Active vibration control of a smart structure

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Abstract

Mechanical vibrations induced in flexible structures such as beams and plates have been researched through various optimization functions and control theories to optimize transient response dynamic characteristics. Piezoelectric materials sensors/actuators have been used to reduce and control these vibrations.
However, stiffened plate has not been investigated and it is a main and important part of aerospace and light weight mechanical structures. The optimal placement of PZT sensor/actuator pairs on a stiffened plate to determine the optimal controller design and to optimise cost and system response have not studied yet. So, the objective of this project is to find the optimal location of sensor/actuator pairs and design optimal controller for beam, plate and stiffened plate to suppress mechanical vibrations theoretically and experimentally. The finite element method will be use to model the structure with piezoelectric material. The genetic algorithm will apply to find optimal placement of a number of piezoelectric sensor/actuator pairs, with objective functions of minimize control force, displacement, velocity, and the error. However, velocity feedback and linear quadratic regulator closed loop control will apply to study the transient dynamic response for the first six modes.

1. Introduction

The development of high strength to weight ratio of mechanical structures are attracting engineers to build light weight aerospace structures to combine loading capacity and fuel consumption, as well as to build tall buildings and long bridges. However, the flexible light weight structures lead to more complicated vibration problems. Traditionally, vibrations have been controlled using passive techniques. Passive vibration reduction is achieved by adding mass damping and stiffness in suitable locations on a structure to reduce vibrations. However these techniques lead to increased structure weight and low response. So, the mechanical vibrations induced in light weight aerospace and large-scale flexible structures have attracted engineers to investigate and develop materials to suppress these vibrations. The development of piezoelectric material has been used as sensors and actuators to control vibrations because it has attractive properties, such as low mass, high actuating force and fast response. The locations of sensor/actuator on a structure have been effected on the performance of control system. So, the positions of sensors and actuators play an important role, a misplaced of sensors and actuators causes lack of observability, controllability and spillover[1]. Optimal placement of sensors actuators has been investigated using finite element method to model structure and piezoelectric materials while various objective functions and genetic algorithm are used to optimize the locations. Then, transient response analysis for beams and plates are studied using output state, velocity feedback closed loop and optimal control theories. In addition, final results have been achieved experimentally or by ANSYS package. In this proposal, beams, plates and plates stiffened by beams will be investigated to design and optimize cost of controller system. Since, the first six fundamental natural frequencies have more effect than higher frequencies; therefore, the control will design according first natural frequencies. In addition, the optimization will depend on adding four sensor/actuator pairs to the structure. The methodology to do this project can achieve through the following points; firstly, Finite element method will be applied to model the structure including piezoelectric materials. Secondly, genetic algorithm will be used to find the optimal location of sensor/actuator pairs on the structure by build a MATLAB computer program to optimize sensor/actuator pairs, control force, system response, and output error for the first six modes. Thirdly, negative gain velocity feedback and linear quadratic optimal control will be applied to study transient response of the system.

2. Literature review

Mechanical vibration problems induced in a flexible beam and plate have been studied. Flexible structures including piezoelectric sensors/actuators are modeled using finite element method while genetic algorithm is used to optimize the location of piezoelectric element depending on varies control theories as objective function. However, controller design optimization is depended on the type of control schemes as well as numbers, locations and kinds of piezoelectric sensors and actuators.

Controllability is used as an objective function to find optimal placement of sensors and actuators in flexible beam and plate [2-7]. The system is controllable if and only if can measure and control all the output actuators and it is impossible to design a controller for uncontrollable or unobservable system. However , modal controllability theory and controllability Grammian are used to find the optimal placement of sensor/actuator in a plate because it gives the degree of controllability of the system [7] .The location of piezoelectric actuator is depended on the vibration modes in the modal analysis as well as controllability index which is a singular value of control matrix [2]. He has studied optimal size (length) and location of piezoelectric pair on to host beam by applying controllability techniques as objective function. Controllability, absorvability and spillover avoidance are considered as an objective function to find suitable location of piezoelectric sensors and actuators of a cantilever composite plate. It found a significant vibration reduction for the first three modes [3]. Additional spatial controllability constrain are considered as un objective function to find the optimal placement of sensors/actuator pairs on a thin plate while maintaining modal controllability and obsevability of selected vibration modes[6]. Optimal placement and sizing of piezoelectric actuator in a uniform beam is studied to realize the control objective of vibration suppressing for a uniform beam [8]. It has found that the linear quadratic regulator control measure more effective than controllability measure to optimize size and placement of piezoelectric actuator for a uniform beam. Maximization of energy dissipation has been considered as objective function to optimize sensor/actuator locations, and feedback gain for a cantilever beam [9]. Linear quadratic regulator LQR has been taken as an objective function to find optimal location of sensor/actuator pairs on a Euler-Bernoulli beam and plate. Stiffness and mass of piezoelectric element has been included and multi-input multi-output (MIMO) are considered in control scheme , it was found that LQR control scheme required less input voltage to actuators than negative velocity feedback as well as the control results showed that constant gain negative velocity feedback more significant than displacement feedback [1]. The researcher [9] emphasized that displacement closed feedback control has insignificant effect compared to velocity closed feedback and LQR optimal control theory , due to increasing of the system damping and reach to maximum when the input voltage to actuators reach peak point. Also, LQR required less voltage feeding to actuators than constant gain velocity feedback to control vibration and it considered more costly. It could be conclude that linear quadratic regulator better and more costly than negative velocity feedback control and modal controllability as an objective function.

