An Improved PSO Algorithm and Its Application to Structural Damage Detection

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Abstract

The particle swarm optimization (PSO) algorithm with inertial weight is improved by adopting the sigmoid function and applied to structural damage identification in frame structures in this paper. The theoretical background on the improved PSO (IPSO) algorithm is introduced, a suite of four famous benchmark functions are then employed for evaluation on the IPSO performance. Some comparative studies on single and multi-damage of a 2-story rigid frame structure have been carried out. The illustrated results show that the IPSO algorithm can significantly improve the performance of the conventional PSO algorithm. IPSO can locate and identify structural damage with higher accuracy.

1. Introduction

Structural damage detection is a vital step of the structural health monitoring (SHM) and structural services during their lifetime, which is often converted into constrained optimization problem. It has been given considerable attention in the past decades [1]. However, there is no universal agreement as to the optimum method for using measured vibration data for damage detection, location or quantification in the literatures [2]. One of difficulties is that the traditional gradient-based methods are easily led to local rather than global minimum. Therefore, the scientists are trying to explore some new approaches for structural damage detection.

Particle swarm optimization (PSO) is an evolutionary computation technique developed by Kenney and Eberhart in 1995 [3]**.** PSO has been developed through a simulation of social behaviors such as fish schooling and bird flocking. PSO is implemented by a very simple algorithm and uses only basic arithmetic operation. It doesn't require gradient information of the objective function and has been improved to be an efficient method for global optimization problems [4]. As a promising method for the constrained optimization problem in the structural damage detection, PSO is easy to produce a premature convergence problem at the last phase of iteration. For an acceptable solution to this problem, an improved particle swarm optimization (IPSO) algorithm is proposed by adopting the sigmoid function for adjusting particle positions. A suite of four benchmark functions are adopted to evaluate the proposed IPSO performance. Some numerical simulation on single and multiple damage of a 2-story rigid frame structure have also been carried out for further evaluation on the IPSO. Some acceptable results are achieved.

2. Theoretical background

According to the PSO theory, each particle is determined by its position and velocity. It moves towards its best previous position, which is called *pbest*, and towards the best position of the whole swarm, which is called *gbest*, in the following equation.

Supposing the swarm of p particles, the search space is d -dimensional, and then the position of i-th particle can be represented by a d-dimensional vector, $x_i = (x_{i1}, x_{i2} \cdots x_{id})$, $i = 1, 2, \cdots, p$. The velocity of the particle can be represented by another vector, $v_i = (v_{i1}, v_{i2} \cdots v_{id})$, $i = 1, 2, \cdots, p$. The best previous visited positions of the particle is denoted as $pbest_i = (pbest_{i1}, pbest_{i2}, \cdots pbest_{id})$, $i = 1, 2, \cdots p$. The velocity and the position of the particle are updated according to the following two equations:

$$v_{ij}^{k+1} = v_{ij}^{k} + c_1 \times r_1 \times (pbest_{ij} - x_{ij}^{k}) + c_2 \times r_2 \times (gbest_j - x_{ij}^{k}) \quad (1)$$

$$x_{ij}^{k+1} = x_{ij}^{k} + v_{ij}^{k+1} \quad (2)$$

Where $i = 1, 2, \cdots p$, $j = 1, 2, \cdots d$. Both r_1 and r_2 are random numbers, which uniformly distributed between [0, 1]. Both c_1 and c_2 are the cognitive and social scaling parameters, which are a positive constant such that $c_1 = c_2 = 2$ [5]. During each iteration, the velocity should be no more than $v_{\max}$, and the position should be ranged between $[x_{\min}, x_{\max}]$.

2.1. PSO with inertial weight

The parameter $v_{\max}$ has proved to be crucial, because large values may result in particles moving past good positions, while small ones could result in insufficient

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exploration of the search space. In order to balance global and local exploration, an inertial weight was incorporated. Thus, in the latest version of PSO, the velocity is updated as follows [6],

$$v_{ij}^{k+1} = w \times v_{ij}^{k} + c_1 \times r_1 \times (pbest_{ij} - x_{ij}^{k}) + c_2 \times r_2 \times (gbest_j - x_{ij}^{k}) \tag{3}$$

Where w is called inertial weight. The inertial weight w is considered critical for the PSO convergence behavior. In order to reach quick convergence and good result, it modified according the following equation,

$$w = w_{\max} - iter \times \frac{w_{\max} - w_{\min}}{iter_{\max}} \tag{4}$$

Where, $w_{\max}$ is the inertial weight, $w_{\min}$ the final weight, $iter$ the current iteration number and $iter_{\max}$ the maximum iteration number respectively. A large inertial weight facilitates global exploration while a small one tends to facilitates local exploration. Suitable selection of the inertial weight can balance the global and local search. In this paper, the PSO refers to the PSO with inertial weight w and $w_{\max}$ =0.9, $w_{\min}$ =0.4.

