Mathematical Model Comparison between Robot-Manipulators and Shear-Buildings Subjected to Base Excitation

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Abstract

The aim of this paper is to contribute to the rapprochement of two important autonomous domains of research: robot manipulator dynamics and earthquake engineering. The application of some methodologies of modeling, identification and control, related to robot manipulator problems, can be applied to the solution of earthquake engineering problems (e.g., health monitoring and tailoring of oscillating behavior). A common language between both domains of research is necessary, and this common language must include specific classes of models. We present then a comparison between mathematical models of robot manipulators (open kinematic chains) in lagrangian generalized coordinates and those commonly used in structural dynamics for low and mid-rise shear buildings. Four different models are presented, two corresponding to robot manipulators, with rigid arms and helicoidal springs and dampers at the ends of the arms, and two models corresponding to buildings with flexible columns and rigid beams and diaphragms with small lateral displacements. The comparison results propose the use of a robot manipulator model with the helicoidal spring and damper on top of the mass for each arm, simulating with good accuracy, the behavior observed in the shear building models.

1 Introduction

The present paper is based on the necessity to mathematically model civil structures, in order to apply fault detection techniques from the robotics point of view. The main idea is to derive adequate models for these techniques and, at the same time, reproduce the physical behavior of flexible structures. For a structural engineer, flexible structures are systems with mechanical properties that include "periodic vibrations with dominating few frequencies, resonances, and natural modes of vibrations" [2]. From the control engineer point of view, flexible structures are "linear systems with oscillatory properties characterized by a strong amplification of a harmonic signal for certain frequencies, a system with weakly correlated states, and which complex conjugate poles have small real parts" [2]. The interrelation between these two views of a flexible structure is through mathematical models. These models consider their nonlinear characteristics, and when the justification of small lateral displacements is introduced, the linear models commonly used in structural dynamics are obtained.

Following is the development of four mathematical models, that is, four different ways to conceive a civil structure from the control and structural points of view. In all of these models, the development is limited to three degree-of-freedom (DOF) system on a sliding base. This tries to simulate the behavior of a three-story building subjected to earthquake excitation. The Euler-Lagrange modeling philosophy is used, first obtaining the kinetic energy T and the potential energy V. The aim of the research is to derive robot-manipulator models which have a similar behavior to those of shear-building models. Those robot models which better represent a building structure, will be used with fault detection techniques from robotics theory and then relate the results to a building structure. The scope of this paper is limited to the derivation of the robot models.

2 Model Description

Four models are presented. Figure 1 shows Model 1, which is the usual robot model in control engineering [5]. It is based on the concept of an inverted pendulum on which gravity is acting, and in this case, rotational springs and dampers concentrated on the lumped masses are added, in order to represent the stiffness and damping of the building's vertical elements (columns). The mass is considered to be lumped at the slabs for each floor in a building, thus the lumping of the mass of the robot arms. This model tries to simulate a robot with rotational servos at the joints of the arms. From the nonlinear model, a linear model is derived considering small rotations, as it occurs in civil structures. Model 2 (Figure 2) is similar to Model 1, yet the springs and dampers are not at the lumped masses, but on top of them. This little variation tries to reproduce the behavior of the columns of a building, which present inelastic rotations at the joints adjacent to the slabs when close to damage, not inside the slabs. Also a linear model is derived from the nonlinear, considering small rotations.

Model 3 is the usual structural engineering model for low-rise shear-buildings (Figure 3). A real building has many DOFs, but in practice is simplified considering the shear behavior, that is, the total displacement of a floor is only due to the lateral displacement and the vertical displacements from the rotations of at the base of the columns are neglected [6]. Nevertheless, even though the vertical displacements of the masses are neglected, the effect of the gravity is considered through the P-Δ effect, which is a second order moment generated by the weight of each level when displaced horizontally. It is important to notice that the model derived for this system is already linear, thus, unnecessary to linearize. On the other hand, keeping the methodology used in models 1 and 2, the generalized coordinate system is also used. Model 4 (Figure 4) is similar to model 3 in all accounts, except that the non-generalized coordinate system is used, that is, the different coordinates are not independent of the variations of the other coordinates. This coordinate system is in terms of the displacements of each level with respect to a fixed point and considered as absolute. This variation is considered because it is the structural dynamic's theory typical procedure. Also, a linear model is derived and no further development is needed.

