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Experimental studies on damage detection of beam structures with wavelet transform

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ABSTRACT

Experimental studies are reported for crack detection of a beam structure under a static displacement with the spatial wavelet transform. An invisible perturbation in the deflection profile of the beam at the crack position due to its existence would be induced. Such a small perturbation will be discerned or amplified through a wavelet transform such that a detection of crack location becomes possible practically. To realize this, the static profile of a cracked cantilever aluminum beam subjected to a static displacement at its free end is analyzed with Gabor wavelet to identify the crack. The damage detection of the beam with different crack depths is conducted. The spatial wavelet transform is proven to be effective in identifying the damage area even when the crack depth is around 26% of the thickness of the beam.

1. Introduction

Damages, such as cracks and delaminations, are inevitable in aerospace, aeronautical, mechanical and civil engineering structures during their service life. To ensure the structure integrity and prevent the structure damages from the deterioration at an alarm rate, advanced structure health monitoring (SHM) techniques are required and have been widely studied during last few dedicates (Bornn, Farrar, & Park, 2010; Li, Zhu, Zhao, & Hub, 2009; Sun, Chaudhry, Rogers, Majmundar, & Liang, 1995; Tua, Quek, & Wang, 2004; Wang, Wong, & Cheng, 2009). Damage detection techniques were provided by researchers, including the crack detection using lamb wave actuated by piezoelectric actuator (Tua et al., 2004) and using vibration energy flow (Li et al., 2009; Wang et al., 2009). Moreover, as a useful tool for signal processing to extract information from different kinds of data, wavelet transform was widely employed by researchers for damage detection (Chang & Chen, 2003; Deng & Wang, 1998; Gokdag & Kopmaz, 2009; Haase & Widjajakusuma, 2003; Kim & Melhem, 2004; Li et al., 2009; Rucka & Wilde, 2006; Zhong & Olutunde, 2007).

From early 1990's, time-frequency analysis using wavelets was found to be feasible to provide more detailed information about non-stationary signals, whereas the traditional Fourier transform cannot obtain (Newland, 1994a, 1994b). Newland applied a wavelet analysis to the study of vibration of buildings caused by underground trains to detect the perturbation signal at the damaged area. Characteristics of representative vibration signals under the wavelet transformation were examined by Hou, Noori, and Amand (2000) for structure health monitoring. However, the noises in vibration signal, which may reduce the accuracy and effectiveness of damage detection using the classic time-frequency analysis, are inevitable especially for the vibration of some complex aeronautical and mechanical structures, such as airframe and vehicle chassis. Identifying the crack location especially for small cracks is another challenge for the time-frequency analysis method.

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To provide an alternative to the classical wavelet analysis based on the time-frequency signals, Liew and Wang (1998), Wang and Deng (1999) proposed a damage detection technique using spatial wavelet analysis. A perturbation on the deflection profile at the crack position would be induced owing to the discontinuity of gradient at the crack in structures. Such a small perturbation, which is invisible, will be discerned or amplified through a wavelet transform such that a detection of crack location becomes possible practically. Under both static and dynamic loading conditions, the numerical simulated deflection or displacement responses were analyzed with wavelet transform, and the presence of the crack was detected by a sudden change in the spatial variation of the transformed response in Wang and Deng's paper (1999). Following this method, many studies on damage detection using spatial wavelet transform for different structures were finished through numerical simulations. Lam, Lee, Sun, Cheng, and Guo (2005) provided a numerical simulation estimating the location and extent of a crack in a beam structure using the spatial wavelet transform and the Bayesian approach. Based on the wavelet-transformed displacement responses, the probability densities of different crack locations and extents were calculated. A distributed two-dimensional continuous wavelet transform algorithm was developed by Huang, Meyer, and Nemat-Nasser (2009) to detect the damage area on a plate structure. Their method was demonstrated by an example based on the deflection field of a simply supported plate subjected to static transverse loads. A finite element model (FEM) of the plate was built to obtain the deflection solutions. In addition to the deflection field, mode shapes were widely used for the damage detection using the spatial wavelet transform as well. Chang and Chen (2003) used wavelet transform to analyze the mode shapes with the Timoshenko beam model. Using the same method, they studied the damage detection of a multiple cracked beam (Chang & Chen, 2005). Both the positions and depths of multi-cracks were estimated by the spatial wavelet transform.

