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Abstract

This chapter is devoted to the description of several methods found in the literature that were developed and applied to solve the problem of structural damage identification. Because of the economical and safety advantages of its solution, the damage identification problem has been attracting the attention of the Civil, Mechanical and Aeronautical Engineering communities for several decades. The present survey is mainly focused on the inverse problems methodology and only deals with methods based on structural dynamic characteristics. In this survey, the methods are categorised in chronological order of appearance in the literature and grouped by the dynamic characteristics (e.g. natural frequencies and mode shapes) and the mathematical techniques (e.g. least squares) in which they are based. Since each method is, typically, only appropriated to certain type of structures, their characteristics are also described. The nature of the data used (numerical, experimental or numerical-experimental) to perform the damage identification is also pointed out.

Keywords: damage identification, model updating, structural dynamics, inverse problems.

1 Introduction

Since there is a considerable scientific and technical interest in the resolution of the structural damage identification problem, the number and variety of methods developed by different researchers over the years is very high. This interest is attested by the large number of bibliographic reviews [1, 2, 3, 4, 5] dedicated to this subject. Doebling at al. [6] and Sohn et al. [7] wrote technical reports about structural health monitoring, in which the damage identification problem is extensively surveyed. The existence of such a large body of work is connected, among other reasons, to the non availability of a universal method which can be applied to all kinds of structures and damages types.

Rytter [8] proposed a classification of damage identification methods, establishing several levels. Rytter's levels can be defined by the following questions: (1) Is the structure damaged? (2) Where is this damage located? (3) After locating the damage, can one quantify the severity of it? and (4) The remaining service life of the structure can be predicted? In this chapter the surveyed methods are those dealing with level 1, 2 and 3 and is in this context that the word identification should be understood. For a review of level 4 methods, the reader is referred to [9].

For a more comprehensive reading of this survey, the authors decided to fix the nomenclature presented in the referenced literature. This means, for instance, that in some papers the damaged structure dynamic characteristics are typed along with a subscript letter, e.g. f_d for frequency, and in others with a superscript character, e.g. f^d. Here, the damaged structure dynamic characteristics will be written with a tilde over the letter, e.g. $\tilde{f}$. Furthermore, in the present survey, the description of the methods is often done by replacing the authors original data definitions and denominations by the equivalent ones in the context of damage identification problems.

This chapter begins with the survey of publications that directly use structural dynamic characteristics, such as natural frequencies and antiresonances, mode shapes and frequency response functions (FRF). Methods which use flexibility or compliance and damping are subsequently addressed. Model updating methods are also surveyed, since they are deeply related to damage identification. Indeed, such methods can be viewed as damage identification methods by considering that the numerical data (for instance, the finite element model results) and the experimental data are, respectively the undamaged and damaged structure dynamic characteristics.

2 Frequencies and antiresonances

Frequencies: Methods based on changes in frequency were the subject of a thorough review by Salawu [1]. According to Doebling et al. [2, 6], these can be subdivided in two categories: (i) forward problem and (ii) inverse problem methods. In general, the first type of methods does not allow the damage localisation and its exact quantification, since frequencies are global structural properties. Under certain conditions, the second type of methods allow the calculation of damage parameters, such as the crack length.

Cawley and Adams [10] showed that the ratio of the frequency changes in two modes is only function of the damage location. These researchers applied this methodology to an aluminum rectangular plate and to a trapezoidal carbon fibre reinforced plastic plate, by means of experimental techniques and the finite element method. Their test results showed that the proposed method can be used to detect, locate and roughly quantify damage.

Inspired by the use of the MAC (see eq. (2)) in modal analysis, Williams et al. [11]

and Messina et al. [12] proposed the Damage Location Assurance Criterion $DLAC$, and the Multiple Damage Location Assurance Criterion $MDLAC$ defined, respectively, by

$$DLAC\,(j) = \frac{\left|\widetilde{\mathbf{\Delta f}}^T \mathbf{\Delta f}_j\right|^2}{(\widetilde{\mathbf{\Delta f}}^T \widetilde{\mathbf{\Delta f}})(\mathbf{\Delta f}_j^T \mathbf{\Delta f}_j)} \text{ and } MDLAC\,(\mathbf{\Delta D}) = \frac{\left|\widetilde{\mathbf{\Delta f}}^T \mathbf{\Delta f}\right|^2}{(\widetilde{\mathbf{\Delta f}}^T \widetilde{\mathbf{\Delta f}})(\mathbf{\Delta f}^T \mathbf{\Delta f})}, \tag{1}$$

The analytical and the measured natural frequencies vectors are denoted by $\mathbf{\Delta f}$ and $\widetilde{\mathbf{\Delta f}}$, respectively, j defines the location, and $\mathbf{\Delta D}$ is the damage vector, function of $\mathbf{\Delta f}$. The highest values of $DLAC$ and $MDLAC$ determine the predicted damage(s) location(s). Experimental investigations in single damaged steel and composite frames [11] and numerical simulations of multiple damaged truss structures [12] were performed.

Antiresonances: The use of antiresonances was proposed by Afolabi [13], based on the fact that, unlike natural frequencies (resonances), those characteristics depend on the location of the measurement site. From the numerical studies performed on a three degrees of freedom beam model, Afolabi concludes that as the point of measurement gets closer to the location of the defect, fewer and fewer antiresonances are shifted from their original values. Therefore, if the measurement site corresponds to the location of the defect, all the antiresonances are exactly as they were in the undamaged state.

