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Optimal sensor placement for modal identification of structures using genetic algorithms—a case study: the olympic stadium in Cali, Colombia

A. Cruz · W. Vélez · P. Thomson

Abstract Adequate sensor placement plays a key role in such fields as system identification, structural control, damage detection and structural health monitoring of flexible structures. In recent years, interest has increased in the development of methods for determining an arrangement of sensors suitable for characterizing the dynamic behavior of a given structure. This paper describes the implementation of genetic algorithms as a strategy for optimal placement of a predefined number of sensors. The method is based on the maximization of a fitness function that evaluates sensor positions in terms of natural frequency identification effectiveness and mode shape independence under various occupation and excitation scenarios using a custom genetic algorithm. A finite element model of the stadium was used to evaluate modal parameters used in the fitness function, and to simulate different occupation and excitation scenarios. The results obtained with the genetic algorithm strategy are compared with those obtained from applying the Effective Independence and Modal Kinetic Energy sensor placement techniques. The sensor distribution obtained from the proposed strategy will be used in a structural health monitoring system to be installed in the stadium.

Notation

m number of vibration modes included in the analysis
N number of measurable degrees of freedom
M Mass matrix without rotational inertia elements
Φ Modal matrix, only m mode shape vectors included

A. Cruz (✉) · W. Vélez · P. Thomson
Earthquake Engineering, Aeolic Engineering and Smart Structures Group (G7),
Escuela de Ingeniería Civil y Geomática, Universidad del Valle, Cali, Colombia
e-mail: aliosha15@univalle.edu.co

W. Vélez
e-mail: whvelez@univalle.edu.co

P. Thomson
e-mail: pethomso@univalle.edu.co

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1 Introduction

This paper deals with the adequate selection of sensor placement for modal identification of structures. A dynamic characterization of a structure can only be performed if sufficient experimental information is available, and this in turn requires sensors to be correctly placed in order to provide the identification process with meaningful data.

The modal properties (natural frequencies, mode shapes and modal damping ratios) of large civil structures such as stadia, bridges, tall buildings and offshore platforms can be experimentally identified with reasonable ease. The modes corresponding to the low spectral range normally provide sufficient information for describing the dynamic behavior of the structure under service conditions. The present work focuses on determining the locations where a pre-defined limited number of sensors should be placed on a structure in order to capture a data set that provides information for modal identification.

Several methods have been proposed for the selection of sensor positions, all of which attempt to maximize the capabilities of the identified modal data to fulfill a given requirement. Meo and Zumpano (2005) compared the capabilities of various methods by assessing the results of their use in the modal identification of a bridge. Papadimitriou (2004) presented a method based on the concept of Information Entropy as a measure of effectiveness. Li et al. (2007) analyzed the relation between the Effective Independence (EI) Method and the Modal Kinetic Energy (MKE) Method. Pickrel (1999) presented a pretest analysis strategy that includes specification of the minimum required number of accelerometers according to the purpose of the modal test.

Genetic Algorithms have been introduced in various fields of applied science and engineering since they first appeared in 1975, including structural dynamics and structural engineering applications. Schoenaur and Xanthakis (1993) presented the concept of *Behavioural Memory* for Genetic Algorithm based constrained optimization problems and applied the technique in the design of planar and three dimensional trusses. Coello et al. (1996) introduced a method for optimal design of reinforced-non-prismatic concrete columns that uses GAs to select optimal design parameters. Levin and Lieven (1998) presented the use of Genetic Algorithms for dynamic finite element model updating problems. Yoshida and Dyke (2005) used Genetic Algorithms to find optimal locations for control devices in a vibration control strategy. Bedrossian (1998) developed GA based strategies for both optimal selection of sensors for modal identification and vibration control actuators.

The existing methods may be classified according to the metric used to establish the best sensor arrangement out of a set of possible locations. The EI method attempts to select a set of positions which make the identified mode shape vectors as linearly independent as possible while retaining enough information for dynamic behavior characterization. The MKE method selects positions throughout the structure which are highly responsive in order to maximize the signal-to-noise ratio of the acquired signals. The present work illustrates the use of genetic algorithms to find an optimal set of locations from a randomly generated initial population of candidate positions. The fitness function specifies positions which maximize natural frequency identifiability and mode shape vector independence under different occupation and excitation scenarios.

The problem to be solved in this study is a combinatorial optimization problem. The combinatorial problem of interest is that of choosing N_s sensor locations from a predefined number of possible locations, N. The number of possible combinations of N_s sensors is given by the expression

$$C = \frac{N!}{N_s!(N - N_s)!} \tag{1}$$

Finite element models of large structures usually have thousands, tens of thousands, and even hundreds of thousands of degrees of freedom. Thus, the number of possible combinations is quite large, and the use of genetic algorithms is therefore justified.

