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Short communication

The new frequency response functions for structural health monitoring

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ABSTRACT

New monitoring techniques, with the frequency response functions based on the higher order spectra, are proposed and developed for the monitoring of structure non-linearity and signal non-Gaussianity due to damage and estimating the harmonic phase coupling of signals from structures for cases of the phase coupled interferences of a structure excitation.

The proposed techniques are generalisations of the classical frequency response functions for the higher order spectral analysis for real valued transforms. It is shown that the proposed techniques provide an essential effectiveness gain for the detection of non-linearity due to fatigue damage in comparison with the classical HOS for the case of the phase coupled interferences of a structure excitation.

1. Introduction

For the monitoring of structure non-linearity due to damage and the non-Gaussianity of signals from structures due to damage and estimating of the harmonic phase coupling of signals from structures, an input signal (e.g. vibration excitation, acoustical excitation, etc.) excites the structure in question and the structure resonance oscillations are processed by the higher order spectra (HOS) [1,2]. This approach has been widely investigated for stationary and non-stationary signals [1–25].

The classical HOS, the bispectrum (bicoherence) and tispectrum (tricoherence) [1] has been widely employed for stationary signals for the detection of symmetrical and asymmetrical non-linearities in structures and machinery, including fatigue damage detection [1,3,5,9,11–13].

HOS has been also investigated for non-stationary signals. The Wigner higher order spectra [14] have been proposed for transient signals with a linear variation of the instantaneous frequency in time. The wavelet bispectrum [15,16] has been proposed for impulses, quasi-periodic and chaotic signals. The chirp-Fourier and the chirp-Wigner higher order spectral analyses [8,24] have been proposed for transient signals with linear and nonlinear polynomial variation of the instantaneous frequency in time. Hanssen and Scharf [17] have developed an important theory of higher order spectral analysis for non-stationary stochastic processes belonging to the harmonisable class. Dandawate and Giannakis [18] and Gardner and Spooner [19] have developed an important theory of higher order spectral analysis for higher order cyclostationary processes. These works [17–19] could be fairly easily generalized to generic non-stationary processes.

Parameter estimation using nonstationary HOS was investigated [14,20,21] for the finite sum of the exponentially damped sinusoidal signals with constant frequencies (i.e. signals with amplitude non-stationarity) in additive white or coloured Gaussian noise. Detection by transient HOS of an unknown deterministic transient signal in broad-band additive stationary noise was investigated in [22,23]. A matched filtering and HOS were combined [25] for the detection of transient deterministic or random non-Gaussian signals on a background of zero-mean Gaussian noise with an unknown covariance sequence.

The vibration excitation produced by exciters (e.g. electrodynamical shakers, rotating eccentric mass (REM) exciters, etc.) normally consists of the main harmonic at the excitation frequency and phase coupled higher harmonics of the excitation frequency (e.g. the quadratic coupled, the cubic coupled, etc.) due to exciter non-linearities. These higher harmonics are interferences which are transferred to the structure resonance oscillations.

The main disadvantage of the HOS in the considered case is that the HOS estimates are increased for both undamaged and damaged structures due to these phase coupled interferences of an excitation. Therefore, these interferences reduce the effectiveness of the damage detection/diagnosis of the structure. These interferences also reduce the effectiveness of the estimation of the signal non-Gaussianity due to damage and the harmonic phase coupling of the signals.

The traditional approaches for eliminating interferences from the excitation are based on the classical frequency response function [26] and higher order frequency response functions

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[27–29]. However, these important and effective approaches are not suitable for structure monitoring by the HOS.

The problem is to create new HOS techniques for the monitoring of structure non-linearity and signal non-Gaussianity due to damage and estimating the harmonic phase coupling of signals suitable for cases of the phase coupled interferences of a structure excitation. It is important to solve this problem for metal and composite engineering structures.