2.2. Improved PSO

At the last phase of PSO calculation, as all the particles move towards *pbest* and *gbest*, the position of all particles would be near to the *pbest* and *gbest*, the last two items of the equation (3) are also close to zero. At the same time, w close to $w_{\min}$, which is often close to 0, the first item of the equation (3) is also close to zeros, which means the deviation of velocity is limited. Then the deviation of position is also limited. Therefore, a so-called premature convergence problem will easily be caused.

For this situation, the particle position is adjusted for easily exploring a global solution at the last stage of the PSO, an improved PSO (IPSO) algorithm is proposed as follows. When the iteration numbers of the algorithm reaches to some target value, e.g., $iter = 0.3 iter_{\max}$ the particle position is changed to the following one,

$$x_{ij}^{k+1} = x_{ij}^{k} + \alpha v_{ij}^{k+1} - (C3 + \alpha) \cdot (f_i - f_{iter\min}) \tag{5}$$

$$\alpha = 1/(1 + \exp(-(f_i - f_{iter\min}))) \tag{6}$$

Where, α is a sigmoid function, a typical neuronal non-linear transfer function that helps make outputs reachable. f_i is the fitness value of the *iter* number for the *i-th* set of particle, $f_{iter\min}$ is the minimum fitness value, $i = 1, 2, \cdots p$. From Equation (6), the adjustment coefficient α ranges between [0, 1]. When $C3 = -\alpha$, the third term in Equation (5) is equal to zero. The second term in Equation is less than the v_{ij}^{k+1} , if α is not equal to one. Therefore, the new position of particle, x_{ij}^{k+1} , is forced to decrease. Meanwhile, the *x* is limited to a range between [$C4 \cdot x_{\min}, x_{\max}$], where C4 is less than one. C4=0.9, C3=1 are accepted in this paper.

3. Benchmark function studies

In order to evaluate the IPSO performance and to compare it with the conventional PSO with inertial weight, a suite of famous four benchmark functions are employed as shown in Table 1.

Table 1. Benchmark functions

Function name	Function expression	Global minimum
Sphere	$f_1(x) = \sum_{i=1}^{30} x_i^2$	0
Rosenbrock	$f_2(x) = \sum_{i=1}^{10} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	0
Rastrigin	$f_3(x) = \sum_{i=1}^{10} (x_i^2 - 10\cos(2\pi x_i) + 10)$	0
Griewank	$f_4(x) = \frac{1}{400} \sum_{i=1}^{30} x_i^2 - \prod_{i=1}^{30} \cos(\frac{x_i}{\sqrt{i}}) + 1$	0

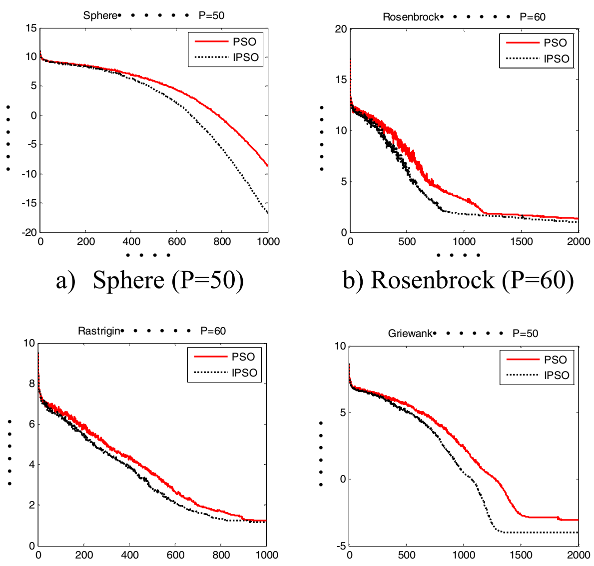
Table 2. Comparison on PSO and IPSO results