The Pascual Guerrero Olympic Stadium, located in the city of Cali, Colombia, was used for validation and application of the proposed method. This work is part of a major project on human–structure interaction of public venues, currently carried out by the Universidad del Valle and the Colombian National Institute for the Development of Science and Technology, Colciencias.

2 Stadium structure

The Pascual Guerrero Olympic Stadium is one of the most important sports scenarios in the city of Cali. It has been used for over 50 years and has experienced various structural modifications to increase its capacity. The northern grandstand was retrofitted in 2000 due to its structural deterioration. The stadium has a capacity of 48000 spectators, and a number of different sporting, musical and religious events are held there on a regular basis. In 2005, the Universidad del Valle conducted a study of the stadium's structural vulnerability and proposed a structural upgrade such that the structure would conform with the Colombian Building Code NSR-98 (Escuela de Ingeniería Civil y Geomática, Universidad del Valle 2005). Vibration serviceability aspects were considered as well due to complaints by occupants regarding discomfort and fear caused by high vibration levels.

The stadium has four grandstands, which are built as independent structures. A reinforced concrete space frame provides seismic load resistance to each of them, and the stands are supported by concrete beams spanning between frames. The southern grandstand rests on 13 two-storey reinforced concrete frames and has cantilevers in the four plane directions. The western grandstand has 15 three-storey reinforced concrete frames, cantilevers in four directions, and a concrete roof built with beams in two directions. Figure 1 shows the western

Fig. 1 Western grandstand

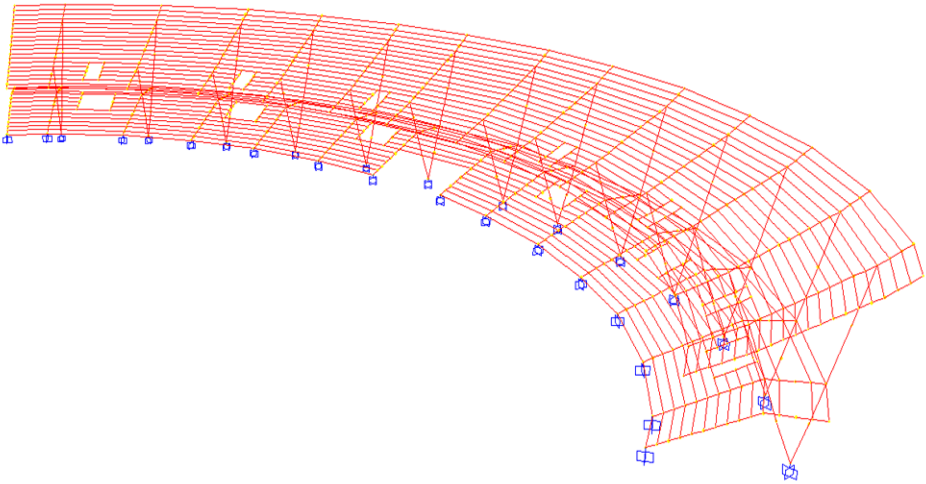

Fig. 2 Finite element model of southern grandstand

grandstand. Twenty accelerometers will be installed in the western grandstand and twelve in the southern grandstand.

3 Finite element models

The stadium structure was modeled using the finite element software SAP 2000 (Computers and Structures 2000). The spatial (mass and stiffness) and modal (eigenvalues and eigenvectors) matrices calculated with SAP 2000 were exported to MATLAB (The Mathworks, Inc. 2005) for use with the custom genetic algorithm developed for this project.

The southern grandstand model contains 1221 beam elements and 710 nodes, 28 of which are fixed base supports, for a total 4092 degrees of freedom. Figure 2 shows the finite element model of this structure. The western grandstand has 2975 beam elements and 1599 nodes that include 47 fixed boundaries. The model has 9312 degrees of freedom. Figure 3 shows the finite element model of this grandstand.

The modal properties of these structural models are the target modal quantities in the identification. The first five natural frequencies and their corresponding mode shapes are shown in Tables 1 and 2, and Figs. 4 and 5, respectively.