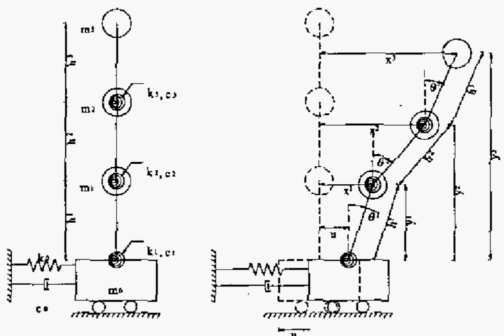


Figure 1: Model 1

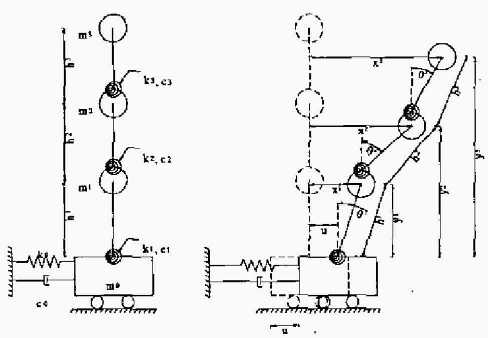


Figure 2: Model 2

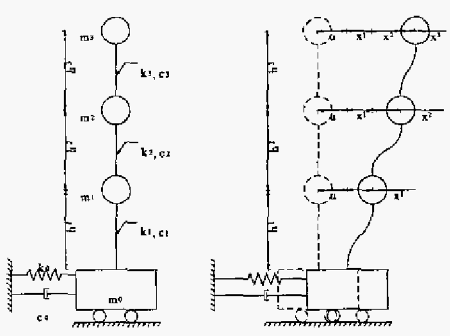


Figure 3: Model 3

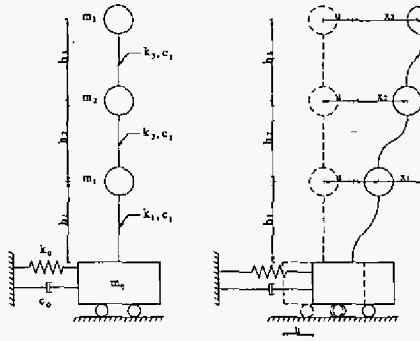


Figure 4: Model 4

Finally, models 3 and 4 also represent the type of models with lumped masses joined together through springs and dampers in a horizontal axial array, such that the stiffness and damping values are equal to those provided by the columns on each floor and assuming that the stiffness of the beams are infinite with respect to those of the columns, as depicted in Figures 5 and 6, [6, 7].

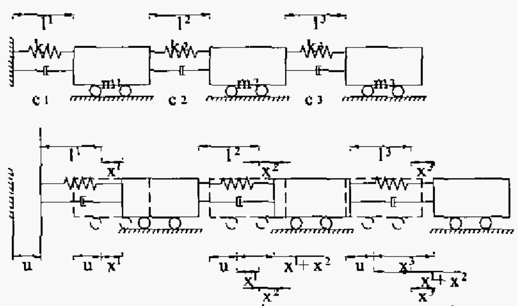


Figure 5: Model 3 horizontal configuration

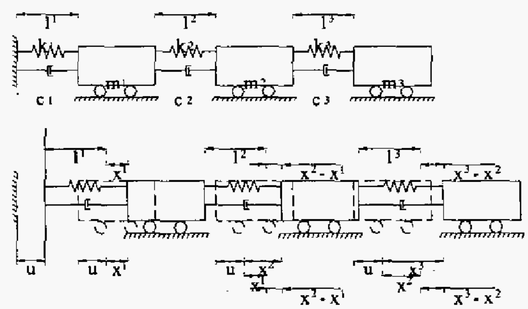


Figure 6: Model 4 horizontal configuration

3 Mathematical models

The development of the models is presented to derive the mathematical expressions that will allow us to simulate and compare the models. In every case, the Euler-Lagrange method is used [3, 4].

3.1 Model 1. Springs and dampers at the masses

Equation (1) describes the linearized system in matrix form.

$$\begin{bmatrix} \sum_{i=0}^{3} m_i & \left(\sum_{i=1}^{3} m_i\right)h_1 & \left(\sum_{i=2}^{3} m_i\right)h_2 & m_3h_3 \\ \left(\sum_{i=1}^{3} m_i\right)h_1 & \left(\sum_{i=1}^{3} m_i\right)h_1^2 & \left(\sum_{i=2}^{3} m_i\right)h_1h_2 & m_3h_1h_3 \\ \left(\sum_{i=2}^{3} m_i\right)h_2 & \left(\sum_{i=2}^{3} m_i\right)h_1h_2 & \left(\sum_{i=2}^{3} m_i\right)h_2^2 & m_3h_2h_3 \\ m_3h_3 & m_3h_1h_3 & m_3h_2h_3 & mm_3h_3^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} +$$

$$\begin{bmatrix} c_0 & 0 & 0 & 0 \\ 0 & c_1+c_2 & -c_2 & 0 \\ 0 & -c_2 & c_2+c_3 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} +$$

$$\begin{bmatrix} k_0 & 0 & 0 & 0 \\ 0 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \left(\sum_{i=1}^{3} m_i\right) g h_1 \theta_1 \\ \left(\sum_{i=2}^{3} m_i\right) g h_2 \theta_2 \\ m_3 g h_3 \theta_3 \end{Bmatrix} \tag{1}$$

3.2 Model 2. Springs and dampers adjacent to the masses

Equation (2) describes the linearized system in matrix form.

$$\begin{bmatrix} \sum_{i=0}^{3} m_i & \left(\sum_{i=1}^{3} m_i\right)h_1 & \left(\sum_{i=2}^{3} m_i\right)h_2 & m_3h_3 \\ \left(\sum_{i=1}^{3} m_i\right)h_1 & \left(\sum_{i=1}^{3} m_i\right)h_1^2 & \left(\sum_{i=2}^{3} m_i\right)h_1h_2 & m_3h_1h_3 \\ \left(\sum_{i=2}^{3} m_i\right)h_2 & \left(\sum_{i=2}^{3} m_i\right)h_1h_2 & \left(\sum_{i=2}^{3} m_i\right)h_2^2 & m_3h_2h_3 \\ m_3h_3 & m_3h_1h_3 & m_3h_2h_3 & m_3h_3^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} +$$

$$\begin{bmatrix} c_0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} + \begin{bmatrix} k_0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} =$$

$$\begin{Bmatrix} 0 \\ \left(\sum_{i=1}^{3} m_i\right) g h_1 \theta_1 \\ \left(\sum_{i=2}^{3} m_i\right) g h_2 \theta_2 \\ m_3 g h_3 \theta_3 \end{Bmatrix} \tag{2}$$