Function	Population		Maximum	Minimum	Average
Sphere	P=50	PSO	4.616e-4	4.165	1.911e-4
		IPSO	3.489e-7	3.579e-9	5.885e-8
	P=100	PSO	2.784e-5	1.542e-6	1.043e-5
		IPSO	1.944e-9	7.85e-11	6.11e-10
Rosenbrock	P=60	PSO	26.2049	0.0227	3.8550
		IPSO	23.5113	0.2434	2.7233
	P=90	PSO	22.1917	0.074	4.2954
		IPSO	23.5113	0.0026	2.4228
Rastrigin	P=60	PSO	4.9748	0.9950	3.3971
		IPSO	5.9697	0.9950	3.1839
	P=90	PSO	3.9798	8.711e-9	2.0258
		IPSO	4.9998	0	1.9912
Griewank	P=50	PSO	0.2905	1.475e-12	0.0474
		IPSO	0.1449	0	0.0182
	P=100	PSO	0.1213	9.104e-15	0.0220
		IPSO	0.0734	0	0.0110

IPSO and PSO are used to calculated and compared under some conditions. The maximum velocity $v_{\max}$ is set to be $(x_{\max} - x_{\min})/2$. The maximum iteration number $iter_{\max}$ equals to 1000. Each experiment is executed 20 times and then their average values are provided by minimum, maximum and average values as shown in Table 2. The searching progress of the average values is illustrated in Figure 1, in which, the horizontal coordinates is for the maximum iterations, the vertical one for the logarithmic fitness values of functions.

It can be found from Table 2 that both PSO and IPSO results are approaching to the global minimum values when the population size is increased. The IPSO values are much closer to the global minimum ones than the PSO, which shown that the IPSO is better than the PSO.

In Figure 1, it can be seen that the IPSO goes down more sharply than the PSO under the same conditions. The IPSO can reach to lower value than the PSO, which shows that the IPSO converge more quickly, or say, it is more efficient than the PSO.



a) Sphere (P=50) b) Rosenbrock (P=60)

c) Rastrigin (P=60) d) Griewank (P=50)

Figure 1. Iteration processes for both PSO and IPSO

4. IPSO application to structural damage detection

4.1 Objective function

The structural damage detection can be converted into a constrained optimization problem based on the changes of dynamic characteristics of structures as well as the finite element models of structures. In mathematical model, the objective functions can be defined as follows

$$f(x)=\sum_{i=1}^{s}\left(\ (1-MAC(\Phi_i^h,\Phi_i^d))+ER(\omega_i^h,\omega_i^d)\ \right) \tag{7}$$

$$ER(\omega_i^h,\omega_i^d)=\left|\frac{\omega_i^h-\omega_i^d}{\omega_i^h}\right|\times 100\% \tag{8}$$

$$MAC(\omega_i^h,\omega_i^d)=\frac{\left|\Phi_i^{hT}\Phi_i^d\right|^2}{\left|\Phi_i^{hT}\Phi_i^d\right|\left|\Phi_i^{hT}\Phi_i^d\right|},\ (i=1,2,\cdots,s) \tag{9}$$

Where, (ω_i^h,Φ_i^h) and (ω_i^d,Φ_i^d) indicate frequency and mode shape corresponding to the healthy structure and the damaged structure respectively. s is the number of modes. x is the damage indicator representing the damage location and extent. In order to avoid singularity of the eigen-problem calculation, x is limited between [0, 0.9], where 0 indicates the structural is intact while 0.9 indicates the element stiffness is reduced by 90%. Then the damage detection is converted into the following constrained optimization problem:

$$\min f(x)=\sum_{i=1}^{s}\left(\ (1-MAC(\Phi_i^h,\Phi_i^d))+ER(\omega_i^h,\omega_i^d)\ \right) \tag{10}$$
$$where,\quad 0\le x\le 0.9,\quad i=1,2,\cdots,s$$

It is calculated through the conventional PSO algorithm and the IPSO algorithm proposed in this paper for evaluation on their validity.

4.2 Numerical simulation

A 2-story rigid frame structure is used to study structural damage detection based on both the PSO and the IPSO algorithms, some numerical simulations are discussed in this paper. Figure 2 shows the finite element model of the frame and the corresponding size of both the column and the beam. The numbers in the rectangular indicate the finite element number and the others near to the frame indicate the node number. The frame was modeled by eighteen 2-dimmensional beam elements with equal length. The properties of the materials are as follows: the elastic modulus for both the vertical column and the horizontal beam, E_c = E_b = 2.0×10^{11} N/m^2, the cross section inertial moments I_c = 0.0000126m^4, I_b = 0.0000236m^4, the cross section areas, A_c=0.00298m^2, A_b=0.0032m^2, material densities, d_c=8590kg/m^3, d_b=7593kg/m^3, where the subscripts c and b represent the column and the beam respectively.