The purposes of this paper are:

- to propose new monitoring techniques, the frequency response functions based on the HOS for the monitoring of structure non-linearity due to damage and signal non-Gaussianity due to damage and estimating the harmonic phase coupling of structure signals for cases of the phase coupled interferences of a structure excitation
- to compare by experiments the proposed techniques with the classical HOS.

2. The new frequency response functions based on the HOS

To overcome the above mentioned disadvantage of the classical HOS, novel monitoring techniques are proposed here: the frequency response functions based on the HOS. For estimating the proposed functions, the following step should be undertaken: the synchronous time domain output and input (i.e. excitation) signals should be divided into overlapping segments by the internal time window, $m = 1, \ldots, M$, M defines the total number of overlapping segments in the signals; the number of segments and level of overlapping should be the same for the output and input signals.

The generic expression of the proposed frequency response functions based on the classical HOS (i.e. using the Fourier transform) for the mth segment is as follows:

$$FRF_m(f_1, f_2, \ldots, f_{n-1}) = \frac{H_{om}(f_1, f_2, \ldots, f_{n-1})}{H_{im}(f_1, f_2, \ldots, f_{n-1})}, \tag{1}$$

where H_{pm} are the *non-averaged* HOS of order n of the output ($p = o$) and input ($p = i$) signals respectively, $n = 3, 4, \ldots$, $H_{pm}(f_1, f_2, \ldots, f_{n-1}) = X_{pm}(f_1)X_{pm}(f_2)\cdots X_{pm}(f_{n-1})X^*_{pm}(f_{n\Sigma})$, $X_{pm}(f_j)$ is the Fourier transform at frequency f_j and segment duration Δt_m for the output and input signals, $j = 1, n-1$, $f_{n\Sigma}$ is the accumulated frequency, $f_{n\Sigma} = \sum_{j=1}^{n-1} f_j$, * is a symbol of the complex conjugate.

Normally, the HOS related techniques should be averaged over time [1]. The time averaging of the new techniques is proposed as follows:

$$FRF_a(f_1, f_2, \ldots, f_{n-1}) = \frac{\frac{1}{M}\sum_{m=1}^{M} H_{om}(f_1, f_2, \ldots, f_{n-1})}{\frac{1}{M}\sum_{m=1}^{M} H_{im}(f_1, f_2, \ldots, f_{n-1})}. \tag{2}$$

So, the proposed averaged frequency response function is the ratio of the two averaged HOS: the averaged HOS of the structure output and the averaged HOS of the input (i.e. excitation).

The non-averaged and averaged frequency response functions (1)–(2) are complex valued, estimated by the Fourier transforms of the output and input signals at n frequencies and depend on $(n-1)$ frequencies. For orders three and four, the frequency response functions (1)–(2) are based on the bispectrum and the trispectrum respectively.

The physical significance of the proposed frequency response functions is that they provide a measure of the structure's output HOS in response to the structure's input HOS. The importance of the proposed functions is that they eliminate the influence of the phase coupled interferences of a structure excitation on structure non-linearity and signal non-Gaussianity detection/diagnosis and phase coupling estimation in a similar way as the classical frequency response function [14] eliminates the influence of the higher harmonics-interferences of a structure excitation. If the phase coupled interferences appear in an excitation, both the HOS of the output and the input signals will increase due to these interferences; thus, the frequency response functions (1)–(2) will be non-essentially affected by these interferences for linear and nonlinear structures.

The proposed techniques cannot be used if the excitation is a sum of non-phase coupled signals (including Gaussian signals) because the denominator in Eq. (1) will be zero in this case, thus leading to a singularity. However, one does not need to use the proposed techniques in this case because the considered excitation does not produce interference in the HOS of the output signal. The classical HOS should be employed in this case. It is highlighted here that the proposed techniques have been developed and should be employed only for cases of the phase coupled interferences of a structure excitation.

