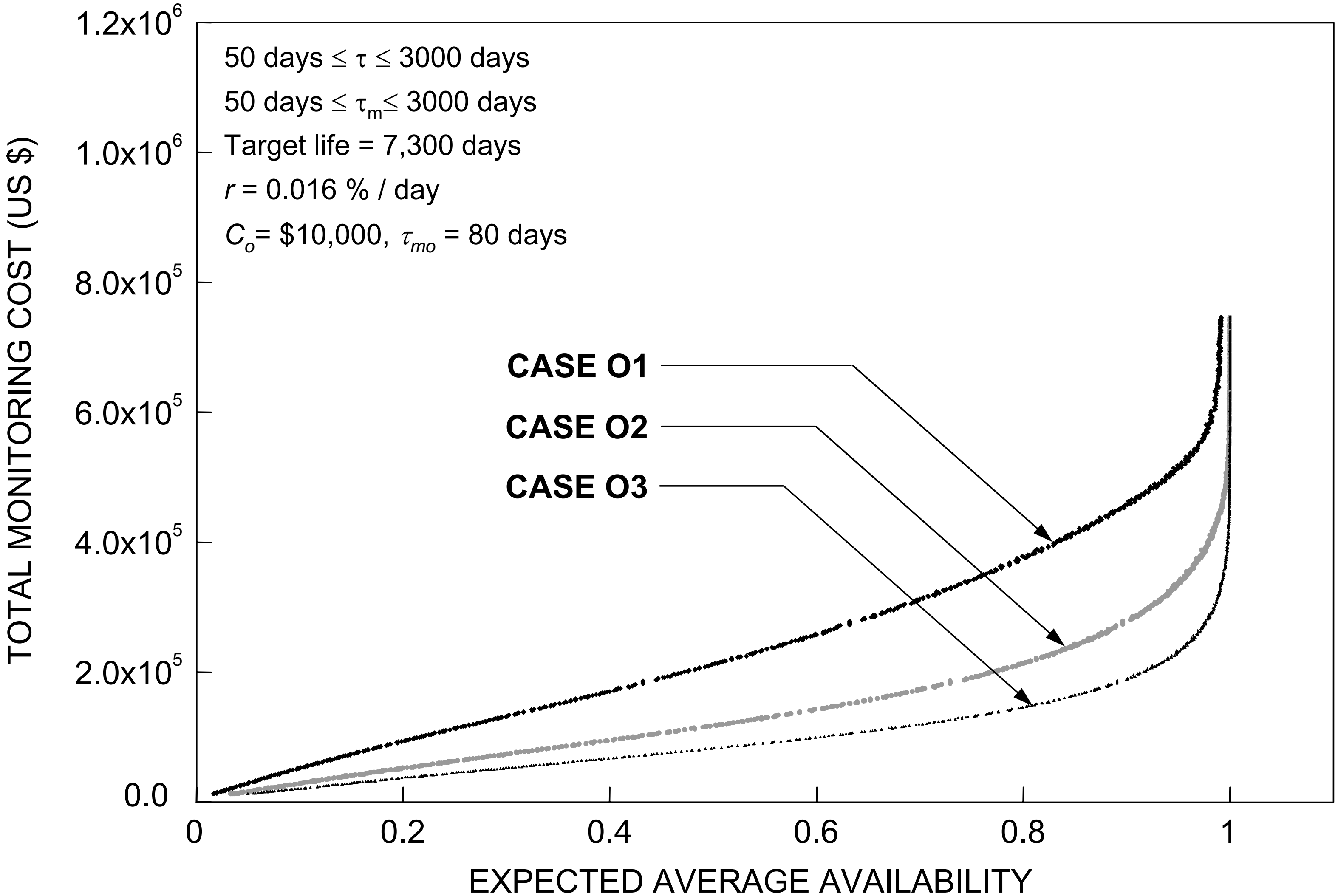

