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## Structural damage assessment using FRF employing particle swarm optimization



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#### ABSTRACT

This paper evaluates the use of Frequency Response Function (FRF) with the help of Particle Swarm Optimization (PSO) technique, for structural damage detection and quantification. The robustness and efficiency of the above method has been established after comparing results between the two methods namely Genetic Algorithm (GA), and PSO, considering natural frequencies as response quantities. The performance of these methods has been evaluated for beam and plane frame structures with various damage scenarios. FRF based damage detection technique is employed subsequently along with PSO. It is observed that the use of FRF as response of damaged structure has led to better accuracy, since it contains data related to mode shape in addition to natural frequencies.

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#### 1. Introduction

The early detection of structural damage is one of the major challenges in the aerospace, civil and mechanical industries. The early damage detection helps in reducing the down time, to evaluate the safety and to prevent catastrophic events. Also it plays a major role in real time monitoring and reporting. Damage may be defined as any deviation in the structure's original geometric or material properties that may cause undesirable stresses, displacements, or vibrations in the structure. Damage in the structure may be caused due to various reasons, such as: deterioration and degradation of structures, fatigue failure, cracks inside the material due to manufacturing defect, accidental damage, earthquake in civil structures, flutter effect in aerospace, mechanical vibrations, etc.

A number of researchers have proposed several methods in literature during last two decades to detect and assess damage in various types of structures. Yuen [1] examined changes in the mode shape and mode-shape-slope parameters. Farhat and Hemez [2] minimized the norm of the Residual Force Vector (RFV) by updating both stiffness elemental and mass parameters in a sensitivity-based algorithm. Yang [3] proposed a numerical technique for structural damage detection using modal residual force criteria and matrix disassembly technique. Pandey and Biswas [4] identified the damage in beam-type structures using changes in the flexibility matrix of the structure. Salawu and Williams [5] using a comprehensive literature review, established some conclusion about damage detection through changes in frequency. In general frequency changes alone do not necessarily imply the damage existence.

Sampaio et al. [6] used a FRF curvature method for detection, localization, and extent of damage in the bridge structure. Lee and Shin [7] used FRF data measured from the damaged structure as the input data. Link [8] in his dissertation presented the method of using FRF in structural damage detection in frame. He used Orthogonal matching pursuit for damage location. Nozarian and Esfandiari [9] presented an element level structural damage identification technique using FRF of the

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structures. Golafshani et al. [10] applied the FRFs for damage detection of shear building. He used concept of a minimum rank perturbation theory to detect the damage location. Salehi [11] used both imaginary and real part of FRF shapes in damage detection of a beam. The damage is detected using Gapped Smoothing Method (GSM) using the change in FRF shapes due to damage.

Another challenge in damage detection is to use efficient optimization tool to overcome computational time and cost. The use of different soft computing tools like Neural Network (NN), GA, PSO, and some hybrid algorithms proved to be effective in handling the inverse problem of damage assessment. Wu et al. [12] were among the very first to use NN in damage detection employing back-propagation algorithm in their neural network architecture with one and two hidden layers to detect damage states in a three-story frame. NN was utilized to assess overall damage at each floor in composite frames and in a steel structure caused by seismic loading [13]. Jiang et al. [14] performed structural damage detection by integrating data fusion and probabilistic neural network. Mares and Surace [15] presented a method of location and quantification of the extent of the damage with GA using residual force method, which is based on conventional modal analysis theory. He et al. [16] presented a method based on GA for the solution of the inverse problems pertaining to the single crack detection in a shaft configuration. Perera and Torres [17] presented GA based damage detection and assessment methodology based on the changes in frequencies and mode shapes of vibration of a structural system. Gomes and Silva [18] used a real coded GA tool and modal sensitivity analysis technique for damage assessment by changes in natural frequencies. Sahoo and Maity [19] proposed one hybrid neuro-genetic algorithm in order to automate the design of neural network for damage detection in different type of structures. Majumdar et al. [20] used ant colony optimization technique to assess damage in the truss structures from changes in natural frequencies.