Studies on structural health monitoring using the spatial wavelet transform were mainly based on numerical simulations. Experimental demonstrations of the effectiveness of the spatial wavelet transform on damage detection were also studied by researchers. The most widely used experimental methods for detection of damage in structures were based on the vibration mode shapes (Cao & Qiao, 2008; Gokdag & Kopmaz, 2009; Rucka & Wilde, 2006). Gokdag and Kopmaz [19] conducted the wavelet transform on the experimental mode shapes of a damaged beam to prove the feasibility of their numerical simulation. Rucka and Wilde (2006) used numerous sensors, which were mounted on the surfaces of the specimens, to obtain the mode shapes of the damaged structures. Wavelet transform was then applied to analyze the mode shapes from the experiments to identify the crack locations. A technology of taking synergistic advantages of stationary wavelet transform and the continuous wavelet transform were proposed to improve the robustness of abnormality analysis of mode shapes in damage detection by Cao and Qiao (2008). To reduce the quantity of the sensors used for the experiment and to facilitate the operation, another experimental method for damage detection using spatial wavelet transform based on the dynamic response of a damaged structure subjected to a moving load applied was developed (Umesha, Ravichandran, & Sivasubramanian, 2009; Zhu & Law, 2006). Since the noise effects in the experimental studies were still obvious, Rucka and Wilde (2008) developed another experimental method using digital camera to get the deflection field of a cracked cantilever beam under bending. The damage detection effects were obvious from their findings when the crack depth was larger than 50% of the thickness of the cracked beam.

In this paper, a high resolution laser profile sensor is employed to measure the deflection profile of a cracked cantilever aluminum beam subjected to a static displacement at its free end. De-noise techniques are introduced to make the detection more efficient. The smoothed static profile of the cracked beam is analyzed with Gabor wavelet to identify the crack. Based on the experimental study, the spatial wavelet transform is proven to be effective to identify the damage area even when the crack depth is around 26% of the thickness of the beam.

2. Principle of spatial wavelet analysis

Wavelet has been wildly used to analyze signals of time domain (Kim & Melhem, 2004). For the wavelet analysis of signals of spatial domain, we can simply replace time with a spatial coordinate by considering a spatial signal $f(x)$ distributed over $[l1, l2]$, where x refers to the spatial coordinate (Wang & Deng, 1999). A spatial signal of displacements of a structure subjected to a static or dynamic loading can be measured over a domain of interest.

For the special wavelet transform analysis, let $\psi(x)$ be a complex-valued function localized in the space domain x. $\psi(x)$ is the mother wavelet, from which wavelet are generated by translation and dilation. The translation from mother wavelet can be defined as (Wang & Deng, 1999)

$$\psi_{a,b}(x) = 2^{a/2}\psi(2^a x - b), \tag{1}$$

where a and b are the dilation (scale) and translation (position) indices, respectively.

For a spatial signal $f(x)$ distributed over $[l1, l2]$, its wavelet transform is given by

$$c_{ab} = \int_{l1}^{l2} f(x)\overline{\psi_{a,b}(x)}dx, \tag{2}$$

where the over-bar indicates the complex conjugate of a function and c_{ab} is called the wavelet coefficient for the wavelet $\psi_{a,b}$ at the scale a and the position b.

The mother wavelet must have a zero mean value and satisfy the admissibility condition shown below,

$$\int_{l1}^{l2} |\psi^*(x)|^2 \frac{d\omega}{|\omega|} < \infty, \tag{3}$$

where $\psi^*(x)$ is the Fourier transform of $\psi(x)$, which is given by

$$\psi^*(x) = \int_{-\infty}^{+\infty} \psi(t)e^{-i\omega t}dt. \tag{4}$$

The wavelets used in the research, the Gabor wavelets, are generated from the Gabor function below (Kishimoto, 1995),

$$\psi(x) = \frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{(\omega_0/\gamma)^2}{2}x^2 + i\omega_0 x\right], \tag{5}$$

where ω_0 and γ are positive constants. To satisfy the admissibility condition of the mother wavelet, the two positive constants in the Gabor function, ω_0 and γ, are set as 2π and $\pi\sqrt{(2/\ln 2)}$ (Wang & Deng, 1999).