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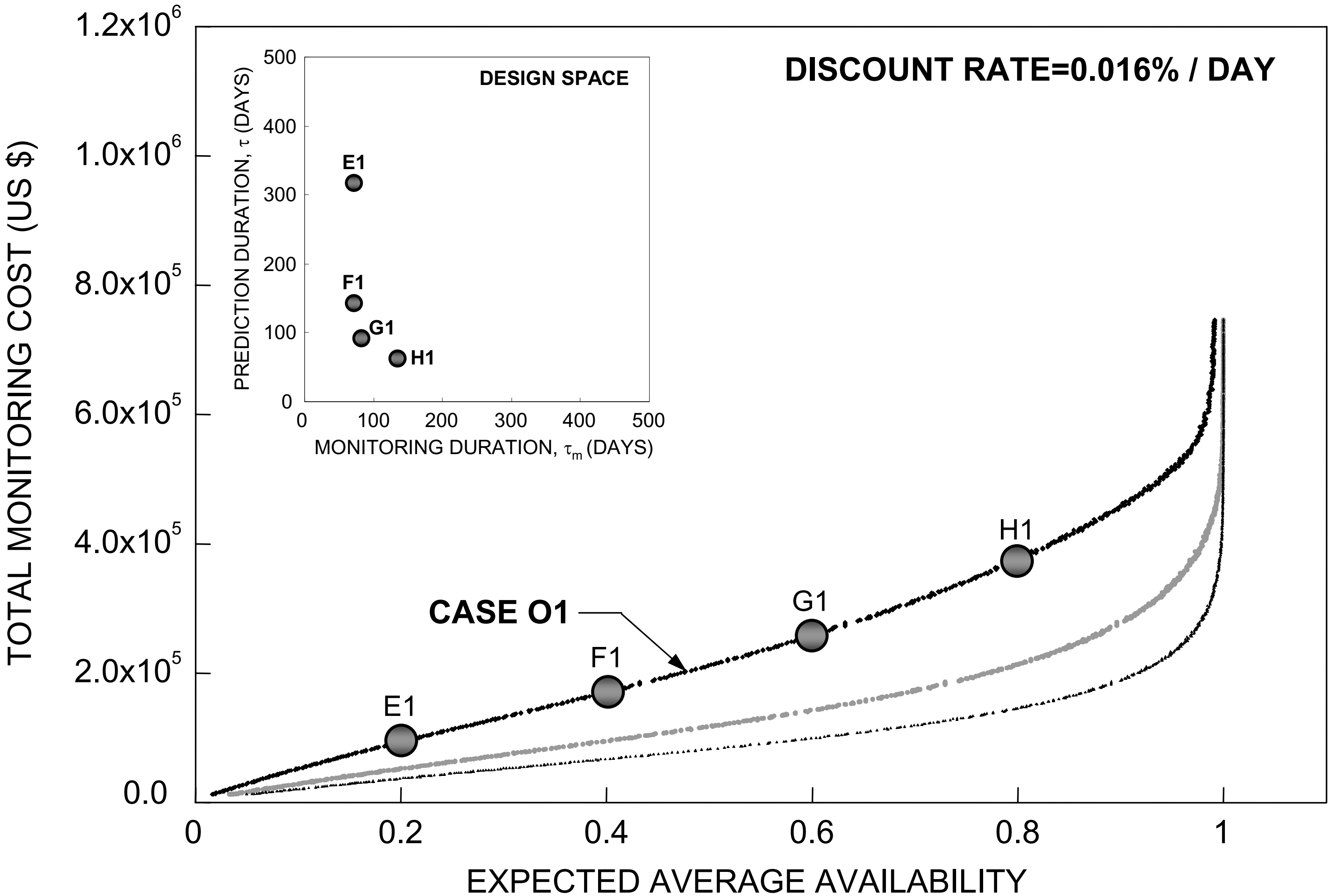