It is known [1,3] that the dependence of the variance of the HOS estimates on the signal power spectral density can result in misleading HOS estimates. In order to avoid a misleading interpretation, the HOS are always normalised [1,3]. Although the proposed frequency response functions are already ratios of products of the Fourier transforms, these functions depend on the two different power spectral densities of the input and the output signals; thus, the misleading interpretation of the techniques could be due to variations of both power spectral densities. Therefore, the techniques should be also normalised.

The two normalised frequency response functions based on the well developed normalisation methods [1,3] are proposed as follows:

$$frf_1 = \frac{h_{o1}}{h_{i1}}, \tag{3}$$

$$frf_2 = \frac{h_{o2}}{h_{i2}}, \tag{4}$$

where h_{pk} are the normalised averaged HOS of the output and the input signals, $k = 1, 2$,

$$h_{p1} = \frac{\sum_{m=1}^{M} H_{pm}}{\sqrt{\sum_{m=1}^{M} |X(f_1)X(f_2)\cdots X(f_{n-1})|^2 \sum_{m=1}^{M} |X(f_{n\Sigma})|^2}}, \tag{5}$$

$$h_{p2} = \frac{\frac{1}{M}\sum_{m=1}^{M} H_{pm}}{\sqrt{\frac{1}{M}\sum_{m=1}^{M} |X(f_1)|^2 \frac{1}{M}\sum_{m=1}^{M} |X(f_2)|^2 \cdots \frac{1}{M}\sum_{m=1}^{M} |X(f_{n-1})|^2 \frac{1}{M}\sum_{m=1}^{M} |X(f_{n\Sigma})|^2}}. \tag{6}$$

So, the normalised frequency response functions are ratios of the normalised HOS of the output and input signals. For orders three and four, Eq. (3), (5) and (4), (6) describe the normalised frequency response functions based on the

- bicoherence and tricoherence respectively
- skewness and kurtosis respectively.

The normalised frequency response functions (3)–(4) could not be used for order two, because the normalised classical HOS for this order for both output and input are unconditionally equal to unity. It should be noted that the normalised classical HOS for this order also could not be used due to the same reason.

The physical meaning of the normalisations of the proposed frequency response functions is that these normalisations allow

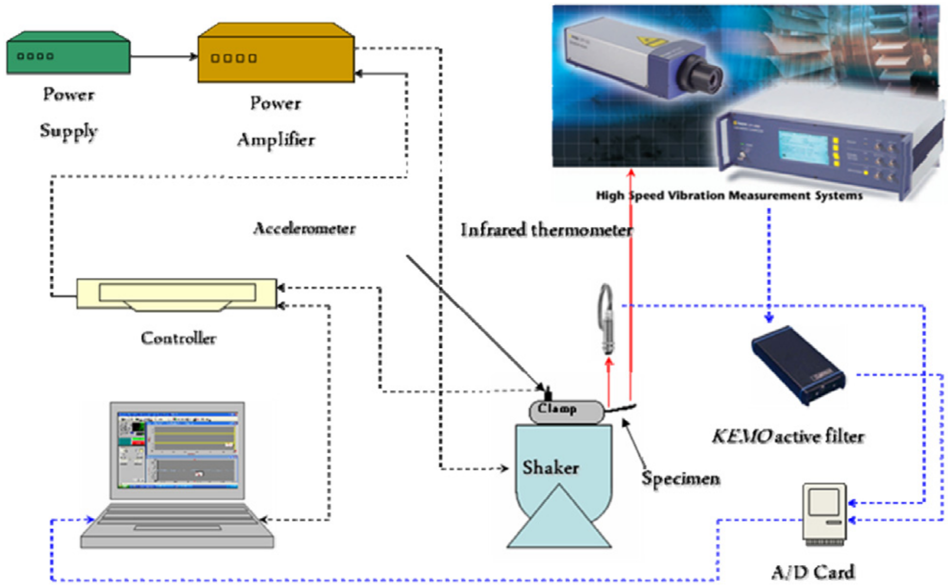

Fig. 1. The schematic of a shaker test.

avoidance of the misleading interpretation of the proposed techniques (1)–(2) due to variations of the power spectral density of both the input and output signals.