The PSO algorithm was first proposed by Kennedy and Eberhart [21]. As compared to other robust design optimization methods PSO is more efficient, requiring fewer number of function evaluations, while leading to better or the same quality of results [22]. Furthermore, it is a population-based algorithm, so it can be efficiently parallelized to reduce the total computational effort. Recently, the PSO has been proved useful on diverse engineering design applications such as logic circuit design [23], control design [24] and also power systems design [25]. Applications in structures had been done in the area of topology optimization [26], and also in structural shape optimization [27], with promising results in such structural design applications. An improved PSO algorithm (IPSO) and its application for structural damage detection have been presented in [28,29]. For multi-objective inverse damage detection problems application of PSO and GA has been evaluated [30]. In another study, Begambre and Laier [31] proposed a hybrid Particle Swarm Optimization—Simplex algorithm (PSOS) for structural damage identification using frequency domain data. A two stage structural damage detection using PSO and adaptive neuro-fuzzy inference system has been proposed [32]. A Multi-Stage Particle Swarm Optimization (MSPSO) for accurately detecting the structural damages has been introduced by Seyedpoor [33]. Kanovic et al. [34] presented a generalized particle swarm optimization algorithm with application to fault detection in electrical machines.

In this paper a robust and efficient method for damage detection has been presented, where FRF is used as input response in objective function, and PSO algorithm is used to predict the damage. FRF is used to detect damage, as it adds an advantage by including both natural frequency and modal characteristics of the structure. The results are compared between PSO and GA with frequency as input response [35]. Efficacy of these tools has also been tested on beam and plane frame structures up to three elements damage using first six natural frequencies and FRF's. Natural frequencies and FRF's for these structures (both damaged and undamaged) have been computed using FE program written in MATLAB environment. It is observed from the study that PSO is robust and competent algorithm than GA, for multi damage detection and identification. Also the advantage of using FRF over natural frequency in locating and quantifying damage has been presented with the help of numerical results.

#### 2. Mathematical backgrounds

#### 2.1. Frequency response function

A frequency response function expresses the structural response to an applied force as a function of frequency. The response can be displacement, velocity, or acceleration. The relationship between Force and Response can be represented by the following equations

$$X(\omega) = H(\omega).F(\omega) \tag{1}$$

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \tag{2}$$

where  $H(\omega)$  is FRF,  $X(\omega)$  is response in frequency domain and  $F(\omega)$  is external force expressed in frequency domain. Here the FRFs are taken as the ratio of Fourier Transforms of the time domain response and input forces.

Equation of motion for multi degree of freedom system without damping in frequency domain can be expressed as

$$[M]\{\ddot{X}(\omega)\} + [K]\{X(\omega)\} = \{P(\omega)\}\tag{3}$$

where [M] and [K] are the mass, and stiffness matrices of the structure respectively;  $\{P(\omega)\}$  is the vector of externally applied loads; and  $\{X(\omega)\}$ ,  $\{\dot{X}(\omega)\}$  and  $\{\ddot{X}(\omega)\}$  are displacement, velocity and acceleration vectors in frequency domain respectively. The FRF for n-DOF system in modal domain without damping is given by

$$H_{jk}(\omega) = \sum_{r=1}^{n} \frac{\phi_{jr} \cdot \phi_{kr}}{(\omega_r^2 - \omega^2)} \tag{4}$$

 $\phi_{jr}$  is the jth element of rth structural mode shape,  $\omega_r$  is the rth resonance frequency,  $\eta_r$  is the rth modal damping loss factor,  $\omega$  is working frequency. The kth FRF shape is defined as the deflection of the structure in degrees of freedom considered due to a unit harmonic excitation at kth degree of freedom. And n is the number of natural frequencies considered for calculation of FRF.

#### 2.2. FE formulation

The FE model for a beam has been developed using two dimensional, two nodded elements, having two degree of freedom  $(\delta_y, \theta_z)$  per node. Considering the beam rigidity (*EI*), and Area (*A*) as constant along the element, the element stiffness and element mass matrices are given by

$$[K]_{e} = \frac{EI}{l^{3}} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^{2} & -6l & 2l^{2} \\ -12 & -6l & 12 & -6l \\ 6l & -2l^{2} & -6l & 4l^{2} \end{bmatrix}$$
 (5)

$$[M]_{e} = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^{2} & 13l & -3l^{2} \\ 54 & 13l & 156 & -22l \\ -13l & -3l^{2} & -22l & 4l^{2} \end{bmatrix}$$
 (6)