3.3 Model 3. Shear behavior and generalized coordinates

Equation (3) describes the system in matrix form.

$$\begin{bmatrix} \sum_{i=0}^{3} m_i & \left(\sum_{i=1}^{3} m_i\right) & \left(\sum_{i=2}^{3} m_i\right) & m_3 \\ \left(\sum_{i=1}^{3} m_i\right) & \left(\sum_{i=1}^{3} m_i\right) & \left(\sum_{i=2}^{3} m_i\right) & m_3 \\ \left(\sum_{i=2}^{3} m_i\right) & \left(\sum_{i=2}^{3} m_i\right) & \left(\sum_{i=2}^{3} m_i\right) & m_3 \\ m_3 & m_3 & m_3 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} +$$

$$\begin{bmatrix} c_0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u \\ x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ \left(\dfrac{\sum_{i=1}^{3} m_i}{h_1}\right) g x_1 \\ \left(\dfrac{\sum_{i=2}^{3} m_i}{h_2}\right) g x_2 \\ \left(\dfrac{m_3}{h_3}\right) g x_3 \end{Bmatrix} \tag{3}$$

3.4 Model 4. Shear behavior and non-generalized coordinates

Equation (4) describes the system in matrix form.

4 Simulations

To observe which of the developed robot models is the most convenient, such that the fault detection techniques are to be applied on it, and to better represent the behavior of a building structure subjected to base excitation, the following comparisons are presented. The simulations are performed using Dymola software [1]. The structure consists of three floors, and each floor is simulated with a 3.00 x 3.00 m slab with a 1.00 ton/m² uniform load acting on it. The slabs are joined together by one concrete column with a cross-section of 0.40 x 0.40 m and 3.00 m height each. All the units were converted to the SI system for software compatibility. Each model is subjected to an initial displacement of 0.01 m and free vibration, also, the stiffness and damping properties are the same for all models. It is not easy to characterize damping in civil structures, therefore, some approximate techniques using modal damping and critical damping values for a specific material are considered.

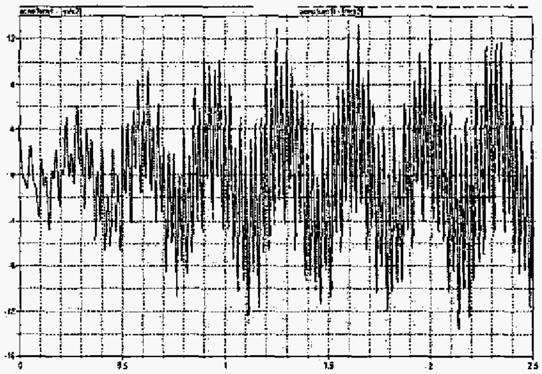
$$
\begin{bmatrix} \sum_{i=0}^{3} m_i & 0 & 0 & 0 \\ m_1 & m_1 & 0 & 0 \\ m_2 & 0 & m_2 & 0 \\ m_3 & 0 & 0 & m_3 \end{bmatrix}
\begin{Bmatrix} \ddot{u} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} +
\begin{bmatrix} c_0 & 0 & 0 & 0 \\ 0 & c_1 + c_2 & -c_2 & 0 \\ 0 & -c_2 & c_2 + c_3 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix}
\begin{Bmatrix} \dot{u} \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} +
\begin{bmatrix} k_0 & 0 & 0 & 0 \\ 0 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix}
\begin{Bmatrix} u \\ x_1 \\ x_2 \\ x_3 \end{Bmatrix} =
$$

$$
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \left(\frac{\sum_{i=1}^{3} m_i}{h_1} + \frac{\sum_{i=2}^{3} m_i}{h_2} \right) & -\left(\frac{\sum_{i=2}^{3} m_i}{h_2} \right) & 0 \\ 0 & -\left(\frac{\sum_{i=2}^{3} m_i}{h_2} \right) & \left(\frac{\sum_{i=2}^{3} m_i}{h_2} + \frac{m_3}{h_3} \right) & -\left(\frac{m_3}{h_3} \right) \\ 0 & 0 & -\left(\frac{m_3}{h_3} \right) & \left(\frac{m_3}{h_3} \right) \end{bmatrix}
\begin{Bmatrix} u \\ gx_1 \\ gx_2 \\ gx_3 \end{Bmatrix} \quad (4)
$$