Submitting Eq. (5) into Eqs. (1) and (2), the Gabor wavelet transform of the spatial signal, $f(x)$, can be determined within a space domain from $l1$ to $l2$,

$$c_{a,b} = \int_{l1}^{l2} f(x)\overline{2^{a/2}\frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{(\omega_0/\gamma)^2}{2}(2^a x - b)^2 + i\omega_0(2^a - b)\right]}dx. \tag{6}$$

In practical applications, the continuous signal displacement function $f(x)$ will be simulated by a number of discrete data $f(x_n)$ (n = 1, 2, 3, ...). For the discrete signals, Eq. (6) can be transformed to be,

$$g_{a,b} = \sum_{n=1}^{N} f(x_n)\overline{2^{a/2}\frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{(\omega_0/\gamma)^2}{2}(2^a x_n - b)^2 + i\omega_0(2^a - b)\right]} \cdot \Delta x, \tag{7}$$

where x_n is the discrete data between $l1$ and $l2$, N is the total number of data, Δx is the size discretization, $g_{a,b}$ is the Gabor wavelet coefficient of the discrete spatial signal $f(x_n)$ corresponding to a scale factor 'a' and a certain position 'b' that is between $l1$ and $l2$. If there is any perturbation at location 'b' of a spatial signal, it will be discerned or amplified by the corresponding wavelet coefficient (Wang & Deng, 1999).

3. Finite element simulations of damage detection using wavelet analysis

A FEM model is first built by ANSYS 10.0 to simulate the bending of a cracked cantilever beam structure. The left end of the beam is fixed at the coordinate origin. The length and thickness of the beam are set to be 20 cm and 0.3 cm, respectively. A crack with the depth of 0.15 cm is located from 10 cm to 10.12 cm on the upper side of the beam along its length direction. The beam is made from aluminum with a Young's Modulus of $7 * 10^{10}$ N/m^2. Element type Plate 42 is used to mesh the two dimensional beam structure. Fig. 1 shows a finite element meshing. A high meshing density is applied around the crack area such that the stress/strain singularity at the crack position is able to be accurately revealed.

An upward transverse displacement of 2 cm is applied at the free end of the cantilever beam. The deflection of the beam is shown in Fig. 2(a). The displacements of the nodes in Y direction at the lower edge of the cantilever beam (deflection) are selected as the spatial signal f-$FEM(x)$, where x is the X coordinate of the chosen nodes.

The technical computing software Matlab R2007a is employed for the data processing and wavelet transform in our study. Spatial signal f-$FEM(x)$ is linearly interpolated to be 400 nodes for the convenience in calculations. Using the Gabor spatial wavelet analysis, the wavelet coefficients of the spatial signal of the deflection profile from FEM can be calculated by Eq. (7), where the measurement region is from 0 cm to 20 cm.

Fig. 2(b) shows the wavelet coefficients at different locations around the crack area with the scale factor of 9. Obviously, the discontinuity of the gradient of the displacement at the crack position can be revealed through the wavelet transform effectively, and hence the crack position can be detected accordingly.

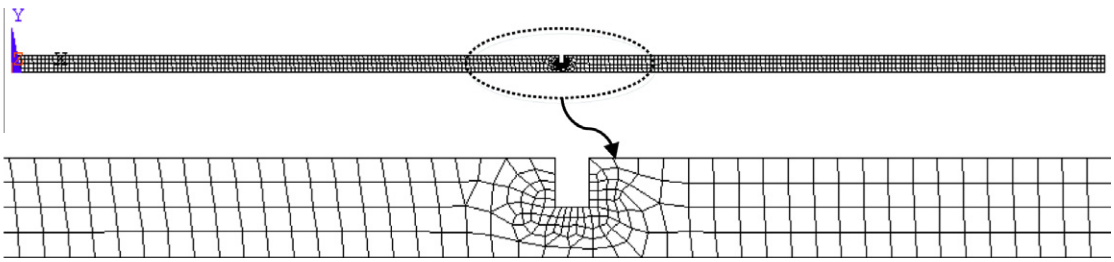

Fig. 1. Finite element meshing of a cracked cantilever beam.