Bamnios et al. [14] developed a method in which, by monitoring the change of the first antiresonance as a function of the measuring location along the beam, a jump in the slope of the plot in the vicinity of the crack occurs. They also concluded that in the vicinity of the crack the first antiresonance tends to coincide with the first natural frequency. By following up this work, Dharmaraju and Sinha [15] used the variation in four antiresonances along the length of a free-free aluminum beam for three cases (no crack, one crack, and two cracks), concluding that, theoretically, the sharp change in the slope of antiresonance plot at crack location may be observed, but realisation in the measurements is difficult.

To avoid the non-uniqueness of the damage location problem, which occurs in symmetrical beams, Dilena and Morassi [16] developed a method which only needs the first natural frequency and the first antiresonance. They presented an analytical solution of axial and bending vibrating beams differential equations, with which satisfactory results of damage identification were obtained. The experimental tests on cracked steel beams, however, showed that the noise and the modelling errors on antiresonances are usually strongly amplified with respect to cases in which only frequency data are used. Therefore, those researchers stated that damage identification techniques based on antiresonance data should be carried out with some caution.

3 Mode shapes and frequency response functions

Mode shapes: Another class of methods uses mode shapes, namely by comparing undamaged mode shapes $\mathbf{q}_1, \mathbf{q}_2, \ldots$ and damaged mode shapes $\widetilde{\mathbf{q}}_1, \widetilde{\mathbf{q}}_2, \ldots$ using the Modal Assurance Criterion (MAC) and the Coordinate MAC (COMAC), defined, respectively, by

$$MAC\left(\mathbf{q}_i, \widetilde{\mathbf{q}}_j\right) = \frac{\left|\mathbf{q}_i^T \widetilde{\mathbf{q}}_j\right|^2}{\left(\mathbf{q}_i^T \mathbf{q}_i\right)\left(\widetilde{\mathbf{q}}_j^T \widetilde{\mathbf{q}}_j\right)} \text{ and } COMAC\left(k\right) = \frac{\left(\sum\limits_{j=1}^{m} \left|q_{kj} \widetilde{q}_{kj}\right|\right)^2}{\sum\limits_{j=1}^{m} \left|q_{kj}\right|^2 \sum\limits_{j=1}^{m} \left|\widetilde{q}_{kj}\right|^2}, \qquad (2)$$

where i e j denote the mode shape number and k the degree of freedom.

Kim at al. [17] presented a method for numerical damage detection and localisation on aluminum plates by combining these two criteria and named it Total MAC. Sujatha et al. [18] applied the MAC to compare the changes caused by progressive damage in glass reinforced polyester laminated beams.

The $MAC\left(\mathbf{q}_i, \mathbf{q}_j\right)$ value is mathematically equivalent to the square root of the virtual work of internal forces $W_{ij} = \mathbf{q}_i^T \mathbf{I} \mathbf{q}_j$ associated to the i-th mode shape along the displacement field of mode j, as pointed out by Petryna et al. [19]. Therefore, these researchers proposed an index defined as

$$D_{VEB} = 1 - \det \frac{\widetilde{\mathbf{W}}}{\mathbf{W}} \qquad (3)$$

for damage quantification and applied it to a reinforced concrete beam and a 3-span slab bridge.

The changes in lumped strain energy is proposed by Carrasco et al. [20] as a damage localisation parameter, since differences in modal deformations are observed in the vicinity of a damaged element. This reasoning is also the foundation of Doebling et al. [21] method, who showed that a strategy of mode selection, based on the damaged structure maximum strain energy, produces better damage location results than a strategy based on the undamaged structure maximum strain energy or on minimum frequency. Both Carrasco et al. [20] and Doebling et al. [21] analysed truss-like structures.

Stubbs and Osegueda [22, 23] developed and applied a method to beams, which is based on the relation among fractional changes of the eigenvalues $\mathbf{z}$, the stiffness parameters $\boldsymbol{\alpha}$, and the mass parameters $\boldsymbol{\beta}$, such that

$$\boldsymbol{\alpha} = \left(\mathbf{S}^{(\mathrm{K})}\right)^+ \left(\mathbf{z} + \mathbf{S}^{(\mathrm{M})} \boldsymbol{\beta}\right) \text{ with } S_{ie}^{(\mathrm{K})} = \frac{\widetilde{\mathbf{q}}_i^T \mathbf{K}_e \widetilde{\mathbf{q}}_i}{k_i} \text{ and } S_{ie}^{(\mathrm{M})} = \frac{\widetilde{\mathbf{q}}_i^T \mathbf{M}_e \widetilde{\mathbf{q}}_i}{m_i}, \qquad (4)$$

where $(\ )^+$ stands for the generalised inverse and $\mathbf{S}^{(\mathrm{K})}$ and $\mathbf{S}^{(\mathrm{M})}$ are, respectively, the sensitivities of the frequency changes with respect to the stiffness and mass. In eq.

(4) $\mathbf{K}_e$ and $\mathbf{M}_e$ are the stiffness and mass matrices of the e-th element and $\widetilde{\mathbf{q}}_i$ and $\mathbf{q}_i$ are, respectively, the damaged and undamaged i-th mode shapes. The sensitivities of the frequency changes relative to the stiffness can also be computed using the mode shape curvatures. This method was also applied by Stubbs et al. [24] to large space structures modelled as equivalent continua with simulated construction defects, while Crema et al. [25, 26] used it for numerically simulated damage localisation in isotropic and orthotropic beams which are divided into macro-elements.

Frequency response functions (FRF): Some researchers propose the use of FRF, such an Choudhury and He [27], who suggest the use of a vector defined by the differences of the undamaged and damaged structures FRF for damage localisation. These researchers perform an expansion of the measured receptances using several techniques. The quantity of data thus obtained is much larger than that of the natural frequencies and mode shapes. A 12 degrees of freedom mass-spring system and a truss structure were numerically and experimentally studied, respectively.