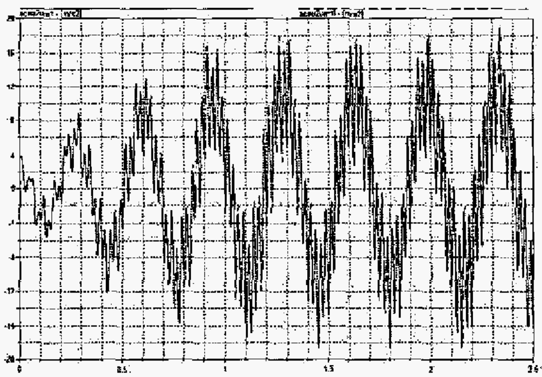
The procedure used here to compute the damping values c_i for each floor is the one used in modal analysis, which consists on computing the values of modal damping through the critical damping ratio for each material, and then transform the modal damping matrix into the physical coordinates damping matrix. To compute the rotational stiffness of the helicoidal springs, in order to represent a lateral stiffness, it is assumed that the rigid bar connected at the bottom must allow a lateral displacement equal to that of the flexible bar. It is convenient to express the rotational stiffness in terms of the geometric properties and material of the bar, instead of the helicoidal spring, in order to better understand how it works. The comparisons are in terms of the absolute lateral accelerations, due to the fact that these are usually recorded for structural health monitoring. Models 1 and 2 give angular accelerations, thus lateral accelerations are later computed. Model 3 gives relative accelerations to the different levels, when summed consecutively, absolute accelerations are obtained. Finally, model 4 gives absolute lateral accelerations, therefore, no further computation is needed.

4.1 Comparison between Model 1 (nonlinear) and Model 1 (linearized)

Figure 7 shows the lateral acceleration response for each floor with no damping. The nonlinear model is in solid line, while the linearized model is in dashed line. It could be observed that the response of both models is practically the same, with a better agreement at the top floors.



a)



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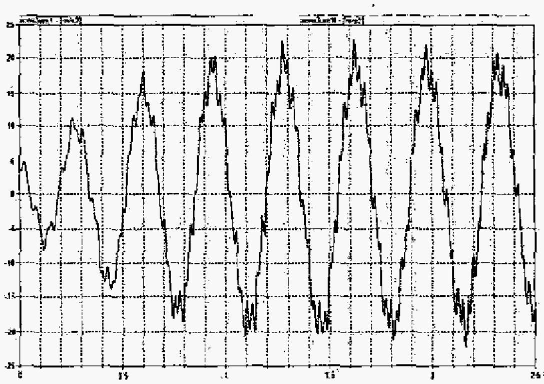
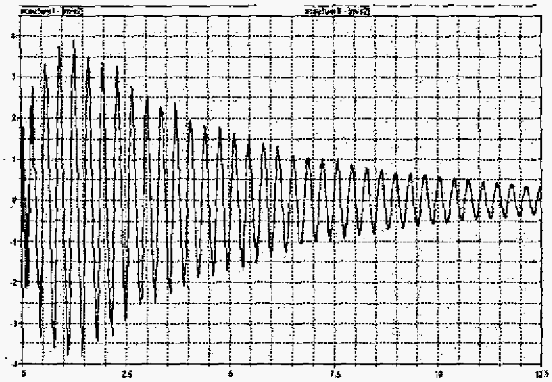


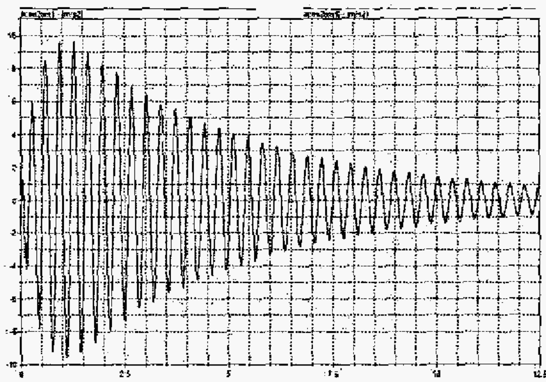
c)

Figure 7: Simulation responses for lateral acceleration without damping. a)Level 1, b)Level 2 and c) Level 3

Figure 8 shows the lateral acceleration response considering modal damping. It could be observed that at the beginning of the simulation, the responses are practically the same, yet at the end of the simulation, the responses differ 10% in average. In all cases, the linearized model gives larger amplitudes.



a)



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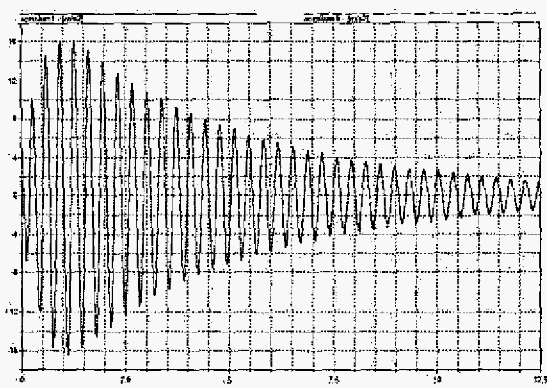
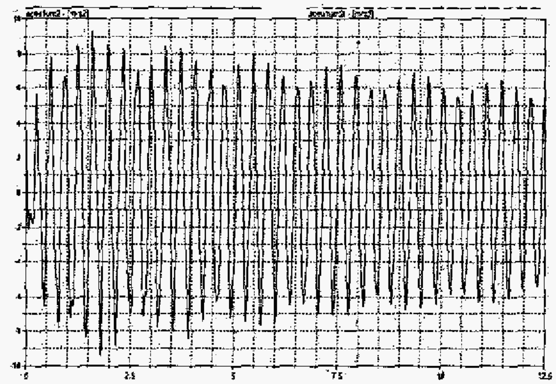
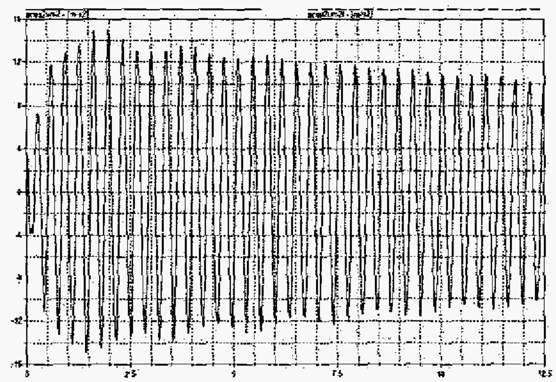


c)