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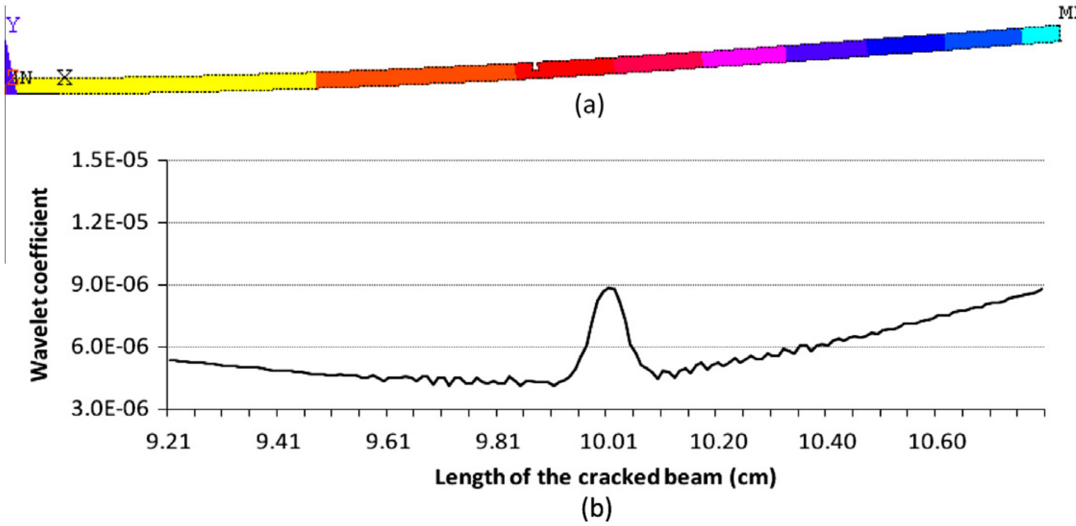

Fig. 2. (a) The deflection of the cracked cantilever beam from FEM; and (b) the corresponding wavelet coefficients of the crack area after transform.

4. Experimental studies of damage detection using wavelet analysis

4.1. Experimental set up

In the following experimental studies of the crack detection using the spatial Gabor wavelet transform, a high resolution laser profile sensor (CrossCheck laser profile sensor model CC3100-30) is employed to measure the deflection profile of a cracked cantilever aluminum beam subjected to a static displacement at its free end. Fig. 3 shows the experimental set up. The length and thickness of the cantilever aluminum beam are 55 cm and 0.3 cm, respectively. A transverse displacement of 3.5 cm is applied at its free end. The crack with the width of 0.1 cm is located on the lower surface of the beam structure. The upper surface of the crack area is paint with black ink to avoid the measurement error because of the reflection of layer beam shot at the bright surface of the aluminum beam.

The specifications and dimensions of the laser profile sensor are given in Fig. 4 and Table 1, respectively. It can be seen that the measurement range of the laser sensor in width direction is feasible from 2.5 cm to 3.5 cm when the distance between the sensor and the measured surface changes from 8.5 cm to 11.5 cm. In our experiment studies, the distance between the laser sensor and the surface of the bending aluminum beam is set to be between 10 cm and 10.5 cm, respectively. Thus, the valid width measurement range will be around 3.0 cm, which will cover the crack area. X-offset of the deflection output of the laser sensor is set as the starting position of the measurement range from the fix end of the cantilever beam.

4.2. Deflection profile of the bending beam and the de-noise process

Fig. 5 illustrates the deflection profile around the crack area measured by the laser sensor, when a crack with a depth of 0.15 cm (50% of the beam thickness) is located at a location of 36 cm from the fixed end of the cantilever beam. There are totally 1280 laser points shot on the surface of the aluminum beam. Based on the discrete reflection signal from the laser points, the wavelet analysis will be conducted to identify the crack position.

However, the signal from the laser spots shot on the surface of the aluminum beam may be disturbed by the vibration of the experiment base or environmental noises. To erase the noises from the original signal of the deflection profile for a better detection purpose, a smoothing algorithm provided by Matlab R2007a, ‘rloess’, is imported in the data processing before the wavelet transform. The method assigns zero weight to data outside six mean absolute deviations. From Fig. 6, it can be seen that the original signal is smoothed effectively for a possible detection with wavelet transform.

4.3. Damage detection using wavelet analysis

Based on the calculation shown by Eq. (7), the crack position can be discerned by conducting the smoothed discrete deflection signals from the laser profile sensor with wavelet transform. To avoid any experimental errors usually happened at the measurement border, the mid 900 signals out of 1280 signals from the profile laser sensor are selected.