Park and Park [28] presented an index which compares a unit vector with the changes in the incomplete experimental FRF. This localisation method does not require a model, but it is frequency dependent, since not all frequency ranges give good results. To overcome this problem, Park and Park [28] developed a technique to select the ranges where their method works satisfactorily. The method was applied to a numerical model of a steel truss an to an experimental model of a steel plate. In this case, the experimental load is very high, since the total number of measurements to construct the model is 900.

Using also a subset of vectors from the full set of FRF for a few frequencies and calculating the stiffness matrix and reductions in explicit form, Hwang and Kim [29] showed numerically that the damage in a cantilever beam and a helicopter rotor blade can be located and characterised.

The combined use of the measured changes in the first three natural frequencies and the corresponding amplitudes of the measured acceleration FRF was suggested by Owolabi et al. [30], with which they experimentally predicted the locations and depths of cracks in aluminum beams.

Other methods: There are other methods which are directly related to mode shapes and FRF, since they need information contained in these dynamic characteristics to identify the damage.

Abdo and Hori [31] stated that the rotation of mode shapes may be feasible to be measured in the near future, due to the introduction of the scanning laser Doppler vibrometer. Hence, they investigated the changes in the rotations of mode shapes, using a finite element model of a steel plate. Based on this investigation, Abdo and Hori [31] concluded that, with a good choice of the mode shapes, the damage location can be accurately located even with only one mode.

Based on the estimation of a node position through the detection of the phase angle of the inertance response function between two points of a beam axis, Dilena and Morassi [32] proposed that the localisation of a crack be made by monitoring the node shift. This shift is due to the presence of damage. Experimental tests in steel beams with transversal saw cuts supported the analytical predictions.

Liu et al. [33, 34] observed the existence of damage by analysing the FRF of undamaged and cracked circular hollow section beams and frame-like welded structure. Their studies showed that the crack presence causes a coupling between longitudinal, bending and torsional vibrations and, therefore, new peaks appear on FRF plots.

Spatial derivatives of mode shapes and FRF: As an alternative to the changes in mode shapes and FRF, some researchers propose methods which use spatial derivatives of these dynamic characteristics, namely their curvatures.

Using finite element models of simply supported and cantilever beams, Pandey et al. [35] showed that the absolute changes in the mode shape curvatures allow the localisation of damage, while the use of changes in mode shapes or the MAC and the COMAC do not. In Pandey et al. [35] work, the curvature mode shapes are obtained numerically by using a central finite difference approximation:

$$q_k^{''} = \left(q_{k+1} - 2q_k + q_{k-1}\right)/h^2, \tag{5}$$

where h is the length of the elements.

Based on a similar procedure, but using experimental measurements, Salawu and Williams [36] compared the changes in mode shape curvatures and changes in mode shapes methods efficiency. They also showed that, typically, the changes in curvatures are not good damage indicators when one uses experimental data, being necessary a mode selection, since only some modes correctly identify and locate the damage.

Stubbs et al. [37] presented a method for damage localisation and quantification, based on an index, which for a certain beam mode i is defined by

$$\beta_{ei} = \frac{\left\{\int_e \left[\widetilde{\mathrm{q}}_i''(x)\right]^2 \mathrm{d}x + \int_0^L \left[\widetilde{\mathrm{q}}_i''(x)\right]^2 \mathrm{d}x\right\} \int_0^L \left[\mathrm{q}_i''(x)\right]^2 \mathrm{d}x}{\left\{\int_e \left[\mathrm{q}_i''(x)\right]^2 \mathrm{d}x + \int_0^L \left[\mathrm{q}_i''(x)\right]^2 \mathrm{d}x\right\} \int_0^L \left[\widetilde{\mathrm{q}}_i''(x)\right]^2 \mathrm{d}x}. \tag{6}$$

where x refers to the longitudinal coordinate, e denotes the element and L is the beam length. By improving this method and applying it to a continuous beam, Stubbs and Kim [38] proved possible to localise and quantify damage only knowing one damage mode shape and a finite element model of the undamaged structure. These authors further improved this method by including in it crack location and size models, based on the use of undamaged natural frequencies and modes and damaged natural frequencies, along with the computation of linear elastic fracture mechanics energy release rate. By applying this method, Kim and Stubbs [39] located and determined the size of a crack in a beam with relatively few modes.

An extension to plate like structures of Stubbs et al. [37] method was presented by Cornwell et al. [40, 41]. These researchers stated that it is possible to locate areas with a stiffness reduction of 10% using relatively few modes. A damage index for plates, similar to the one in eq. (6), was also proposed by Yam et al. [42]:

$$(\beta_e)_{\mathrm{cms}} = \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\int\int_{\Omega_e}\left\{\left[\widetilde{\mathrm{q}}''_{ij}(x,y)\right]_e^2 - \left[\mathrm{q}''_{ij}(x,y)\right]_e^2\right\}\mathrm{d}x\mathrm{d}y}{\max\limits_{l\subset[1,N]}\left\{\int\int_{\Omega_l}\left[\mathrm{q}''_{ij}(x,y)\right]_l^2\mathrm{d}x\mathrm{d}y\right\}}, \quad e\subset[1,N], \tag{7}$$

where cms stands for curvature mode shape, e and l denote elements and (m,n) define the orders of modes selected for analysis. In their work another index is presented, where the mode shape curvatures are replaced by the strain frequency response function. Li et al. [43] defined and applied other indices, this time based on bending moments and residual strain mode shapes.

