



Scientific Background on the Nobel Prize in Physics 2015

## NEUTRINO OSCILLATIONS

compiled by the Class for Physics of the Royal Swedish Academy of Sciences

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# Neutrino oscillations

## Introduction

The discovery that neutrinos can convert from one flavour to another and therefore have non-zero masses is a major milestone for elementary particle physics. It represents compelling experimental evidence for the incompleteness of the Standard Model as a description of nature. Although the possibility of neutrino flavour change, i.e. neutrino oscillations, had been discussed ever since neutrinos were first discovered experimentally in 1956, it was only around the turn of the millennium that two convincing discoveries validated the actual existence of neutrino oscillations: in 1998, at Neutrino'98, the largest international neutrino conference series, Takaaki Kajita of the Super-Kamiokande Collaboration presented data showing the disappearance of atmospheric muon-neutrinos, i.e. neutrinos produced when cosmic rays interact with the atmosphere, as they travel from their point of origin to the detector. And in 2001/2002, the Sudbury Neutrino Observatory (SNO) Collaboration, led by Arthur B. McDonald, published clear evidence for conversion of electron-type neutrinos from the Sun into muon- or tau-neutrinos. These discoveries are of fundamental importance and constitute a major breakthrough. Neutrino oscillations and the connected issues of the nature of the neutrino, neutrino masses and possible CP violation among leptons are today major research topics in particle physics.

The history of the neutrino goes back to 1914, when J. Chadwick first demonstrated that the  $\beta$ -spectrum from the decay of a radioactive element was continuous, as opposed to the  $\alpha$ - or  $\gamma$ -spectrum [1]. This seemed to imply a missing particle – or even possibly, as it was thought at the time, a breakdown of energy conservation. In 1930, W. Pauli postulated a solution to this enigma in terms of a new constituent of the atomic nucleus: an electrically neutral, weakly interacting, spin- $\frac{1}{2}$  fermion with mass similar to the electron. In analogy with the proton, Pauli suggested that this particle be named the *neutron* [2]. When Chadwick in 1932 discovered a much more massive, neutral, strongly interacting particle similar to the proton, which could sensibly bear that name [3], E. Fermi proposed instead the name *neutrino* for Pauli's elusive particle, concluding that it might conceivably be massless [4]. Although these early papers already contained ideas on how one might measure the neutrino mass, the smallness of this quantity poses serious experimental difficulties, and still today only upper limits exist for the masses of the three known neutrino flavours.

Pauli did not expect that his hypothetical new particle would ever be observed. However, in the early 1950s, F. Reines and C.L. Cowan Jr., encouraged by B. Pontecorvo, set up a decisive experiment at the Savannah River nuclear reactor in South Carolina, demonstrating that (anti)neutrinos produced in the reactor processes sometimes interacted with protons in the detector medium, each reaction resulting in a neutron and a positron which could be registered (so-called inverse  $\beta$ -decay). This was the long awaited unambiguous proof of the existence of the neutrino, and in June 1956, just two years before Pauli's death, Reines and Cowan could send a telegram informing Pauli of their discovery. Reines [5] shared the Nobel Prize in Physics 1995 with M.L. Perl.

Today we know that nuclear reactors produce – not neutrinos – but electron anti-neutrinos. But – what distinguishes neutrinos ( $\nu$ ) from anti-neutrinos ( $\bar{\nu}$ )? As opposed to electrons and positrons, neutrinos carry no electric charge and so a new concept was needed to make the distinction. In 1953, E.J. Konopinski and H.M. Mahmoud [6] postulated, based on lack of experimental evidence for certain decay processes, that particles like  $e^-$ ,  $\mu^-$  and  $\nu$  possessed a

new quantum number, a lepton number  $L = +1$ , whereas  $e^+$ ,  $\mu^+$  and  $\bar{\nu}$  were assigned  $L = -1$  (the muon had been discovered in 1937; tauons were unknown at the time). All other particles had  $L = 0$ . Postulating further that the lepton number would be conserved, the absence of reactions like  $\bar{\nu} + n \rightarrow p + e^-$  could be explained [7].