4 Genetic algorithms

Genetic algorithms are based on Darwin's principle of natural selection and on elements of the science of genetics. A genetic algorithm attempts to solve optimization problems by performing a stochastic—adaptive search which discards non optimal solutions according to a *survival of the fittest* scheme (Bedrossian 1998). The optimization process starts from an initial population or set of possible solutions. New generations are created in each iteration by cross-over, mutation and replication of individuals, assigning more importance to those which produce higher fitness values (objective function values) and thus discarding the rest. Although the process may seem somehow autonomous, several parameters are user defined

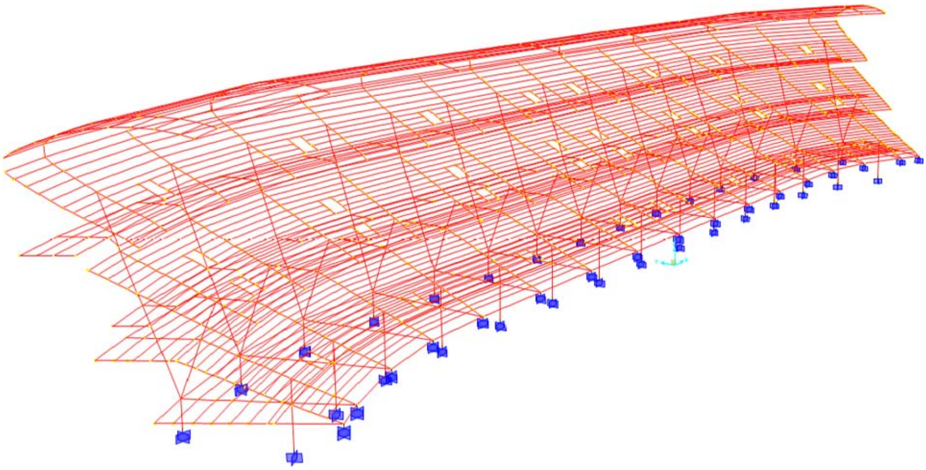

Fig. 3 Finite element model of western grandstand

Table 1 Western grandstand natural frequencies

Mode	Frequency (Hz)
1	0.65
2	0.67
3	0.71
4	0.97
5	1.05

Table 2 Southern grandstand natural frequencies

Number	Frequency (Hz)
1	1.47
2	2.56
3	2.91
4	2.96
5	3.06

and their definition is problem specific. For instance, the population size (the number of points in the search space) has to be defined in such a way that the optimal solution is contained in it and provides the search with enough diversity. The selection process, which discards non-optimal individuals and selects the fittest ones for reproduction, requires the implementation of a selection strategy, a purpose for which several alternatives are available (Bedrossian 1998). Additional parameters, namely the elite count (maximum number of individuals to be cloned into the next generation), the crossover rate (number of offspring to be generated from parents crossing) and the number of generations (iterations) to be produced must be defined *a priori*. It may even be necessary to modify these parameters manually in order to cope with ill conditioning and convergence problems. An adaptive approach could be implemented at the expense of computational efficiency.

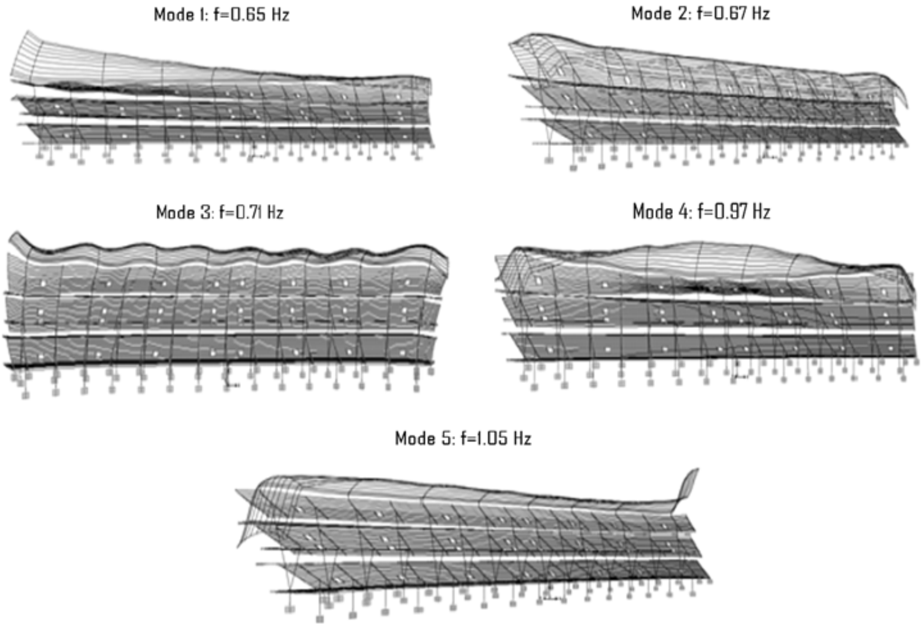

Fig. 4 Western grandstand mode shapes

In this work, GAs are used to solve a combinatorial optimization problem. From (1), there are 9.8982×10^{54} possible combinations of 20 sensors for the western grandstand, and 1.0877×10^{31} combinations of 12 sensors for the southern grandstand. Genetic algorithms are suitable for such a large search space.

