Structural Safety 31 (2009) 460–474

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

# Lifetime-oriented multi-objective optimization of structural maintenance considering system reliability, redundancy and life-cycle cost using GA

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ARTICLE INFO

*Article history:*
Received 5 August 2008
Received in revised form 15 May 2009
Accepted 30 June 2009
Available online 11 August 2009

*Keywords:*
Structural systems
Redundancy
Reliability
Maintenance
Optimization
Genetic algorithms

ABSTRACT

The need to design and construct structural systems with adequate levels of reliability and redundancy is widely acknowledged. It is as crucial that these desired levels are maintained above target levels throughout the life of the structure. Optimization has served well in providing safer and more economical maintenance strategies. Lifetime maintenance optimization based on system reliability has already been proposed. It is still needed, however, to incorporate redundancy in the lifetime maintenance optimization process. Treating both system reliability and redundancy as criteria in the lifetime optimization process can be highly rewarding. The complexity of the process, however, requires the automation of solving the optimization problem. Genetic algorithms (GAs) are used in this study to obtain solutions to the multi-objective optimization problems considering system reliability, redundancy and life-cycle cost (LCC). An approach to provide the optimization program the ability to optimally select what maintenance actions are applied, when they are applied, and to which structural components they are applied is presented. Two different strategies are proposed. The first strategy has the ability to optimally select mixed maintenance types to apply to different parts of the structure at the same time. This strategy can be used in cases where any combination of different maintenance options can be practically applied to any part of the structure. The application of this strategy on truss structures is shown in a numerical example. The second strategy can be used when a limited number of possibilities of practical maintenance options are available. The application of this strategy to bridge structures is shown in a numerical example. The greatest advantage of the proposed approach (both strategies) is its ability to avoid the application of maintenance interventions to structural components that are not critical.



## 1. Introduction

The ability to anticipate and react to disfunctions of structures well in advance saves financial resources and even human lives. Maintenance interventions are generally scheduled to prevent or react to these structural disfunctions. The practice of scheduling maintenance interventions has emerged in recent decades as the numbers of deficient structural systems, including buildings and bridges, are in continuous increase. Decision makers in charge of maintaining and improving the quality of civil infrastructure under uncertainty must have the most accurate and optimum maintenance strategies to base their decisions upon, in order to wisely take the best advantage of the very limited funds and resources available. Providing these optimum maintenance strategies has been a task that attracted many researchers in recent years, and several methods have been proposed. These methods vary in their goals, constraints and solution methodologies, but one objective remains the same: provide the least expensive and most efficient maintenance plan during the lifetime of deteriorating structural systems.

Maintenance actions can be categorized as preventive and essential. The objective of the former is usually to delay the occurrence of reaching a threshold of the performance, whereas the latter is usually intended to improve the performance when the threshold is reached. Preventive maintenance may result in stopping the deterioration of the performance or slowing down the rate of deterioration and it may be applied at regular or irregular time intervals [1].

Despite the variety of the available maintenance optimization methods presented in the literature, the main design variable among them is the time of application of a predetermined type of maintenance to a, sometimes predetermined, part of the structure. Some methods are single-objective based and solved enumeratively. Most do not allow the application of mixed maintenance types (i.e., essential and/or preventive) to different parts of the structure at the same time. To the best knowledge of the authors, none has included the system redundancy in the maintenance optimization process under uncertainty despite its importance. However, improving the redundancy of structural systems was

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doi:10.1016/j.strusafe.2009.06.005

one of the criteria of a multi-objective optimization problem targeting the initial design of structural systems [2].

Most structural performance indicators are expected to change over time, and the goal of a maintenance strategy is to plan interventions that improve or maintain the performance at acceptable levels. Hence, the main ingredients of a maintenance optimization method are the chosen performance indicators, their predictive models, the selected types of maintenance, the optimization problem formulation, and the solution technique.

The selected types of performance indicators may vary between strength and serviceability. It is most likely, and preferable, that the performance measures are probabilistic in order to quantify and take into account both the aleatory and epistemic uncertainties [3]. These measures may be reliability-based, such as the system-reliability index [4] and the lifetime failure probability [5], or simulation-based, such as the safety and condition indices [6,7]. The ability of the performance measures and their predictive models to accurately interpret the effects of applying maintenance interventions is necessary. This ability may be limited to maintenance actions applied to the entire structure or may incorporate the effects of maintenance actions on any parts of the structure.

Performance is treated as constraints in single (minimum cost) objective maintenance optimizations [4,5]. Scheduling essential maintenance actions in this minimum cost optimization problems has been performed in enumerative search schemes [4,5]. The advances achieved in the field of multi-objective genetic algorithms [8] have paved the road for solving multi-objective maintenance optimization problems considering both essential and preventive maintenance. Multi-objective optimization problems have been formulated and solved where performance is treated as objective to be improved [6,7,9].

The objective of this study is to propose a novel maintenance optimization approach that integrates the system reliability and redundancy as objectives in addition to the LCC objective. This approach is able to optimally and automatically select what maintenance actions are applied, when they are applied, and to which structural components they are applied. The greatest advantage of the proposed approach is its ability to avoid the application of maintenance interventions to non-critical structural components.

## 2. System reliability, system redundancy and LCC

As indicated in the previous section, structural performance measures are numerous. Two system performance measures are selected for this study, namely system reliability and system redundancy. However, it is emphasized that this proposed approach is applicable with any other structural performance measures as long as they are mathematically defined. System reliability provides a measure of performance that accounts for the interaction between the components that form the structural system and accurately estimates the overall safety of the system. Computing the system reliability requires knowledge of both aleatory and epistemic uncertainties associated with the loads, resistances, and the model that captures the overall system performance. Due to these uncertainties, a structure may fail in any one of its possible failure modes. For this reason, all possible failure modes have to be taken into account when the probability of system failure is calculated. Therefore, the time-variant probability of system failure $P_{f(sys)}(t)$ of a structure with $n$ possible failure modes is

$$P_{f(sys)}(t) = P[\text{any } g_i(t) < 0] \quad i = 1, 2, \ldots, n \tag{1}$$

where $g_i(t)$ = time-variant system performance function with respect to failure mode $i$. The probability of damage occurrence to the system is required for system redundancy analyses. The occurrence of damage is usually described as being the result of first yielding or failure in the system. Therefore, the time-variant probability of first member failure occurrence (also called damage occurrence) $P_{f(dmg)}(t)$ to a system with $m$ components is

$$P_{f(dmg)}(t) = P[\text{any } h_j(t) < 0] \quad j = 1, 2, \ldots, m \tag{2}$$

where $h_j(t)$ = time-variant performance function with respect to damage of component $j$.

System redundancy can be defined as the availability of system warning before the occurrence of structural collapse. A commonly adopted sign of warning is the occurrence of damage to the system. Despite the abundance of redundancy measures in the literature to date [10–15] a standard redundancy measure has yet to be specified. The time-variant redundancy index used in this study is defined as [16]

$$RI(t) = \frac{P_{f(dmg)}(t) - P_{f(sys)}(t)}{P_{f(sys)}(t)} \tag{3}$$

where $RI(t)$, $P_{f(dmg)}(t)$, and $P_{f(sys)}(t)$ are the redundancy index, probability of damage occurrence to the system, and probability of system failure at time $t$, respectively. Further details can be found in [16].

The LCC in this study comprises the discounted costs of application of the maintenance interventions for a given maintenance strategy. The initial (design and construction) cost may be added, although it has no effect on the results of the optimization problem since it is the same for any maintenance strategy. The total discounted LCC considered is calculated as

$$\text{LCC} = C_0 + \sum_{i=1}^{n} \frac{Cm_i}{(1+v)^{t_i}} \tag{4}$$

where $C_0$ = initial cost of the structure, $Cm_i$ = undiscounted cost of applying maintenance at intervention $i$, $t_i$ = time of application of maintenance intervention $i$, $v$ = discount rate of money, and $n$ = number of maintenance interventions.

The term $Cm_i$ is calculated according to the first strategy, which allows the application of maintenance to any selected component, as

$$Cm_i = \sum_{j=1}^{q} \sum_{k=1}^{m} (C_{MAINT})_{j,k} \times M_{i,j,k} \quad i = 1, 2, \ldots, n \tag{5}$$

where $(C_{MAINT})_{j,k}$ = cost of applying maintenance type $j$ to component (or group of components) $k$, $M_{i,j,k}$ = a binary code that takes a value 1 if maintenance type $j$ is applied to component (or group of components) $k$ at intervention $i$ and a value of 0 otherwise, $m$ = the number of components (or group of components) in the structure, and $q$ = number of maintenance types. According to the second strategy, which allows using a limited number of practical maintenance options, the term $Cm_i$ is assigned a value equal to the cost of the maintenance option at intervention $i$ selected by the program.

## 3. Genetic algorithms for multi-objective optimization

An advanced GA called non-dominated sorting GA with controlled elitism, NSGA-II [17], has gained much popularity since its birth. The computational cost involved when solving optimization problems associated with this study can be high. This is attributed to the time of evaluating some objective functions and constraints at the design points and not to the GA operations. It is known that GAs are random and memoryless in nature. New design points are generated through random processes and some of which, or very similar ones, may have already been found in previous generations. In fact, design points may even get selected for the mating pool but do not have the chance to participate in the genetic operations and they remain unchanged in the offspring populations. In addition, the nature of the genetic operations (i.e., cross over and mutation) tends to produce offspring individuals that are similar to their parents. Two individuals are deemed similar if the differences between the values of each of their corresponding design variables are less than a specified tolerance. Hence, new generations may have same or similar design points that had their objective functions already been evaluated in previous generations.

For practical purposes, the difference between solutions of two similar design points may be deemed insignificant. For example, two maintenance strategies having identical maintenance applications except one where the same maintenance type is applied at time $t_1 = 21.33$ years in the first and $t_2 = 21.34$ years in the second are practically the same maintenance strategy. Therefore, it would be a waste of time and resources to perform the computations for evaluating the objectives at both design points.

In this study, a database with a book-keeping subroutine that prevents GAs from evaluating the objectives and constraints for similar design points is added to NSGA-II in an effort to reduce its computational cost. A flowchart of the added book-keeping subroutine (algorithm) is shown in Fig. 1. This algorithm is run every time the objective functions of a new population need to be evaluated. If a new individual has a similar one in the book-keeping database (BK matrix), a large value is assigned to each of the objective functions of the new individual to ensure that it will die off in the selection process within the generation. This is similar to the approach of handling constraints in GAs using the penalty function method. It is suggested that the assigned values are the result of multiplying an appropriate violation coefficient by the objective values of the similar point from the BK matrix. If a new individual does not have a similar one in the BK matrix, the objectives of the new individual are calculated and also appended to the BK matrix.