The lepton number concept was further refined introducing individual lepton numbers for the muon and the electron. The distinction helped explain absence of radiative muon decays  $\mu \rightarrow e + \gamma$  and suggested that the neutrinos produced in pion decay  $\pi^+ \rightarrow \mu^+ + \nu$  ( $\pi^- \rightarrow \mu^- + \bar{\nu}$ ) together with muons (called muon-neutrinos,  $\nu_\mu$ ) were distinct from those originating from nuclear  $\beta$ -decay  $p \rightarrow n + e^+ + \nu$  ( $n \rightarrow p + e^- + \bar{\nu}$ ) (electron-neutrinos,  $\nu_e$ ) [8].

The existence of the muon-neutrino was established experimentally at Brookhaven National Laboratory in 1962 using  $\bar{\nu}$ 's from  $\pi^-$  decays and demonstrating occurrence of the reaction  $\bar{\nu} + p \rightarrow \mu^+ + n$  and absence of  $\bar{\nu} + p \rightarrow e^+ + n$ . If  $\nu_e$  and  $\nu_\mu$  were indistinguishable, the rates of both reactions should be equal [9]. L.M. Lederman, M. Schwartz and J. Steinberger were awarded the Nobel Prize in Physics 1988 for this discovery.

The assumption that neutrinos are massless and characterised by distinct, individual lepton numbers was incorporated into the theory of electro-weak interactions and subsequently into the Glashow-Weinberg-Salam Standard Model (S.L. Glashow, S. Weinberg and A. Salam, Nobel Prize in Physics 1979). The model further incorporates the fact that weak interactions violate parity by only allowing left-handed neutrinos (and right-handed anti-neutrinos) to participate in weak interactions. Right-handed neutrinos and left-handed anti-neutrinos have never been observed and – if they exist – do not interact via the known interactions. They are therefore called “sterile”. The handedness (chirality) of the neutrino is consistent with the measured neutrino helicity,  $h = -1$ , within the experimental uncertainties, just as expected for a massless particle [10].

The Standard Model was completed during the late 1970s, incorporating in the lepton sector also a third lepton, the tauon, discovered at Stanford Linear Accelerator Laboratory in 1975 [11] (Nobel Prize in Physics 1995 to M.L. Perl, shared with F. Reines). Twenty-five years later, the corresponding tau-neutrino was observed directly for the first time in the DONUT experiment, published in 2001 [12].

The Standard Model, including the quantum field theory of strong interactions (QCD) and the unified theory of electromagnetic and weak interactions, turned out to be an extremely successful description of matter at the fundamental level. A crucial accomplishment was the discovery by the ATLAS and CMS Collaborations at CERN of the predicted fundamental Higgs boson, necessary for the mass generating mechanism [13] (Nobel Prize in Physics 2013 to F. Englert and P.W. Higgs for their theoretical contributions). The predictions of the Standard Model have been verified in precision experiments, most notably at the Large Electron Positron Collider LEP at CERN. Based on the measurement of the so-called invisible  $Z^0$  width, these experiments established the number of light neutrinos to be three – consistent with the effective number of relativistic degrees of freedom determined in cosmology [71].

In the Standard Model, lepton numbers are conserved, neutrinos are massless and neutrino flavours do not oscillate. However, the conjecture that neutrino oscillations might exist is not new: already in 1957, Pontecorvo suggested the possibility of  $\nu \leftrightarrow \bar{\nu}$  oscillations, in analogy to the phenomenon of  $K^0 \leftrightarrow \bar{K}^0$  oscillations [14]. Following the discovery of  $\nu_\mu$  in 1962, Z.

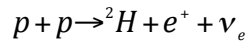
Maki, M. Nakagawa and S. Sakata discussed the possibility that the two known flavours were a mixture of two neutrino mass eigenstates [15]. However, the first phenomenological model for  $\nu_e \leftrightarrow \nu_\mu$  mixing and oscillations was worked out by Pontecorvo [16], later improved by Gribov and Pontecorvo [17] as a possible – although generally questioned – solution to the solar neutrino problem.

## Solar neutrinos

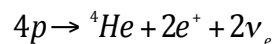
Thermonuclear fusion reactions in the solar core produce energy – and neutrinos. The solar neutrino problem refers to the observation that compared to theoretical predictions, the flux of neutrinos from the Sun measured on Earth appears anomalously low. This problem persisted for more than 30 years before it was finally resolved by measurements with the heavy water detector at the Sudbury Neutrino Observatory (SNO).