Investigation of active vibration control in stiffened plates have been considered a significant study in aerospace structures , the wings and fuselage consist of a skin with a group of stiffened ribs, piezoelectric stiffener pass through any path in plate is studied and used velocity feedback control to study transient response of the plate. He has found that gain actuation effects tend to excite a higher- order modes and need to emphasis by other researcher [10]. Vibration reduction in orthotropic plate using piezoelectric stiffener-actuators have studied and it has found that the control time is reduced in case of increasing voltage applied to piezoelectric actuators stiffener[11]. However, the method of increasing voltage applied to actuators is could be considered most costly and may need heavy equipments to provide enough voltage to actuators and this method could be unreliable for aerospace structures. Therefore it needs to optimize the controller cost by optimization the locations of sensor/actuator and could be the using of stiffened beam instead of stiffened piezoelectric line is better and need to study and research to know the differences.
In this project, a square thin cantilever plate and stiffened plate by one beam on the edges of plate as well as two, three, and four beams are studied. Firstly, natural frequencies and mode shapes have found by Ansys package. Optimization of sensor/actuator pairs using linear quadratic regulator control theory and genetic algorithm will be applied as a second step. Transient response of beam and plate theoretically and experimentally will be studied on the first year of my research as well as the results will compare with the sumilink simulation.

3. Methodology

Optimal controller design to suppress mechanical vibration and dynamic response optimization for a flexible structure will be investigated theoretically and experimentally. The problem will be considered as a multi-input multi-output control system. Finite element method will be applied to model a mechanical structure including mass and stiffness of piezoelectric element. The first six modes will be controlled and genetic algorithm will be used to find optimal location of four sensor/actuator pairs. Linear quadratic regulator will be taken as an objective function to optimise piezoelectric locations, controller cost, speed response error reduction. Then dynamic response of the structure will be studied experimentally by applying closed loop negative gain velocity feedback and LQR optimal control theory while theoretically Newmark integration method will be applied to study transient response of the system. Dynamic response of the structure will be compared with Ansys package and sumlink simulation.

4. Objective of the project

1. Finite element will be used to model beam, plate and plate stiffened by beams including piezoelectric materials.
2. The effects of stiffeners on dynamic response, frequencies and mode shape will be studied.
3. The optimisation of sensor/actuator pairs will be investigated using genetic algorithm.
4. The effects of stiffeners on the numbers and locations of piezoelectric sensor/actuator pairs will be researched.
5. Optimal controller will be designed to satisfying the requirements of light weight structures including optimization of speed response and controller cost.

5. Materials

Experimental study is required equipments and materials. Firstly, beam and plate as well as stiffened plates by beams are needed with standard and known martial mechanical properties and fixations. Structural materials and fixation will be designed and curried out in mechanical department workshop. Piezoelectric sensor/actuator pairs are required including twenty five pairs as a first needing. In addition, it has been required equipments to measure and control vibrations including laptop computer with software, data question, amplifier, interface, oscilloscope, channel selector, and power source.

6. Modelling

Piezoelectric senor/actuator materials behaviour is modelled by equations (1) and (2) including two mechanical and electrical variables [12] reported by Gandhi M.V. and Thompson B.S. in 1992.   .

$$\{\sigma\} = [c]\{\varepsilon\} - [e]\{E\} \qquad 1$$

$$[D] = [[e]]^T\{\varepsilon\} + [\epsilon]\{E\} \qquad 2$$

Where:

{σ}= stress vector
[D]= electric flux density vector
{ε}= elastic strain vector
{E}= electric field intensity vector
[c]= elasticity matrix
[Є]= dielectric matrix
[e]= piezoelectric stress matrix

Finite element equations are modelled for piezoelectric structures using variational principles, structural system global equation motion with n degree of freedom can be written as eq.(3) [12] reported by Tzou H.S. and Tseng C.I.