The time added to the NSGA-II by this book-keeping algorithm may be insignificant with proper programming. Programming this algorithm using vectorization and memory pre-allocation capabilities in programming engines such as MATLAB produces a significantly faster implementation. In addition, the time saved by the algorithm is far larger than the time that may be spent on the evaluation of unnecessary objective functions.

NSGA-II with the proposed book-keeping algorithm is tested on a bi-objective optimization example from ([8], p. 19). A population of 100 is used and the program is let to run for 100 generations. A book-keeping tolerance of 0.01 is used for both design variables. For this example, Fig. 2 shows the number of times the objective functions are evaluated at each generation using NSGA-II with and without book-keeping. The decrease in the number of objective evaluations is clear as the GA progresses. This is due to the fact that the book-keeping matrix is growing, which increases the chance of finding similar individuals in later generations. The total number of objective evaluations with book-keeping is only 48% of the total number of objective evaluations without book-keeping. It is worth noting that the Pareto-optimal set was obtained for this simple example within the first 20 generations, where 85% of the total number of objective evaluations without the book-keeping was performed.

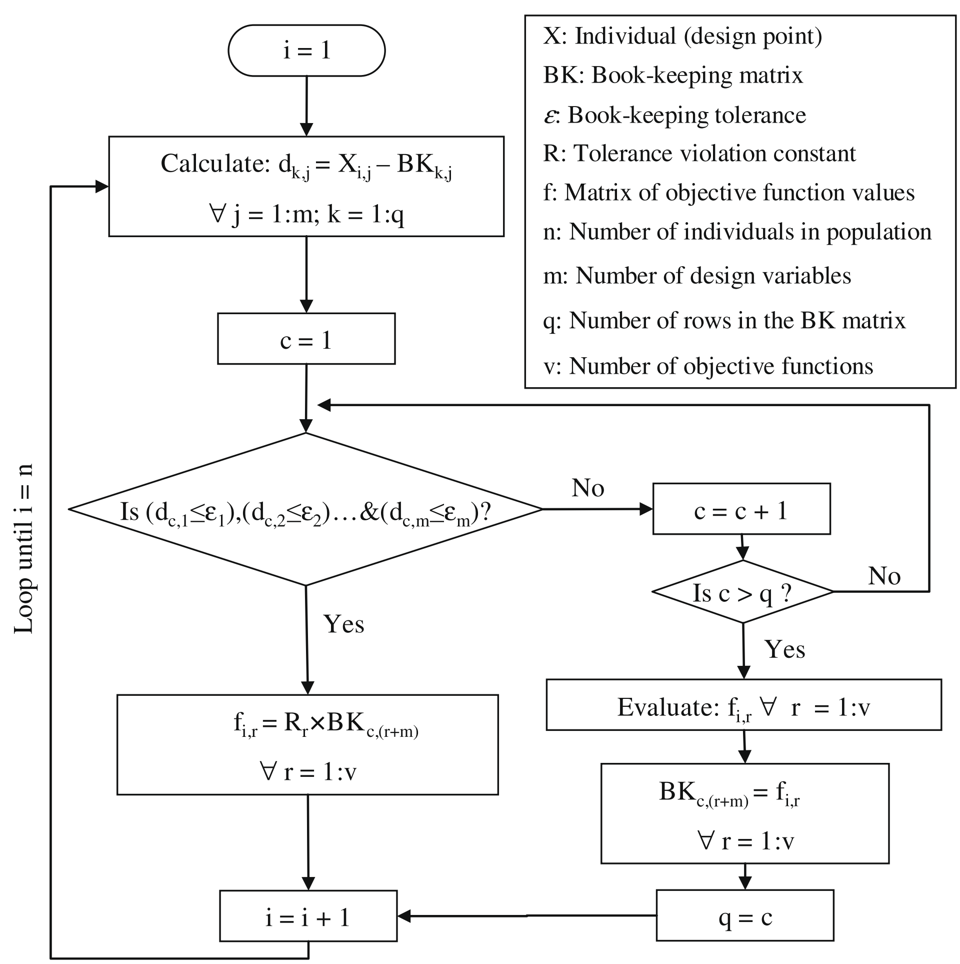


**Fig. 1.** Book-keeping algorithm for NSGA-II.

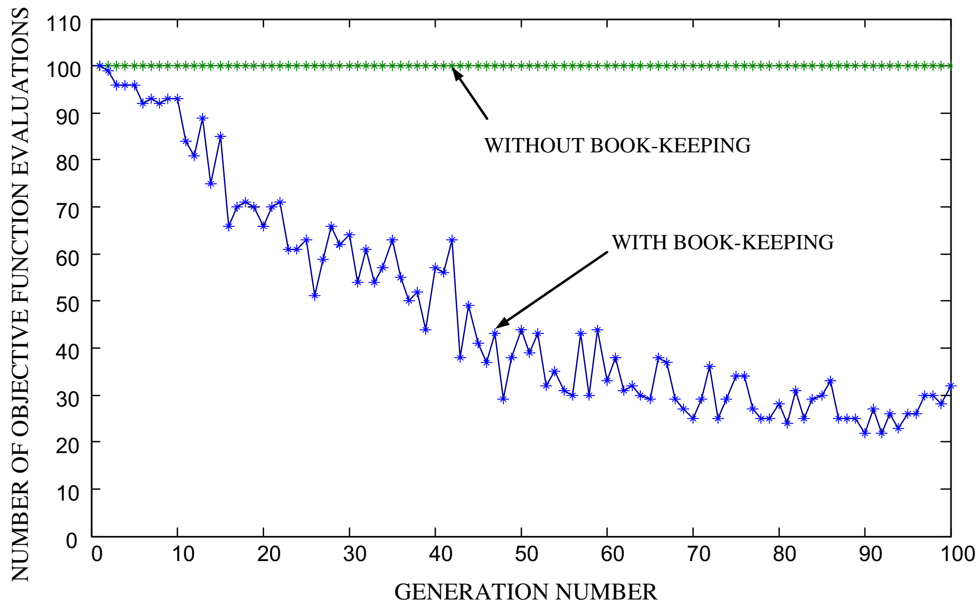


**Fig. 2.** Comparison of the number of objective evaluations with and without the book-keeping algorithm.

The penalty function method for handling constraints in GAs states that constraint violations are magnified and added to the values of the objective functions at the design point violating the constraints. Therefore, even if a constraint is violated, its objective functions are still evaluated. Some constraints are composed of simple explicit mathematical relations of the design variables. If these constraints are violated at a given design point, it may be time saving to assign a large value, proportional to the constraint violation, to the corresponding penalty function without resorting to evaluating the objective functions.

## 4. Probabilistic optimization approach

This study proposes an automated probabilistic maintenance optimization approach that is able to optimize the maintenance schedule and maintenance types as well as select the components at each intervention to maintain or replace. Two strategies are presented. The first strategy is to introduce base $n$ coded design variables, maintenance codes (MCs), that describe the type of maintenance applied to each structural component (or group of components), where $n$ = the number of available maintenance types (including no maintenance). Each bit in the maintenance code (MC) represents a component (or group of components) from the structure and the value of that bit represents the identification number (id) of a type of maintenance. A value of zero in a bit means no maintenance is applied to that component (or group of components) represented by this bit. For example, MC = [1 0 2] is a base three maintenance code (three maintenance types) with three bits (three structural components) design variable which specifies that maintenance type 1 is performed on component 1, no maintenance is performed on component 2, and maintenance type 2 is performed on component 3. A corresponding application time (continuous) design variable $t$ specifies the time at which MC is applied. This approach can adopt any maintenance types as long as their effects on the performance indicators and costs are defined. Different maintenance types are considered in the examples presented in this paper such as member replacement, and painting of steel members. Further details are given in the examples.

The MCs are treated in the genetic operations as follows. Cross over is implemented the same as in binary coded cross over operations. Mutation is also similar to the binary mutation. The difference is that once a bit is selected for mutation, it takes an integer value generated from a uniform distribution in the range between 0 and $n$ [18]. Recall that the book-keeping algorithm requires calculating the difference between the values of the corresponding design variables of the individuals. Therefore, the decimal equivalents of the MCs are used in the book-keeping algorithm. The decimal equivalent of a base $n$ code with $m$ bits BN = $[b_1\, b_2 \cdots b_m]$ is calculated as

$$\text{Decimal equivalent} = b_1 \times n^0 + b_2 \times n^1 + \cdots + b_m \times n^{m-1} \tag{6}$$

where $b_i$ = value of the $i$th bit.

The second strategy treats MC as an integer design variable that represents the id number of a predefined maintenance option. Clearly, the first formulation is broader and more cost efficient. However, some of the results obtained by the first formulation may be impractical. For example, a strategy that requires the replacement of several scattered girders without the replacement of the deck is practically impossible to implement. In addition, only a limited number of maintenance or repair options may be available for a certain structure. The second formulation is designed to treat these cases.

A book-keeping tolerance of zero is used for the MCs (their decimal equivalents) in this study to ensure that only individuals with the same corresponding MCs from the BK matrix are considered in the book-keeping algorithm. The book-keeping tolerance used for the continuous (time of maintenance application) design variables in this study is 0.01 years.

## 5. Problem formulation

As mentioned earlier, the structural performance indicators used in this study are the system reliability and system redundancy. These performance indicators are introduced in the optimization process as criteria. The worst value reached for each performance indicator throughout the lifetime of the structure is the value of one of the objective functions. The goal of the optimization is to improve these (worst) values. In other words, one objective is to minimize the maximum (worst) value reached for the probability of system failure throughout the service life $P_{f(sys),max}$ and the other objective is to maximize the minimum (worst) value reached for the redundancy index throughout the service life $RI_{min}$. However, in order to ensure a minimum acceptable level of safety provided by any solution, the value of $P_{f(sys),max}$ obtained in any solution is not allowed to exceed an allowable value $P_{f(sys),allowable}$. If a design solution does in fact have its $P_{f(sys),max}$ greater than $P_{f(sys),allowable}$ that solution is treated as an infeasible solution. Similarly, an allowable value for redundancy $RI_{allowable}$ is imposed. The third objective is the LCC required to apply maintenance activities in order to achieve these values of $P_{f(sys),max}$ and $RI_{min}$.