A model for the energy production in the Sun evolved over a large part of the 20th century. The so-called Solar Evolutionary Model was first proposed by Schwarzschild in 1957 [18]. It described the Sun's development from a protostar  $\sim 4.5$  Gy ago to the present era, fitting the known values of luminosity, mass and radius. Since 1963, many improvements to the solar model have been made, prominently by John Bahcall and collaborators who coined the label “Standard Solar Model” (SSM) to describe a “solar model that is constructed with the best available physics and input data” [19]. Several SSMs have been constructed over time – some of the recent ones are called BSB06 [20] and BPS08 [21].

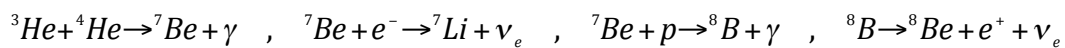
Pontecorvo realised early on that neutrinos with their tiny interaction cross sections would be the perfect probe of the interior of a star, allowing verification of the mechanism of energy production [22]. In the Sun, the main energy generating mechanism is the fusion of hydrogen to helium, driven by the weak process



The neutrino generating reactions in the so-called pp-chain and in the CNO-cycle, involving carbon, nitrogen and oxygen, can be summarised as



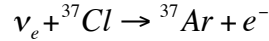
The total energy release is 26.73 MeV. In secondary branches, neutrinos are produced through electron capture by  ${}^7\text{Be}$  and in the  $\beta$ -decay of  ${}^8\text{B}$ , in the sequence of processes



The  ${}^3\text{He}$  nuclei are a byproduct of the pp-chain. Figure 1 shows the expected flux of solar neutrinos at Earth's surface as a function of the neutrino energy, in total  $6.5 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$ . The electron capture on  ${}^7\text{Be}$  produces neutrinos of two distinct energies.

The first solar neutrino detector was constructed in the 1960s, deep underground in the Homestake gold mine in Lead, South Dakota, by Raymond Davis Jr (Davis shared the 2002 Nobel Prize in Physics with M. Koshihara and R. Giacconi). Using 615 t of tetrachloroethylene

(more than  $2 \times 10^{30}$  chlorine atoms), Davis aimed to detect the inverse  $\beta$ -decay process initiated by solar neutrinos from  $^8\text{B}$



The  $\beta$ -decay of  $^8\text{B}$  produces neutrinos with energies up to 15 MeV, well above the threshold (0.814 MeV) for the capture on chlorine. According to SSM predictions, small contributions can also be expected from  $^7\text{Be}$  and the p-e-p process, see figure 1. On the other hand, the much more copious neutrinos from the pp-chain have maximum energies of 0.420 MeV and do not contribute.