The proposed techniques can be also used for non-stationary signals by employing the appropriate time–frequency transforms (e.g. the chirp-Wigner transform [24], etc.). This could be done by substituting time–frequency transforms for the Fourier transform in Eqs. (1)–(6).

In the second-order case, the proposed techniques (1) reproduce the squared classical frequency response function only for the real valued transforms (e.g. the Wigner distribution, the chirp-Wigner transform [30], etc.). For the complex valued transforms (e.g. the Fourier transform, etc.) the proposed techniques (1) reproduce the ratio of squared magnitudes of transforms for output and input signals in the second-order case. Thus, the proposed techniques (1) are generalisations of the classical frequency response function for the higher order spectral analysis only for the real valued transforms.

3. Experimental validation of the frequency response functions based on the HOS

To demonstrate that the proposed techniques can effectively detect structure non-linearity due to fatigue damage in the cases of the phase coupled interferences of the excitation, a shaker test with fatigue damaged and undamaged rectangular beams was performed. The schematic of the shaker test is shown in Fig. 1 and the test rig is shown in Fig. 2.

A Data Physics (UK) shaker Signal Force V400 was driven by a Data Physics Signal Star Vector controller, using a Kistler 8202A10 accelerometer mounted on the shaker's table and a closed loop control. Controller software was installed on a Sony laptop. Signal conditioning of the accelerometer data was performed using an Endevco 133 signal conditioner and analogue active anti-aliasing filter Kemo PocketMaster 1600 (the filter attenuation in the transition band is 80 dB/oct, the cut-off frequency of the filter is 4 kHz). Resonance vibrations from the beams were captured by the high speed laser vibration measurement system Polytech OFV-5000 with the OFV-505 optical head. The temperature of the beams was captured by a Raytek CI-3A infrared noncontact temperature sensor.

Fig. 2. Test rig with shaker.

The vibration and temperature data were recorded using an analogue active anti-aliasing filter Kemo PocketMaster 1600 and National Instruments data acquisition A/D card NI-6251 and stored on a Sony laptop. The sampling frequency was 10 kHz and the cut-off frequency of the filter was 4 kHz.

The stationary random single frequency sine vibration excitation (distortion is 0.01%, signal/noise ratio is 92 dB) from a shaker with a constant acceleration amplitude 5 g, a constant frequency tuned to the beam resonance frequency and a random initial phase excited the resonance oscillations of the first (i.e. bending) vibration mode of beams. The shaker excitation also consists of the phase coupled higher harmonics of the excitation frequency (i.e. interferences).

Twenty eight stationary signals: i.e. 14 signals from the damaged beam and 14 signals from the undamaged beam were tested for damage (i.e. non-linearity) detection. The beam size