Similarly for plane frame structure, two nodded beam element having three degree of freedom  $(\delta_x, \delta_y, \theta_z)$  per node is considered, and the corresponding element stiffness and mass matrices in global coordinate are given by

$$[K]_e = [T]^{-1}[k]_e[T] \tag{7}$$

$$[M]_{a} = [T]^{-1}[m]_{a}[T]$$
 (8)

where

$$[k]_e = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0\\ 0 & \frac{12El}{l^3} & \frac{6El}{l^2} & 0 & -\frac{12El}{l^3} & \frac{6El}{l^2}\\ 0 & -\frac{6El}{l^2} & \frac{4El}{l} & 0 & \frac{6El}{l^2} & \frac{2El}{l}\\ \frac{EA}{l} & 0 & 0 & \frac{EA}{l^2} & 0 & 0\\ 0 & -\frac{12El}{l^3} & -\frac{6El}{l^2} & 0 & \frac{12El}{l^3} & -\frac{6El}{l^2}\\ 0 & \frac{6El}{l^2} & \frac{2El}{l} & 0 & -\frac{6El}{l^2} & \frac{4El}{l} \end{bmatrix};$$

$$[m]_e = \frac{\rho A l}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22 l & 0 & 54 & -13 l \\ 0 & 22 l & 4 l^2 & 0 & 13 l & -3 l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13 l & 0 & 156 & -22 l \\ 0 & -13 l & -3 l^2 & 0 & -22 l & 4 l^2 \end{bmatrix}; \quad [T] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where l is the length of element, and  $\rho$  is the mass density of the material used, C and S are cosine and sine of the element angle  $\theta$  considered from positive x-axis respectively.  $[k]_e$  and  $[m]_e$  are the element stiffness and mass matrices calculated in elemental coordinate system respectively. [T] is the transformation matrix for converting elemental matrices from local coordinate system to global coordinate system.

#### 2.3. Damaged FE model

Considering one damage parameter per structural element, the updated stiffness matrix of an element can be described as

$$[K]_{ed} = (1 - \infty)[K]_e$$
 (9)

where  $[K]_e$  undamaged element stiffness matrix,  $[K]_{ed}$  damaged element stiffness matrix, and  $\alpha$  is the damage ratio or index, which is always between 0 and 1.  $\alpha$  is equal to zero and one indicates no damage and completely damaged in the corresponding element respectively. This study assumes the linear behaviour of the structure throughout, even after inflicting 85% damage locally to an element.

#### 2.4. FRF based objective function

Damage parameters are obtained by formulating an optimization problem in which the error between the computed and actual receptence function of damaged structure is minimized. The error function for a given structural test is given by

$$E_{q}(\alpha) = \sum_{a=1}^{R} \sum_{p=1}^{M} \frac{|H_{ak}(\omega_{p}, \alpha) - H_{ak}^{m}(\omega_{p})|}{\max(H_{ak}^{m}(\omega_{p}))}$$
(10)

where  $\alpha$  is the variable indicating the vector of damage ratio for elements in the structure, k is the excitation degree of freedom, p is the indices for the excitation frequencies. R is the total number of responses considered in the structure, M is the total number of excitation frequency, and  $H_{ak}^m$  is the actual receptence and  $H_{ak}$  is the computed receptence. The damage parameters are often obtained by solving objective function given below:

Minimize 
$$\frac{1}{N} \sum_{q=1}^{N} E_q(\alpha)$$
, Subject to  $0 < \alpha < 1$  (11)

where *N* is the number of runs performed on the structure. In this paper the responses are calculated using FE model, and hence *N* is kept equal to one throughout.

In the damage detection algorithms (GA/PSO), we have optimized to determine the element number as well as its magnitude of stiffness reduction, i.e., location and extent of damage. All the elements are considered as having possible reduction in stiffness, hence we have that many number of variable as that of number of elements in the structure. Then algorithm will assess the damage (actual reduction of stiffness) in the elements, while trying to optimize to the response of the damaged structure. The response of the damaged structure which is input to the damage detection algorithm has been generated using FE code, where the location and extent were known for the response simulation. The nature of the damage considered here is the overall stiffness reduction of that particular element.

#### 2.5. FRF with noise

In practical measurements of FRF of structures, there can be a possibility of errors, which leads to noise in FRF curve. In order to account for these errors in the measurements; normally distributed random noise can be added to the simulated FRF data with zero mean and a variance of 1. The FRF including noise is obtained from the FRF without noise using the following equation:

$$\overline{H}_{ak} = H_{ak}(1 + 2 * N_L(rand - 0.5))$$
 (12)

where  $\overline{H}_{ak}$  is akth measured FRF with noise is the akth measured FRF without noise,  $N_L$  is the noise level (e.g., 0.05 relates to a 5% noise level) and rand is the random number between 0 and 1.