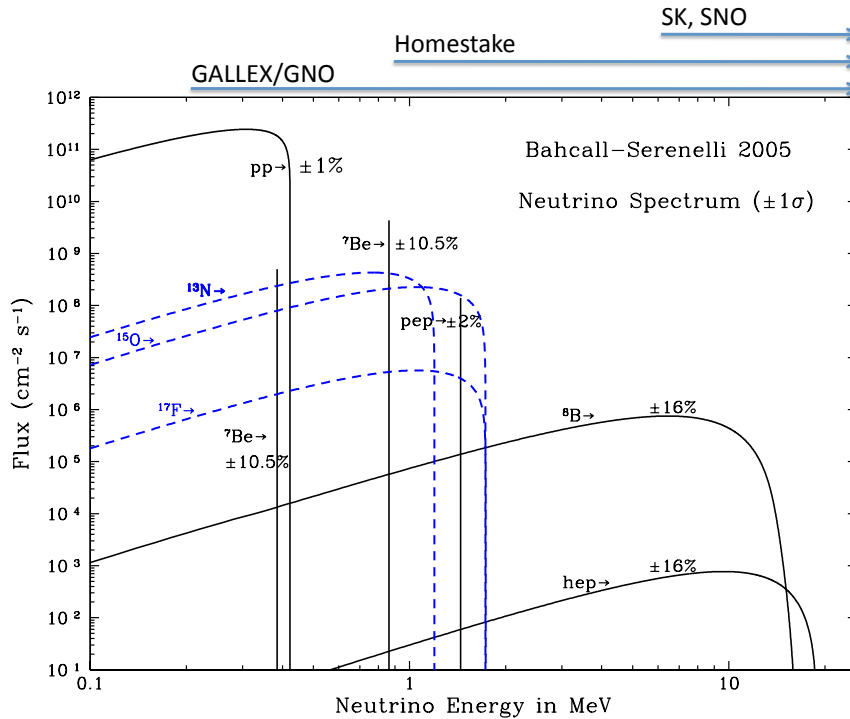


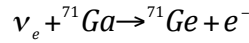
Figure 1: Neutrino fluxes (with percentage uncertainties) as predicted by the Bahcall-Serenelli solar model (BS05) [38], in  $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$  ( $\text{cm}^{-2} \text{s}^{-1}$  for the lines). The arrows above the diagram indicate the energy ranges accessible to experiments. [From J.N. Bahcall's web site <http://www.sns.ias.edu/~jnb/> with arrows added above the graph.]

Every second day, on average, one  $^{37}\text{Ar}$  atom was produced in the Homestake detector by a solar neutrino. The radioactive Ar atoms were extracted every couple of months, approximately the time required to reach equilibrium between  $^{37}\text{Ar}$  production and decay (the half-life of  $^{37}\text{Ar}$  is about 35 days). The first results from Davis' experiment appeared in 1968 [23] indicating an observed flux much lower than the theoretical expectation. The final results were published in 1998 [24]. The average value of the solar neutrino rate obtained by Homestake after more than 25 years of almost continuous measurement is

$$2.56 \pm 0.16 \text{ (stat)} \pm 0.16 \text{ (sys) SNU}$$

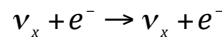
about 30% of the theoretically predicted  $8.5 \pm 0.9$  SNU [21]. (One Solar Neutrino Unit, SNU, corresponds to one reaction per  $10^{36}$  target atoms per second.)

When the solar neutrino problem was first identified it was expected that it would be solved through measurement of the dominant flux of solar neutrinos from the pp-chain. This flux was eventually measured in the 1990s and in the first decade of the new millennium by other radiochemical experiments, GALLEX/GNO in the Gran Sasso Laboratory in Italy [25, 26] and SAGE located in the Baksan Neutrino Observatory, Russia [27]. All three experiments studied the reaction



with a threshold of 0.233 MeV. In all cases the observed solar rate in this energy range was at about 50% of the SSM prediction [21], a discrepancy at the  $5\sigma$  level. Hence, the solar neutrino problem persisted.

In 1989, the Kamioka Observatory in Japan reported their first results of measurements of the solar neutrino flux (see [28] for a summary). The Kamioka Nucleon Decay Experiment (Kamiokande), a water Cherenkov detector located in the Mozumi mine near Kamioka in Japan, about 1,000 m underground, was originally built to search for nucleon decay but could measure the solar neutrino flux from  ${}^8\text{B}$  through the elastic scattering reaction



where  $x$  stands for  $e$ ,  $\mu$  or  $\tau$ . The reaction is sensitive to all three neutrino flavours through processes mediated by neutral weak bosons ( $Z^0$ ). However, due to additional contributions from processes mediated by charged weak bosons ( $W$ ), the cross-section for electron-neutrinos is about six times larger than that for muon- or tau-neutrinos. By registering the pattern of Cherenkov photons generated when the final electron moves with superluminal speed through the water-filled detector, the direction and energy of the incident neutrino could be determined. The angular distribution of the events clearly pointed to the direction of the Sun, figure 2. The average measured flux of  ${}^8\text{B}$  neutrinos was again much lower than expected, at the level of about 50% [29]. Similar and even more precise results were later obtained by the next generation Super-Kamiokande experiment [30].