The design variables considered are the maintenance codes ($MC_i$) and their application time variables ($t_i$). Different numbers of ($MC_i$) and ($t_i$) are considered in the examples (i.e., different numbers of maintenance applications). It is desired not to apply maintenance anytime in the first or last few years of the service life of a structure. Thus, the application time variables are only allowed to vary from the year $t_{min}$ to the year $t_{max}$. In addition, a minimum time span of two years between any two consecutive maintenance applications is required.

Accordingly, the formulation of the problem may be stated as follows:

- *Find:*

$$\text{The times of applications of maintenance}: t_1, t_2, \ldots, t_n \tag{7a}$$

$$\text{The maintenance codes for each application}: \mathrm{MC}_1, \mathrm{MC}_2, \ldots, \mathrm{MC}_n \tag{7b}$$

- *To achieve the following three objectives:*

$$\text{Minimize } P_{f(sys),max} \tag{7c}$$
$$\text{Maximize } RI_{min} \tag{7d}$$
$$\text{Minimize } C_T = LCC \tag{7e}$$

- *Subject to the constraints:*

$$P_{f(sys),max} \leqslant P_{f(sys),allowable} \tag{7f}$$
$$RI_{min} \geqslant RI_{allowable} \tag{7g}$$
$$t_{\min} \leqslant t_i \leqslant t_{max} \tag{7h}$$
$$t_j - t_i \geqslant 2 \text{ years} \tag{7i}$$

where $i$, $j = 1, 2, \ldots n$, and $n$ = the number of maintenance applications.

It is the common trend in maintenance scheduling optimization that essential maintenance actions are justified only at the time a performance threshold is reached. The design variable in this case is the performance threshold. In addition, preventive maintenance is applied prior to reaching any performance threshold and its application time is the design variable. Application time design variables are used for both essential and preventive maintenance. The reason is the presence of other design variables (the MCs) that are associated with the application time design variables. Due to the MCs, a maintenance intervention may be purely preventive, purely essential, or mixed. It is the duty of the optimization program to find the application times that provide the optimum values for the objective functions. Therefore, if an MC requires a pure essential maintenance, it is expected that the optimization program provides its application times at the time the worst value of one of the performance indicators $P_{f(sys),max}$ or $RI_{min}$ is reached.

## 6. Application examples

### 6.1. Truss example

The first strategy of the proposed approach is implemented in this illustrative example to find optimum maintenance strategies for the five-bar steel truss shown in Fig. 3. The random variables considered are the time-variant resistances of the bars, $R_1(t), R_2(t), \ldots, R_5(t)$ and the time-variant applied load $P(t)$. The mean of the time-variant random resistance of bar $i$ at time $t$, $\mu_{Ri}(t)$, is related to its cross-sectional area as follows

$$\mu_{Ri}(t) = A_i(t) \times (\mu_{Fy})_i \tag{8}$$

where $(\mu_{Fy})_i$ = mean of the random yield stress $Fy$ of bar $i$, where $Fy$ is assumed to be lognormal and constant over time. Mean material yield stresses of 25 and 12.5 kN/cm$^2$ are assumed for tension and compression, respectively, for the five bars. An initial cross-sectional area of 3 cm$^2$ and an initial coefficient of variation of 10% of the initial resistances of all bars are assumed. The vertical, horizontal and diagonal bars have the cross-sectional areas $A_1$, $A_2$ and $A_3$, respectively, and they are assigned to groups 1, 2 and 3, respectively. The time-variant applied load is assumed as a lognormal random variable with an initial mean of 20 kN and initial standard deviation of 4 kN.

It is assumed that the degradation in resistance is due primarily to a continuous section loss over time. The mean of the resistance of bar $i$ decreases annually by the deterioration rate $DR_i$ = 0.03% per year while its standard deviation increases by the same rate. The mean of the applied load annually increases by the rate $LIR$ = 1% per year. The coefficient of variation of the applied load is $COV(P)$ = 20% and is assumed constant throughout the life of the structure. Therefore, the standard deviation increases with time by the same rate of increase of the mean of the applied load. Statistical independence is considered among the resistances of the bars, and between the resistances and the applied load. The limit state functions are formulated using the incremental method [19]. The probabilities of system failure and damage occurrence are found as the upper Ditlevsen bound [20] using the software CALREL [21]. A service life of 50 years and an initial cost of $C_0$ = $5250 are assumed. A discount rate of $v$ = 2% is used. The truss is allowed to be repaired between the years $t_{min}$ = 5 and $t_{max}$ = 45. An allowable maximum probability of system failure of $P_{f(sys),allowable}$ = 0.005 and an allowable minimum redundancy index of $RI_{allowable}$ = 10 are considered.

The details of this example progress in three cases. Case 1 aims to establish a benchmark for the results obtained by the proposed approach, where the problem is treated in the conventional main-

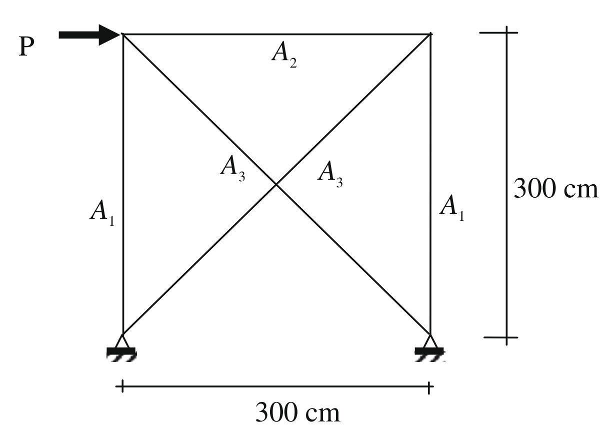


**Fig. 3.** Five-bar truss. Geometry and loading.

tenance scheduling optimization manner. In other words, the goal is to find only the time of application of a prescribed maintenance type to a prescribed part of the structure, namely replace the truss. Case 2 introduces a maintenance code (MC) with one maintenance type, namely replace a group of bars, and one intervention is considered. Case 3 extends the previous case to introduce an additional type of maintenance, namely painting of a group of bars, and up to three maintenance interventions are allowed.

#### 6.1.1. Case 1: one replacement only

Replacing the truss restores the resistances of all bars to their initial values. The undiscounted cost of replacing the truss is equal to the initial cost, i.e., $5250. It is worth noting that $P_{f(sys)}(t)$ and $RI(t)$ are not restored to their initial values due to the annual increase in the applied load. A single design variable is considered in this case, namely the time to replace the entire structure $t$ (years).

The problem is solved by simultaneously optimizing all three objectives in Eqs. (7c)–(7e). The obtained Pareto-optimal set for this tri-objective problem is shown in Fig. 4. Table 2 shows the details of three selected optimum solutions $S_1$, $S_2$ and $S_3$.

#### 6.1.2. Case 2: one selective bar group replacement only

The design variables in this case are (a) a maintenance code MC with a base 2 (binary variable) and three bits, and (b) its corresponding application time (continuous) variable $t$. Each bit of MC represents one of the three bar groups considered. Each group has its bars replaced if the corresponding bit in MC takes a value of 1 and not replaced if the value is 0. The value of the design variable $t$ represents the time at which MC is applied. The undiscounted cost of replacing the bars of each group is shown in Table 1. Replacing the bars in a group restores the resistances of the bars of that group to their initial values.

The problem is solved by simultaneously optimizing all three objectives (see Eqs. (7c)–(7e)). Fig. 5 shows the Pareto-optimal set obtained for this case and superimposed on the Pareto-optimal set of Case 1. Clearly, the new proposed approach has found less expensive solutions with the same $P_{f(sys),max}$ and $RI_{min}$ than the solutions found by Case 1. Projections of the results presented in Fig. 5 in the bidimensional space are presented in Fig. 6. Six solu-

**Table 1**
Undiscounted costs of applying the types of maintenance on the bar groups of the five-bar truss example and their effects on the deterioration.

| Maintenance type | MC value | Cost for group $A_1$ ($) | Cost for group $A_2$ ($) | Cost for group $A_3$ ($) | Effect on deterioration |
|---|---|---|---|---|---|
| Do nothing | 0 | 0 | 0 | 0 | Nothing |
| Replace bars | 1 | 1800 | 900 | 2550 | Area is restored to original |
| Paint bars | 2 | 360 | 180 | 510 | Area is unchanged for two years |

**Table 2**
Selected solutions from the tri-objective Pareto-optimal set of Case 1.

| Pareto solution | Time $t$ (years) | Maximum lifetime system reliability $P_{f(sys),max}$ | Minimum lifetime system redundancy $RI_{min}$ | LCC ($) |
|---|---|---|---|---|
| $S_1$ | 39.10 | 0.0017 | 33.4 | 7635 |
| $S_2$ | 42.00 | 0.003 | 25.5 | 7529 |
| $S_3$ | 44.70 | 0.005 | 19.9 | 7410 |