#### 2.6. Frequency based objective function

The objective function based on just the natural frequencies is given as

$$F(x) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f_i^* - f_i)^2}$$
 (13)

where  $f_i^*$  = damaged frequencies obtained by initial FE analysis,  $f_i$  = calculated frequencies from simulation of the structural dynamic behaviour using FE analysis, n = no of input response parameters (natural frequencies), and in the context of this paper n = 6.

#### 3. Optimization algorithms

In the present study basic models of GA [35] and Particle Swarm Optimization (PSO) have been tested and compared for their efficacy in detecting single element, two elements and three elements damage in beam and plane frame structures.

#### 3.1. Particle swarm optimization

In PSO algorithm, each solution is based on concept of the bird flock and is referred to as a particle. In this framework the birds, besides having individual intelligence, also develop some social behaviour and coordinate their movement towards a destination.

Initially, the process starts from a swarm of particles, in which each of them generated randomly contains a solution to the problem, and then one searches the optimal solution by iteration. The ith particle is associated with a position in an S-dimensional space, where S is the number of variables involved in the problem; the values of the S variables which determine the position of the particle represent a possible solution of the optimization problem. Each particle i is completely determined by three vectors namely its current position  $X_i$ , its best position reached in previous cycles  $Y_i$ , and its velocity  $V_i$ . Thus,

Current position,  $X_i = (x_{i1}, x_{i2}, ..., x_{is})$ Best previous position,  $Y_i = (y_{i1}, y_{i2}, ..., y_{is})$ Flight velocity,  $V_i = (v_{i1}, v_{i2}, ..., v_{is})$ .

This algorithm simulates a flock of birds which communicate during flight. Each bird looks at a specific direction (its best ever attained position  $Y_i$ ), and later, when they communicate among themselves, the bird which is in the best position is identified. With coordination, each bird moves also towards the best bird using a velocity which depends on its present velocity. Thus, each bird examines the search space from its current local position, and this process repeats until the bird possibly reaches the desired position. Note that this process involves as much individual intelligence as social interactivity; the birds learn through their own experience (local search) and the experience of their peers (global search).

In each cycle, one identifies the particle which has the best instantaneous solution to the problem; the position of this particle subsequently enters into the computation of the new position for each of the particles in the flock. This calculation is carried out according to

$$X_i' = X_i + V_i' \tag{14}$$

where the primes denote new values for the variables, and the new velocity is given by

$$V'_{i} = w * V_{i} + C_{1} * rand_{1} * (Y_{i} - X_{i}) + C_{2} * rand_{2} * (Y_{i}^{*} - X_{i})$$

$$(15)$$

where  $C_1$  and  $C_2$  are two positive constants which are called learning factors or rates;  $rand_1$  and  $rand_2$  are two independent random numbers between 0 and 1; w is a factor of inertia in order to control the impact which the histories of velocities has on the current velocity. The w factor may vary from one cycle to the next as shown in Eq. (15).

$$w = w_{max} - Gen * (w_{max} - w_{min})/Gen_{max}$$

$$\tag{16}$$

As w controls the balance between global and local search, it is recommended to decrease the value of w linearly with time, usually in a way to first emphasize global search and then, with each cycle of iteration to prioritize local search. Finally,  $Y_i^*$  is the best present solution of all  $Y_i$  solutions.

The particles propagate through the solution space and are influenced by the best solution which was previously found individually, as well as the best particle of the entire swarm. Thus, in Eq. (14), the second term represents the cognition or intrinsic knowledge of particle i, since it compares its current position  $X_i$  with its best previous position  $Y_i$ . The third term in this equation represents the social collaboration between the particles: it measures the difference between the current position  $X_i$  and the best solution of the entire system found up to the moment  $Y_i^*$ .

To control any change in the particle velocities, we introduce the respective upper and lower limits:

$$V_{min} < V_i < V_{max} \tag{17}$$

Once the current position is calculated, the particle directs itself towards a new position. A schematic representation of the PSO algorithm is shown below in Fig. 1.

The termination condition for the algorithm is when either the maximum number of iterations is reached or the objective function assumes a preset value. In this work, the algorithm searches for a solution until a certain number of iterations is reached without any further improvement of the best cost obtained.