5 Genetic algorithm for optimal sensor placement

The algorithm designed, implemented and used for the present study starts from a randomly generated initial population which is mathematically expressed as a matrix with p rows, each containing integer entries that specify a measurement degree of freedom, and q columns that correspond to the number of transducer locations sought. Reproduction individuals are selected in a roulette round in which elite genes may be repeated. Once the selection is completed, crossover and mutation operations are executed upon these individuals and fitness function evaluation is performed. The fitness function is defined by the expression

$$J = \frac{1}{a} p(\Phi, M, \Gamma) + g(\Phi) \tag{2}$$

The first term in (2) evaluates the effectiveness of the trial sensor distribution for natural frequency identification, and its optimum value is unity. The effectiveness is quantified by calculating the mass contribution to the system response of the trial DOFs from the expression

$$\begin{aligned} \sigma_{N\times m} &= \Phi_{N\times m}[\Phi^T_{m\times N} M_{N\times N} \Gamma_{N\times 1}]_{m\times m} Mn^{-1}_{m\times m} \\ Mn_{m\times m} &= \Phi^T_{m\times N} M_{N\times N} \Phi_{N\times m} \end{aligned} \tag{3}$$

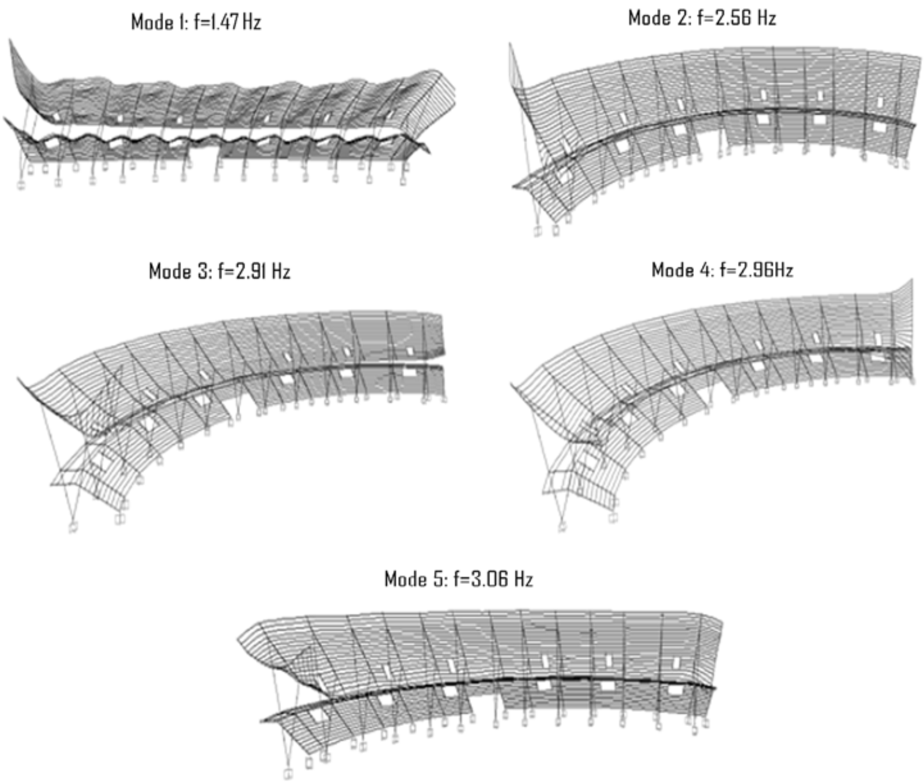

Fig. 5 Southern grandstand mode shapes

The σ matrix is normalized so that the elements in each column are contained in the range [0, 1]. The factor $p(\Phi, M, \Gamma)$ is generated by selecting the maximum value along each row of σ indicated by the trial vector of locations, and averaging these values. The parameter a is the average of the maximum values along the m columns of σ. The second term in (2) evaluates the effectiveness of the selected measurement degrees of freedom for identification of independent mode shape vectors. Since these vectors are usually used for modal correlation between analytical and experimental modes of vibration, linear independence is critical in achieving a successful mode pairing (Friswell and Mottershead 1993). The term is obtained by generating a matrix of MAC (Modal Assurance Criterion) values for the eigenvectors defined at the DOFs specified by each individual. The matrix MAC numbers are defined by the expression

$$\mathrm{MAC}_{jk} = \frac{\|\{\Phi\}_j^T\{\Phi\}_k\|^2}{(\{\Phi\}_k^T\{\Phi\}_k)(\{\Phi\}_j^T\{\Phi\}_j)} \tag{4}$$