In order to simulate the structural damage cases, a single and multi-damage of the frame are specified. First five natural frequencies and mode shapes for both the healthy and damage structures are taken into consideration during the damage detection calculation.

For both the PSO and the IPSO algorithms, the population size P is set to be 250 for the single damage cases and 500 for the multi-damage cases. Each case is executed 20 times to get an average value.

Further, in order to simulate more realistic cases with contaminated noise, the mode shapes are corrupted with a normally distributed random noise with different levels of 5% and 10% for both single and multi-damage cases.

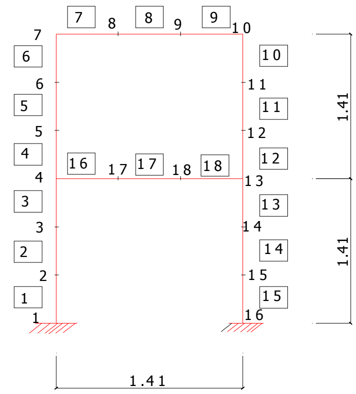


Figure 2. FEM model of frame (Unit: m)

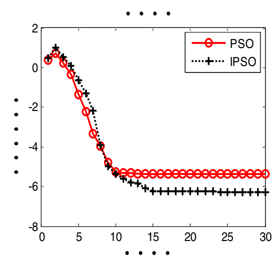
4.2.1 Single damage detection. For the single damage case, the stiffness of the 17th element is set to be reduced by 3%, 5%, 10% and 15% respectively. Table 3 shows comparison between damage detection results from both PSO and IPSO algorithms respectively. Where, capital letter NL indicates Noise Level contaminated in the mode shapes, e.g. NL=5, which means 5% noise is contaminated into the mode shapes. Damage extent symbol 3@17 indicates 3% reduction of stiffness at element 17. Same recognition can be found in the following tables.

Table 3. Single damage detection results

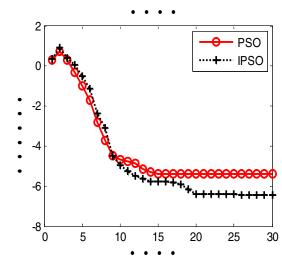
True Damage (%)		Damage Detection (%)	
extent	NL	PSO	IPSO
3@17	0	1.5@17, 1.8@18, 0.6@4, 0.6@6	2@17, 0.6@18, 0.2@4, 0.4@6
	5	2.1@17, 2@18	1.5@17, 0.7@18
	10	0.6@17, 5@6	1.4@17
5@17	0	4.5@17	5@17
	5	4.5@17	5@17
	10	1.5@17, 3@18	2.8@17, 1@6
10@17	0	8.1@17, 0.8@18	10@17
	5	8.2@17, 2.5@18,1@4	10@17
	10	8@17, 1@18	10@17
15@17	0	13.5@17, 2.5@18	15.5@17
	5	14@17, 1@18	15%17
	10	13@17, 1.5@18	15@18

It can be seen from Table 3 that a) the structural damage detection under four cases is precise if without noise (NL=0), the higher the damage extent is, the better the damage detection results are, and the IPSO results are much better than that the PSO results. b) The noise obviously affect the detection results for both PSO and IPSO, but IPSO is effective than PSO under same noise level. c) For the case of minimum 3% damage, the PSO is failed, but IPSO can provide valued reference, which can be referred to location of damage. d) For 5% damage case without noise or with 5% noise, both PSO and IPSO can give excellent location of damage. For 10% noise case, IPSO can provide a valued reference but PSO cannot locate damage. e) For cases higher than 10% damage, both PSO and IPSO can give acceptable results. Both of them can locate damage but also detect damage extent. Meanwhile, IPSO are all better than PSO.