The PSO parameters used in this study are set as follows after some trails to suit the problem: Initial w = 0.7;  $c_1 = 2.1$ ;  $c_2 = 1.9$ ;  $w_{max} = 0.9$ ;  $w_{min} = 0.2$ ;  $V_{max} = 0.75$ . Population size is kept 50 throughout, but maximum number of generations will vary for different damage scenario, and is mentioned under numerical example section.

#### 4. Numerical examples with results

The formulation for structural damage detection using FRF and PSO outlined above is applied on two different types of structures, such as cantilever beam and plane frame. The comparison of the same has been done, between damage detection using frequency and PSO, and using frequency and GA [35]. First six natural frequencies and corresponding FRF's are used as

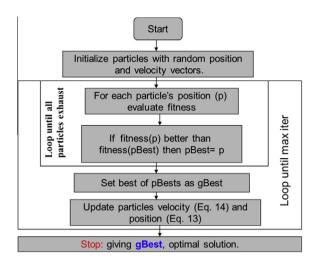


Fig. 1. Schematic representation of PSO algorithm.

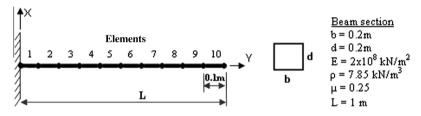


Fig. 2. Cantilever beam.

input response in predicting different damage scenarios (single, two, three elements damage). Input responses for damaged structures are obtained using FE programs developed in MATLAB environment.

#### 4.1. Cantilever beam

The cantilever beam considered for the assessment of damage is shown in Fig. 2. The dimensions and material properties of the beam used are also cited in same figure. The response of the beam is evaluated assuming negligibly small damping present in the structure. The performance of PSO in detecting a single, two and three elements damage cases of beam using first six natural frequencies and FRF's are studied and compared with GA and they are discussed below. FRF analysis has been carried out by applying vertical unit force at node 10, with frequency range of 50–14,000 Hz at 50 Hz frequency increment.

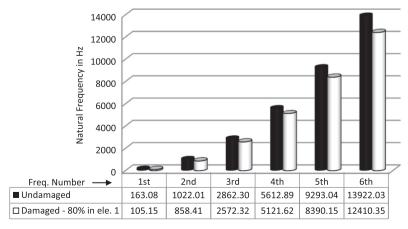


Fig. 3. Natural frequencies for undamaged beam and damaged beam.

This range is selected based on the first six natural frequencies. And the frequency increment is chosen after studying its feasibility in getting results of considerable accuracy with minimum iterations. Responses are considered at nodes 7, 8, and 9 in vertical degrees of freedom for all the cases under this section. The first six natural frequencies for undamaged beam and damaged beam with 80% stiffness reduction in element 1 are shown in Fig. 3. Also the FRF curves for undamaged and damaged beams are shown in Fig. 4.

In case of single element damage, 80% damage has been considered in element 1, and the predicted damage using frequencies and FRF's are compared with PSO and GA are as shown in Fig. 5. The maximum generation of 50 is used for both GA and PSO for comparison purpose.

Fig. 6 shows the performances of PSO and GA in detecting two elements damage cases using first six natural frequencies and FRF's for cantilever beam. The maximum number of generations used for both GA and PSO is 300. First element is damaged by 90% and fifth element is damaged by 70%.

The comparisons between the accuracies of PSO and GA in predicting three element damages for beam are also presented in Fig. 7. Maximum 400 generations are used in this case for both GA and PSO. The first element is damaged by 90%, fifth element is damaged by 70%, and eighth element is damaged by 55%.

From all the above cases it can be observed that PSO is quite capable of predicting correct damage using first six natural frequencies as input response parameters. Even though GA also predicts the damaged element, but it gives false idea of dam-

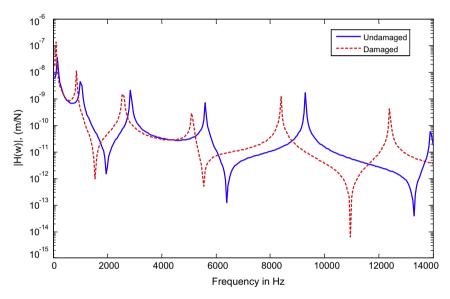


Fig. 4. FRF at node 7 in vertical direction, for undamaged beam and damaged beam (80% damage in element 1).