Ratcliffe and Bagaria [44] developed a method named “gapped smoothing method”. In it, the damaged mode shape experimental curvatures are interpolated at beam positions x_{i-2}, x_{i-1}, x_{i+1} e x_{i+2}. The proposed damage index is defined as the square difference of curvature values computed using the polynomial at x_i and the corresponding experimental value. Undamaged structure models or data are not required in this method. A successful location of delamination in a glass-reinforced epoxy beam was experimentally accomplished by these researchers. This method was extended to plate-like structures, using not only mode shape data, but also frequency dependent operating displacement shape data, by Yoon et al. [45]. These researchers applied the extended gapped smoothing method to a plate finite element model, as well as to composite plates with experimentally induced multiple delaminations and to a large composite hull structure.

A method, which allows the prediction of damage location and severity based on the determination of bending stiffness and torsion stiffness, through the computation of mode shape curvatures (second order derivative) and torsion rates (first order derivatives), respectively, was developed by Maeck and Roeck [46]. In order to obtain reasonable derivative computations, a preliminary data smoothing technique is carried out. This method was applied to a reinforced concrete beam and the results showed that the torsion stiffness decreases less than the bending stiffness.

Maia et al. [47] and Sampaio et al. [48] developed a damage localisation method using FRF curvatures of beams. These researchers stated that this method main advantages are its simplicity and the fact that it does not require any modal analysis for mode shapes or natural frequencies identification. Hence, considering Nf frequencies and Ne applied forces, an index for each point k can be defined:

$$FRF\_MSC_k = \sum_{l=1}^{Nf}\sum_{j=1}^{Ne}\left|\widetilde{a}''_{kj}(\omega_l) - a''_{kj}(\omega_l)\right| \quad \text{for} \quad k=1,\ldots,Ng. \tag{8}$$

In reference [48] is also presented an index similar to the one in eq. (6), but where the mode shape curvatures are replaced by the FRF curvatures. A comparison among methods which use mode shapes, FRF and their derivatives was performed by Maia et al. [49] and the conclusion is that the methods which use derivatives present better damage localisations.

An improvement on the mode shape curvature method was presented by Dutta and Talukdar [50]. Since it is necessary to compute accurately as possible the dynamic parameters, these researchers propose the control of the discretisation errors by using an adaptive finite element model. By applying this method to numerically simulated data, multiple damage in a simple supported and continuous beam was localised. As in other works presently surveyed, Dutta and Talukdar [50] also observed the superior performance of mode shape curvatures over mode shapes alone in damage localisation.

Hamey et al. [51] and Lestari and Qiao [52] used mode shape curvatures to identify delamination, impact and saw-cut damages in carbon/epoxy composite beams and core-faceplate debonding and core crushing in fiber glass reinforced polymer honeycomb sandwich beams. In their first work, the researchers also used the FRF curvatures method. The dynamic characteristics were obtained with piezoelectric sensors, which allow the direct measurement of curvatures. Hence, it is not necessary to perform any kind of differentiation.

A performance comparison of differences between translations, differences between slopes (rotations) and differences between curvatures methods for impact damage localisation in a carbon fibre reinforced epoxy rectangular plate was reported by Araújo dos Santos et al. [53]. The mode shapes experimental acquisition was performed using double pulse TV holography with acoustic excitation and the rotations and curvatures are obtained by numerical differentiation of mode shapes translations using a differentiation/smoothing technique. These researchers found that the best localisations are achieved by selecting the most changed mode and by applying the curvatures differences methods.

Lestari et al. [54] formulated an analytical relationship between damaged and undamaged beams, in which the mode shape curvatures are related to the stiffness loss. They applied it to estimate the extent of damage in carbon/epoxy laminated composite beams with surface-bonded piezoelectric sensors, used to directly acquire the mode shape curvatures. Delamination, impact and saw-cut damage scenarios were simulated.

The computation of the curvatures by numerical differentiation is identified by Guan and Karbhari [55] as the main cause for the poor performance of modal curvature methods under sparse and noisy measurements. Therefore, instead of eq. (5), these researchers proposed the use of a more accurate differentiation equation:

$$q_k^{''} = \left(-q_{k+2} + 16q_{k+1} - 30q_k + 16q_{k-1} - q_{x-2}\right)/12h^2. \tag{9}$$

In Guan and Karbhari [55] work, the unknown modal rotations are found using a penalty-based minimisation approach. Their method is numerically and experimentally applied to aluminum beams.

As a final remark, it can be stated that it seems to be a consensus among several researchers to whom methods based on spatial derivatives of mode shapes or FRF, namely curvatures, are more sensitive to damage than those based on frequencies, mode shapes or FRF alone.

4 Flexibility and compliance

A fourth class of methods is based on the structure flexibility or compliance. A method relying on the differences between undamaged and damaged structures flexibilities was presented by Pandey and Biswas [56, 57]. Their work showed, through numerical and experimental examples in beams, that these differences allow the localisation of damage using only two or three natural frequencies and mode shapes, since the higher changes in flexibility are located in the damaged area. In reference [57], Pandey and Biswas reported the method application to the localisation of two saw cuts. An advantage of this method is the non requirement of an analytical model of the structure. The success of this method, using only few lower natural frequencies and mode shapes, is related to the rapid convergence of the flexibility matrix $\mathbf{G}$, as shown in the following equation:

$$\mathbf{G} = \sum_{i=1}^{n} \frac{1}{\omega_j^2} \mathbf{q}_j \mathbf{q}_j^T . \tag{10}$$

Lu et al. [58] performed multiple damage localisations in beams by applying Pandey and Biswas [56, 57] method and a method where the flexibility curvatures are approximated by central finite differences:

$$g_i^{''} = \left(g_{i-1,i-1} - 2g_{i,i} + g_{i+1,i+1}\right)/h^2, \tag{11}$$

where $g_{i-1,i-1}, g_{i,i}, g_{i+1,i+1}$ are diagonal elements of the flexibility matrix and $g_i^{''}$ is the i-th element of the flexibility curvature vector, being h the element length (cf. eq. (5)). The method allows the damage localisations without knowing the flexibility curvature shapes of the undamaged structure, since the multiple damages induce well defined peaks in the flexibility curvature shapes. The numerical tests showed that, for damage positions not-well separated or when different positions are damaged to different extents, the flexibility curvatures present better results than the flexibilities themselves. Lu et al. [58] also modified the $MDLAC$ index proposed by Messina et al. [12] (eq. (1)), but replaced the absolute frequency change by the relative frequency change.