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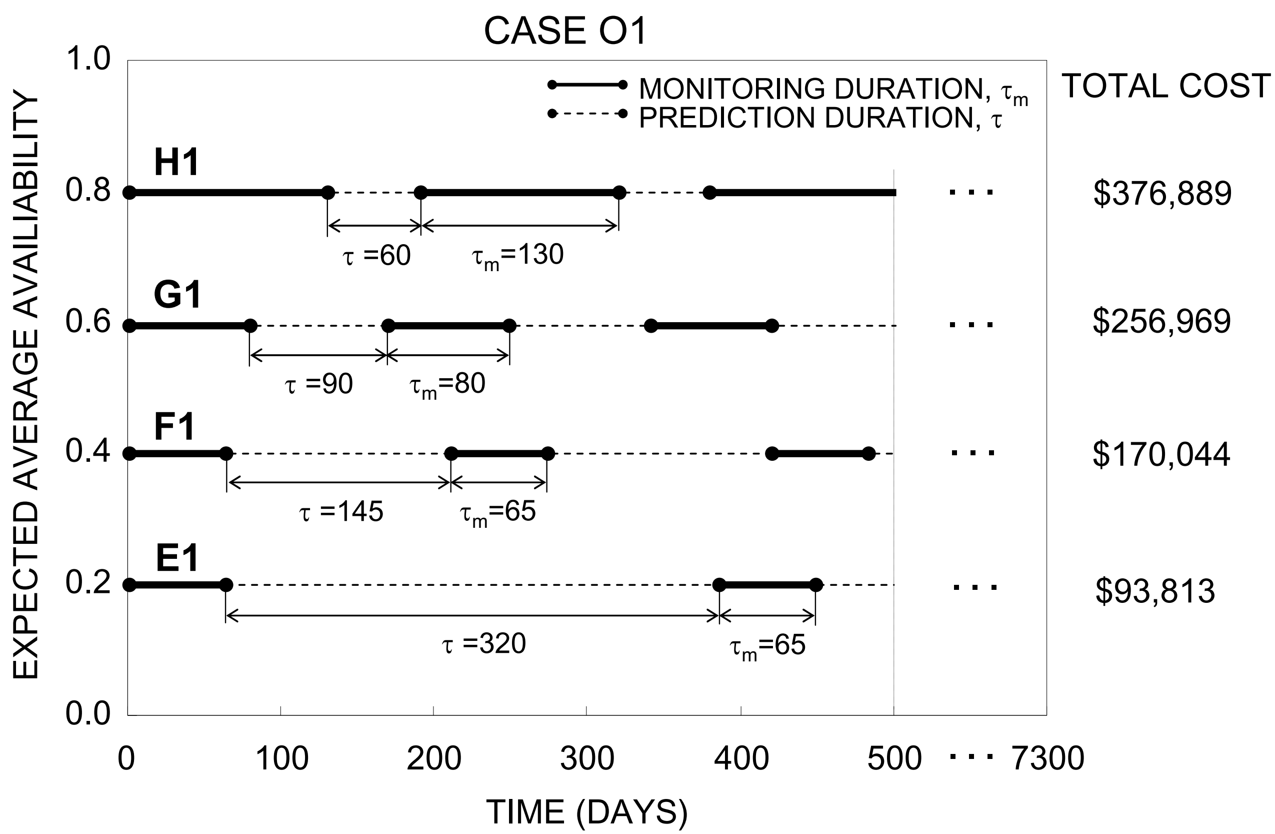