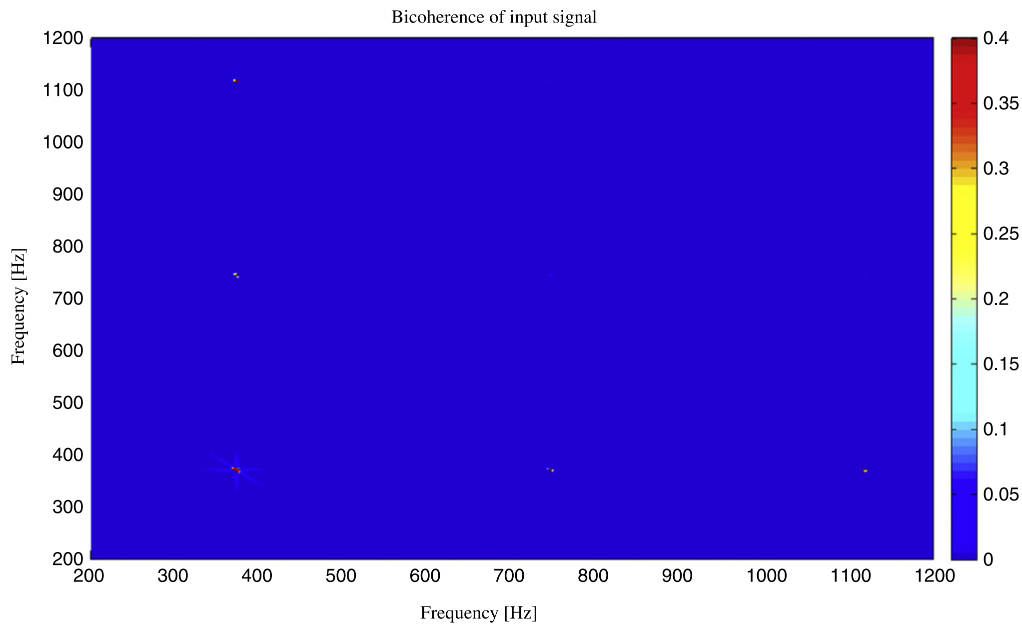

Fig. 3. The classical bicoherence of the shaker excitation.

is: 112 mm × 40 mm × 4.5 mm. The resonance frequencies of the first mode of the damaged and undamaged beams are 373 Hz and 279 Hz respectively.

The classical bicoherence of the shaker excitation is shown in Fig. 3, the frequency resolution is 8 Hz (i.e. segment size is 0.125 s), the duration of excitation is 15 s, the segment overlapping is 50% and the internal time domain window is the Hamming window. Relatively high values of bicoherence components at the fundamental–fundamental harmonics (i.e. bicoherence component B11), the fundamental-second harmonics (i.e. bicoherence component B12) and the fundamental-third harmonics (i.e. bicoherence component B13) of the excitation could be seen from this Figure. These values characterise the relatively high level of quadratic phase coupling of the shaker excitation. For example, the mean value of the bicoherence component B12 at the fundamental-second harmonics of the excitation is 0.39.

It is well known that fatigue damage introduces an additional level of structure non-linearity. The proposed frequency response functions (3)–(4) of order 3 based on the classical bicoherence and skewness respectively and the classical bicoherence and skewness of the beam resonance oscillations have been employed for detecting additional levels of beam nonlinearity due to fatigue damage. The bicoherence and skewness components at the fundamental and second harmonics have been employed in both cases.

The following parameters have been used for estimating the frequency response functions (3)–(4) and the classical bicoherence and skewness: the frequency resolution is 8 Hz (i.e. segment size is 0.125 s), duration of stationary input (i.e. a system excitation) and output signals is 15 s, the segment overlapping is 50%, the internal time domain window is the Hamming window and the sampling frequency is 10 kHz.

The non-dimensional mean/standard deviation values of the magnitudes of the proposed frequency response functions (3) of order 3 at the fundamental and second harmonics for this test are 0.98/0.002 and 2.54/0.098 for the undamaged and damaged beams respectively. The dependencies of the selected component of new functions vs. signal number are shown in Fig. 4 for damaged and undamaged beams. Relatively stable behaviour of the selected component is observed. The Fisher criteria of detection

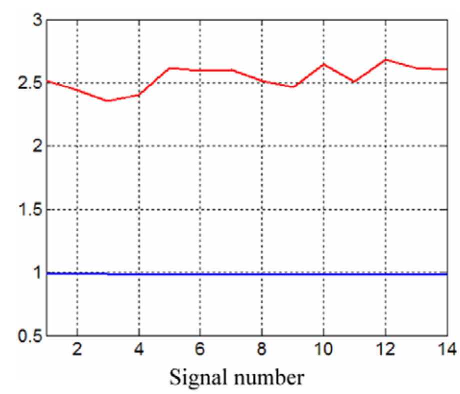

Fig. 4. The normalised frequency response functions based on the bicoherence vs. signal number; red and blue curves are for the damaged and undamaged beams respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

effectiveness [31] are 255 and 87 for the technique (3) and classical bicoherence. It is known [31] that features with higher values of the Fisher criterion provide better detection effectiveness.