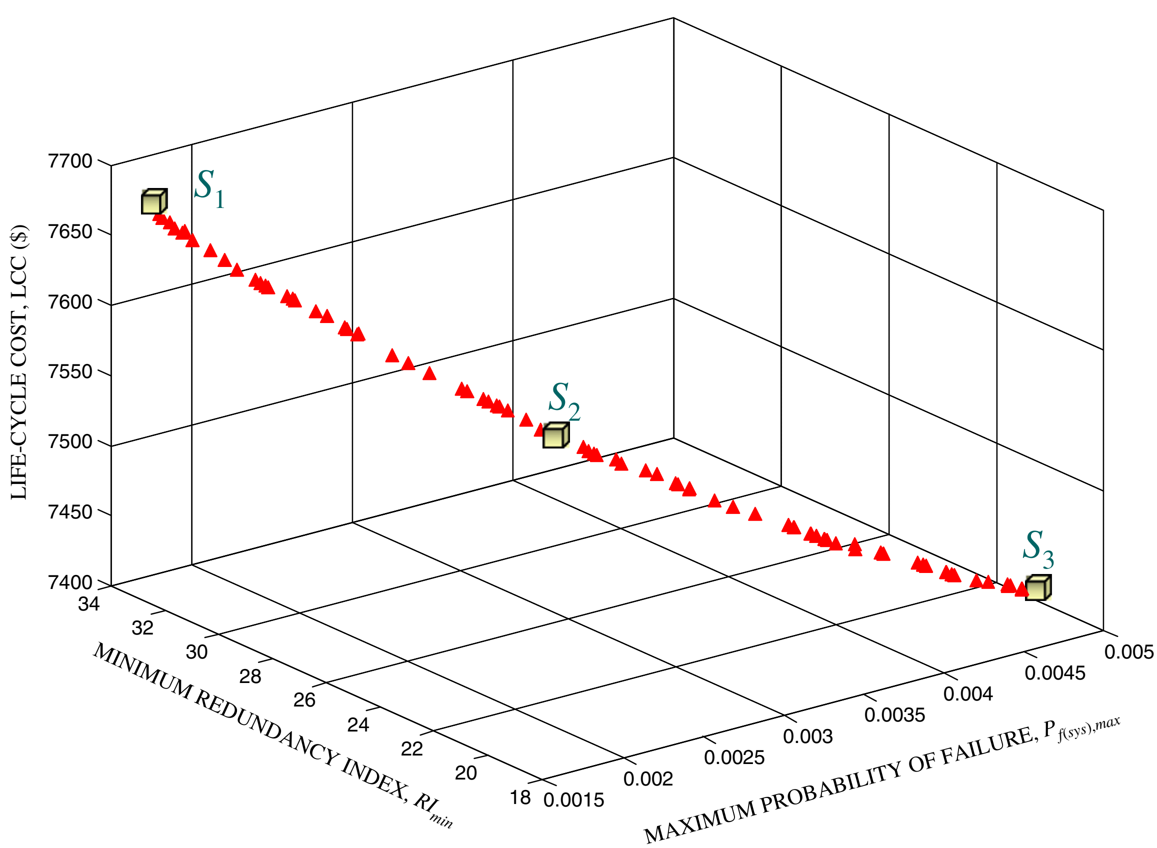


**Fig. 4.** Pareto-optimal set for Case 1 for the five-bar truss.

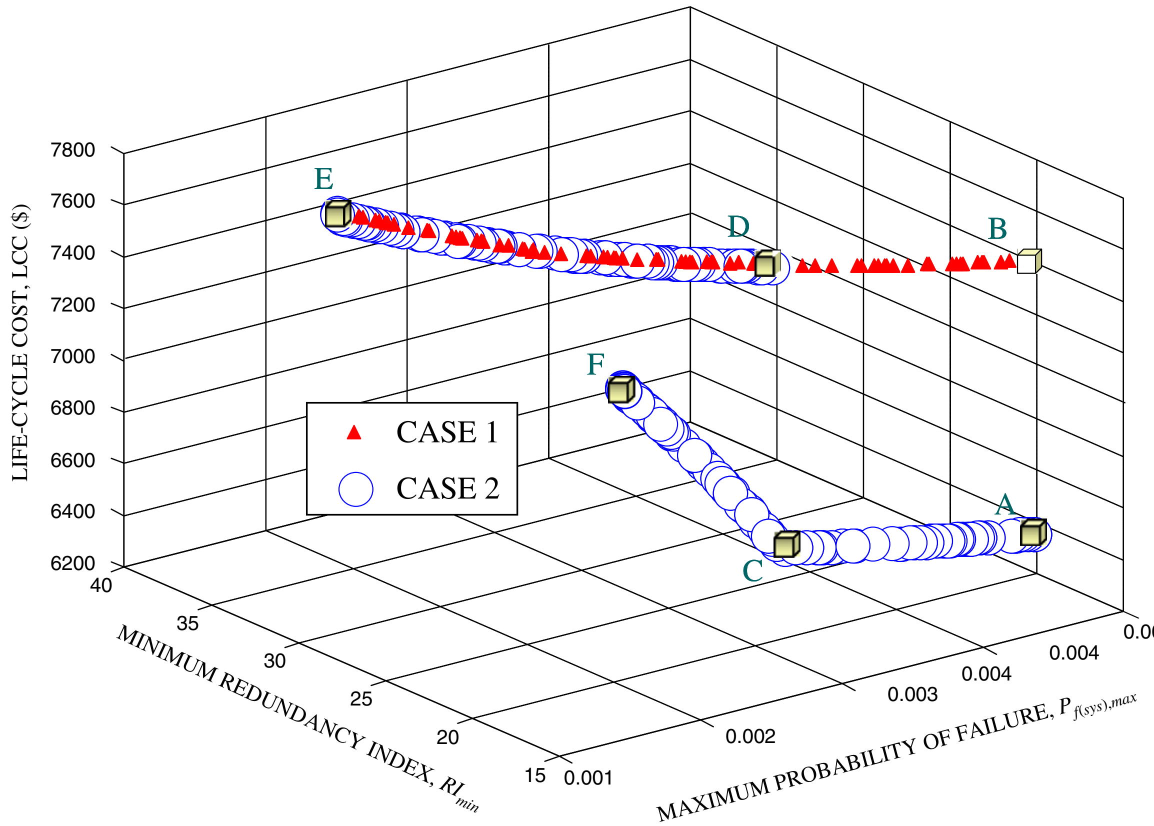


**Fig. 5.** Pareto-optimal sets for Cases 1 and 2 for the five-bar truss.

tions selected from the Pareto-optimal sets of Cases 1 and 2 in Figs. 5 and 6 are compared in Table 3.

Table 3 shows that solutions A and B provide the same $P_{f(sys),max}$ and $RI_{min}$ and are applied at the same time $t$ = 44.70 years. In fact, these $P_{f(sys),max}$ and $RI_{min}$ values are reached at $t$ = 44.70 years in both solutions A and B. Clearly any maintenance action applied at that time should improve both $P_{f(sys),max}$ and $RI_{min}$. The challenge that the program faced was to find the best selection of bar groups to replace in order that these $P_{f(sys),max}$ and $RI_{min}$ values may not be reached again until at least the end of the service life, $t$ = 50 years. The program determined that replacing the horizontal and vertical bars (groups 1 and 2) achieves such a goal. In other words, the program determined that replacing the diagonal bars (group 3) is not necessary. Therefore, replacing them will only result in unnecessary expenses. Without any doubt, solution B has the advantage of providing a longer service life, but for a predefined service life of 50 years, solution A is superior. Solution C differs from solution A in that it is applied earlier, resulting in a higher LCC (due to the discount rate effect) but better redundancy and reliability. It is clear that the $P_{f(sys),max}$ and $RI_{min}$ values obtained by solution C are the best that can be achieved with MC = [1 1 0], i.e., without replacing the diagonal bars. Solution D shows that further improvement in $P_{f(sys),max}$ cannot be achieved without replacing all bars. For this reason a jump exists in the LCC solutions from C to D and the Pareto-optimal curve is discontinuous between these solutions. Solution E provides the best possible value for $P_{f(sys),max}$ but with the highest LCC. Solution F shows that a higher $RI_{min}$ than that of solution E can be achieved with a lower LCC. However, $P_{f(sys),max}$ for solution F is higher than that of solution E. Solution C has a lower $P_{f(sys),max}$ than solution F but $RI_{min}$ is also lower. It is worth noting that the solutions in Fig. 6 are not the solutions of the corresponding bi-objective optimization problems. Clearly, some solutions in Fig. 6 are dominated by others in the bi-objective sense, which prevents their set from being a Pareto-optimal set. The reason is that for optimization problems with more than two objectives, two solutions may have the same value of one objective but different values for the others. One of the two solutions may be dominated by the other with respect to two of the objectives but non-dominated with respect to any of the other objectives. These two solutions may both, however, belong to the Pareto-optimal set. Some of the solutions in Figs. 5 and 6 indeed have this property. For example, solutions between C and F are dominated by some solutions between A and C in Fig. 6a. However, the solutions between C and F dominate some solutions between B and E in Fig. 6b. The reason for this is explained next.

The problem is investigated enumeratively, with a one year increment of $t$. This is shown in Fig. 7. Each point on the figure presents a maintenance strategy, where the bar groups 1 and 2 of the truss are replaced at time $t$ (MC = [1 1 0]). The maximum probability of failure throughout the life of the structure $P_{f(sys),max}$ and the minimum redundancy index $RI_{min}$ are plot vs. the time $t$ in Fig. 7. The figure shows that the time of application of solution C ($t_C$ = 42.69 years) is the application time that provides the minimum $P_{f(sys),max}$, whereas the time of application of solution F ($t_F$ = 37.90 years) is the application time that provides the maximum $RI_{min}$. However, the time of application of solution A ($t_A$ = 44.70 years) is the application time that provides the minimum LCC. Clearly, reliability and redundancy have different properties, and the time of applying a type of maintenance that best improves the reliability is different from the time of applying the same type of maintenance that best improves the redundancy. As a result, the two performance indicators become conflicting between the times $t_C$ and $t_F$ and non-conflicting otherwise. In other words, as $t$ increases, the reliability improves and the redundancy worsens only between $t_C$ and $t_F$, otherwise they improve or worsen simultaneously. Evidently, the solutions between $t_A$ and $t_F$ are part