The damage detection of the crack with the depth of 0.15 cm (50% of the beam thickness) located at 36 cm from the fixed end of the cantilever beam is first studied. Fig. 7 shows the calculated wavelet coefficients of the smoothed deflection profile around the crack area of the un-polished aluminum beam with the wavelet scale of 8.5. Although small perturbations can be

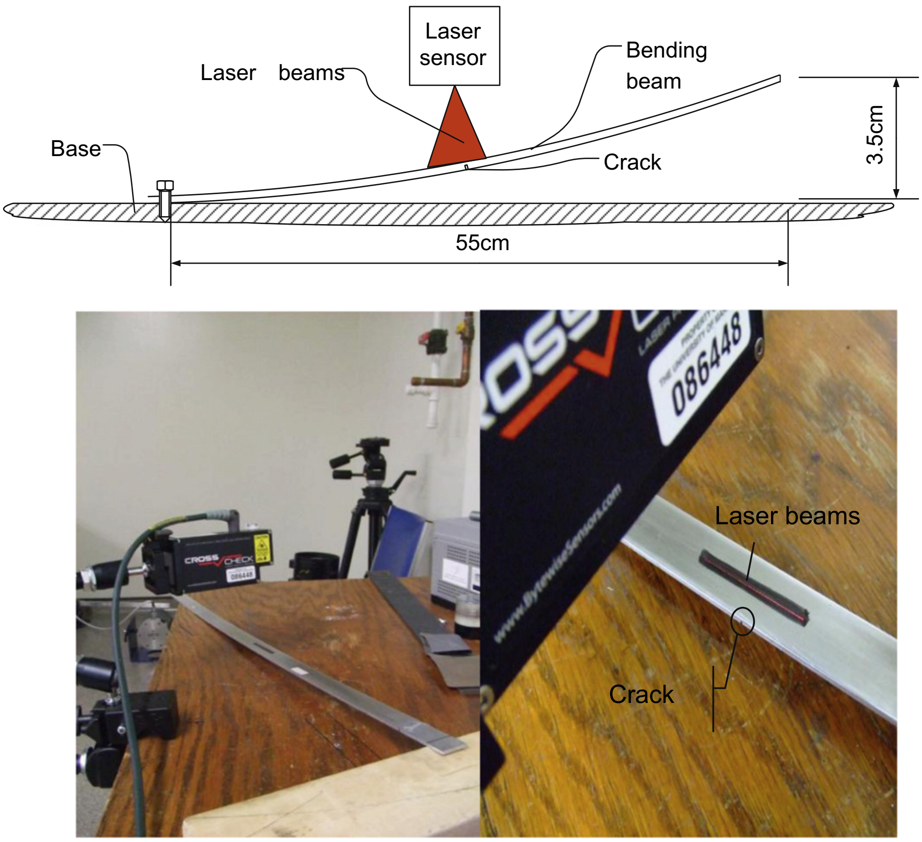

Fig. 3. Experimental set up.

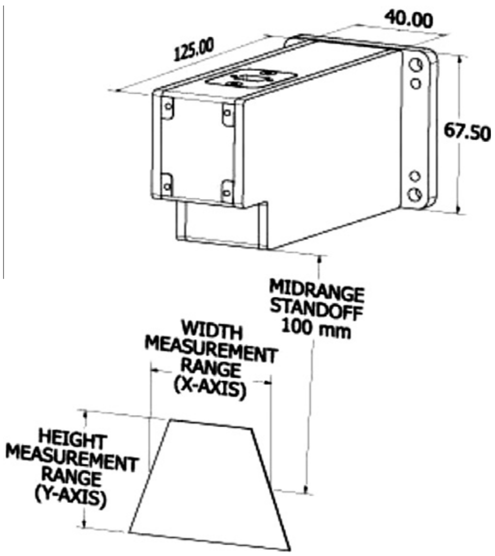

Fig. 4. The dimensions of CrossCheck laser profile sensor model CC3100-30. (Dimensions are in mm.)

seen at two regions from 34.85 cm to 35.05 cm and 35.15 cm to 35.50 cm in Fig. 7, an apparent perturbation at the third region from 35.74 cm to 36.03 cm is intransigently un-changeable when we change the laser beam to the different positions along the width direction of the beam structure for other attempts. The crack position can thus be confirmed to be at the

Table 1
The specifications of the CrossCheck Laser Profile sensor model CC3100-30.