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \ddot{u} \\ \ddot{\phi} \end{pmatrix} + \begin{bmatrix} C_{uu} & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \dot{u} \\ \dot{\phi} \end{pmatrix} + \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi}^T & K_{\phi\phi} \end{bmatrix}\begin{pmatrix} u \\ \phi \end{pmatrix} - \begin{pmatrix} F_u \\ F_\phi \end{pmatrix} \qquad 3$$

$$C_{uu} = \alpha K_{uu} + \beta M_{uu}$$

Where:
u= denotes structural displacement

ϕ=denotes electric potential

M_{uu} = Structural mass matrix
C_{uu} = structural damping matrix
K_{uu} = structural stiffness matrix
$K_{u\phi}$ = piezoelectric coupling matrix

The general dynamic equation (3) is represented a global dynamic equation for a mechanical structural including piezoelectric sensor/actuator pairs. Firstly, it will be solved as a free vibration to get natural frequencies and mode shapes of a structure. Secondly, genetic algorithm will be applied to optimize locations for number of piezoelectric sensor/actuator pairs to control six modes of vibrations. The solution of equation (3) for flexible structures and for n degrees of freedom is considered much costly and complex to solve. In reality, only few first modes m $\ll n$ are affected on the vibration of mechanical structures. Therefore, it is better to write this equation in term of modal coordinate in the next section.

7. State space modelling

The state space model is considered as a time domain function for modelling, analysing and designing a wide range of control systems. State space is modelled a multi-input multi-output, non linear and time variant systems, as well as various controller design schemes can be applied [13]. Equation (3) can be converted to first order differential equations and in form of state space model by the following steps:

$$M_{uu}\ddot{u} + C_{uu}\dot{u} + K_{uu}\,u = -K_{u\phi}\,\phi + F_u \qquad 4$$

Equation (4) represent the dynamic equation of motion for a mechanical structure including piezoelectric stiffness, assume that the state vector $\{X\}$ as;

$$\{X\} = \begin{pmatrix} X1 \\ \dot{X}2 \end{pmatrix}$$

Assume that $u = X1 \; and \; \dot{u} = X2$, so, $\dot{X}1 = X2$ and $\dot{X}2 = \ddot{u}$ arrange the state vector in matrix form.

$$\begin{pmatrix} \dot{X}1 \\ \dot{X}2 \end{pmatrix} = \begin{bmatrix} 0 & I \\ -M_{uu}^{-1}K_{uu} & -M_{uu}^{-1}C_{uu} \end{bmatrix} \begin{pmatrix} X1 \\ X2 \end{pmatrix} - \begin{bmatrix} 0 \\ M_{uu}^{-1} K_{u\phi} \end{bmatrix} \{\Phi\} + M_{uu}^{-1}F_u \qquad 5$$

If, equation (5) is compared with state space equation $X = A\dot{X} + BU$, it can found that:

$$[A] = \begin{bmatrix} 0 & I \\ -M_{uu}^{-1}K_{uu} & -M_{uu}^{-1}C_{uu} \end{bmatrix} = \quad \text{state matrix}$$

$$[B] = \begin{bmatrix} 0 \\ M_{uu}^{-1} K_{u\phi} \end{bmatrix} = \quad \text{input matrix to the actuator}$$

State space equations (5) can be converted from coupled nodal coordinate equations to global modal uncoupled equations by applying:

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2\xi\Omega \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} - \begin{bmatrix} 0 \\ \Lambda^T K_{u\varphi} \end{bmatrix} \{\varphi\} \qquad 6$$

Where $\Omega^2 = M_{uu}^{-1} K_{uu}$ and $2\xi\Omega = M_{uu}^{-1} C_{uu}$

$X = \Lambda x$, $X = nodal\ coordinate$, x = global model coordinate and Λ= modal matrix.

Equation (6) represent uncoupled modal state space equation, it could be apply control theories for a number of modes because the first frequencies of dynamic system which represent the dominate modes of vibrations.

8. Controllability and absorvability

Control theories and controller design is investigated for a mechanical structure when the structure is controllable and absorvable. “Controllability represents the ability of a control input to control or change all the state variables of a system” [14].

$$[P] = ([B], [A][B], [A]^2[B], \ldots\ldots, [A]^{n-1}\ [B]) \qquad 7$$

The rank of controllability matrix P should the same order of order of the system [14] reported by Ogata, 1990. Also for absorbability can represented by following equation.

$$[P] = ([C], [A][C], [A]^2[C], \ldots\ldots, [A]^{n-1}\ [B])^T \qquad 8$$

Equations (7) and (8) must be applied for each system and to emphasize that the rank of [P] matrix equal the rank of [A] matrix and if not it will be impossible to design a controller for that system.

9. Controller design

Figure (1) represents closed loop state feedback control, the general equation to determine feedback gain controller for closed loop state feedback and output feedback control will be written in this section.