The mean value of the signal/noise ratio for the proposed frequency response function based on the bicoherence at the fundamental and second harmonics is 0.6 for the undamaged case. This value is low; therefore, the proposed techniques essentially suppress phase coupled interferences of the excitation.

The non-dimensional mean/standard deviation values of the proposed frequency response functions (4) of order 3 at the fundamental and second harmonics for this test are 0.98/0.004 and 1.94/0.052 for undamaged and damaged beams respectively. Dependencies of the selected component of new functions vs. signal number are shown in Fig. 5 for damaged and undamaged beams. Relatively stable behaviour of the selected component is observed.

The Fisher criteria of detection effectiveness [31] are 333 and 73 for the technique (4) and classical skewness.

Finally, the dependencies of the Fisher criteria of detection effectiveness vs. the frequency resolution for constant signal duration were obtained (Fig. 6), where the continuous and dashed curves are for the normalised frequency response functions based

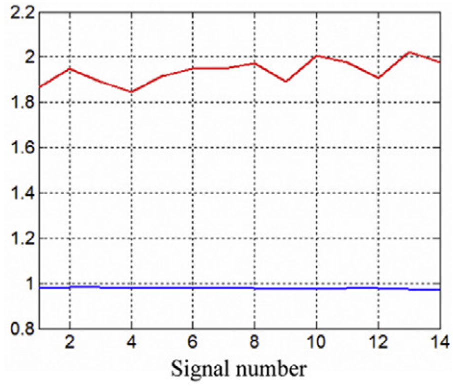

Fig. 5. The frequency response functions based on the skewness vs. signal number; red and blue curves are for the damaged and undamaged beams respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

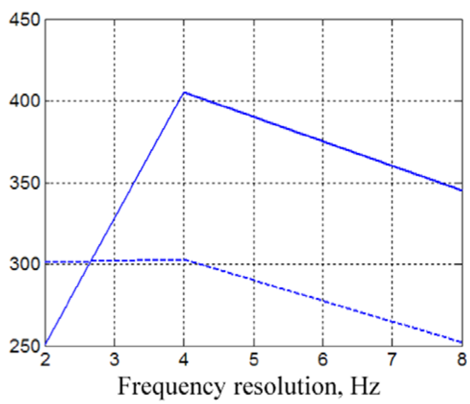

Fig. 6. The Fisher criteria vs. the frequency resolution.

on the skewness and bicoherence respectively. The optimum frequency resolution of 4 Hz is observed for both of the frequency response functions.

Thus, the proposed technique provides an essential effectiveness gain (i.e. 2.9 and 4.6 times using the proposed frequency response functions based on bicoherence and skewness respectively) in comparison with the classical HOS and, therefore, is more effective for damage (i.e. non-linearity) detection in cases of the phase coupled interferences of structure excitation.

4. Conclusions

1. New monitoring techniques, with the frequency response functions based on the HOS, are proposed and developed for monitoring of structure non-linearity and signal non-Gaussianity due to damage and estimating the harmonic phase coupling of signals from structures for cases of the phase coupled interferences of a structure excitation.
2. Time averaging and normalisation of the techniques are also developed.
3. The proposed techniques are generalisations of the classical frequency response functions for the higher order spectral analysis for real valued transforms.
4. It is shown by experiment that the proposed techniques provide an essential effectiveness gain (i.e. 2.9 and 4.6 times) for the detection of non-linearity due to fatigue damage in comparison with the classical HOS for the case of the phase coupled interferences of a structure excitation.

The proposed techniques could be used in mechanical and electrical engineering, telecommunications, underwater acoustics, etc. for cases of the phase coupled interferences of a structure/system excitation.

Further investigation should be focused on comparison of the proposed normalisations (3)–(4) of the frequency response functions.

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