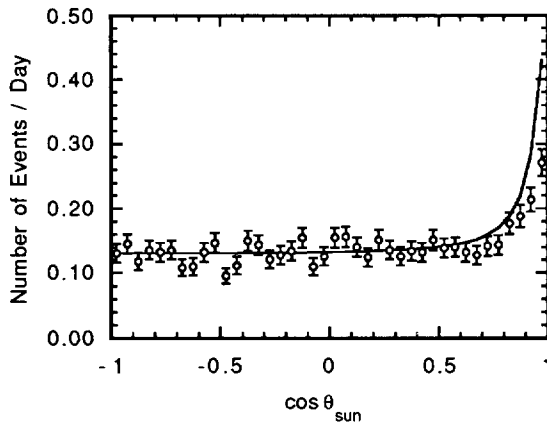


Figure 2: Angular distribution of events with respect to the Sun, Kamiokande [29].

By the end of the millennium, it had become quite obvious that the discrepancy between the solar flux measurements and the SSM predictions could not be explained away by large

uncertainties. The solar models had improved, in large part thanks to J.N. Bahcall and his collaborators' indefatigable efforts<sup>1</sup>. It could finally be shown that model predictions agree well with the independent helioseismological observations, which provide information on the speed of sound and the density of matter in the interior of the Sun [19]. On the other hand, all measurements consistently pointed to a deficit of solar neutrinos. The only consistent explanation appeared to require neutrino oscillations: some of the electron-neutrinos produced in the solar core might change flavour during propagation, becoming muon- or tau-neutrinos which are not detected by the radiochemical experiments on Earth and, due to differences in cross section, only partly detected by (Super) Kamiokande. Conclusive evidence for this scenario required simultaneous and efficient detection of all neutrino flavours and was provided by the SNO solar neutrino experiment, which started observations in 1999.

The key feature of SNO was the use of heavy water, allowing simultaneous measurement of the relative rate of neutrino-deuteron reactions forming two protons (possible only for electron-neutrinos), and neutrino-deuteron reactions resulting in a proton and a neutron (possible for all neutrino flavours). The ratio would indicate if any transformation of solar electron-neutrinos to other types was taking place. Seizing the opportunity to loan a large quantity of heavy water from Canada's reserves, a proposal to build a unique neutrino observatory deep underground in the Creighton Mine (owned by INCO Ltd) in the town of Walden near Sudbury in Ontario, Canada, was presented in October 1987 by a collaboration of scientists from Canada, US and UK. At that time, Davis' experiment had been running for 20 years, consistently showing a deficit in the flux of  $^8\text{B}$  solar neutrinos. Two interpretations were widely discussed: the SSM could be wrong – for instance, a temperature in the Sun's interior that was lower than anticipated would result in a decreased production of  $^8\text{B}$  and a lower expected flux of electron-neutrinos. Alternatively, the  $^8\text{B}$ -neutrinos, produced with the flux anticipated by the SSM, could change in transit into other neutrino flavours. Actually, in 1986 Mikheev and Smirnov [31] had proposed a mechanism, which would enhance neutrino conversion in solar matter based on a theory initially developed by Wolfenstein [32]. If this mechanism were at work, the Sun would not only produce electron-neutrinos but also transform a large fraction of them into muon-neutrinos and tau-neutrinos.

The SNO Collaboration was first established in 1984 with G. Ewan from Queen's University, Kingston, Canada, and H. Chen from University of California, Irvine, USA, as co-spokespersons. In 1985 they were joined by D. Sinclair from Oxford University as UK co-spokesperson. Since 1990, the collaboration has been led by A. B. McDonald from Princeton University, USA, who, moving to Queen's, became the first director of the Sudbury Neutrino Observatory.

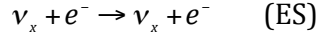
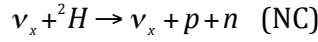
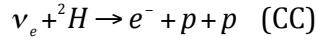
The SNO heavy water Cherenkov detector, consisted of 1,000 t ultra-pure heavy water ( $\text{D}_2\text{O}$ ) in an acrylic sphere, 12 m in diameter. The volume was monitored by 9,500, 20 cm in diameter, photomultiplier tubes mounted on a geodesic support structure, and surrounded by ultra-pure  $\text{H}_2\text{O}$  as a shield against radioactive decays in the support structure and the surrounding rock. The overburden of rock shielded the instrument from cosmic rays.

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<sup>1</sup> A personal recollection, and an explanation of the solar neutrino problem in Bahcall's own words, can be found at the web site *nobelprize.org*.