Y-axis	Height range	30 mm
	Height resolution	0.003 mm
	Height accuracy	0.015 mm
X-axis	Mid length range	30 mm
	Near/far length range	25/35 mm
	Length resolution	0.010 mm
	Length accuracy	0.030 mm

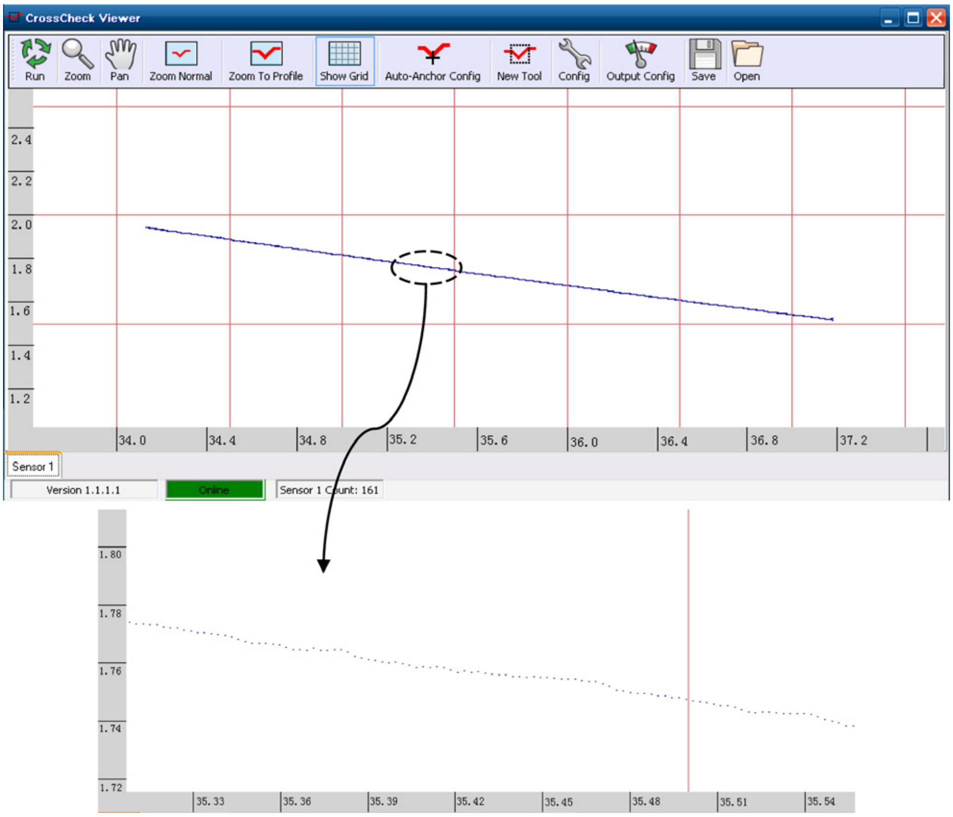

Fig. 5. Deflection profile of the bending aluminum beam measured by the laser profile sensor. (Dimensions are in cm.)

region from 35.74 cm to 36.03 cm. To find a more accurate crack position, a secondary wavelet transform is carried out. From wavelet coefficients in the secondary wavelet transform of Fig. 7, it can be seen that the perturbation of deflection profile is exactly located from 35.96 cm to 36.05 cm, where the 0.1 cm width crack occurs. The other two small perturbations at regions of 34.85 cm–35.05 cm and 35.15 cm–35.50 cm are owing to noises, which are considered as a result of the unpolished surface of the aluminum beam. To improve the effectiveness, the surface of the aluminum beam is polished with the wet-sanding method. Fig. 8 shows the surface conditions before and after polishing. The small surface curvatures and scratches, which may affect the reflection of laser beams, are erased by the surface treatment. The wavelet coefficients from the smoothed deflection profile around the crack position of the polished aluminum beam with wavelet scale 8.5 are shown in Fig. 9. Although small noises in the wavelet coefficients still can be found in Fig. 9, the perturbation at the crack is much more obvious compared to the one in Fig. 7 showing the efficiency of surface polishing treatment. From the secondary wavelet transform, the crack is perfectly found to be at 36.02 cm (+−0.04 cm). Another crack with the depth of 0.2 cm (65% of the beam thickness) located at 50 cm from the fixed end of the polished aluminum beam structure is also effectively detected by the wavelet transform with the wavelet scale of 8.5. Polishing surface and smoothing signals are critical in the detection of crack location of the beam.