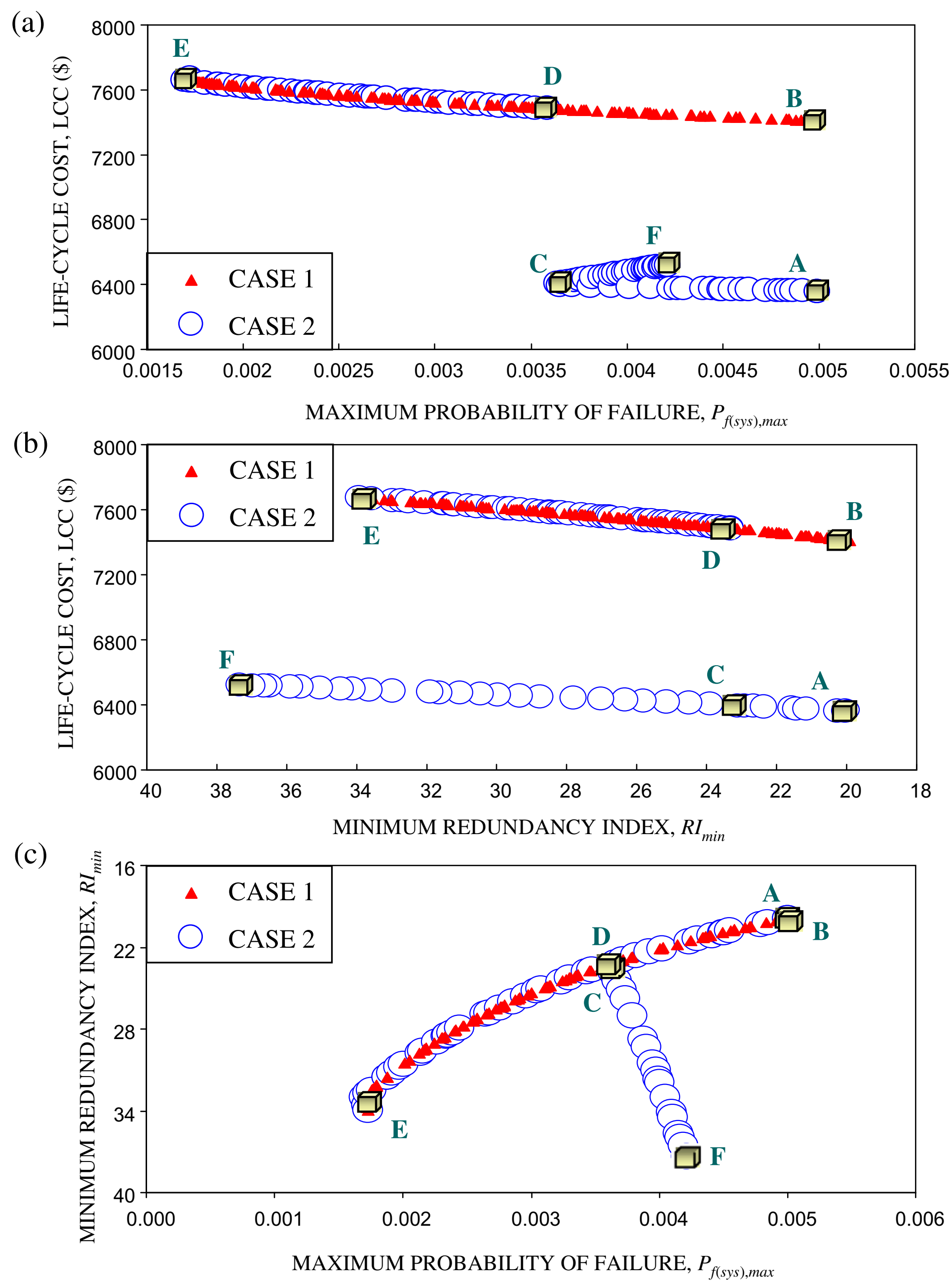


**Fig. 6.** Projections of the results of Fig. 5 in the bidimensional spaces: (a) $P_{f(sys),max}$, and LCC; (b) $RI_{min}$ and LCC and; (c) $P_{f(sys),max}$, and $RI_{min}$.

**Table 3**
Selected solutions from the tri-objective Pareto-optimal sets of Cases 1 and 2.

| Pareto solution | Case | Maintenance code MC | Time $t$ (years) | Maximum lifetime system reliability $P_{f(sys),max}$ | Minimum lifetime system redundancy $RI_{min}$ | LCC ($) |
|---|---|---|---|---|---|---|
| A | 2 | 1 1 0 | 44.70 | 0.005 | 19.9 | 6360 |
| B | 1 | 1 1 1 | 44.70 | 0.005 | 19.9 | 7410 |
| C | 2 | 1 1 0 | 42.69 | 0.0036 | 23.9 | 6405 |
| D | 1 and 2 | 1 1 1 | 42.96 | 0.0036 | 23.3 | 7485 |
| E | 1 and 2 | 1 1 1 | 39.22 | 0.0017 | 32.9 | 7658 |
| F | 2 | 1 1 0 | 37.90 | 0.0042 | 37.4 | 6520 |

of the Pareto-optimal set of the bi-objective optimization problem: maximize $RI_{min}$ and minimize LCC. However, only solutions between $t_A$ and $t_C$ are part of the Pareto-optimal set of the bi-objective optimization problem: minimize $P_{f(sys),max}$ and LCC simultaneously.

The Pareto-optimal sets of the bi-objective optimization problems are easily recognized in Fig. 6a and b. Results of bi-objective optimization problems are commonly plot so that the direction of improvement of both objective functions is towards the origin of the reference axes at the lower left corner of the graph. The

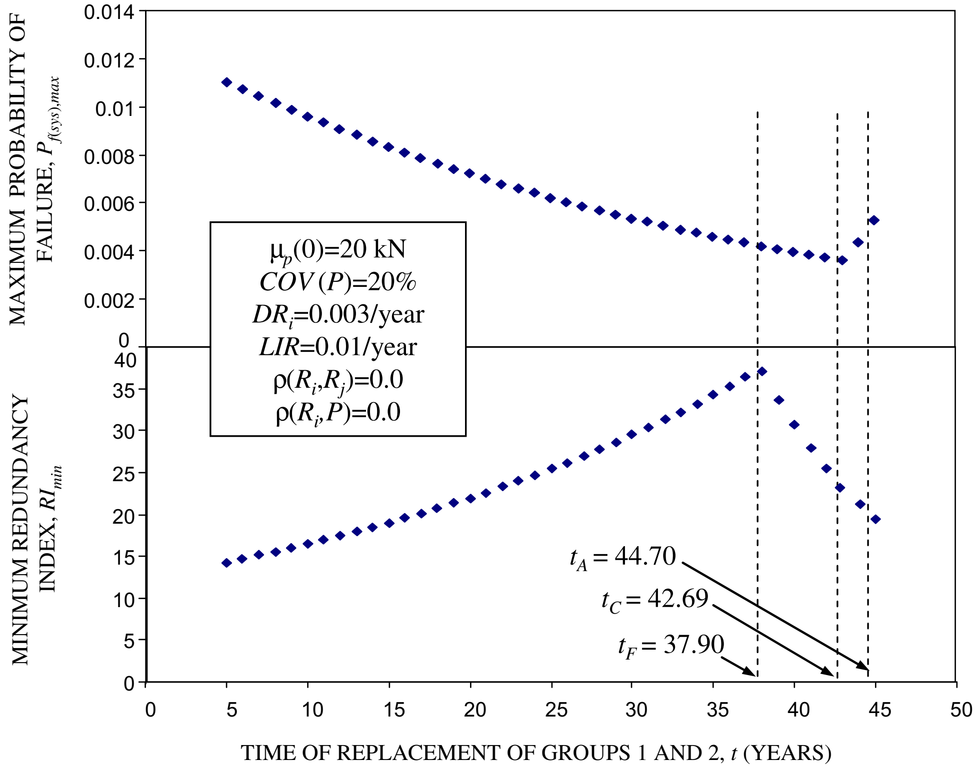


**Fig. 7.** Variation of $P_{f(sys),max}$, and $RI_{min}$ with the time of replacing bar groups 1 and 2 of the five-bar truss, $t$.

Pareto-optimal set, therefore, comprises all solutions on the lower left boundary of the solution set. Hence, the Pareto-optimal set of the bi-objective optimization problem: minimize $P_{f(sys),max}$ and minimize LCC for Case 2 is the set of solutions in Fig. 6a between A, and C, and between D and E. In addition, the Pareto-optimal set of the bi-objective optimization problem maximizing $RI_{min}$ and minimizing LCC for Case 2 is the set of solutions in Fig. 6b between A and F. However, a clear Pareto-optimal set of the bi-objective optimization problem minimizing $P_{f(sys),max}$ and maximizing $RI_{min}$ is difficult to obtain since the two objectives are not conflicting in the entire domain. It is clear in Fig. 6c that it is possible to improve both $P_{f(sys),max}$ and $RI_{min}$ simultaneously in the solutions between A and E and not in the solutions between C and F. In fact, the solutions between C and F exist in Fig. 6c because they are non-dominated in the solution of the bi-objective optimization problem maximizing $RI_{min}$ and minimizing LCC simultaneously (see Fig. 6b).

### *6.1.3. Case 3: three selective bar group replacement or bar painting*

A new maintenance type is additionally considered in this case, namely painting of a group of bars. It is assumed that this action stops the deterioration of the painted bar group for two consecutive years from the time of painting (i.e., the resistance remains unchanged during these two years). The deterioration rate resumes at the conclusion of these two years. The undiscounted cost of painting the bars of each group is shown in Table 1. The maintenance code design variable MC now has a base 3. Thus each bit (group of bars) of MC may have a value of 0 (do nothing), 1 (replace), or 2 (paint). The value of a corresponding continuous design variable $t$ represents the time at which MC is applied. In order to allow up to three maintenance actions, three maintenance code design variables are used ($MC_1$, $MC_2$, $MC_3$) with their corresponding application time design variables ($t_1$, $t_2$, $t_3$), respectively. Solutions with the number of maintenance applications less than three are possible to obtain with this formulation. This can be obtained if the program provides one or more maintenance codes with the value of $MC_i$ = [0 0 0].

The problem is solved first by simultaneously (a) optimizing objectives 1 and 3 (see Eqs. (7c) and (7e)) and also (b) optimizing objectives 2 and 3 (see Eqs. (7d) and (7e)). The resulting Pareto-optimal set of the bi-objective optimization problems (a) and (b) are shown in Fig. 8a and b, respectively. Superimposed on the figure are the corresponding bi-objective Pareto-optimal sets of Cases 1 and 2. It is evident that Case 3 provides some solutions that are less expensive for the same $P_{f(sys),max}$ and $RI_{min}$ values than Cases 1 and 2. In addition, solutions of Case 3 provide $P_{f(sys),max}$ and $RI_{min}$ values that have not been obtained in the previous cases. Furthermore, the overlapping parts of Case 3 over parts of Cases 1 and 2 prove that solutions of Cases 1 and 2 were also obtained by Case 3. In other words, the program was able to identify solutions that require less number of maintenance applications and only a specific type of maintenance when they were optimum.