SNO detected  $^8\text{B}$  solar neutrinos via the reactions



CC stands for “charged current” reactions mediated by the charged weak boson ( $W$ ). The CC reactions are sensitive only to  $\nu_e$  and provide the flux  $\phi(\nu_e)$ . In this case the electron carries off most of the energy and a measurement of the electron energy spectrum provides information on possible distortions of the  $\nu_e$  spectrum due to oscillations. NC (“neutral current”) reactions are mediated by the neutral weak boson ( $Z^0$ ) and sensitive to all neutrino flavours. They measure the total neutrino flux,  $\phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$ . Finally, the elastic scattering (ES) reactions, although occurring for all three flavours, are predominantly sensitive to  $\nu_e$  since the interaction cross-sections for  $\nu_\mu$  and  $\nu_\tau$  are about six times smaller.

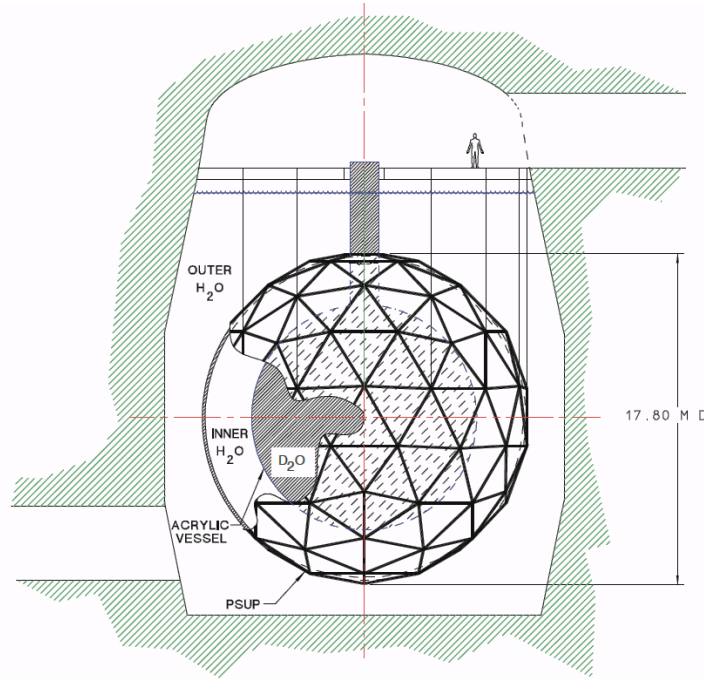


Figure 3: Layout of the SNO detector, from [33].

The direction of the final electron produced in an ES reaction gives the direction of the neutrino, which is used to confirm that the neutrinos actually come from the Sun. The NC reaction can be identified by observing the  $\gamma$  rays from the capture of the final neutron in deuterium, albeit with a fairly low detection efficiency. Therefore, a second phase of the experiment was run with 2 t of NaCl added to the 1,000 t of  $\text{D}_2\text{O}$  which improved the neutron capture efficiency due to the larger neutron capture cross-section of chlorine relative to deuterium. A final, third phase involved purging the NaCl and installing  $^3\text{He}$  neutron counters. In this way, the reliability of the NC measurement could be checked and improved. The first results were published in 2001 and 2002 [34], providing evidence for neutrino flavour conversion and showing that the total flux of  $^8\text{B}$  neutrinos was in agreement with the

solar model prediction. Continued data-taking refined these results. Data-taking was concluded in 2006 and the final results were published in 2013 [35]. The  $^8\text{B}$  neutrino flux from the final fit to all reactions is

$$\phi = \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = 5.25 \pm 0.16(\text{stat})_{-0.13}^{+0.11}(\text{sys}) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

in very good agreement with the theoretically expected  $5.94 (1 \pm 0.11)$  [SSM BPS08] or  $5.58 (1 \pm 0.14)$  [SSM SHP11] (see [36] and references therein).

The flux of muon- and tau-neutrinos deduced from the results shown in figure 4 is

$$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.26 \pm 0.25_{-0.35}^{+0.40}) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

deviating significantly from zero. A comparison with the total  $^8\text{B}$  flux clearly demonstrates that about two thirds of the solar electron-neutrinos changed flavour, arriving at Earth as muon-neutrinos or tau-neutrinos. SNO's ES results are consistent with the results from Super-Kamiokande and with the SNO results above, however by themselves insufficient as evidence for flavour change (figure 4).