The resulting matrix is symmetric, thus only the upper diagonal is required for computations. The $g(\Phi)$ term is calculated by defining a unique independence indicator from the MAC values. This indicator is defined by the expression

$$g(\Phi) = \prod_{row}\left(\prod_{col}\left(1 - \mathrm{MAC}_{ij}^{(\frac{2}{m^2-m})}\right)\right) \tag{5}$$

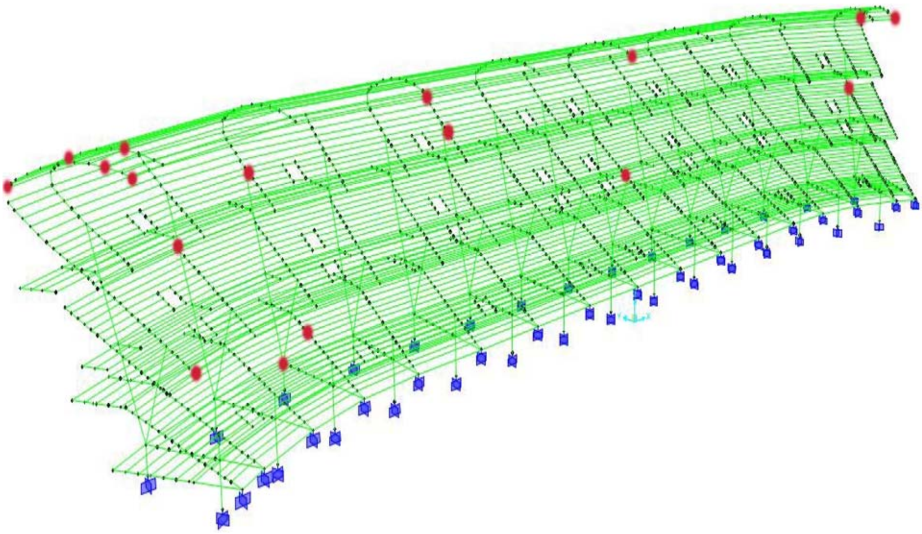

Fig. 6 Sensor positions for the western grandstand—GA method

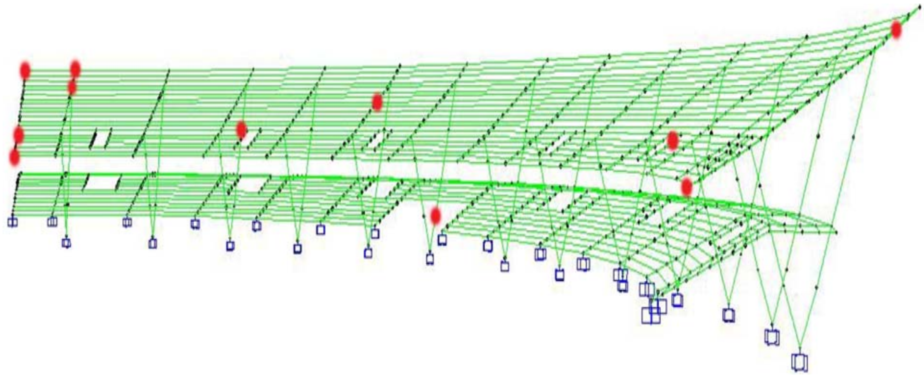

Fig. 7 Sensor positions for the southern grandstand—GA method

where the consecutive, double product indicates it is taken along each of the columns of the MAC matrix, and then calculated along the resulting row of m coefficients. The $g(\Phi)$ term ranges between 0 and 1; the latter corresponds to the optimum value, which indicates no correlation between the eigenvectors, i.e., the evaluated individual is optimum for mode shape identification.

Equation (2) accounts for different occupation cases and seismic effects by modifying the influence vector Γ. For the case of the structure without spectators and subject to seismic excitation, the influence vector multiplies the mass participation coefficients corresponding to in-plane DOFs by 1.0, and gravity direction (Z-direction) by 0.1. For the case of the structure with spectators, four occupation levels are defined as 25, 50, 75 and 100% live load, and the influence vector multiplies the in-plane coefficients by 0.1, and those of the gravity direction by 1.0. These values reflect the fact that the vertical component of an earthquake is generally much less than horizontal components while for the case of crowd excitation the major contribution is in the vertical direction. The final fitness value in each iteration is the average of the values obtained from the five excitation cases; the highest possible value is 2. The process is repeated until a maximum fitness value is attained or as many times as the user judges necessary. This flexibility is allowed due to the fact that no unique stopping criteria exist for genetic algorithms, as opposed to other optimization methods (Bedrossian 1998).