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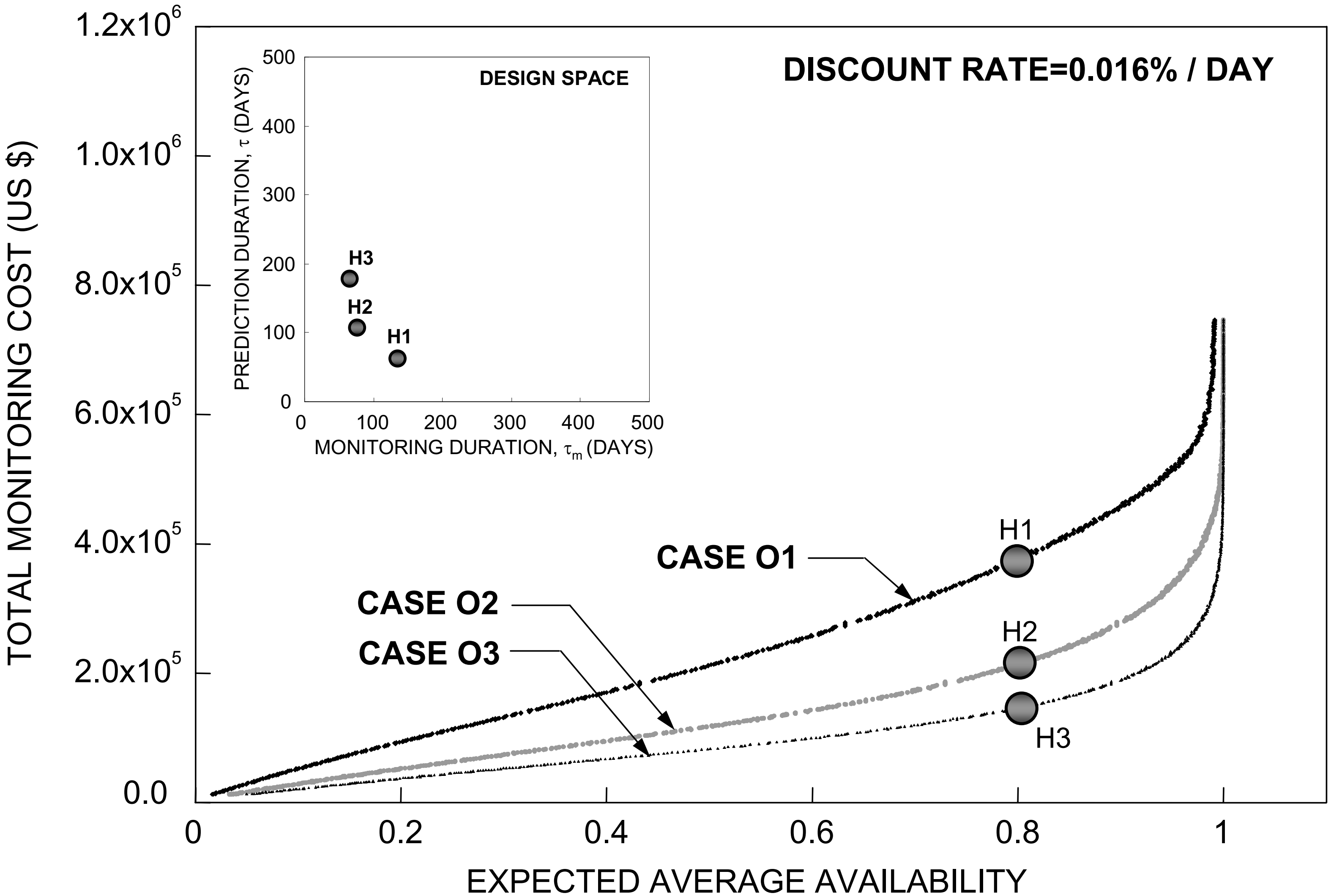

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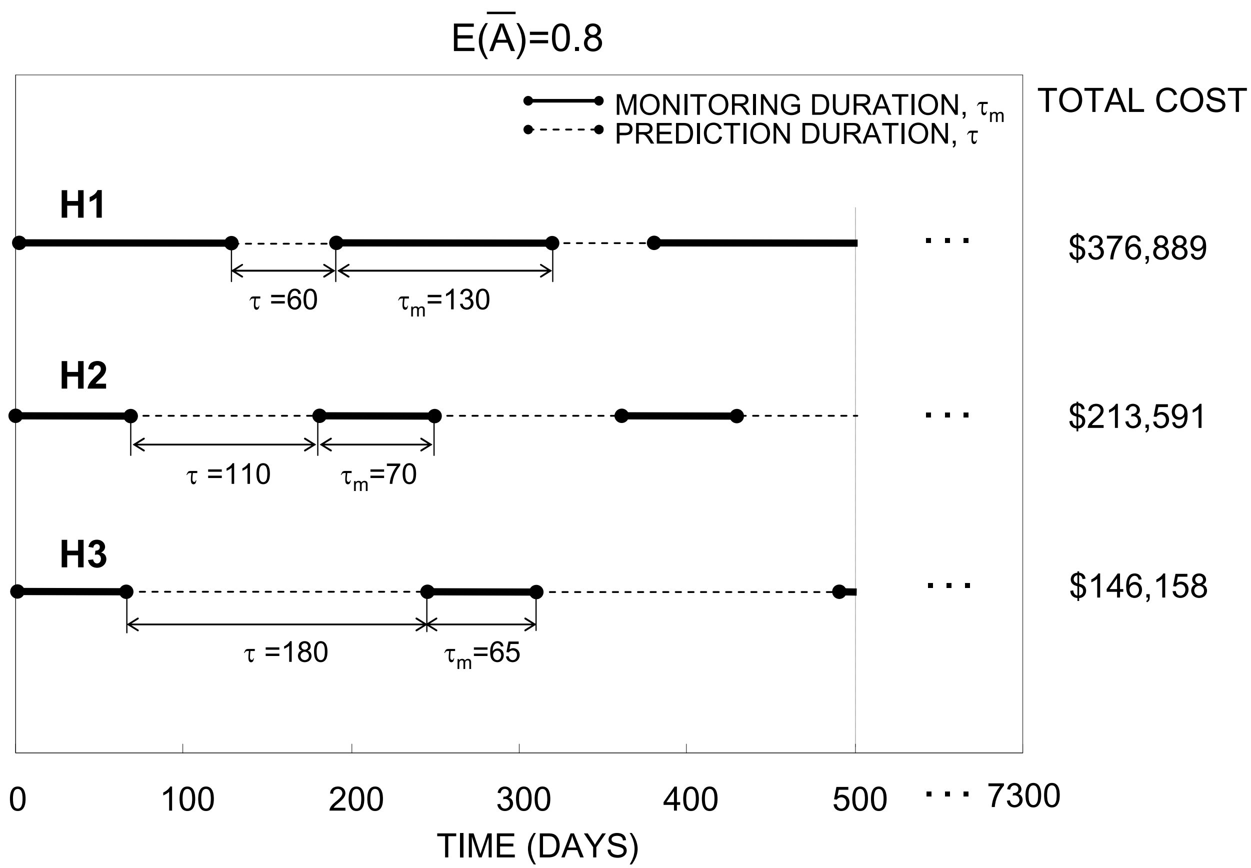
(e)