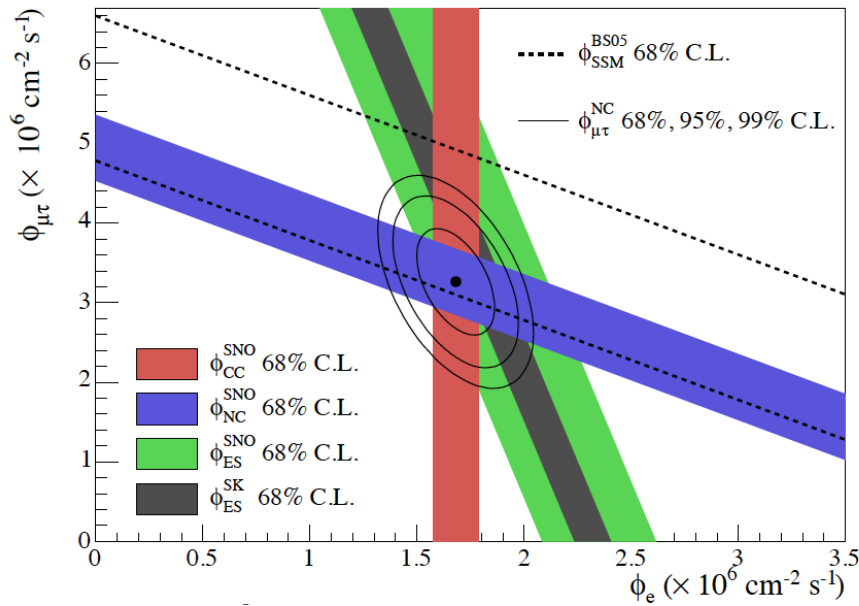


Figure 4: Fluxes of  $^8\text{B}$  solar neutrinos from SNO and Super-Kamiokande. The SSM BS05 [38] prediction is shown as a range between the dashed lines. C.L. stands for confidence level. From [36] and references therein.

The SNO evidence for neutrino flavour conversion was confirmed a year later by the KamLAND reactor experiment. KamLAND (Kamioka Liquid scintillator AntiNeutrino Detector) [39] was proposed in 1994, funded in 1997 and started data-taking in January 2002. The first KamLAND results were published in January 2003 [40] and show clear evidence for disappearance of electron anti-neutrinos, consistent with the expectation from the solar

neutrino results, assuming CPT. A combined fit to KamLAND and solar neutrino results demonstrates a unique solution in terms of oscillation parameters  $\Delta m^2$  and the mixing angle  $\theta$  [41]. A few years later, with increased statistics, KamLAND also showed the expected distortion of the electron anti-neutrino spectrum (see [42] and references therein).

### Atmospheric neutrino oscillations

The Earth is continuously exposed to a flux of cosmic rays from outer space. These particles – mainly protons but with a small admixture of heavy nuclei – interact with atomic nuclei in the atmosphere, creating secondaries including all kinds of hadrons. Specifically, many pions but also kaons are produced which decay into muons and muon-neutrinos. The muons, in turn, decay into electrons, muon-neutrinos and electron-neutrinos:

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}) \quad \mu^{\pm} \rightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_{\mu}(\nu_{\mu})$$

Kaons generate atmospheric neutrinos at somewhat higher energies than pions, and charm particles originating in the cosmic ray interactions contribute at even higher energies.