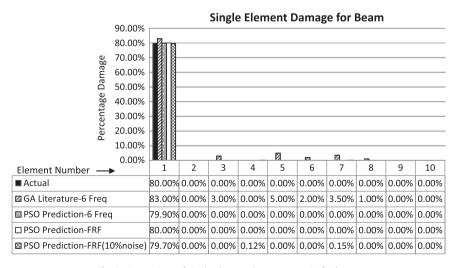


Fig. 5. Comparison of single element damage scenario for beam.

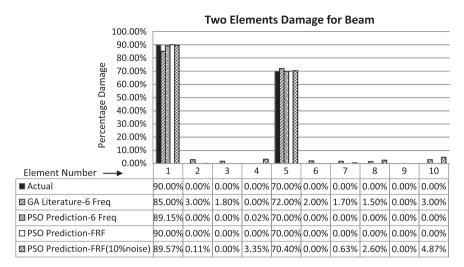


Fig. 6. Comparison of two elements damage scenario for beam.

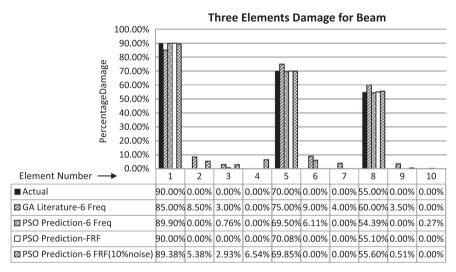


Fig. 7. Comparison of three elements damage scenario for beam.

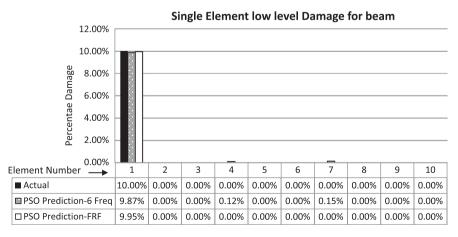


Fig. 8. Single element low level damage scenario for beam.

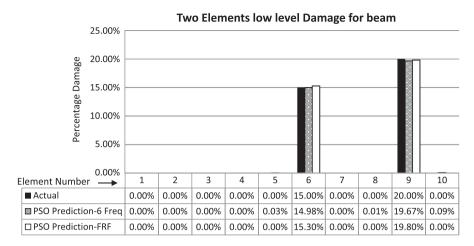


Fig. 9. Two elements low level damage scenario for beam.

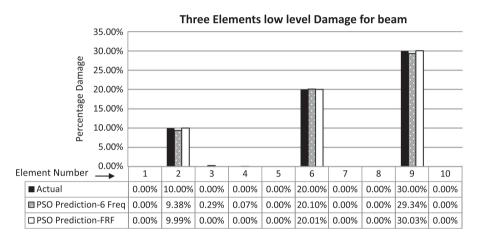


Fig. 10. Three elements low level damage scenario for beam.

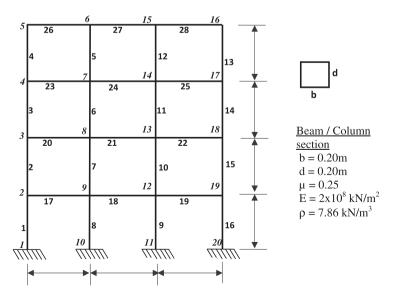


Fig. 11. Two dimensional plane frame structure.

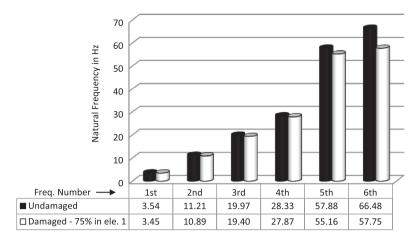


Fig. 12. Natural frequencies for undamaged frame and damaged frame.

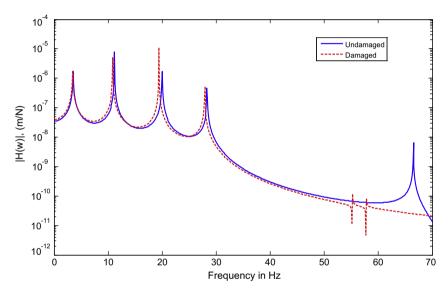


Fig. 13. FRF at node 2 in horizontal direction, for undamaged frame and damaged frame (75% damage in element 1).