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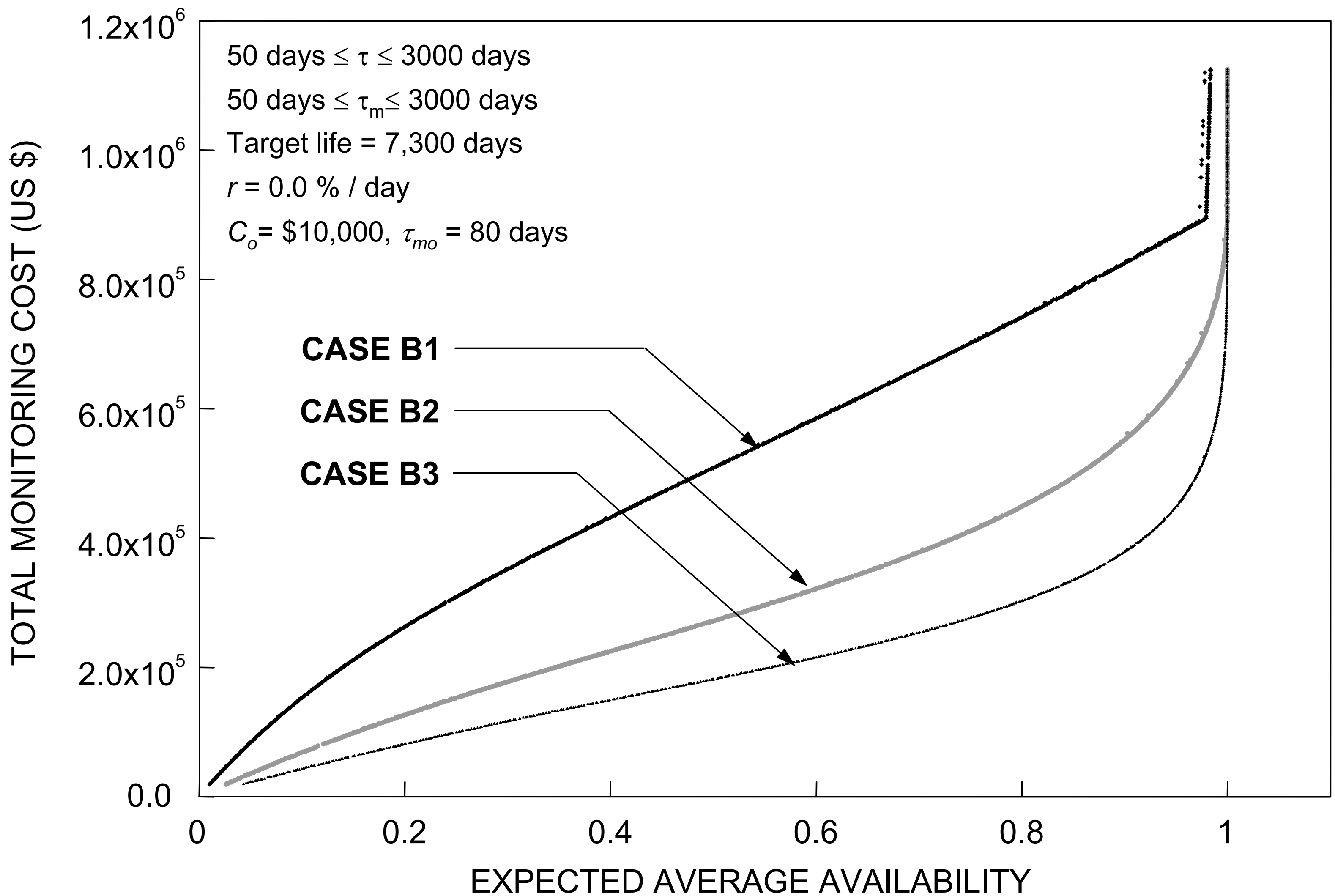