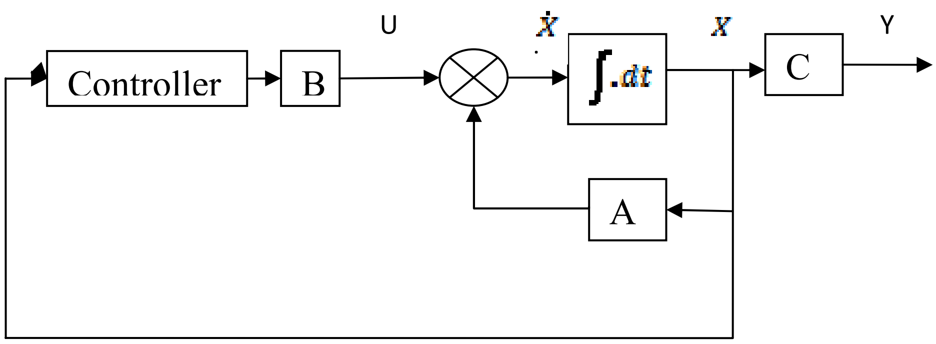


Figure (1) state feedback control block diagram

$$U_{(t)} = -KX_{(t)} \qquad 9$$

$$\dot{X}_{(t)} = AX_{(t)} + BU_{(t)} \qquad 10$$

$$\dot{X}_{(t)} = AX_{(t)} + B(-KX_{(t)}) \qquad 11$$

$$\dot{X}_{(t)} = (A - BK)X_{(t)} \qquad 12$$

$$Y_{(t)} = CX_{(t)} \qquad 13$$

Equation (12) can be solved to find the gain of controller but does not represent the optimal value for the system and it cost time calculations to make balance between the cost and speed response and especially for multi-input multi-output system. This method called pole placement to move poles of the system from unstable to stable region. It easy to put the frequencies or poles in the stable region when assume the value of gain very large and the system go faster to stable region. However the controller will be expensive. In modern control theory, gains system controller is selected depending upon the requirements of control system designer represented by controller cost and speed dynamic response. Controller optimizations are achieved by Linear Quadratic Regulator (LQR) and selection of weighted factors Q and R. Optimise control force and dynamic speed response of the system by minimizing the performance indexing J.

$$J = \int_0^\infty (X^TQX + U^TRU)dt \qquad 14$$

Where, Q and R are positive definite matrices. Substitute equation (9) in eq. (14) and solve Riccati equation (15) for a positive-definite matrix P to determine the optimal gain K [15].

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \qquad 15$$

$$K = R^{-1}B^T P \qquad 16$$

$$U_{(t)} = -KX_{(t)}$$

after finding K, it could find $U_{(t)}$ input control to actuator

Output feedback control is represented by bock diagram (2) and sumilink simulation in appendix B. In reality, output feedback control is given the latest and accurate picture about a system as well as, always the designer had full information about output of the sensors and easy to measure while the designer doesn't had a full knowledge about the state output X [12]. Linear quadratic regulator can be applied for output closed feedback control.

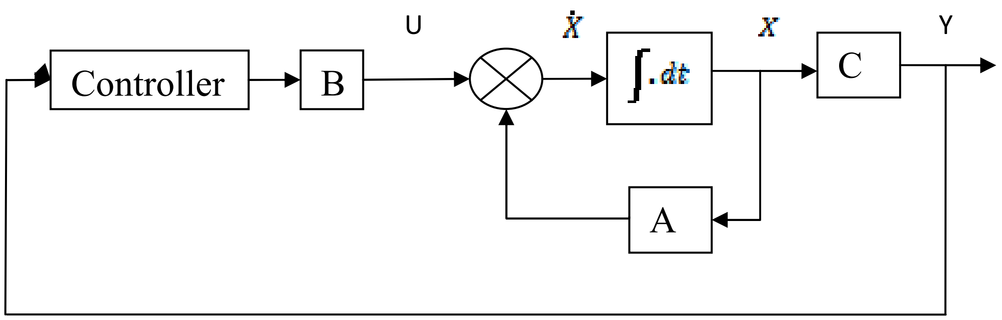


Figure (2) output feedback control block diagram

$$U_{(t)} = -KY_{(t)},\; Y_{(t)} = CX_{(t)},\; \text{hence } U_{(t)} = -KCX_{(t)} \quad 17$$

$$\dot{X}_{(t)} = AX_{(t)} + B(-KCX_{(t)}) = (A - BKC)X_{(t)} \quad 18$$

$$J = \int_0^{\infty} \left(X_{(t)}^T (Q + C^T K^T RKC) X_{(t)} \right) dt$$

Optimum gain K can be obtained for output feedback control by minimizing indexing J subjected to dynamic restriction in eq. (18). The optimization problem to get optimal gain is reduced to three coupled equations [12] reported by Lewise F. and Syrmos V.L., 1995.