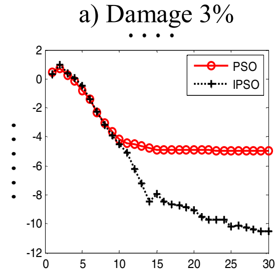
For more clearly comparison on convergence for both PSO and IPSO, Figure 3 illustrates the convergent curves for each case without noises mentioned above, in which, the horizontal coordinates is for the iteration number, the vertical one for the logarithmic values of fitness. It can be seen that IPSO results are all better than PSO. IPSO can get higher accurate result. It can converge more efficiently and more quickly than PSO.



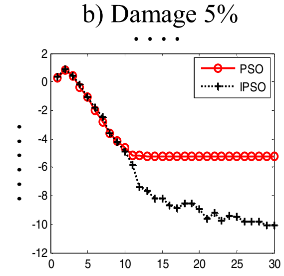
a) Damage 3%



b) Damage 5%



c) Damage 10%



d) Damage 15%

Figure 3. Convergent curves under without noise

4.2.2 Multi-damage detection. For the multi-damage scenarios, stiffness of both the 8th and 17th elements is set to be reduced by different damage extents simultaneously. Four cases are simulated, they are (10%, 10%), (10%, 15%), (15%, 10%), (15%, 15%). For example, case (10%, 15%) means stiffness of 8th element is set to be reduced by 10% but the 17th element reduced by 15% simultaneously. Table 4 shows comparison between the damage detection results from both the PSO and the IPSO respectively.

It can be seen that a) noise obviously affect the detection results for both the PSO and the IPSO, but the IPSO is effective than the PSO under same noise level. b) For the multi-damage case of (10%, 10%), some good results can be predicted by the IPSO and the PSO algorithms respectively under different noise levels except for with noise 10%. For the case of noise 10% under the damage (10%, 10%), although the results from the two algorithms can not exactly indicate the damage extent, especially for 8th element, they are still adopted as an index of damage location. c) If the damage extents at both elements 8 and 17 are not same, the detection results for the damage case at element 8 with higher damage extent are better than ones at element 8 with lower damage extent. d) The above illustrated results demonstrate that the PSO algorithm is valuable for structure damage detection and the IPSO algorithm can get much better results in most cases.

Table 4. Multi-damage detection results

True Damage (%)			Damage Detection (%)
extent	NL	method	
10@8 10@17	0	PSO	10@8, 10@17
		IPSO	10@8, 10@17
	5	PSO	8@8, 10@17, 2@7
		IPSO	8@8, 10@17, 1.9@7
	10	PSO	3.9@8, 10@17, 2.4@5
		IPSO	5.5@8, 10@17, 2.5@7
10@8 15@17	0	PSO	8@8,15@17,2.1@7
		IPSO	8@8, 15@17, 2@2
	5	PSO	8@8, 15@17, 2.1@7
		IPSO	8@8, 15@17, 2@7
	10	PSO	3@8, 11.8@17, 3.5@6, 4.1@18
		IPSO	3.2@8, 12.2@17, 5@7, 4@18
15@8 10@17	0	PSO	13@8, 9@17, 2.5@7, 1.8@18
		IPSO	13.5@8, 10@17
	5	PSO	13@8, 10@17, 1.5@7
		IPSO	15@8, 10@17
	10	PSO	11.8@8, 9.5@17, 4@7
		IPSO	11.8@8, 9@17, 4.5@7
15@8 15@17	0	PSO	15@8, 15@17
		IPSO	15@8, 15@17
	5	PSO	10@8, 15@17, 2@7
		IPSO	12@8, 15@17, 4@7
	10	PSO	10@8, 13@17, 8@7
		IPSO	10@8, 15@17, 6@7

5. Conclusions

The particle swarm optimization (PSO) algorithm with inertial weight is improved by adopting the sigmoid function and applied to structural damage identification problem in this paper. The theoretical background on the improved PSO (IPSO) algorithm is introduced, a suite of four famous benchmark functions are then employed for evaluation of the IPSO performance. The benchmark function tests show that the IPSO is much efficient than the conventional PSO with inertial weight. Further, an objective function for structural damage detection is defined through difference between the analytical and experimental results based on the first five natural frequencies and modal shapes of the frame finite element models. The structural damage detection problem is then converted into a constrained optimization problem and solved by the IPSO as well as the conventional PSO algorithm. Some comparative studies on single and multi-damage of a 2-story rigid frame structure have shown that the IPSO algorithm can significantly improve the performance of the conventional PSO algorithm although they are both affected by the noise. The IPSO can locate and identify structural damage with higher accuracy.

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