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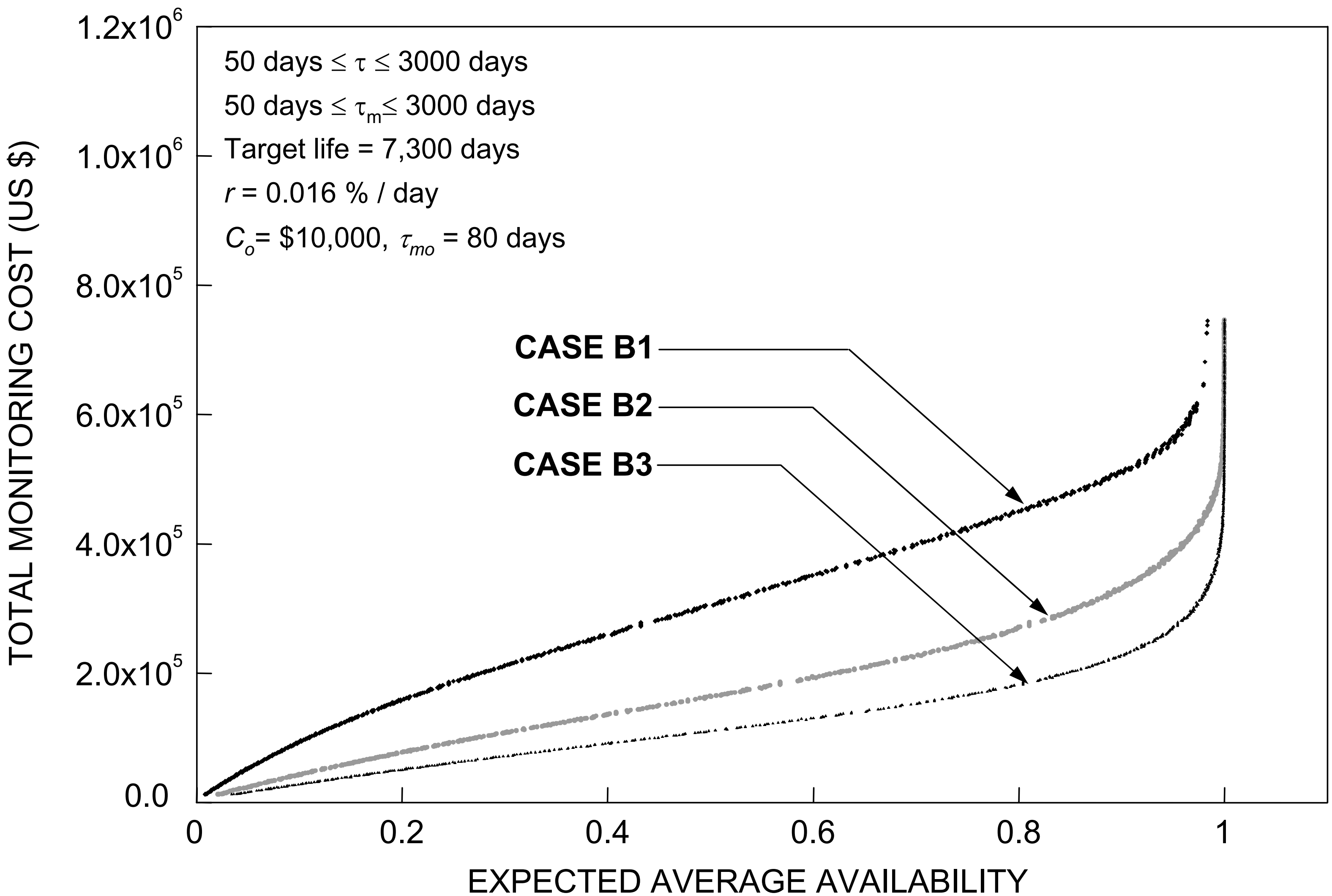