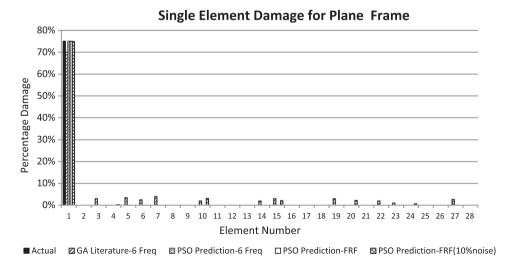


Fig. 14. Comparison of single element damage scenario for plane frame.

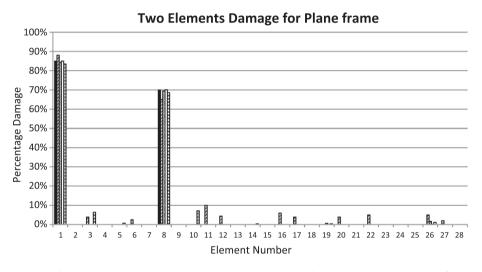
age in other elements. In addition, from Figs. 3–7 it is quite evident that the use of FRF instead of frequency increases the accuracy of damage prediction. Thus it can be concluded that PSO with FRF is performing better than GA with frequency or PSO with frequency as a whole.

All the above different damage scenario cases are also evaluated for the capability of algorithm to detect damage in noisy environment. The PSO algorithm is also able to predict the damage with 10% noise level in FRF (Eq. (12)), with negligible false damages in some of the other elements, as can be seen in Figs. 5–7.

In order to check efficiency of the PSO algorithm for predicting the lower level of damages: single, two and three elements damage cases are also evaluated for beam structure and presented as shown in Figs. 8–10 respectively. Also the maximum generation size of 150, 200 and 400 are used in respective cases. It is observed that PSO with FRF is successfully able to detect the damage and its extent without false damage in other elements except for two elements damage case. It is also to be mentioned that increased generation size reduces the false damage prediction.

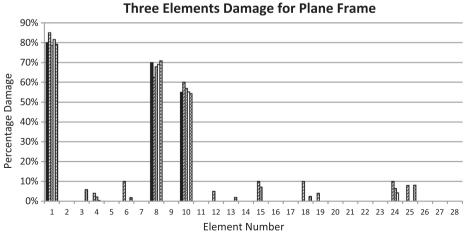
#### 4.2. Plane frame structures

The dimensions and material properties of the plane frame structures used are described in Fig. 11. The performance of the developed algorithm is studied with single, two and three elements damages of the frame. FRF analysis has been carried



■ Actual ② GA Literature-6 Freq ③ PSO Prediction-6 Freq ☐ PSO Prediction-FRF ⑤ PSO Prediction-FRF(10%noise)

Fig. 15. Comparison of two elements damage scenario for plane frame.



■ Actual ☑ GA Literature-6 Freq ☑ PSO Prediction-6 Freq ☐ PSO Prediction-FRF ☑ PSO Prediction-FRF(10%noise)

Fig. 16. Comparison of three elements damage scenario for plane frame.

out by applying horizontal unit force at node 5, with frequency range of 1–70 Hz, at 5 Hz frequency increment. This range is selected based on the first six natural frequencies of the plane frame considered and the frequency increment is chosen after studying its feasibility in getting results of considerable accuracy with minimum iterations. Responses are considered at nodes 2, 3, and 4 in horizontal degrees of freedom. The first six natural frequencies for undamaged frame and damaged frame with 75% stiffness reduction in element 1 are shown in Fig. 12. Also the FRF curves for undamaged and damaged beams are shown in Fig. 13.

The comparison on the performance of assessing single element damage for plane frame structure with 75% damage in 1st element is shown in Fig. 14. Damage was predicted using both PSO and GA, by using first six natural frequencies and FRF's as input responses. Maximum generations of 50 have been used in this case for the accuracy comparison with GA [35]

In case of two elements damage scenario for plane frame, the predicted element damage have been shown in Fig. 15. The 1st element is damaged by 85% and the 8th element is damaged by 70% of stiffness. Maximum generation number has been kept as 300. Fig. 15 shows different percentage damage predicted with respect to each element for various cases, such as GA and six frequencies, PSO and six frequencies, and PSO and FRF.

Fig. 16 shows the performances of PSO and GA for detecting three elements damage cases for plane frame using first six frequencies and FRF. In this case 1st element is damaged by 80%, 8th element is damaged by 70% and 10th element is damaged by 55% of reduction in stiffness. The maximum 500 generations are used in both GA and PSO algorithms for the purpose of their performance.