Using the relationship between the experimental flexibility matrix and the stiffness matrix, it is also possible to define another method, as proposed by Lin [59] in the context of model updating. In fact, the relation

$$\mathbf{E} = \widetilde{\mathbf{G}}\mathbf{K} - \mathbf{I}, \tag{12}$$

where $\widetilde{\mathbf{G}}$ and $\mathbf{K}$ are the experimental flexibility matrix and the analytical stiffness matrix, respectively, allows the localisation of modelling errors, since the the error in each degree of freedom can be measured by the maximum absolute value of the components in the corresponding column of $\mathbf{E}$. The method was applied to numerical examples of modelling errors in truss and beam structures.

Alvin et al. [60] defined the Relative Damage Indicator by

$$\mathbf{RDI} = \left|\mathrm{diag}\left(\widetilde{\mathbf{G}}_{\varepsilon i} - \mathbf{G}_{\varepsilon i}\right)\right|, \tag{13}$$

where $\widetilde{\mathbf{G}}_{\varepsilon i}$ and $\mathbf{G}_{\varepsilon i}$ are the flexibility matrices computed using the i-th strain mode shape. The non zero values in $\mathbf{RDI}$ indicate changes in the flexibility in the corresponding degree of freedom and, therefore, the relative amount of damage. A simple spring-mass was analysed using this method.

In order to determine the damage in plate-like structures, Kim et al. [61, 62] presented an index based on flexural characteristics, since there is a linear relationship between the flexibility curvatures and the flexural damage, i.e.:

$$\boldsymbol{\Theta}\boldsymbol{\beta} = \widetilde{\boldsymbol{\kappa}} \tag{14}$$

where $\boldsymbol{\beta}$ is the unknown damage index vector, $\widetilde{\boldsymbol{\kappa}}$ is the curvature vector. The $\boldsymbol{\Theta}$ matrix represents the undamaged structure curvatures, computed using the modal flexibility vectors. The performance of this method is compared with others and the authors conclude that its results are consistent and even better that those obtained using the curvature method and the damage index.

Based on compliance at a given point of a structure, Choi et al. [63] proposed the index

$$\beta_{ie}^{\mathrm{c}} = \frac{\widetilde{s}_e}{s_e}, \tag{15}$$

where s_e and $\widetilde{s}_e$ are, respectively, factors related with mode i and element e undamaged and damaged compliances. Taking into account n modes, these researchers defined the composed index $\beta_e^{\mathrm{c}} = \sum_{i=1}^{n} \beta_{ie}^{\mathrm{c}}$. The proposed method presents better damage quantification results in beams, experimentally and numerically analysed, than the strain energy based index of Park et al. [64], defined by

$$\beta_{ie}^{\mathrm{e}} = \frac{k_e}{\widetilde{k}_e} \approx \frac{\left[\widetilde{\mathbf{q}}_i^T \left(k_e^{-1}\mathbf{K}_e\right)\widetilde{\mathbf{q}}_i + \widetilde{\mathbf{q}}_i^T\mathbf{K}\widetilde{\mathbf{q}}_i\right]\mathbf{q}_i^T\mathbf{K}\mathbf{q}_i}{\left[\mathbf{q}_i^T \left(k_e^{-1}\mathbf{K}_e\right)\mathbf{q}_i + \mathbf{q}_i^T\mathbf{K}\mathbf{q}_i\right]\widetilde{\mathbf{q}}_i^T\mathbf{K}\widetilde{\mathbf{q}}_i}, \tag{16}$$

where k_e and $\widetilde{k}_e$ are, respectively, the undamaged and damaged stiffness parameters of element e. Choi et al. [63] also used the average of the two indices (equations (15) and (16)), thus obtaining damage predictions without errors in three experimental damage scenarios. This method was applied to damage quantification in plate-like structures by Choi et al. [65].

5 Damping

The development of damage identification methods based on structural damping is not so extensive as, for instance, the ones based on natural frequencies and mode shapes, although, according to Chandra et al. [66], among others, damping is more sensitive to damage in composites than stiffness. This relative low number of works using damping is perhaps due to the fact that it is a difficult property to model and obtain.

A method for numerical identification of cracks in a longitudinally vibrating beam with dissipative boundary conditions, modelled with viscous damping, was presented by Dilena and Morassi [67]. These researchers reported that it is possible to locate and quantify cracks in this kind of structures by measuring the changes in a suitable pair of eigenvalues, even in the presence of random noise in the data.

Nokes and Cloud [68] used laser Doppler vibrometry and electronic speckle pattern interferometry to measure modal parameters on a composite beam at high frequencies (up to 10 kHz) with high spatial resolution. They found the damping loss factor to be a sensitive indicator of global material damage. They also found the higher torsion modes to be especially sensitive to local damage.

6 Model updating

An important class of damage identification methods is based on structural matrices updating. The methods included in this class are theoretically analogous to some model updating methods, which are used to locate and quantify modelling errors. In general, the updated matrices are determined by solving an optimisation problem, based on the equations of motion, original matrices and experimental data. By comparing the matrices in the undamaged state with the original ones, it is possible to locate and even quantify the damage. This class of methods can be divided in three groups which use (1) optimal matrix update procedures, (2) dynamic characteristics sensitivities, and (3) minimisation of modal force errors. Usually, by applying a model updating method to damage identification problems, one relates the damage with the structure mechanical proprieties, hence being able to directly quantify it.