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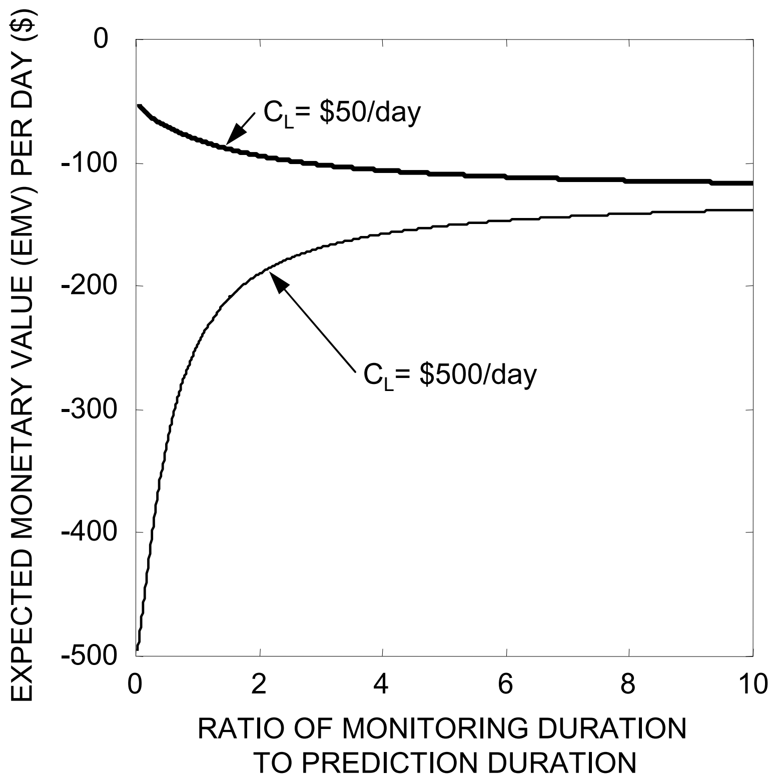

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Figure11b

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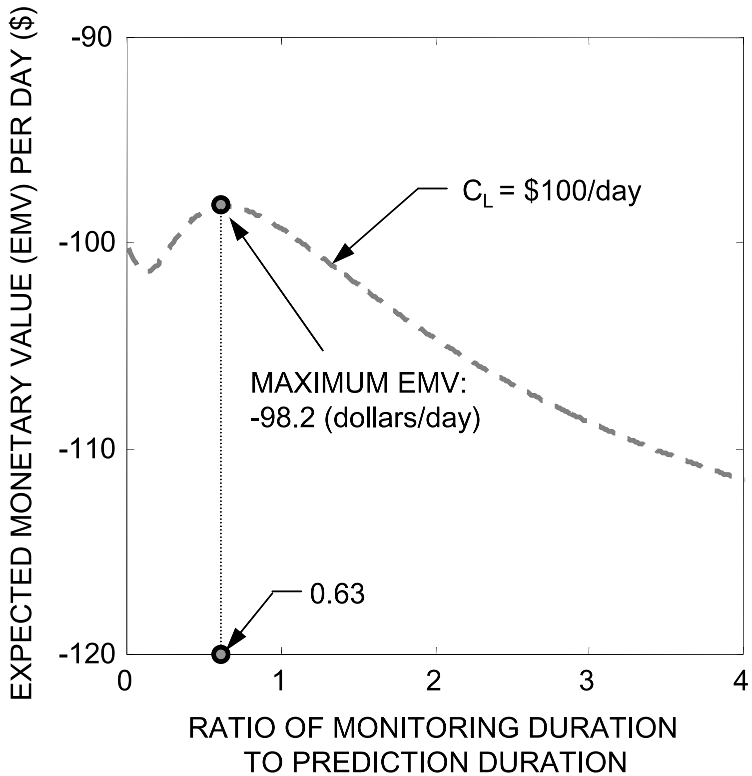