Figure 8: Simulation responses for lateral acceleration with modal damping. a)Level 1, b)Level 2 and c) Level 3



a)



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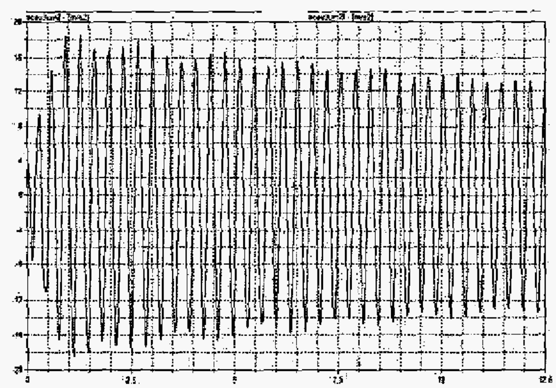


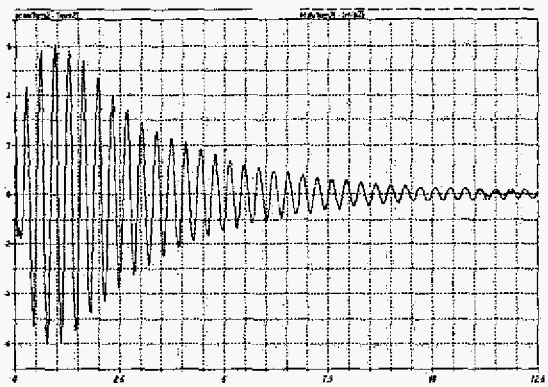
c)

Figure 9: Simulation responses for lateral acceleration without damping. a)Level 1, b)Level 2 and c) Level 3

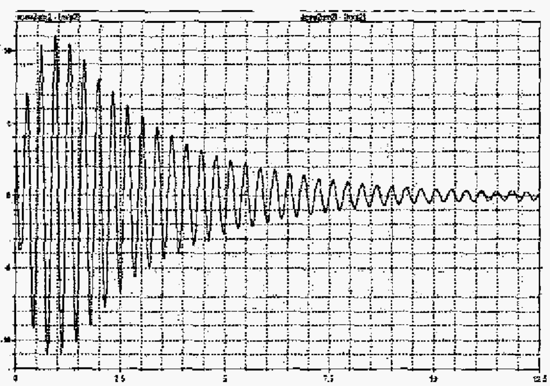
4.2 Comparison between Model 2 (nonlinear) and Model 2 (linearized)

Figure 9 shows the responses for each floor without damping. It could be observed that the responses are identical through out the simulation time and at every floor.

Figure 10 shows the response for each floor considering modal damping. It could be noted that, when damping is considered, at the beginning of the simulation, the responses coincide, yet, after around 7 seconds of the simulation, the amplitudes have a variation of up to 20%. In this case, the nonlinear model gives larger amplitudes. Finally, the responses of the linearized model die out faster than the nonlinear model.



a)



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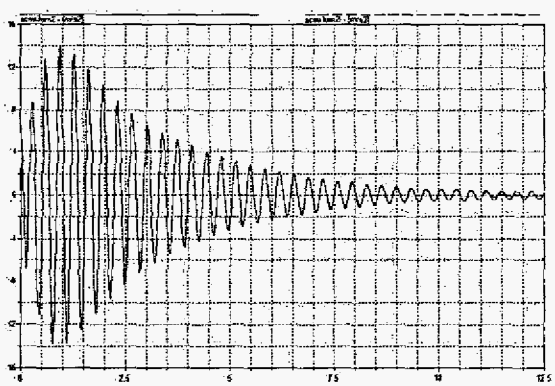
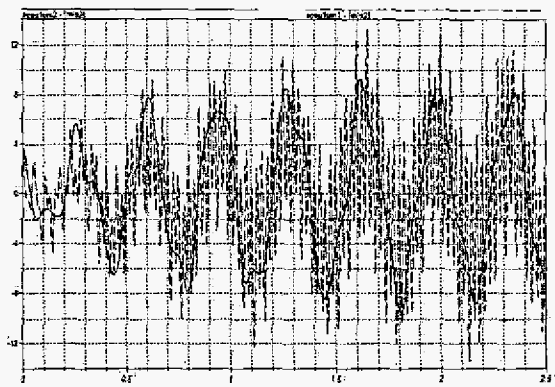
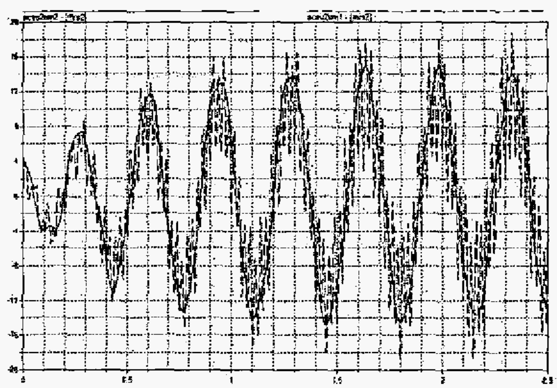


c)