Optimal matrix updating Optimal matrix updating methods use direct techniques to compute the damaged structure matrices. Most of these methods are based on the minimisation of an error matrix between undamaged and damaged matrices.

The methods presented by Berman and Nagy [69], and Kabe [70] are based on the minimisation of the Frobenius norm of the global parameters perturbation matrix, using as constraints null error in the modal force and the matrix symmetry proprieties. In most cases, the results do not keep the structural conectivities and may not have physical meaning. A 508 degrees of freedom structural model was analysed by Berman and Nagy [69], while Kabe [70] studied a eight degrees of freedom mass-spring model.

Another kind of methods relies on the minimisation of the perturbation matrix rank. Kaouk and Zimmerman [71] proposed the minimum rank perturbation theory (MRPT) method for damage localisation. This method is based on the amplitude of the eigenvalues and eigenvectors equation residuals, using the measured eigenvalues and eigenvectors and the stiffness and mass matrices of the undamaged structure. A stiffness modified matrix with minimum rank, which reproduces the measured data, is computed, thus obtaining the damage localisation. The authors applied this method to damage in a truss and a beam using numerical data. This method was also applied by Zimmerman and Simmermacher [72] to model updating of a truss-like structure, using experimental data from modal and static tests.

Liu [73] proposed a method which minimises the norm of the error associated to the eigenvalues and eigenvectors equation, i.e. minimises the norm of the error vector in the modal force, given by

$$\mathbf{e}_i = \left(\Delta\mathbf{K} - \widetilde{\lambda}_i \Delta\mathbf{M}\right) \widetilde{\mathbf{q}}_i. \tag{17}$$

This is done in order to compute the stiffness $\Delta\mathbf{K}$ and mass $\Delta\mathbf{M}$ perturbation matrices. This method was applied to damage identification in trusses.

Sensitivities: Another group of methods uses matrix modifications based on different kinds of sensitivities. In these methods one computes, for instance, the derivatives of the eigenvalues (natural frequencies) or eigenvectors (mode shapes) in respect to physical parameters, yielding a damage parameter vector. In most works here surveyed, such a vector is computed by solving a system of linear equations through the computation of the pseudo-inverse of a sensitivity matrix.

A method for damage location and extent was proposed by Ricles and Kosmatka [74] and is based on the computation of the following matrix:

$$\mathbf{S} = \begin{bmatrix} \frac{\partial \omega^2}{\partial \mathbf{K}} & \frac{\partial \omega^2}{\partial \mathbf{M}} \\ \\ \frac{\partial \mathbf{Q}}{\partial \mathbf{K}} & \frac{\partial \mathbf{Q}}{\partial \mathbf{M}} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{K}}{\partial \mathbf{p}} \\ \\ \frac{\partial \mathbf{M}}{\partial \mathbf{p}} \end{bmatrix}, \tag{18}$$

where ω and $\mathbf{Q}$ denote the natural frequencies and mode shapes, respectively, and $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices of the undamaged structure. These researchers begin by locating the damage using the residual force vector,

$$\mathbf{f}_i = \left(\mathbf{K} - \widetilde{\lambda}_i \mathbf{M}\right) \widetilde{\mathbf{q}}_i, \tag{19}$$

before computing its severity. Therefore, they only include potentially damaged elements in the computation of the sensitivity matrix $\mathbf{S}$. This method was applied to a truss structure.

Lin and He [75] and Lin et al. [76] developed a method named "improved inverse eigensensitivity method (IEM)" in which the sensitivities of the j-th eigenvalue to parameter p_k are defined by

$$\frac{\partial \lambda_j}{\partial p_k} = \mathbf{q}_j^T \frac{\partial \mathbf{K}}{\partial p_k} \widetilde{\mathbf{q}}_j - \widetilde{\lambda}_j \mathbf{q}_j^T \frac{\partial \mathbf{M}}{\partial p_k} \widetilde{\mathbf{q}}_j, \tag{20}$$

while the sensitivities of the j-th eigenvector are given by

$$\frac{\partial \mathbf{q}_j}{\partial p_k} = \sum_{\substack{i=1 \\ i \neq j}}^{Ng} \frac{\mathbf{q}_i \mathbf{q}_i^T}{\widetilde{\lambda}_j - \lambda_i} \left(\frac{\partial \mathbf{K}}{\partial p_k} - \widetilde{\lambda}_j \frac{\partial \mathbf{M}}{\partial p_k} \right) \widetilde{\mathbf{q}}_j - \frac{1}{2} \mathbf{q}_j^T \frac{\partial \mathbf{M}}{\partial p_k} \widetilde{\mathbf{q}}_j, \tag{21}$$

where Ng is the analytical model number of degrees of freedom. Considering a Taylor series first order approximation, m natural frequencies and mode shapes and Ngc

measured degrees of freedom, a set of $m \times (Ngc+1)$ linear equations in order to N unknowns can be defined:

$$\sum_{k=1}^{N} \frac{\partial \mathbf{q}_j}{\partial p_k} p_k = \widetilde{\mathbf{q}}_j - \mathbf{q}_j \text{ and } \sum_{k=1}^{N} \frac{\partial \lambda_j}{\partial p_k} p_k = \widetilde{\lambda}_j - \lambda_j \tag{22}$$

for $j = 1, \ldots, m$. The unmeasured degrees of freedom are replaced by the corresponding degrees of freedom of the analytical model. This method was applied to model updating of trusses. Note that que classical inverse method (see e.g. [77, 78]) is obtained by replacing $\widetilde{\lambda}_j$ and $\widetilde{\mathbf{q}}_j$, by λ_j and $\mathbf{q}_j$, respectively, in (20) and (21). In the first work [75] a eight degrees of freedom model and a frame structure were analysed, while in the second [76] a truss structure was studied.