$$A^T P + PA + C^T K^T RKC + Q = 0 \quad 20$$

$$A_C S + A_C{}^T + X_{(0)} = 0 \quad 21$$

$$K = R^{-1} B^T PSC^T (CSC^T)^{-1} \quad 22$$

Where $X_{(0)}$ *is a function of initial state:* $X_{(0)} = X_{(0)} X_{(0)}{}^T$

10. Estimator controller design

Controller design for all natural frequencies of a mechanical structure is complex and costly; on the other hand, the first dominant frequencies have an effect on a structural vibration. In addition, it will be difficult to know all the state variables n. The state variables are estimated by estimator (observer) based on the measurements of the output variables. However, the output of the system (plant) is different from the output of estimator. So, it needs to calculate this error and feed it to estimator after amplifying as shown in figure (3). Equations (23) is represented a general equation of estimator-state feedback control system [15].

$$U_{(t)} = -K\bar{X} \quad \textit{Where } \bar{X} \textit{ estimator state}$$

$$\dot{X} = AX - BK\bar{X} = (A - BK)X + (X - \bar{X})$$

$$\dot{X} = (A - BK)X + BKe$$

$$\dot{e} = (A - K_e C)e$$

$$\begin{bmatrix} \dot{X} \\ \dot{e} \end{bmatrix} - \begin{bmatrix} A - BK & BK \\ 0 & A - K_e C \end{bmatrix} = \begin{bmatrix} X \\ e \end{bmatrix} \quad 23$$

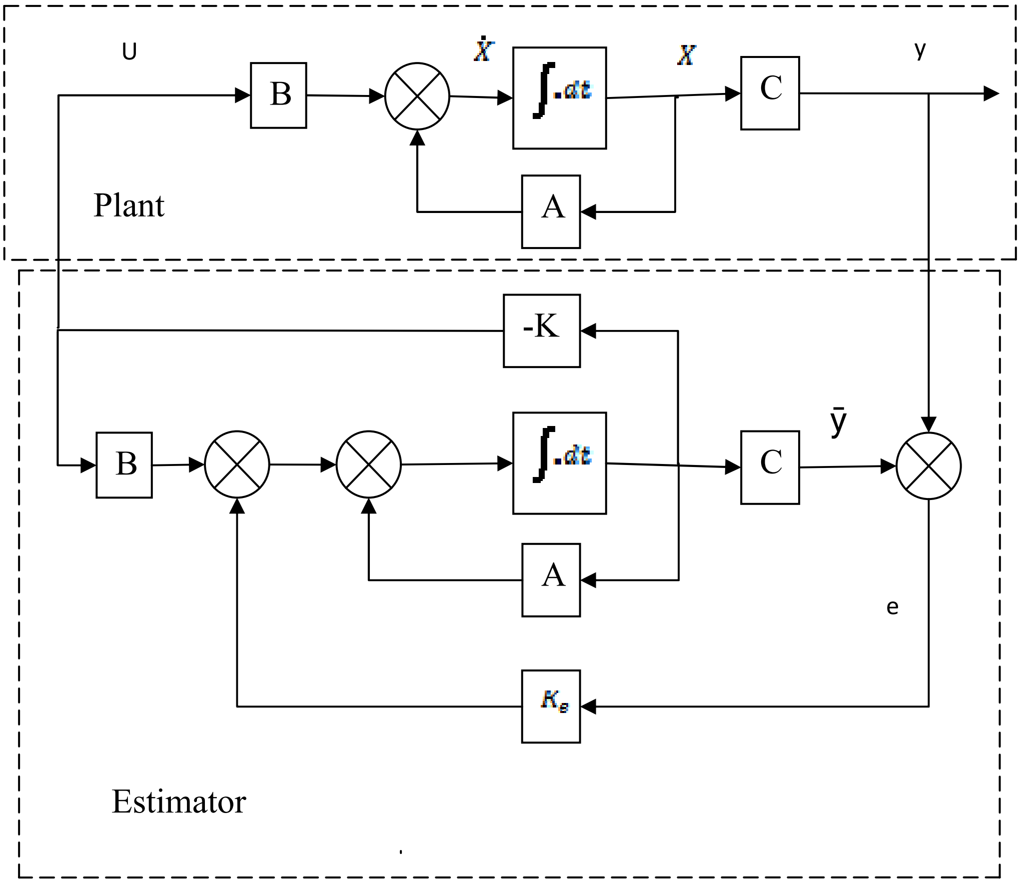


Figure (3) estimator-state feedback control with error feedback

11. Sumilink simulation

The transient dynamic response of the system will be tested by sumilink. Estimator State feedback control with plant and controller are represented by block diagram in figure (4) [16]. The difference "error" between the output of the plant and estimator will be taken as a feedback to the estimator after amplifying it. In the sumlink diagram Gain 8 will be obtained it by applying either pole placement or LQR schemes while gain6 will be obtained from equation (23). If the error converges to zero then the system will be stable and controlled and gain magnification dose not effect on the system, while if it does not converge to zero then the error will diverge and go to infinity and make the system uncontrollable.