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Figure. 11(b)

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Table01

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Table 1. Exceedance Probability with Various Number of Exceedances

Number of exceedances	Case	Exceedance probability, p_e
At least one-time exceedance considering the largest value	O1	$1-exp\left(-\frac{t}{\tau_m}\right)$
At least two-time exceedances considering the largest value	O2	$1-\frac{(t+\tau_m)}{\tau_m}\cdot exp\left(-\frac{t}{\tau_m}\right)$
At least three-time exceedances considering the largest value	O3	$1-\left[1+\frac{t}{\tau_m}+\frac{1}{2}\left(\frac{t}{\tau_m}\right)^2\right]\cdot exp\left(-\frac{t}{\tau_m}\right)$
At least one-time exceedance considering both the largest and the smallest values	B1	$1-exp\left(-\frac{2t}{\tau_m}\right)$
At least two-time exceedances considering both the largest and the smallest values	B2	$1-\left(\frac{t+\tau_m}{\tau_m}\right)^2\cdot exp\left(-\frac{2t}{\tau_m}\right)$
At least three-time exceedances considering both the largest and the smallest values	B3	$exp\left(-\frac{2t}{\tau_m}\right)\cdot\left[\frac{t^4}{4\tau_m^4}+\frac{t^3}{\tau_m^3}+\frac{2t^2}{\tau_m^2}+\frac{2t}{\tau_m}+\left(exp\left(\frac{2t}{\tau_m}\right)-1\right)\right]$

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Table 2. Objective and Design Variable Values Associated with Various Cases as Indicated in Table 1; Discount Rate 0%/day

Case	Objectives		Design variables	
	$E(\bar{A})$	C_M ($)	τ (days)	τ_m (days)
O1	0.2	151,875	2,035	405
	0.4	283,125	1,680	755
	0.6	431,250	1,285	1,150
	0.8	624,375	770	1,665
O2	0.2	82,500	2,215	220
	0.4	155,625	2,020	415
	0.6	236,250	1,805	630
	0.8	354,375	1,490	945
O3	0.2	58,125	2,285	155
	0.4	108,750	2,155	290
	0.6	163,125	2,000	435
	0.8	240,000	1,795	640
B1	0.2	264,375	1,735	705
	0.4	431,250	1,285	1,150
	0.6	583,125	880	1,555
	0.8	740,625	460	1,975
B2	0.2	125,625	2,100	335
	0.4	223,125	1,840	595
	0.6	320,625	1,580	855
	0.8	448,125	1,240	1,195
B3	0.2	80,625	2,220	215
	0.4	148,125	2,040	395
	0.6	215,625	1,860	575
	0.8	301,875	1,630	805

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Table03

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Table 3 Objective and Design Variable Values Associated with Various Cases as Indicated in Table 1; Discount Rate 0.016 %/day

Case	Objectives		Design variables	
	$E(\bar{A})$	C_M (\$)	τ (days)	τ_m (days)
O1	0.2	93,813	320	65
	0.4	170,044	145	65
	0.6	256,969	90	80
	0.8	376,889	60	130
O2	0.2	52,553	500	50
	0.4	93,912	245	50
	0.6	143,645	170	60
	0.8	213,591	110	70
O3	0.2	37,422	890	60
	0.4	67,049	405	55
	0.6	98,874	255	55
	0.8	146,158	180	65
B1	0.2	159,743	160	65
	0.4	258,588	130	115
	0.6	350,900	65	115
	0.8	449,895	50	215
B2	0.2	77,387	375	60
	0.4	135,823	230	75
	0.6	194,741	185	100
	0.8	271,646	155	150
B3	0.2	49,985	515	50
	0.4	90,547	310	60
	0.6	130,143	195	60
	0.8	181,097	205	100

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