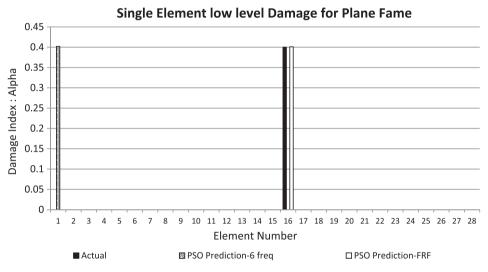


Fig. 17. Single element low level damage scenario for plane frame.

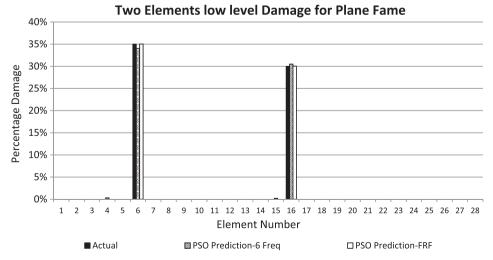


Fig. 18. Two elements low level damage scenario for plane frame.

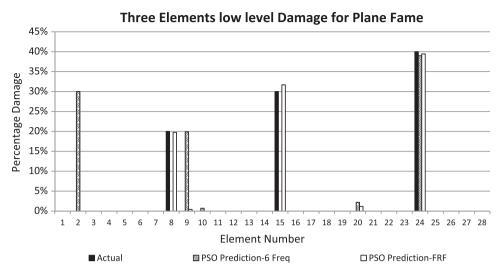


Fig. 19. Three elements low level damage scenario for plane frame.

From the above different damage scenarios for plane frame, the common observations are as fallows. Both GA and PSO algorithms were able to locate the damage in the element. Using GA with frequency has yielded more prediction error in comparison with PSO with frequency. But PSO with FRF is able to locate all the element damages with adequate accuracy. Even though false damages were observed in some elements for all the cases, they are more in case of GA than PSO. The use of PSO with FRF brings down the number of false damages to a considerable extent, as it can be especially observed from Fig. 16.

In case of plane frame also, above damage cases are evaluated in noisy FRF environment of 10%. Though some false damages are identified, the algorithm is able to detect the actual damages with very less error, as can be seen in Figs. 14–16.

The lower level single, two and three elements damage cases are evaluated for plane frame structure and the results are as shown in Figs. 17–19 respectively. Also the maximum generation size of 200, 400 and 700 are used in respective cases. In case of single element and three elements damage, the PSO with frequency as response parameter, predicted damage in other symmetrical elements than the actual damaged element. These kinds of result are expected due to symmetrical position of the elements and they contribute same nature of effect on the overall frequency of the structure, whereas in case of PSO with FRF as response parameter, there is no wrong prediction even due to presence of symmetrical elements, because FRF contains the frequency as well as mode shape information.

#### 5. Conclusions

A simple but robust methodology is presented here to locate and quantify damages in the structures using FRF as input response in PSO algorithm. The algorithm is tested on simple cantilever beam, as well as plane frame structures with different damage scenarios. Accuracy and the efficacy of the present algorithm have been compared with PSO and GA considering frequency as input response. Also the efficiency of algorithm is tested for low level damage and ability to detect damage in noisy environment. The input responses for damaged structure have been obtained from the FE codes developed in MATLAB environment.

It is observed that PSO is more reliable in predicting damages than GA. Moreover PSO is quite easy to implement, which is known for its quick convergence and excellent local search capability. PSO is also consistent in providing solutions over many runs for different damage cases. These observations prove PSO to be a robust and efficient algorithm for damage detection.

PSO with FRF as input response shows better accuracy than PSO with frequencies. This is because FRF contains data related to mode shape in addition to natural frequencies. Unnecessary prediction of false damages is reduced with the help of PSO than GA. The use of FRF can bring down the false damage prediction in the structure.

Damage detection using 10% noisy FRF data has been evaluated for most of the case studies. Algorithm was able to detect the damage in the structures with acceptable accuracy, though negligible false damages in some of the other elements are detected. Low level damage cases up to 10% damage for beam, up to 20% damage for plane frame have been explored and damages away from the support cases are also considered. Numerical case studies reveal that FRF and PSO based damage detection methodology presented here is relatively reliable. Also it is observed that only use of frequency data may lead to damage prediction in other symmetrical elements than the actual damaged element for a symmetric structure. Such wrong prediction can be avoided by the use of FRF as response parameter.

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