6 Results

The algorithm previously described was used to select a set of optimal sensor locations for modal identification of the stadium. An initial population of 100 location vectors was used. The locations selected for the 20 accelerometers to be used in the western grandstand are depicted in Fig. 6 and the 12 locations selected for the southern grandstand are shown in Fig. 7. Tables 3 and 4 show the values assigned to reproduction parameters and the attained fitness values for the two grandstands. These values are very close to the optimal value. Figures 8 and 9 show the progress of the algorithm; they exhibit patterns that suggest the possibility of attaining higher fitness values were more iterations allowed. However, due to the computational time, the obtained values are considered satisfactory as shown in Figs. 8 and 9.

The Effective Independence (EI) and Modal Kinetic Energy (MKE) techniques, which are popular sensor location methods, are used here for comparison with the GA strategy. The EI method attempts to find measurement locations that make experimental mode shape vectors linearly independent, whereas the MKE aims at selecting highly responsive positions (Li et al. 2007). The selected locations are shown in Figs. 10 and 11 for the EI method; Figs. 12 and 13 show the measurement locations selected by the MKE method. Table 5 shows the fitness values that result from substitution in (1) of the location vectors specified by EI and MKE. These fitness values are lower than those obtained thorough the use of the GA strategy.

Table 3 Genetic algorithm parameters and attained fitness value—Western grandstand

Grandstand	Western
Number of accelerometers	20
Iterations	10000

Population size	Elite individuals	Crossover individuals	Mutation individuals	Fitness value J
100	24	50	26	1.9223

Table 4 Genetic algorithm parameters and attained fitness value—Southern grandstand

Grandstand	Southern
Number of accelerometers	12
Iterations	10000

Population size	Elite individuals	Crossover individuals	Mutation individuals	Fitness value J
100	24	50	26	1.9125