Four solutions (i.e., G, H, I, and J) selected from the Pareto-optimal set of Case 3 in Fig. 8 are compared in Table 4. Solution G provides a probability of failure $P_{f(sys),max}$ = 0.0031 which was obtained by the other cases, but solution G is less expensive. The second and third maintenance codes of solution G ($MC_2$ and $MC_3$) prove that the program is able to select mixed maintenance types at the same time. Solution H provides a $P_{f(sys),max}$ that is lower than any $P_{f(sys),max}$ obtained by the previous cases. This solution requires the replacement of the truss, one group of bars at a time, in addition to a painting schedule. It is observed that the $RI_{min}$ obtained by solution G is higher than the one obtained by solution H although solution G is less expensive. Solutions I and J both provide $RI_{min}$ higher than the $RI_{min}$ provided by the solutions obtained in the previous cases. It is observed that the maintenance codes for both solutions I and J do not require applying maintenance actions to the diagonal bars. It was shown in [16] that for this truss system, improving the resistance of the diagonal bars improves the reliability but worsens the redundancy. It is also concluded from comparing solution I with

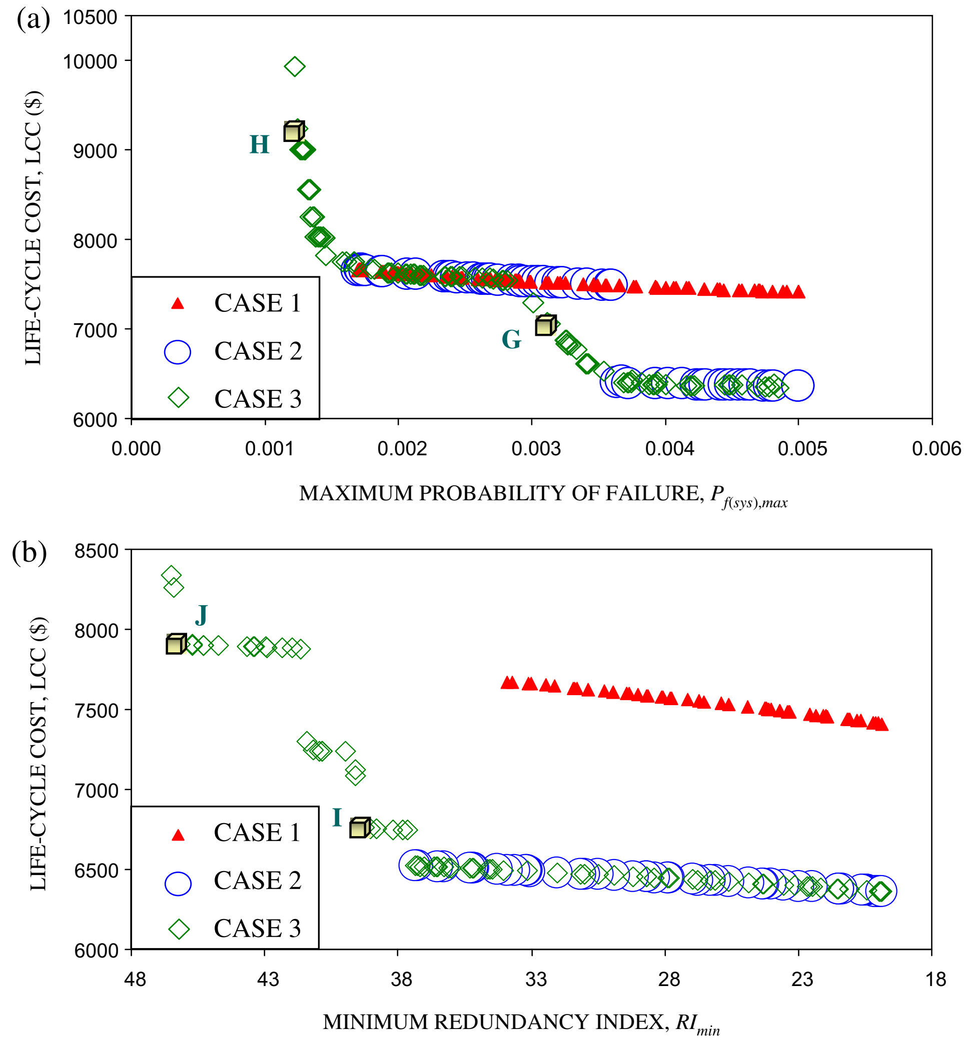


**Fig. 8.** Pareto-optimal sets of Case 3 for the bi-objective optimization problems: (a) minimize $P_{f(sys),max}$ and minimize LCC and; (b) maximize $RI_{min}$ and minimize LCC.

**Table 4**
Selected solutions from the bi-objective Pareto-optimal sets associated with Case 3.

| Pareto solution | Objectives[a] | $MC_1$ | $t_1$ (year) | $MC_2$ | $t_2$ (year) | $MC_3$ | $t_3$ (year) | $P_{f(sys),max}$ | $RI_{min}$ | LCC ($) |
|---|---|---|---|---|---|---|---|---|---|---|
| G | 1 and 3 | 2 0 2 | 40.46 | 0 1 2 | 42.56 | 1 2 0 | 44.99 | 0.0031 | 24.76 | 7054 |
| H | 1 and 3 | 2 0 1 | 38.02 | 0 1 2 | 41.66 | 1 2 2 | 44.82 | 0.0013 | 19.76 | 8256 |
| I | 2 and 3 | 0 2 0 | 18.46 | 2 1 0 | 37.69 | 1 0 0 | 41.29 | 0.0040 | 39.28 | 6762 |
| J | 2 and 3 | 2 0 0 | 22.59 | 1 1 0 | 35.94 | 1 1 0 | 45.00 | 0.0034 | 46.27 | 7908 |

[a] This column shows the two objectives considered in the bi-objective optimization problem that provided this solution. Objectives are numbered as: 1, minimize: $P_{f(sys),max}$; 2, maximize: $RI_{min}$; and 3, minimize: LCC.

solution G that for a little increase in LCC, the reliability is improved but the redundancy is worsened. Many similar trade-offs exist among these solutions and others from the Pareto-optimal sets.

The problem is now solved by simultaneously optimizing all three objectives 1, 2, and 3. Fig. 9 shows the Pareto-optimal set of the tri-objective optimization problem of Case 3. The 10 solutions (A, B, ... ,J) are also shown on the graph. It appears that the solutions of this tri-objective optimization problem belong to a three-dimensional (3D) surface that is the Pareto-optimal surface. Clearly, the LCC increases as $P_{f(sys),max}$ is decreased and/or $RI_{min}$ is increased. With the exception of solution B, which is dominated by solution A, all other selected solutions clearly belong to the tri-objective Pareto-optimal set. Evidently the tri-objective Pareto-optimal set offers a larger selection of solutions than can be offered by the bi-objective Pareto-optimal sets.

### 6.2. Bridge superstructure example

In this example, the second strategy of the proposed approach is applied to the superstructure of the Colorado Bridge E-17-AH. A detailed description of this bridge can be found in [4]. The bridge has three simple spans of equal length (13.3 m) and a total length of 42.1 m. The thickness of the deck consists of 22.9 cm of reinforced concrete and a 7.6 cm surface layer of asphalt. The east–west bridge has two lanes of traffic in each direction. The slab is supported by nine standard-rolled, compact, noncomposite steel girders [4].

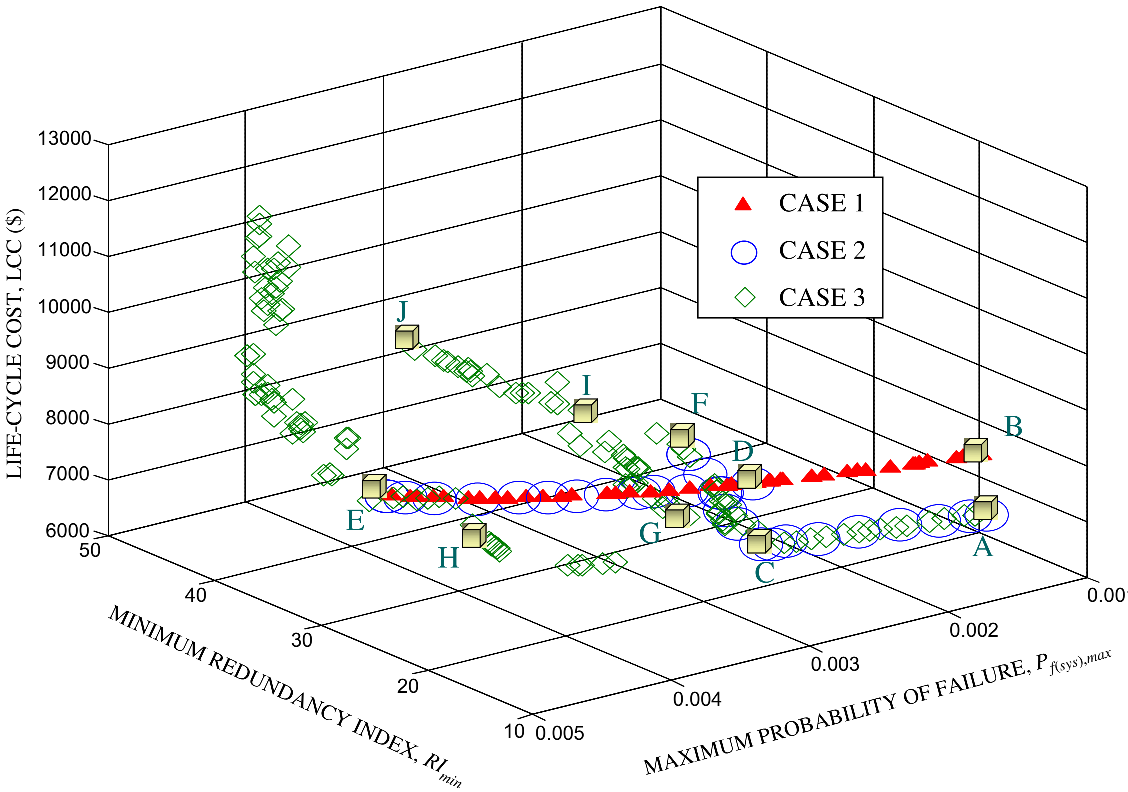


**Fig. 9.** Pareto-optimal set for the tri-objective optimization of Case 3 for the five-bar truss.

The possible failure modes considered in this study are those related to the bridge superstructure and namely the flexural failure of the deck, and the flexural and/or shear failure of the girders. Limit state functions for these failure modes for the bridge deck and for the exterior, interior and exterior–interior girders are formulated in [22]. For example, the limit state equation for the concrete deck flexural failure can be expressed as

$$g(1) = M_{capacity} - M_{demand} = \gamma_{mfc}\left[0.349\lambda_{rebar} f_y d_{eff} - \frac{0.3844(\lambda_{rebar})^2 f_y^2}{244.8 f_c'}\right] - 0.137\lambda_{asph} - 0.471\lambda_{conc} - 4.26\lambda_{trk} = 0 \quad (9)$$

where $M_{capacity}$ = moment capacity, $M_{demand}$ = moment load effect (i.e., demand), $\gamma_{mfc}$ = model uncertainty factor for flexure in concrete, $\lambda_{rebar}$, $\lambda_{asph}$, $\lambda_{conc}$, and $\lambda_{trk}$ = uncertainty factors for the reinforcing steel area in concrete, weight of asphalt, weight of concrete, and weight of truck on bridge, respectively, $f_y$ = yield stress of steel reinforcing in deck (MPa), and $f_c'$ = 28-day compressive strength of concrete (MPa). The other limit state functions and their random variables are all defined in [4].