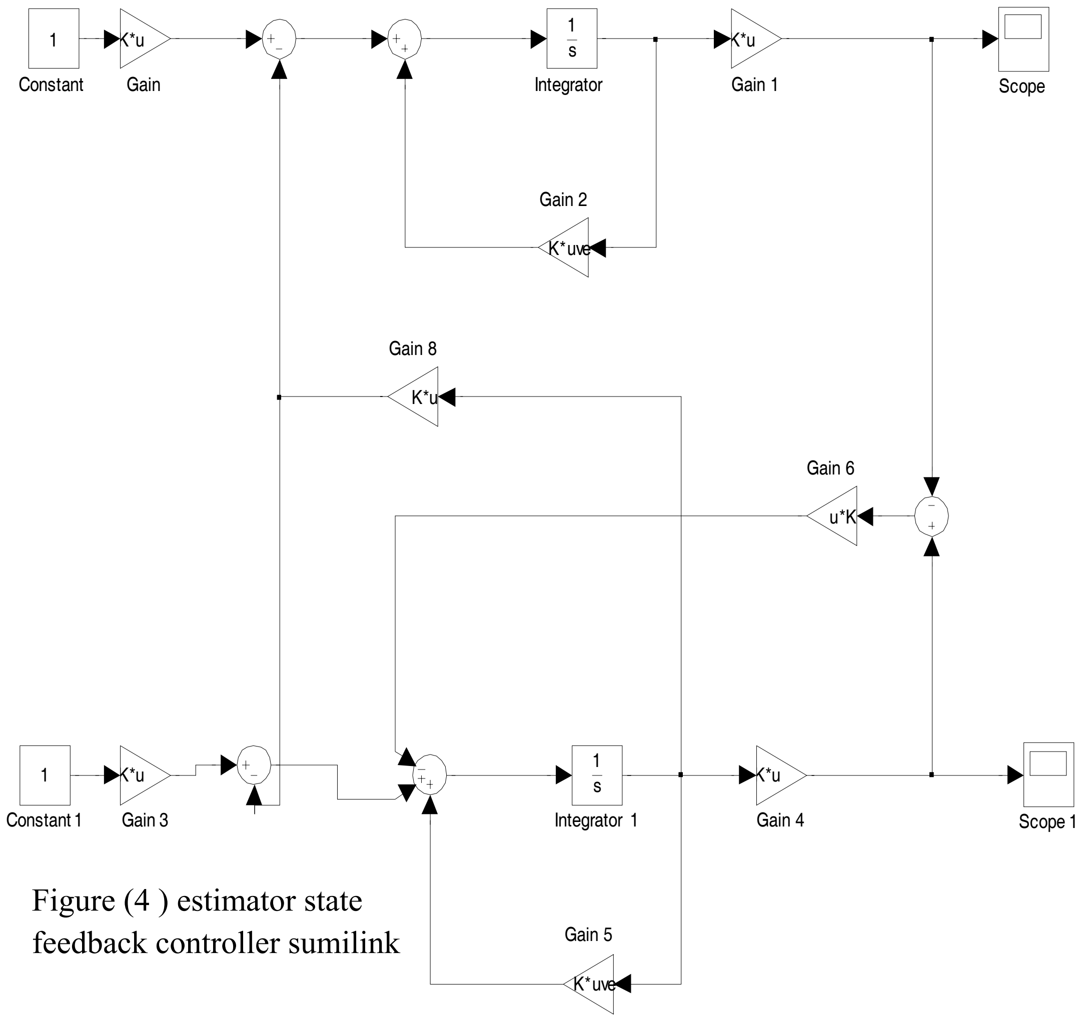


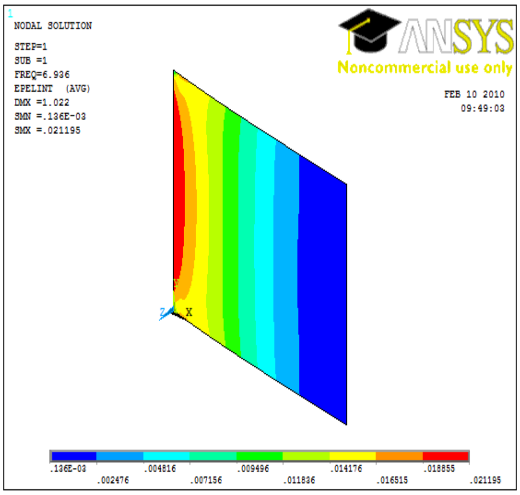
Figure (4) estimator state feedback controller sumilink