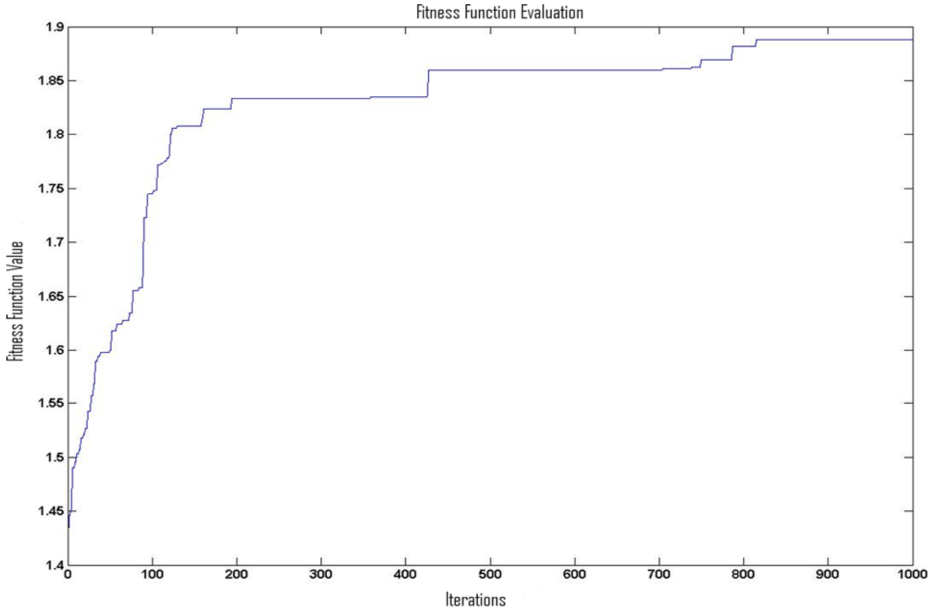

Fig. 8 Iteration progress, western grandstand

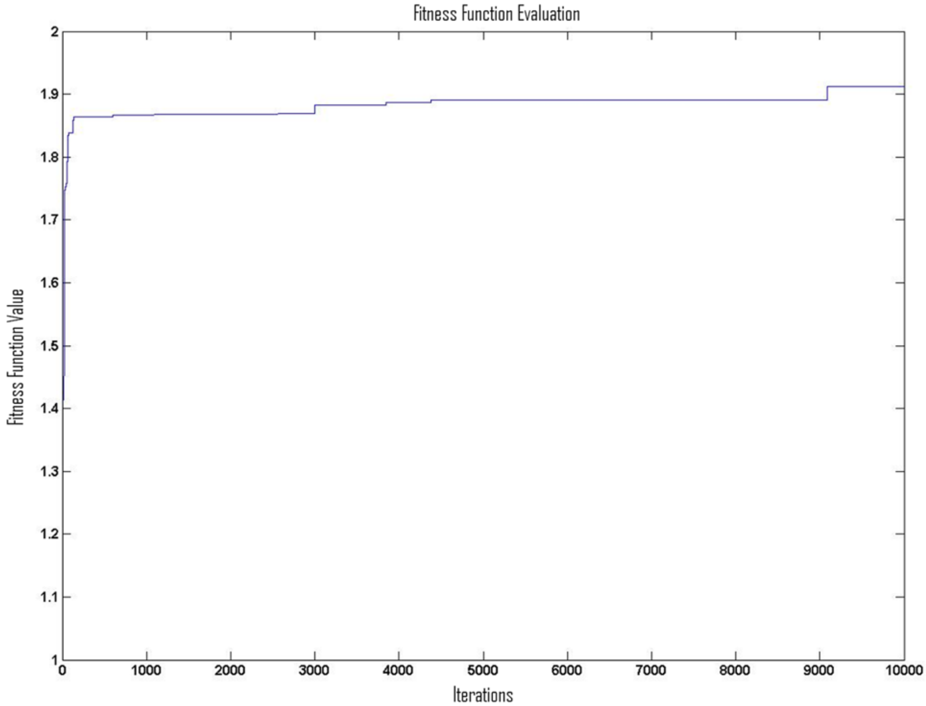

Fig. 9 Iteration progress, southern grandstand

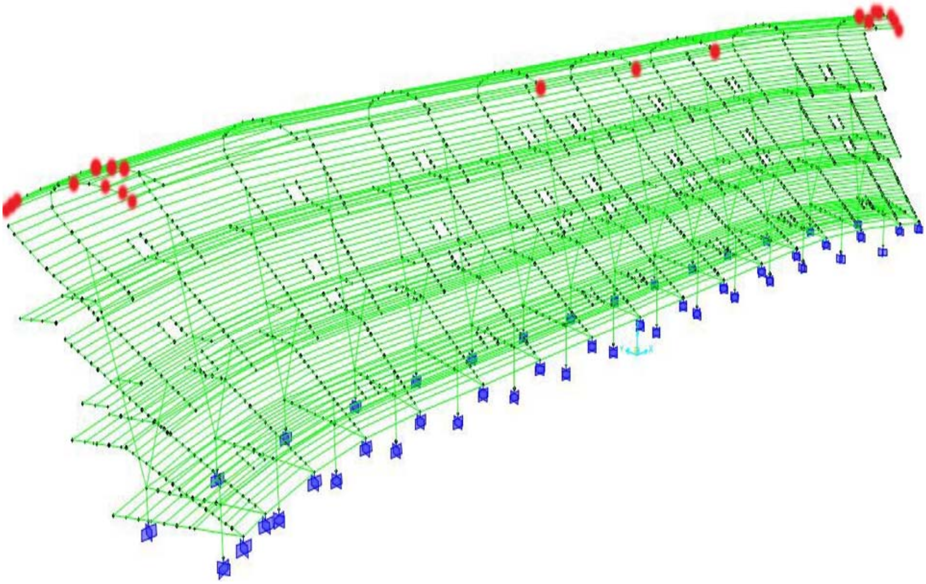

Fig. 10 Sensor positions for the western grandstand: EI method

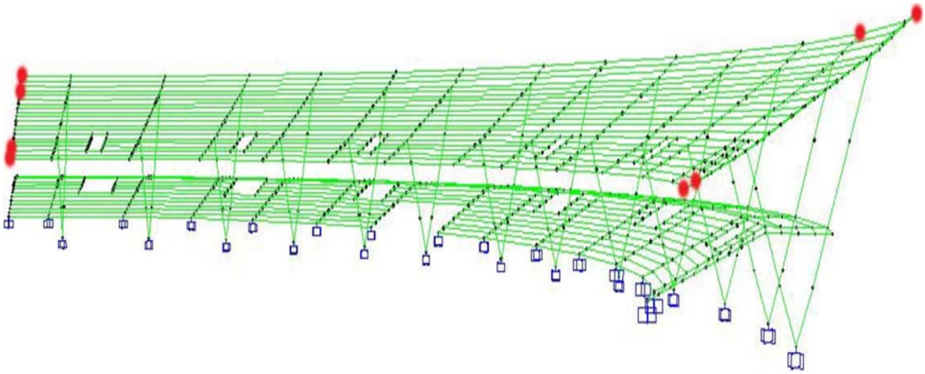

Fig. 11 Sensor positions for the southern grandstand—EI method

7 Discussion

The values of the fitness function attained by the genetic algorithm for both grandstands are very close to the optimum value and are therefore judged satisfactory in light of the number of iterations allowed and from an engineering perspective. When compared with the EI and MKE generated distributions for the stadium, the arrangement obtained with the GA strategy is evenly distributed and thus provides a better description of the dynamic behavior of the structure. This characteristic is ideal for test-analysis modal correlation, where experimental mode shape vectors must be uniquely matched with analytical ones, thus requiring a broad distribution that prevents spatial aliasing from taking place in the process. In contrast, the EI and MKE produced locations concentrated in certain areas of the structure, namely, in the lateral cantilevers. This proximity among sensors may prevent modal analysis methods

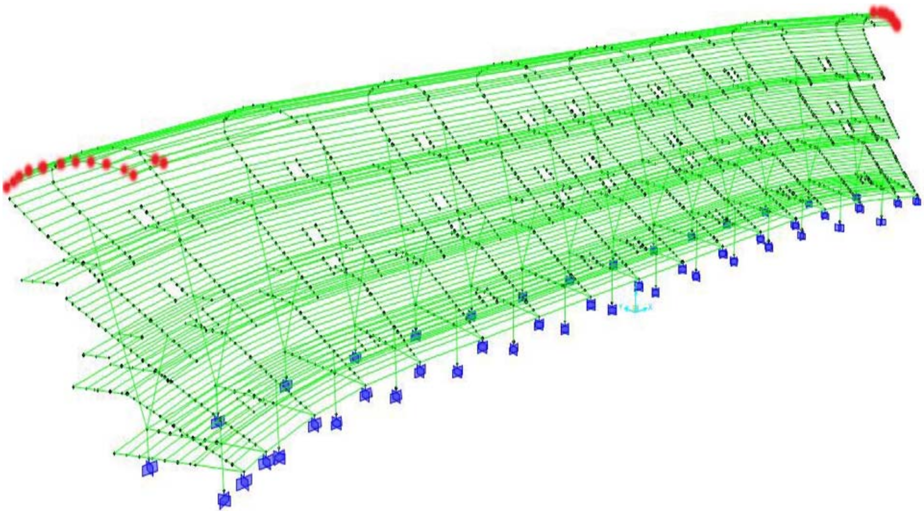

Fig. 12 Sensor positions for the western grandstand—MKE method