Multi-girder bridges are modeled in system reliability analyses as combinations of series and parallel components [4,23]. For the bridge being analyzed, the system failure is assumed to occur by the failure of any two adjacent girders (in flexure and/or shear) or the deck (in flexure). Damage occurrence is described as the failure of any component in any mode (i.e., failure of the deck in flexure or failure of any girder in flexure and/or shear). It was found that it is crucial that each girder has its own limit state functions and resistance random variables in order to accurately quantify the redundancy associated with these models. Statistical independence is assumed between the resistances of the girders and between the resistances of the girders and the deck.

Time effects include the increase in live load and degradation in resistance due to steel corrosion in both the reinforcement of the concrete deck [24] and the steel girders [25]. The models used and their computations are described in details in [22]. The probabilities of failure and first yield are found using the software RELSYS [26] in which the average of the Ditlevsen bounds [20] is the given probability of failure. A service life of 100 years is assumed.

Six repair options are considered to maintain the system reliability and redundancy within acceptable values. Table 5 shows these options and their costs [4]. The actual original (initial) cost of the bridge was $C_0$ = $39,300 [4]. Repairing a component restores its resistance to its initial levels. The choice of repair options to implement is controlled automatically by the genetic algorithm program by using an integer design variable that represents the repair option identification numbers. In other words, the maintenance code design variable MC takes an integer value between 0 and 5. Another real continuous design variable $t$ specifies the time at which this action is implemented. For instance, MC = 3 and $t$ = 36.9 implies that the exterior girders and deck have to be replaced at the year 36.9. Repair actions are allowed in this example to be implemented up to three times. Therefore, three maintenance code variables ($MC_1$, $MC_2$, $MC_3$) and three corresponding application time variables ($t_1$, $t_2$, $t_3$) are used. The bridge is allowed to be repaired between the years $t_{min}$ = 10 and $t_{max}$ = 90. An allowable maximum probability of system failure of $P_{f(sys),allowable}$ = 0.001 is considered.

The problem is solved first by simultaneously (a) optimizing objectives 1 and 3 (see Eqs. (7c) and (7e)) and also (b) optimizing objectives 2 and 3 (see Eqs. (7d) and (7e)). The resulting Pareto-optimal set of the bi-objective optimization problems (a) and (b) are shown in Fig. 10a and b, respectively. Six solutions ($a, b, \ldots, f$) selected from these Pareto-optimal sets are compared in Table 6. The history profiles for the time-variant probability of failure,

**Table 5**
Replacement options and associated repair costs [4].

| Option identification | Option definition | Cost ($) |
|---|---|---|
| 0 | Do nothing | 0 |
| 1 | Replace deck | 225,600 |
| 2 | Replace exterior girders | 229,200 |
| 3 | Replace exterior girders and deck | 341,800 |
| 4 | Replace exterior and exterior–interior girders and deck | 390,000 |
| 5 | Replace superstructure | 487,100 |

redundancy index and LCC associated with three selected solutions (*a*, *e*, *f*) are plot in Fig. 11.

The observations and conclusions that can be drawn from these results are numerous. It is first worth mentioning that solutions from *a* to *b* in Fig. 10a and from *a* to *f* in Fig. 10b all require one replacement of the deck with the application time varying from 86.4 years (solution *a*) to 76.59 years (solution *b*) to 68.15 years (solution *f*). Also, all solutions from *c* to *d* in Fig. 10a require one replacement of the deck, exterior, and exterior–interior girders with application time varying from 76.59 years (solution *c*) to 61.30 years (solution *d*). Solutions *a* and *b* were obtained by both bi-objective optimization problems (a) and (b). On the other hand, solution *f* does not belong to the Pareto-optimal set of Fig. 10a because solution *b* offers similar value of $P_{f(sys),max}$ but for a lower LCC (see Table 6). In fact, solution *f* is the only solution that requires repair prior to reaching $P_{f(sys),max}$, which makes it dominated by solutions that require repair when $P_{f(sys),max}$ is reached. Meanwhile, solutions *c*, *d*, and *e* do not belong to the Pareto-optimal set of Fig. 10b because, as shown by Fig. 11 for solution *e*, they actually reduce the redundancy rather than increase it. This seemingly inconsistent result is justified by the fact that the chance that one of the girders fails before the deck was higher prior to the repair than after, where failure of a girder may not cause the failure of the system while failure of the deck may.

The history profiles in Fig. 11 show that the three selected solutions have their maintenance actions applied at the time of reaching $P_{f(sys),max}$ and/or $RI_{min}$. The other solutions act similarly. This proves the ability of the used mathematical formulation to select solutions where essential maintenance actions are applied only when $P_{f(sys),max}$ and/or $RI_{min}$ are reached. Although up to three repair applications were allowed, none of the solutions required more than two applications and most of them required one. This shows the robustness of the program and efficiency in choosing optimal solutions regardless of the number of MC variables used. The disconnection between solutions *b* and *c* in Fig. 10a is due to the shift from repair type 2 in solution *b* to repair type 4 in solution *c*. Solutions between *d* and *e* vary in both repair options and application times. However, the improvement achieved in $P_{f(sys),max}$ by

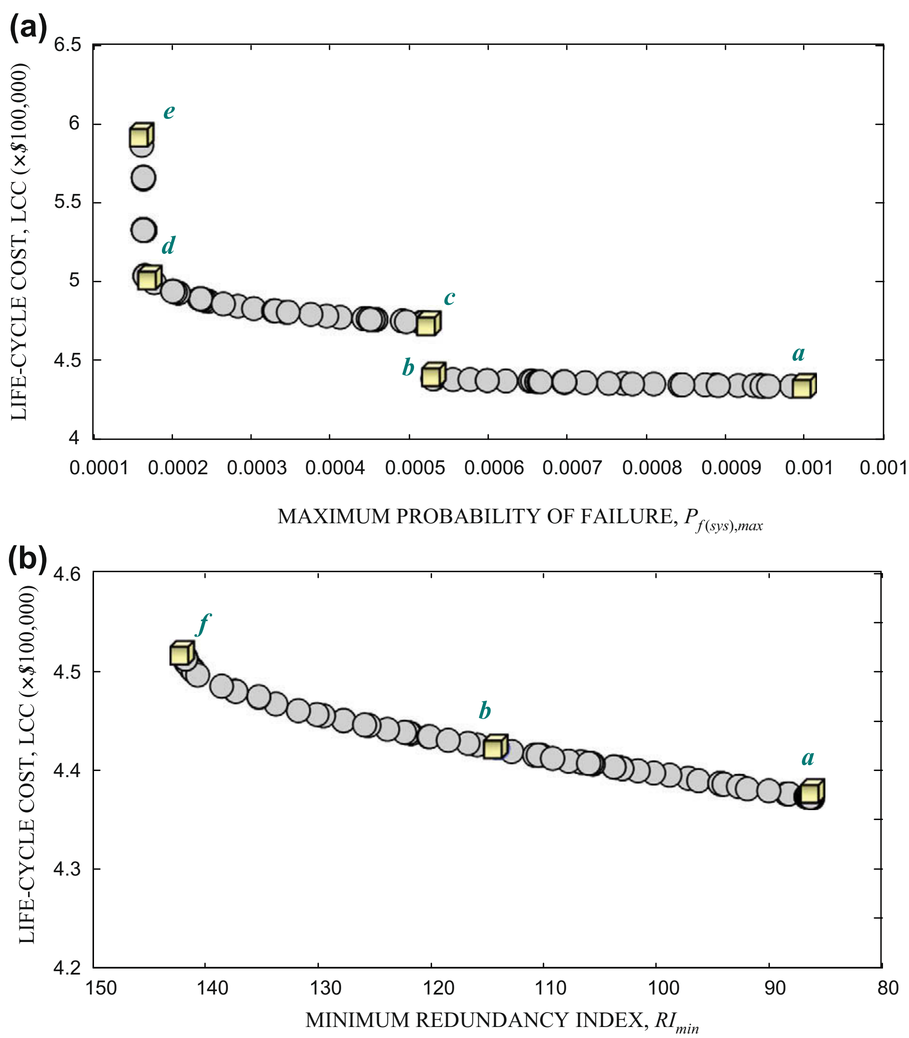


**Fig. 10.** Pareto-optimal sets of for Bridge E-17-AH for the bi-objective optimization problems: (a) minimize $P_{f(sys),max}$ and minimize LCC and; (b) maximize $RI_{min}$ and minimize LCC.

**Table 6**
Selected solutions from the bi-objective Pareto-optimal sets of Case 3 (See Fig. 11 for history profiles of solutions *a*, *e* and *f*).

| Solution | Objectives[a] | $MC_1$ | $t_1$ (year) | $MC_2$ | $t_2$ (year) | $MC_3$ | $t_3$ (year) | $P_{f(sys),max}$ | $RI_{min}$ | LCC ($) |
|---|---|---|---|---|---|---|---|---|---|---|
| *a* | 1 and 3 or 2 and 3 | 2 | 82.03 | 0 | – | 0 | – | 0.001 | 86.4 | 438,158 |
| *b* | 1 and 3 | 2 | 76.59 | 0 | – | 0 | – | 0.00052 | 115.4 | 443,300 |
| *c* | 1 and 3 | 4 | 76.59 | 0 | – | 0 | – | 0.00052 | 89.1 | 478,588 |
| *d* | 1 and 3 | 4 | 61.30 | 0 | – | 0 | – | 0.00016 | 92.0 | 508,836 |
| *e* | 1 and 3 | 4 | 59.25 | 5 | 90.00 | 0 | – | 0.00015 | 87.66 | 595,611 |
| *f* | 2 and 3 | 2 | 68.15 | 0 | – | 0 | – | 0.00052 | 142.3 | 452,447 |

[a] This column shows the two objectives considered in the bi-objective optimization problem that provided this solution. Objectives are numbered as: 1, minimize: $P_{f(sys),max}$; 2, maximize: $RI_{min}$; and 3, minimize: LCC.