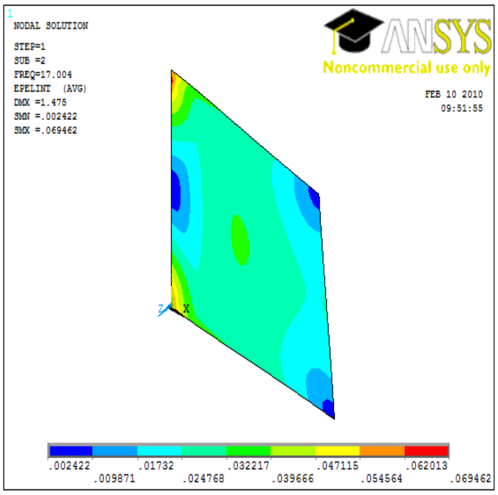
12. Controller optimization

Input matrix [B] to actuators and output matrix from sensors [C] are depended on the positions of the sensor/actuator pairs on mechanical structure space. So, the mechanical structure space has a massive number of positions probabilities for these piezoelectric sensor/actuator as well as solutions ranged from fault locations on the modal nodes to optimal locations which give optimal control design. So, the best technique to solve and find optimal placement in structure space is genetic algorithm and particle swarm. Most optimization problems are widely solved by genetic algorithm[17]. Genetic algorithms are general a fast research method depending on fundamental steps solution, which are chromosome creation , fitness function , the genetic operations making up the reproduction function , termination criteria [7] reported by Goldberg D.E. 1989 and Krishan kumar K. 1992.

13. Results and discussion

A cantilever flat plate has studied to find natural frequencies and mode shapes. Mode shapes and intensity of model strain energy are shown in figures (5) and (6). The results are compared with researcher [1] as shown in table (1). It has found that the distribution of modal strain energy intensity is depended on the mode number. The positions of maximum modal strain energy intensity are different from one mode to another. Fourth mode is more energetic than fifth and sixth mode. So, fourth mode will be taken under investigation to see the effects on controller. One method to suppress and control vibration is to put actuators in these positions to control vibration by energy dissipation but the positions are changed according to the mode number. Therefore, genetic algorithm has been used to optimise actuators positions for flexible structures. The plate is stiffened with one, two, three, and four beams and solved by Ansys package as shown in figure (6). It has found that the natural frequencies are increased and reach to the doubled with four beam stiffener because of the stiffness matrix value increased more than mass matrix. In addition, modal strain energy is decreased with increasing number of stiffeners. Also, it has appeared that the intensity of modal energy positions is varied according to the mode number. In addition, it has found from table (1) that the effects of third and fourth stiffener on the natural frequencies are very limited as compared with the plate stiffened by one and two stiffener. The optimization of piezoelectric sensor/actuator pairs using genetic algorithm and linear quadratic control theory will be applied as a second step for plate and plate stiffened by beams. Third step, it will be made a simulation by sumilink as well as by Ansys package and experimental study to design a controller and optimize transient response of the system.





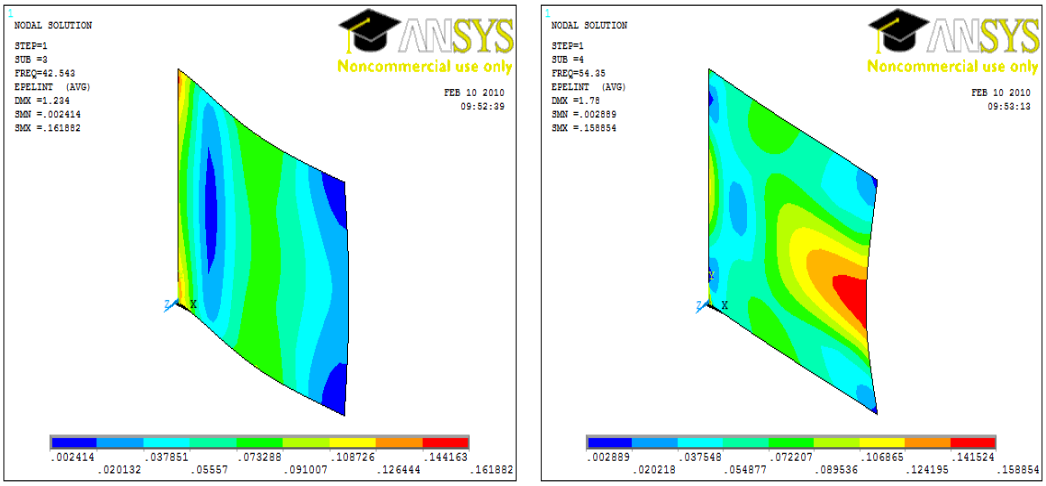


Figure (5) mode shapes and distribution of modal strain energy for the first four natural frequencies of a cantilever flat plate.