The classical inverse method was applied by Titurus et al. [79, 80], along with generic elements or substructure parameterisation for model updating and damage detection. The parameterisation is based on eigenvalues and eigenvectors changes, function of substructures stiffness matrices. Model updating and damage detection was performed by these researchers in thin-walled tubes connected by fillet welds.

In order to locate modelling errors and estimate physical parameters, an extension of an element-by-element sensitivity method [81] was developed by Alvin [82]. The algorithm includes a linearisation of the minimisation problem to increase its convergence and also a ponderation of the errors. Numerical data of a truss structure and experimental data of a tubular welded frame were used to test this method.

Bicanic and Chen [83] presented a damage identification method based on the differences between eigenvectors and a spectral decomposition. This method does not require the eigenvectors of the damaged structure, since they are defined as a linear combination of the undamaged structure eigenvectors, yielding a set of $m \times n$ nonlinear equations with $N + m(n-1)$ unknowns, where m and n are the number of measured eigenvectors and N the number of damage parameters. This set of equations is solved by the Gauss-Newton least-squares technique. These researchers applied their method to damage identification in a truss and a frame.

A method which considers the sensitivities of the orthogonality conditions of the damaged mode shapes was proposed and applied to multiple damage quantification on a laminated rectangular plate by Araújo dos Santos et al. [84, 85]. This methods makes use of the singular value decomposition (SVD) technique and an iterative process of undamaged elements elimination to uniquely solve the following set of $m(m+1)/2$ linear independent equations in order to the N damage parameters $\delta b_e \in [0, 1[$:

$$\sum_{e=1}^{N} \widetilde{\mathbf{q}}_{ie}^T \mathbf{K}_e \widetilde{\mathbf{q}}_{je} \delta b_e = \widetilde{\mathbf{q}}_i^T \mathbf{K} \widetilde{\mathbf{q}}_j - \delta_{ij} \widetilde{\lambda}_j \quad \text{for} \quad j = 1, \ldots, m, \tag{23}$$

where $\widetilde{\mathbf{q}}_{ie}$ and $\widetilde{\mathbf{q}}_{je}$ are the element e displacement vector of the i-th and j-th mode shapes of the damaged structure, respectively. Araújo dos Santos et al. [86, 87, 88] also developed a method based on the mixed sensitivity of the undamaged/damaged structure mode shapes, thus obtaining a larger set of linear equations:

$$\sum_{e=1}^{N} \widetilde{\mathbf{q}}_{je}^{T} \mathbf{K}_e \mathbf{q}_{ie} \delta b_e = \left(\lambda_i - \widetilde{\lambda}_j\right) \widetilde{\mathbf{q}}_j^T \mathbf{M} \mathbf{q}_i \text{ for } i = 1, \ldots, n \text{ and } j = 1, \ldots, m. \tag{24}$$

The two sets of equations defined by (24) and (23) were combined to obtain a new set of $m(m + 2n + 1)/2$ with N unknowns, which are solved simultaneously.

An improvement on these methods was presented by Araújo dos Santos et al. [89, 90], in which the N unknown parameters are obtained using the Non Negative Least Squares (NNLS) and the Bounded Variable Least Squares (BVLS) techniques. The first solves the set of linear equations such that $\delta b_e \geq 0$, while the second bounds the solution in order to compute a physically admissible solution, i.e. $0 \leq \delta b_e \leq 1$. The incompleteness of measured coordinates and the presence of experimental data noise and errors was also discussed.

Based on the method of sensitivities of the orthogonality conditions proposed by Araújo dos Santos [84], Ren and Roeck [91] showed that, in most cases, the use of the Non Negative Least Squares (NNLS) technique, along with a regularisation algorithm with error-based truncation, can lead to satisfactory results. By defining a realistic damage pattern, described by few parameters, and by applying the above method to experimental data, Ren and Roeck [92] successfully quantified the damage in a reinforced concrete beam. The results presented a good correlation with the ones obtained using the direct stiffness calculation method [46]. The influence of incompleteness and errors on the damage identification results of these methods, using a fibre reinforced epoxy rectangular plate model, was studied by Araújo dos Santos et al. [93]. This problem was also addressed by Rahai et al. [94], who defined the unmeasured degrees of freedom in function of the eigenvalues and structural submatrices. An optimisation procedure was applied to overcome the non-uniqueness of the set of equations. Rahai et al. [94] applied this procedure to numerical data of a truss structure.

In the context of model updating, Lin and Ewins [95] proposed the use of frequency response functions (FRF), thus obtaining the equation

$$\mathbf{A}(\omega)\left(-\omega^2 \Delta\mathbf{M} + \Delta\mathbf{K} + i\mathbf{D}\right)[\mathbf{a}(\omega)]_i = [\mathbf{a}(\omega)]_i - [\widetilde{\mathbf{a}}(\omega)]_i\,, \tag{25}$$

relating the analytical FRF matrix $\mathbf{A}(\omega)$, the i-th columns of the this matrix and the corresponding experimental one, $[\mathbf{a}(\omega)]_i$ and $[\widetilde{\mathbf{a}}(\omega)]_i$, respectively, to the changes in the mass $\Delta\mathbf{M}$, stiffness $\Delta\mathbf{K}$ and damping $\Delta\mathbf{D}$ matrices. In the numerical examples of model updating of a truss, Lin and Ewins [95] did not account for the damping and considered that the changes are given by