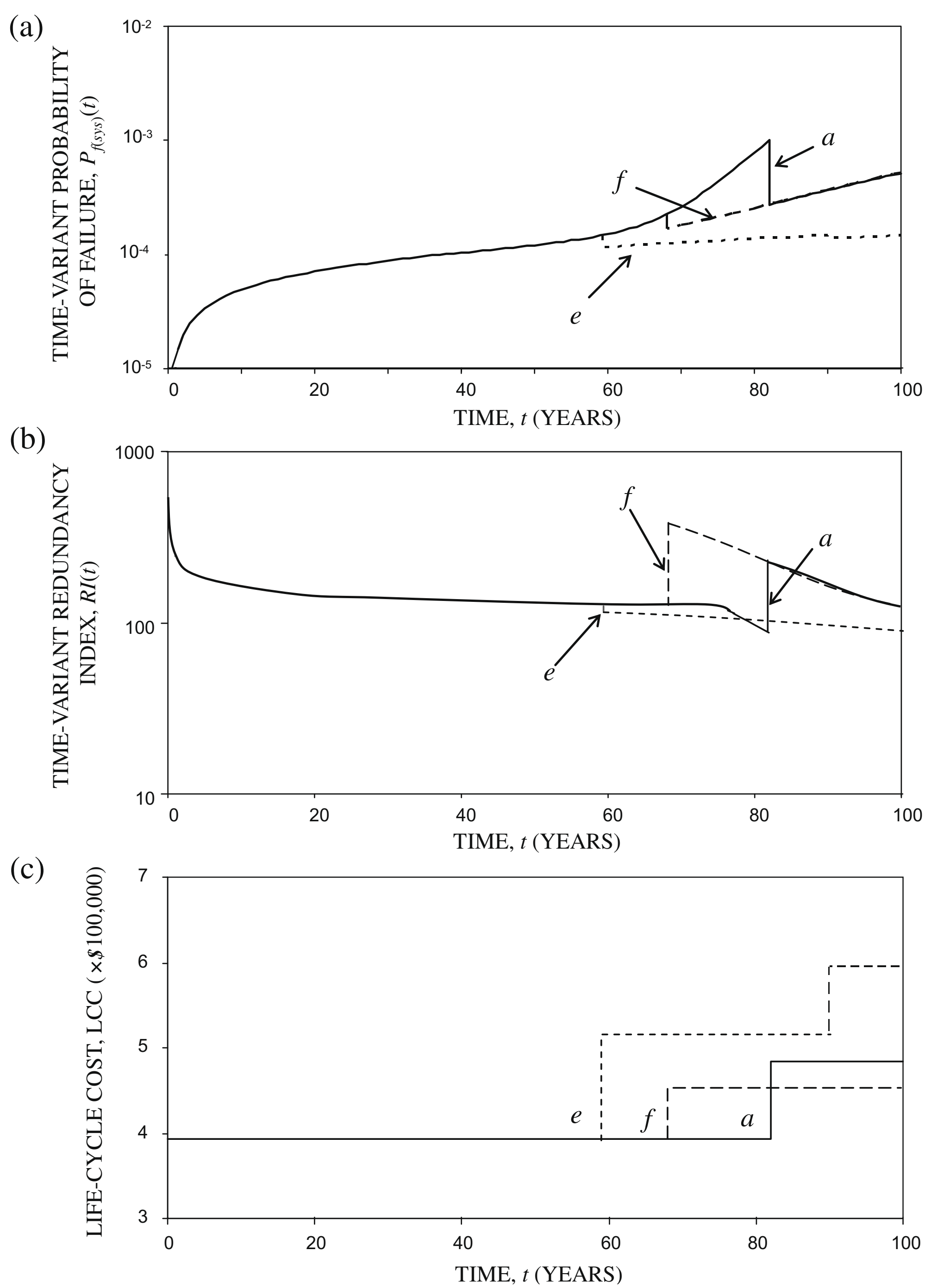


**Fig. 11.** History profiles for: (a) time-variant probability of failure $P_{f(sys)}(t)$; (b) time-variant redundancy $RI(t)$ and; (c) LCC for selected optimum solutions associated with Bridge E-17-AH.

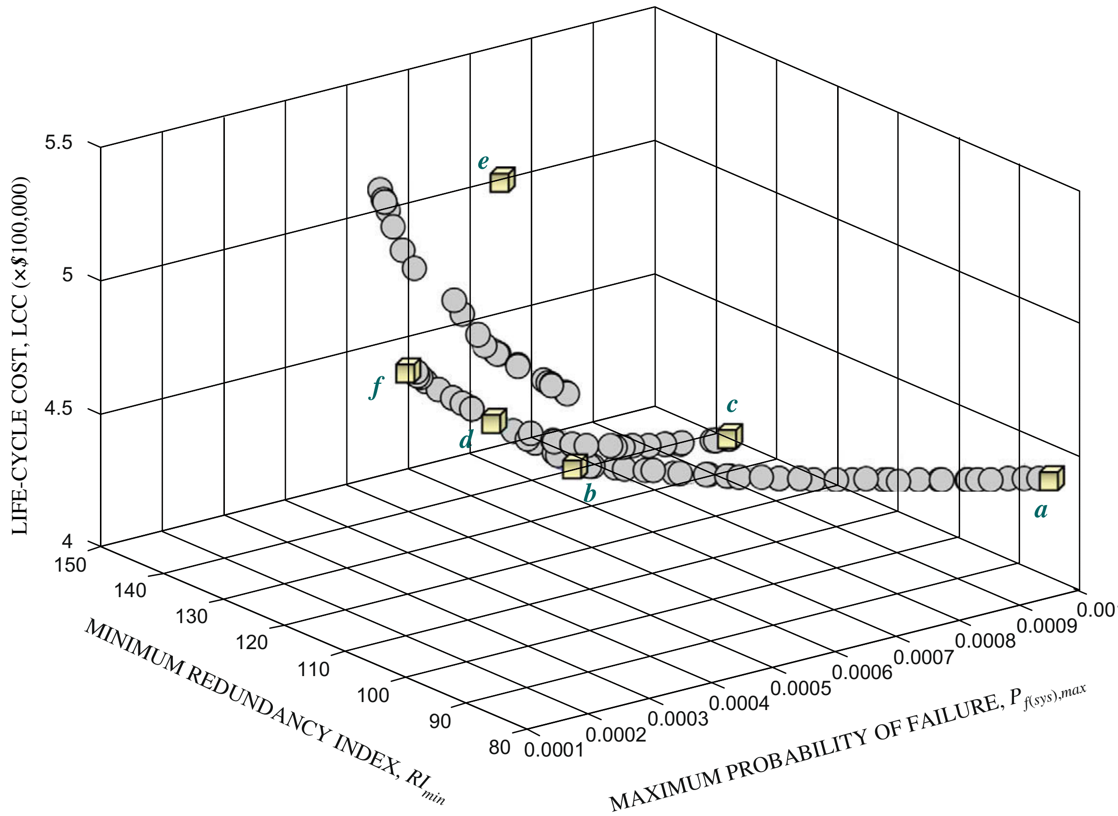


**Fig. 12.** Pareto-optimal set for the tri-objective optimization of Case 3 for Bridge E-17-AH.

solution *e* relative to solution *d* is insignificant compared to the price that has to be paid.

Compromises between these trade-off solutions are clear. For example, solutions *b* and *f* provide the same $P_{f(sys),max}$ but the latter provides higher $RI_{min}$ for a higher LCC. It can be observed that the difference in LCC between solutions *a* and *b* is $5142 and $P_{f(sys),max}$ is improved from 0.001 to 0.00052, whereas the difference in LCC between solutions *b* and *d* is $65,536 (about 13 times the difference between solution a and b) and $P_{f(sys),max}$ is improved from 0.0005 to 0.00016. In other words, improving $P_{f(sys),max}$ from 0.001 to 0.00016 costs about fourteen times improving it to 0.00052. This emphasizes the importance of the presence of such results at the time a choice of a maintenance strategy is made. The merit of investing in any of these solutions can be clearer when this information is available.

Finally, the problem is solved by simultaneously optimizing all three objectives 1, 2 and 3 (see Eqs. (7c)–(7e)). The resulting Pareto-optimal set of this tri-objective optimization problem is shown in Fig. 12. The six solutions $(a, b, \ldots, f)$ are also shown in the figure. Clearly, the LCC increases as $P_{f(sys),max}$ is decreased and/or $RI_{min}$ is increased. The shape of the Pareto-optimal set appears to take multiple disconnected curves. Each curve represents solutions with the same repair option and varying application time. With the exception of solution *e*, solutions $(a, b, \ldots, f)$ clearly belong to the tri-objective Pareto-optimal set. Evidently the tri-objective Pareto-optimal set offers a larger selection of solutions than can be offered by the bi-objective Pareto-optimal sets.

## 7. Conclusions

This paper presented a framework for an automated multi-objective optimization of structural maintenance strategies considering system reliability, system redundancy and the life-cycle cost as criteria. An approach to provide the optimization program the ability to optimally select what maintenance actions are applied, when they are applied, and to which structural components they are applied is presented. Two different strategies are proposed. Both strategies use maintenance code design variables by which the optimization program determines the maintenance options to apply at a time determined by another design variable. The greatest advantage of the proposed approach is its ability to avoid the application of maintenance interventions to structural components that are not critical, which is shown to provide more economy and prove its efficiency.

Genetic algorithms are used to solve the optimization problems. A book-keeping database and algorithm is added to the NSGA-II program in order to prevent the time consuming evaluation of the objective functions at design points already analyzed. A modification to the penalty constraint method is used in the handling of constraints in these problems.

System reliability and system redundancy have different properties and the time of applying a type of maintenance that best improves the system reliability is different from the time of applying the same type of maintenance that best improves the system redundancy. In fact, increasing the resistance of certain components in some structural systems may improve the system reliability but worsen the system redundancy. Therefore, the need to also incorporate the system redundancy in the optimization process is clearly important.

The ability of the optimization program to select the maintenance options to be applied provides economical and efficient solutions. In addition, the program is able to identify solutions that require less number of maintenance applications and only a specific type of maintenance when they were optimum, which shows the robustness of the program and efficiency in choosing optimal solutions regardless of the number of maintenance code variables used. Furthermore, the program is able to select essential and preventive maintenance types to different parts of the structure at the same time.

One of the main advantages of treating the system reliability and system redundancy and the life-cycle cost as criteria in a multi-objective optimization problem is to analyze the interactions between the cost and the system performance. As a matter of fact, it was found by the results of the numerical examples that the difference in life-cycle cost required to improve the worst value of a performance indicator is not always proportional to the improvement achieved. This emphasizes the importance of the presence of such results at the time a choice of a maintenance strategy is made by decision makers.

## Acknowledgments

The support by several Grants from the Commonwealth of Pennsylvania, Department of Community and Economic Development, through the Pennsylvania Infrastructure Technology Alliance (PITA) is gratefully acknowledged. The support of the National Science Foundation through Grant CMS-0639428 to Lehigh University is also gratefully acknowledged. Finally, the support from the Federal Highway Administration under cooperative agreement DTFH61-07-H-0040, and the support from the Office of Naval Research under award N-00014-08-0188 are gratefully acknowledged. The opinions and conclusions presented in this paper are those of the authors and do not necessarily reflect the views of the sponsoring organizations.

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