From previous experimental studies, it is found that the crack, whose depth is equal or more than 50% of the beam thickness, can be evidently detected by the proposed experiment set and spatial wavelet transform. To further show the effectiveness of the damage detection method using the spatial wavelet transform, the detection of a smaller transverse crack with

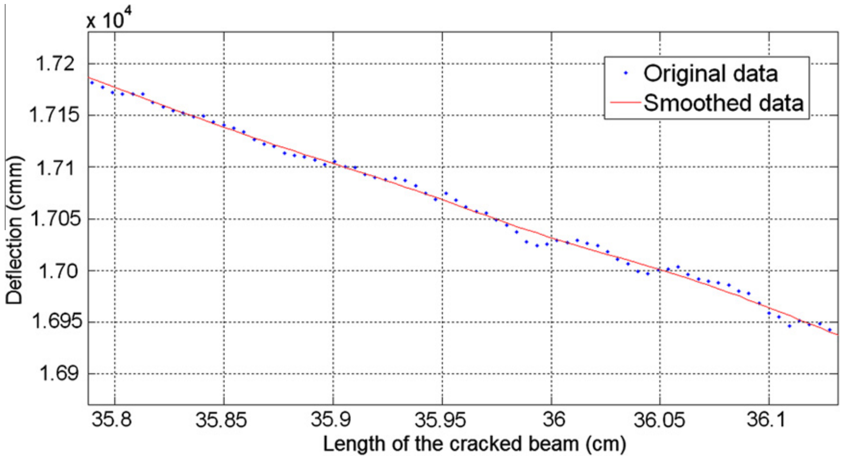

Fig. 6. Un-smoothed and smoothed deflection profiles of the bending aluminum beam.

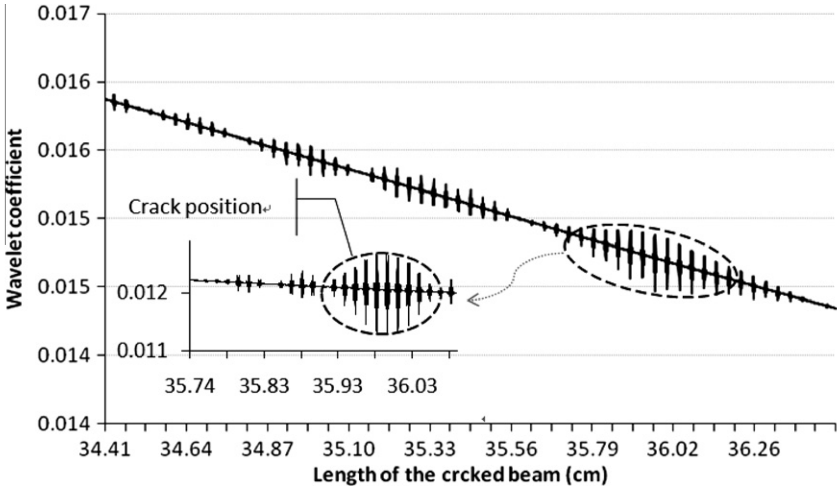

Fig. 7. Wavelet transform of the deflection signal of the crack area of an un-polished beam with a crack of depth of 0.15 cm and width of 0.1 cm located at 36 cm from the fixed end of the cantilever beam.

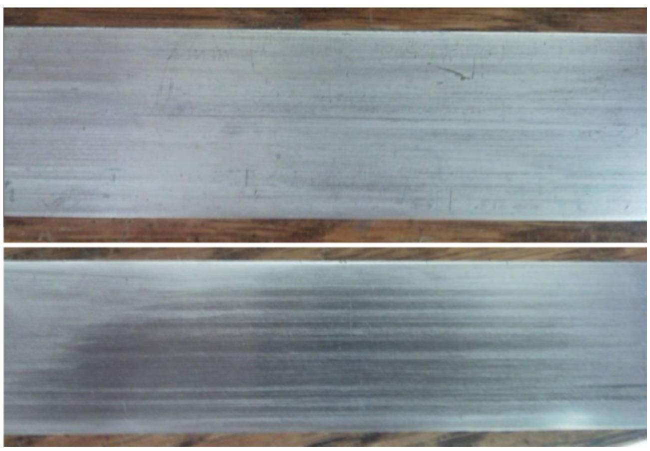

Fig. 8. The aluminum beam surface conditions before and after surface treatment.