$$\Delta\mathbf{K} = \sum_{e=1}^{N} \beta_e \mathbf{K}_e^{\mathrm{e}} + \gamma_e \mathbf{K}_e^{\mathrm{b}} \quad \text{and} \quad \Delta\mathbf{M} = \sum_{e=1}^{N} \beta_e \mathbf{M}_e, \tag{26}$$

where N is the number of elements, being β_e and γ_e the changes in the design variables associated to element e. The matrices in eq. (26) are the extensional stiffness $\mathbf{K}_e^{\mathrm{e}}$, the bending stiffness $\mathbf{K}_e^{\mathrm{b}}$ and the mass $\mathbf{M}_e$ matrices at element level. By computing the values of β_e and γ_e based on data from an incomplete set of degrees of freedom, these researchers applied their method with success to a truss structure widely used in model updating tests. Lin and Ewins (1994) [95] also pointed out that model updating methods based on modal data are particular cases of this method.

The method developed by Lin and Ewins [95] was applied by Wang et al. [96] for the experimental damage detection on a plane 3-bay frame structure. Due to the incomplete measurements, an iterative algorithm was introduced. Using numerical data and actual measurements on a beam and a F-shaped structure, respectively, Modak et al. [97] applied this method and the method proposed by Berman and Nagy [69], concluding that the FRF based method presents better model updating results. Araújo dos Santos et al. [98] presented an application of Lin and Ewins [95] method and a numerical study on the influence of the number of frequencies and modes shapes used in the computation of the FRF, the frequency range, the excitation location and the number of measured degrees of freedom of a laminated plate. In order to obtain better results, a procedure for weighting and deletion of equations was used. Both dynamic and static expansions of the measured degrees of freedom was performed and the set of linear equations was solved using the Bounded Variable Least Squares technique. Maia et al. [99] applied the FRF curvatures method [48, 49] and a four step procedure, which relies on the FRF sensitivities method [95, 98], to the damage identification of an experimentally cracked beam. This last method allowed the quantification of stiffness decrease, i.e. the decrease in Young's modulus.

Modak et al. [100] made a comparison of the eigenvalues and eigenvectors sensitivities method [75, 76] and the FRF sensitivities method [95] and concluded that the latter performs better than the former in the presence of data incompleteness. This comparison was conducted in a fixed-fixed beam. The combined use of the eigenvalues and the FRF sensitivities was presented by Crema and Mastroddi [101] and Agneni et al. [102]. In the former work, a numerical study was performed for damage detection in an aluminum beam, while in the latter an experimental identification of a saw-cut crack in a carbon fibre reinforced beam was carried out.

Abdel Wahab [103] studied the inclusion of the mode shape curvature in the damage identification based on the mode shapes sensitivities algorithm, concluding that the convergence does not improve, altough one knows that the mode shapes curvatures are more sensitive to damage, when directly used (e.g. Pandey et al. [35]). This study was performed using simulated data of a beam model.

A method, that does not require a model, based on the mode shapes sensitivities was proposed by Parloo et al. [104]. In this method the sensitivity of degree of freedom i of the j-th mode shape to a local mass at degree of freedom k is given by

$$\frac{\partial q_{ij}}{\partial m_k} \approx -\lambda_j \frac{q_{kj}^2}{s_j} q_{ij} + \sum_{\substack{r=1 \\ r \neq j}}^{n} \frac{\lambda_j^2}{\lambda_r - \lambda_j} \frac{q_{kr} q_{ir}}{s_r} \tag{27}$$

whereas the sensitivity of degree of freedom i of the j-th mode shape to a change in stiffness between the degrees of freedom p and q is given by

$$\frac{\partial q_{ij}}{\partial k_{pq}} \approx \left(q_{pj} - q_{qj}\right) \sum_{\substack{r=1 \\ r \neq j}}^{n} \frac{1}{\lambda_r - \lambda_j} \frac{q_{pr} - q_{qr}}{s_r} q_{ir}, \tag{28}$$

where s_r are modal scaling factors and n is the number of known mode shapes. These researchers also applied the flexibility change method, the mode shape curvature method, the strain energy method and also used the MAC and $COMAC$. The structures studied were a clamped board and a highway bridge. A comparison of the different methods performance showed that the proposed sensitivity based technique could locate the damage in an earlier state.

Jones and Turcotte [105] developed a method which uses both antiresonant and natural frequencies sensitivities, thus obtaining 48% better correlation to experimental FRF than an update that uses only natural frequencies. These researchers also observed an improvement on the numerical stability of the computations using both sensitivities. An experimental 6 m aluminum truss served as structure to test the method.

Assignment methods: A third group of model updating methods is based on the determination of a pseudo or fictitious controller, through the minimisation of the modal force error. This controller can be perceived as parameter matrix perturbations of the undamaged structure. These methods are usually known as eigenstructure assignment methods, if one uses modal information (Lim and Kashangaki [106] and Cobb and Liebst [107]) or FRF assignment methods (Schulz et al. [108]) if the response model is used. In all of these works the methods were applied to truss-like structures. Koh and Ray [109] also made use of a minimum-gain eigenstructure assignment method in order to maintain a well-conditioned sensitivity matrix and generate independent modal data. These researchers analysed a beam using simulated data.

7 Conclusions

In this chapter, a survey of structural damage identification methods has been presented. This area of research has been very active and the number of journal publications is still growing. In fact, by searching in the internet for "structural damage identification" alone, using a scientific information engine, results in one entry for 1997, while the number of entries for the last year is almost two hundred. Furthermore, a year by year analysis of the collected data shows indeed an exponential trend on the increase of published works. This increase will continue in the foreseeable future, as both numerical and experimental techniques are further improved and incorporated in already existent methods or in new methods to appear.

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