Figure 10: Simulation responses for lateral acceleration with modal damping. a)Level 1, b)Level 2 and c) Level 3



a)



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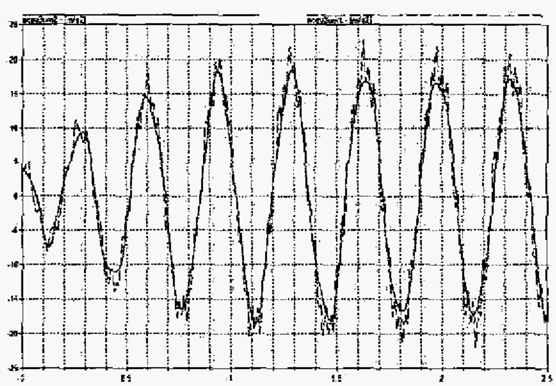


c)

Figure 11: Simulation responses for lateral acceleration without damping. a)Level 1, b)Level 2 and c) Level 3

4.3 Comparison between linearized Models 1 and 2

This comparison is made to later introduce the comparison with linear models 3 and 4. Figure 11 shows the responses for each floor without damping of lateral accelerations. Model 2 is depicted in solid line and model 1 in dashed line. It is noted that the responses differ quite a bit in level 1, while a better agreement is observed in levels 2 and 3. Nevertheless, the response of model 1 tries to follow the response of model 2, both in frequency content and in amplitude, and probably if the response of model 1 should be filtered, the response would coincide acceptably with the response of model 2.

Figure 12 shows the response for each level with modal damping for lateral accelerations. Model 1 is depicted in solid line and model 2 in dashed line. The difference in the behavior is large in all floors. In levels 1 and 2, model 1 shows smaller amplitudes at the beginning of the simulation, and the response of model 2 dies out faster than that of model 1. In level three, model 1 always gives larger amplitudes than model 2.

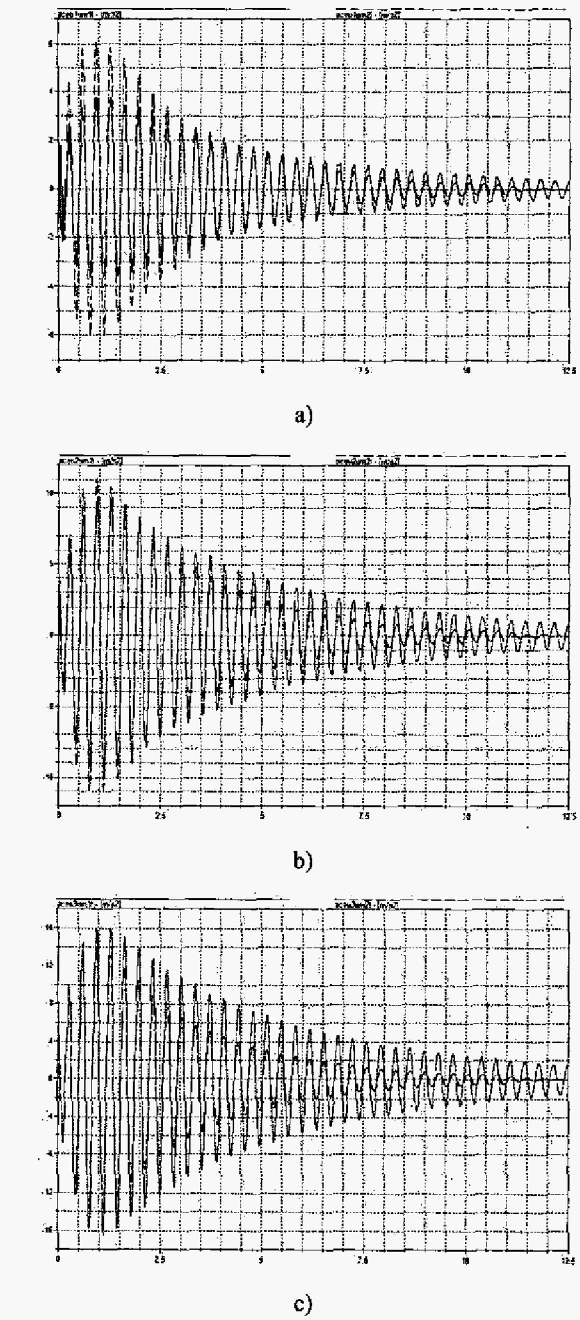


Figure 12: Simulation results for lateral acceleration with modal damping. a)Level 1, b)Level 2 and c) Level 3