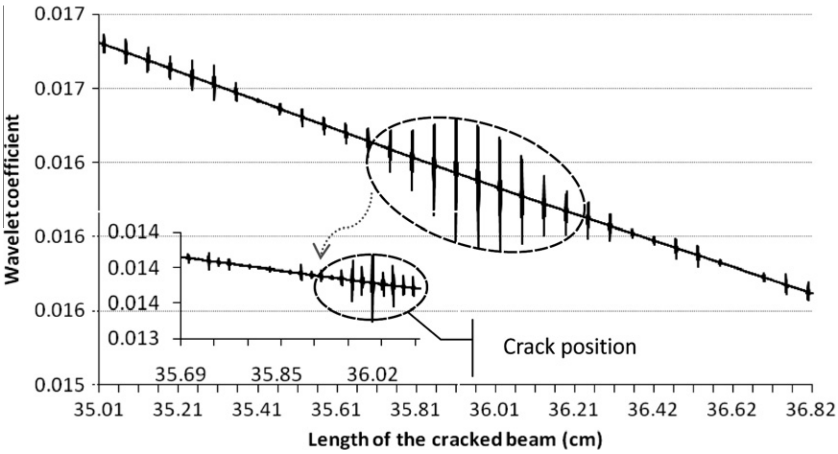

Fig. 9. Wavelet transform of the deflection profile signal of the wet-sanding polished beam with a crack of depth of 0.15 cm and width of 0.1 cm located at 36 cm from the fixed end of the cantilever beam.

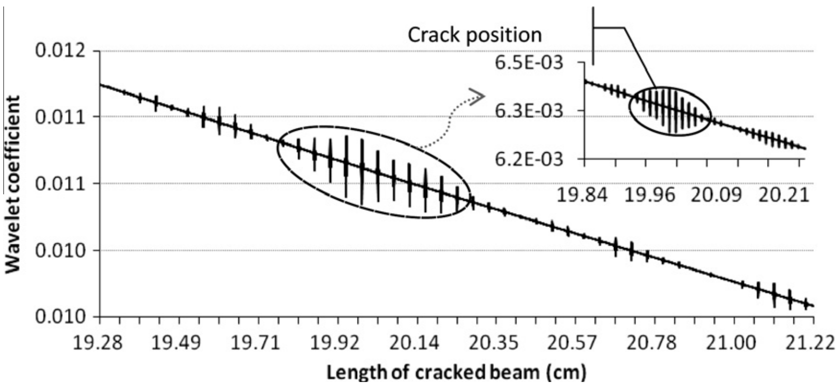

Fig. 10. Wavelet transform of the deflection profile signal of the wet-sanding polished beam with a crack of depth of 0.08 cm and width of 0.1 cm located at 20 cm from the fixed end of the cantilever beam.

depth of 0.08 cm (26.67% of the thickness of the beam) and width of 0.1 cm, which is located at the position of 20 cm from the fixed end of the same beam structure, is conducted. Fig. 10 illustrates the wavelet coefficients after the transform of the deflection profile around the crack area with the wavelet scale of 10. The upper-face of the aluminum beam above the crack area is polished. It is noted that the perturbation of the wavelet coefficients at the crack position is not significant with wavelet scale of 8.5 for the crack with depth of 0.08 cm. Noise signals in the detection of the minor crack with wavelet scale of 8.5 are observable even when the surface of the beam is polished and the original deflection signal is smoothed. However, from Fig. 10, it can be seen that the crack can be located at the position of 19.99 cm (+−0.05 cm) apparently, when the wavelet scale is increased from 8.5 to 10. Although the crack depth is around 26% of the beam thickness, the crack position still can be detected successfully by the spatial wavelet transform method experimentally.

5. Conclusions

The crack detection of a beam structure is experimentally studied using the spatial wavelet transform. In experimental studies, a high resolution laser profile sensor is employed to measure the static deflection profile of a cracked cantilever beam subjected to a static displacement at its free end. Such a static deflection profile, in which the discontinuity of gradient at the crack position is too small to be perceived, is conducted by the Gabor wavelet transform to discern or amplify the perturbation at the crack position. De-noise processes of the original deflection signal given by the laser sensor and the surface polishing treatment of the beam are critical for an effective detection. The detection method using the spatial wavelet transform with Gabor wavelet function is proven to be practically feasible and effective through our experimental studies even for a minor crack with a depth of around 26% of the beam thickness.

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