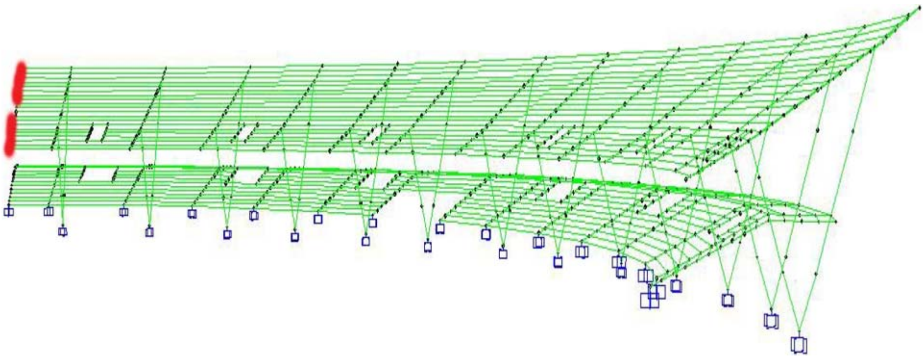

Fig. 13 Sensor positions for the southern grandstand—MKE method

from estimating correct phase parameters and misleading mode shapes would be extracted; modal identification and health monitoring would fail. The fitness values attained by the GA strategy are higher than those obtained by substitution of MKE and EI results in (1). The MKE produces much lower values than EI with a maximum difference of 29%, thus it is considered inadequate for this structure. Although the EI locations produce larger fitness values than those obtained with MKE, their use is not recommended in this case due to the arguments given regarding spatial distribution problems. Since the formulation of the GA strategy reflects the mass variability and earthquake effects which are inherent to structures of the type studied here, the proposed genetic algorithm method is suitable for structures that fall in this category. The sensor distribution obtained from the use of the GA strategy and the FEM model of the stadium will be used in a structural health monitoring system to be installed in the actual structure.

Table 5 Fitness value comparison

Grandstand	Southern	Western
Number of accelerometers	12	20
Method	J	J
Genetic algorithm	1.9125	1.923
Effective independence	1.8641	1.8323
MKE	1.352	1.544

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