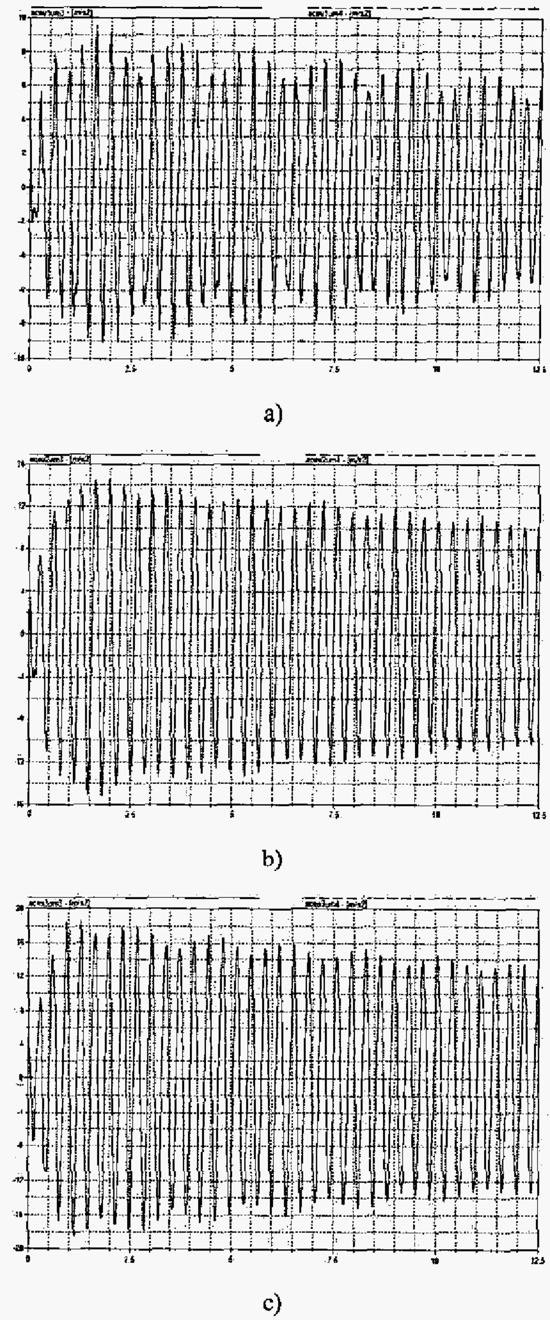
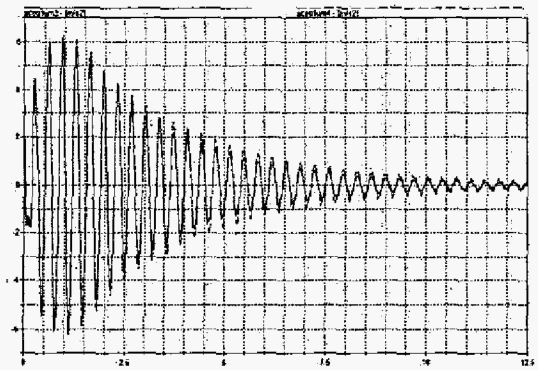


Figure 13: Simulation responses for lateral acceleration without damping. a)Level 1, b)Level 2 and c) Level 3

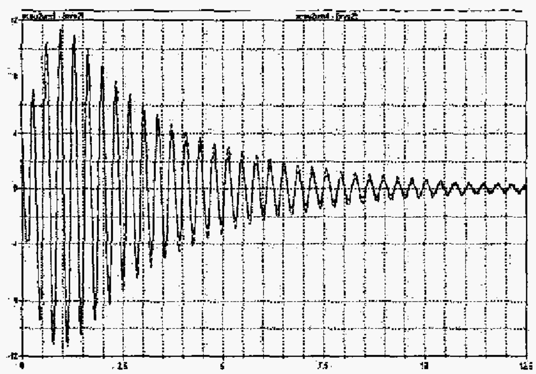
4.4 Comparison between models 3 and 4

Figure 13 shows identical acceleration responses for both models, which was obvious since the only difference between the models is the coordinate system.

Figure 14 shows model 3 in solid line and model 4 in dashed line. It could be observed that with modal damping, model 4 gives larger amplitudes than model 3. In level 1, the difference is around 20% on average, while this difference increases up to 30% on average in level 3. It is important to notice then the importance of the presence of this kind of damping, because in theory both models should behave exactly the same in the presence of any kind of damping.



a)



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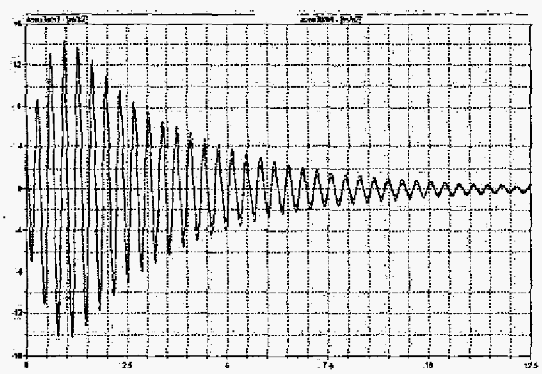
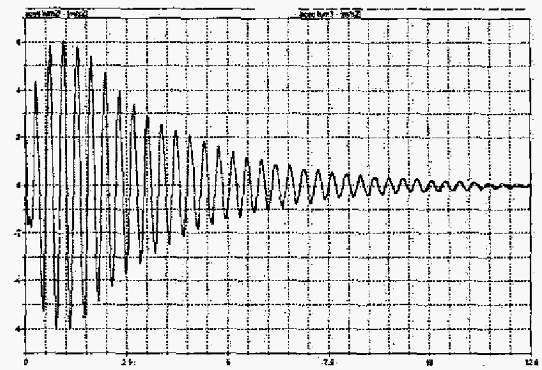
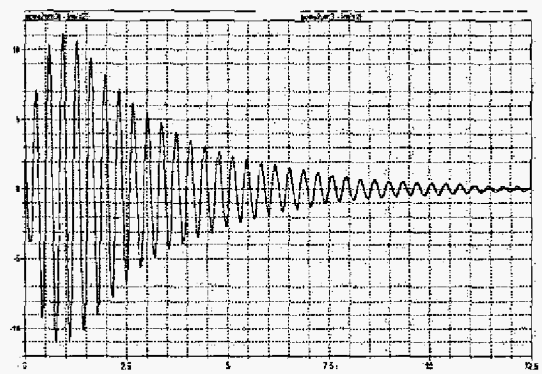