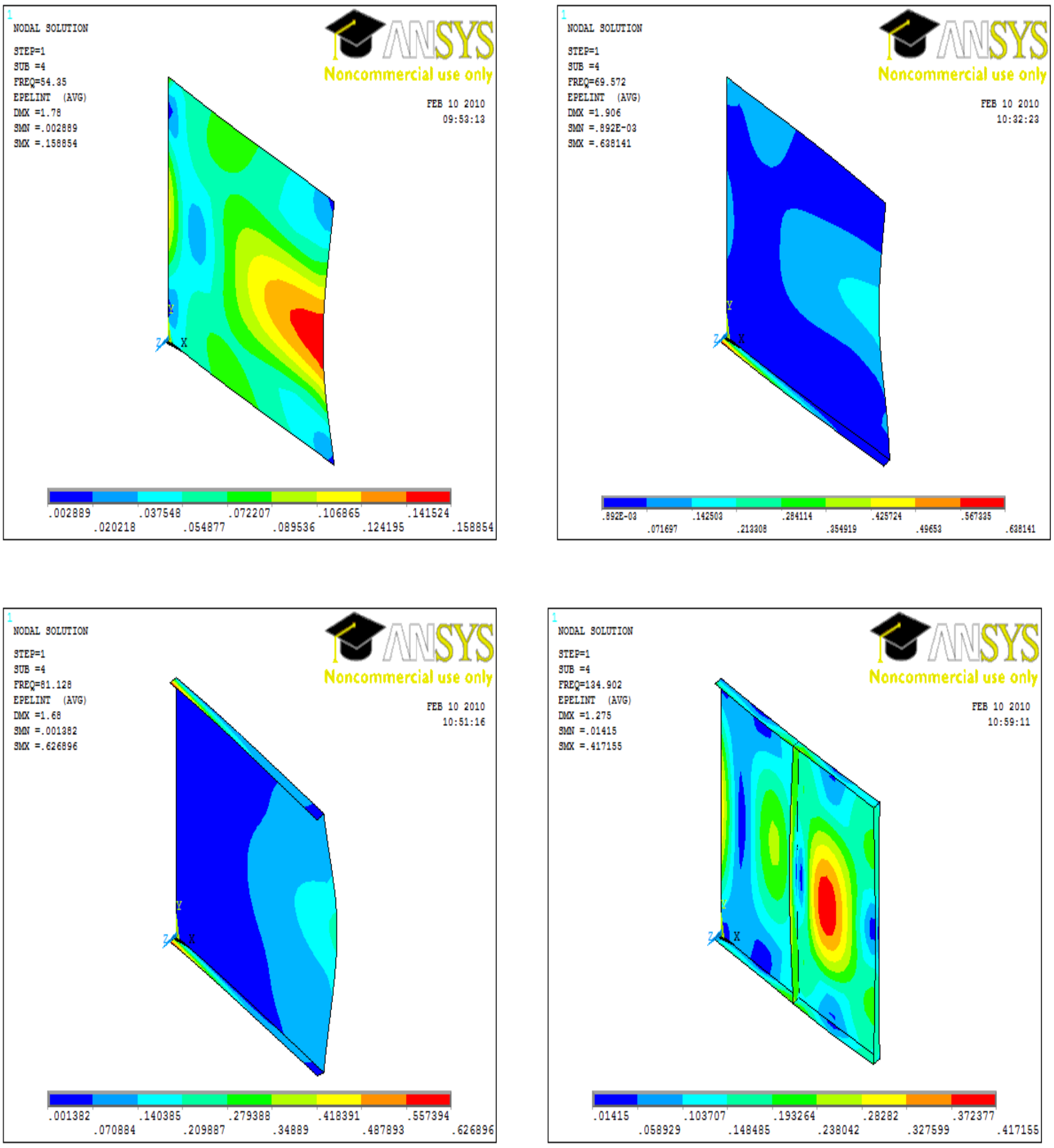


Figure (6)Mode shapes and modal strain energy for the fourth mode of a cantilever flat plate, stiffened plate by one beam, two beams and four beams.

Table (1)

Case	First Frq. Hz	Second freq. Hz	Third freq. Hz	Fourth freq. Hz	Fifth freq. Hz	Sixth freq. Hz
Flat plate	6.9357	17.004	42.543	54.350	61.898	108.36
One stiffener	10.568	33.450	49.313	69.572	87.457	126.08
Two stiffener	21.933	50.049	65.760	81.128	120.58	134.72
Three stiffener	29.380	51.484	63.568	118.26	130.59	183.50
Four stiffener	31.306	52.380	110.03	134.9	188.23	204.63
(Kumar,K.R.) [1] Flat plate	6.935	16.989	42.529	54.337	61.838	108.232

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