c)

Figure 14: Simulation results for lateral acceleration with modal damping. a)Level 1, b)Level 2 and c) Level 3



a)



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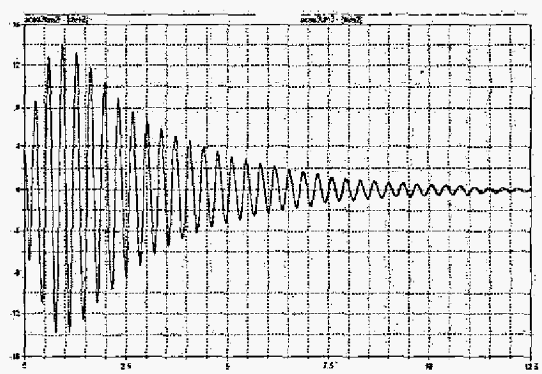

c)

Figure 15: Simulation responses for lateral acceleration with modal damping. a)Level 1, b)Level 2 and c) Level 3

4.5 Comparison between Model 2 (linearized) and Model 3

Figure 15 compares models 2 (linearized) and 3 with modal damping. Model 2 is depicted in solid line and model 3 in dashed line. It could be noted that the responses show very little variation from each other, and it could be considered that the behavior of the robot-manipulator is very similar to that of a shear building. The amplitudes show a variation of around 15% on average in every floor, and the response of model 2 dies out a bit faster than that of model 3 only in level 2.

5 Results analysis

The aim of the simulations is to establish similarities and differences between the robot-manipulator models and shear-building models. The models developed here are to aid in structural health monitoring by means of fault detection techniques used in robotics' theory.

The first comparison is between the behavior of linear and nonlinear models. Model 1 shows close agreement between the linear and nonlinear responses.

Clearly the responses of model 1 are larger than the shear-building models, though the differences are considered within the acceptable range allowed in structural dynamics.

Model 2 shows again some differences between nonlinear and linear variants, the linear model damps out faster than the nonlinear one. Models 3 and 4 are linear and no further computation is needed to linearize them, thus no comparison is presented between nonlinear and linear variants.

The second comparison is between linear models. Models 1 and 2 have an interesting situation from the structural dynamics' point of view, since both have rotational springs and dampers where the stiffness and damping for each level is concentrated. It could be noticed that the response of model 1 shows a more complex behavior than model 2, which does not imply a more precise response in model 1 than in model 2.

As it was stated above, both models have the same tendencies in amplitudes and resonant frequencies, and for the moment, the authors are trying to modify the stiffness and damping of the rotational devices such that the responses have a better agreement.

When models 3 and 4 were compared, the issue of damping was noticed, and due to the fact that modal damping is an approximate method, the differences on the responses could be attributed to it.

Finally, models 2 and 3 were compared and a good agreement was found between a robot-manipulator model and a shear-building model

6 Final comments

Linear models could be used if the input excitation is small, and acceptable responses from the structural dynamic's point of view are observed. No loss in the configuration characteristics of the models is presented and the linear models noticeably simplify the mathematics.

It is important to study in further detail the behavior of model 1 to establish some criteria on how to obtain stiffness and damping coefficients, such that allow for closer responses to those of the rest of the models.

Agreement between the responses of models 2, 3 and 4 allow us to think that model 2 is adequate to represent the behavior of a low-rise shear-building structure.

Simulation as presented here only considers free vibration, which opens the door for further comparisons subjecting the models to different kinds of input excitations, such as harmonic functions or seismic records.

References

[1] Elmqvist, H., Brück, D., Mattsson, S.E., Olsson, H., Otter, M., *Dymola – Dynamic Modeling Laboratory (User's Manual)*, Dynasim AB, 2001.

[2] Gawronsky, W., *Dynamics and Control of Structures, A Modal Approach*, Mechanical Engineering Series, Springer, 1998.

[3] Meisel, J., *Principles of Electromechanical - Energy Conversion*, Mc Graw-Hill, 1966.

[4] Meirovitch, L., *Analytical Methods in Vibrations*, Macmillan Publishing Co., 1976.

[5] Spong, M., Vidyasagar, M., *Robot Dynamics and Control*, Willey, 1989.

[6] Paz, M., *Structural Dynamics, Theory and Computation*, Chapman & Hall, 1997.

[7] Tedesco, J., McDougal, W., Ross, C., *Structural Dynamics, Theory and Applications*, Addison Wesley, 1999.