At low energies ( $\leq 1$  GeV), when most muons decay before hitting Earth's surface, the flux ratio of muon-neutrinos to electron-neutrinos is expected to be  $\sim 2$ , increasing above this value at higher energies [43]. Here “flux” refers to the sum of the fluxes of neutrinos and anti-neutrinos of a specific flavour. The fluxes of atmospheric neutrinos are relatively well understood within theoretical uncertainties of the order of 10-20%. The earliest indications of deviations from the expected 2:1 ratio appeared in the middle of the 1980s [44] – the so-called “atmospheric neutrino anomaly”. At that time, several nucleon decay experiments, prompted by expectations based on Grand Unified Theories, came into operation – for instance the water Cherenkov detectors IMB in Ohio, USA, and Kamiokande in Japan, and the fine grained iron calorimeters Fréjus and NUSEX in the tunnels below the Alps linking France and Italy. Nucleon decay is expected to occur very rarely, necessitating large volume detectors deep underground to keep down the cosmic ray background. However, atmospheric neutrinos penetrate the overburden, interacting in the detectors themselves as well as in the surrounding rock, and this background contribution needs to be determined with great precision. The first reports from IMB [44] indicated an unexpected deficit in the muon-neutrino flux at about 2.5 sigma. Kamiokande, too, reported a deficit [45] whereas NUSEX [46] and Fréjus did not observe any anomalies [47]. Eventually, it turned out that whereas certain IMB data showed deviations from expectation, other IMB data did not [48]. To reduce uncertainties the experimental results were reported in terms of the ratio of data to theoretical expectation

$$R = (N_{\mu} / N_e)_{obs} / (N_{\mu} / N_e)_{theor}$$

With increased statistics, IMB reported  $R \approx 0.54$  [49], while Kamiokande measured  $R \approx 0.60$  [50]. The provenance of the deficit was unclear and interpretations ranged from violation of Lorentz invariance, flavour-changing neutral currents to neutrino decay – and neutrino oscillations.

The compelling evidence in favour of neutrino oscillations was presented by T. Kajita of the Super-Kamiokande Collaboration at the international neutrino conference Neutrino'98 [51]. Super-Kamiokande (SK) is a second generation, 50,000 t water Cherenkov detector, more

than ten times larger than its predecessor Kamiokande in the Mozumi zinc mine. Super-Kamiokande launched its operations in April 1996 and could, after less than two years of data-taking, report the first striking results: a deficit in the number of up-going high energy muon-neutrinos, strongly varying with the zenith angle (i.e. the angle between the neutrino direction and vertical).

Atmospheric neutrinos are produced high in the atmosphere and the flux at the surface of the Earth is expected to be isotropic, independent of the zenith angle. This implies that the observed fluxes of up-going and down-going neutrinos in an underground detector like SK should be equal. A water Cherenkov detector is able to distinguish the electrons and muons produced in the final state of  $\nu_e$  and  $\nu_\mu$  charged current (CC) reactions but cannot distinguish neutrinos from anti-neutrinos. By determining the directions of the final electrons and muons, the directions of the incident neutrinos can be inferred.

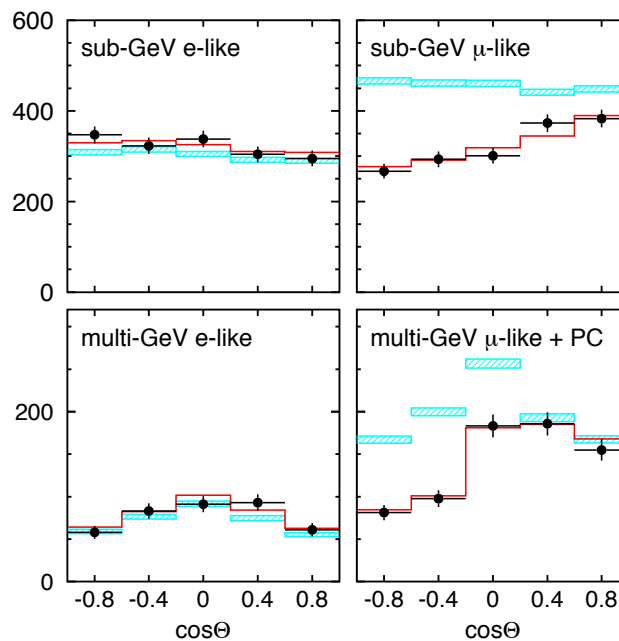


Figure 5: Zenith angle distributions of e-like and  $\mu$ -like events in Super-Kamiokande with momenta above and below 1.33 GeV [52]. The boxes show the expectation assuming no oscillations, whereas the full drawn lines show the results of the best fit.

Figure 5 clearly shows that whereas the flux of electron-neutrinos has almost no zenith angle dependence, the flux of down-going ( $\cos\theta = 1$ ) muon-neutrinos significantly exceeds the flux of up-going  $\nu_\mu$ . This can be simply interpreted in terms of oscillations: neutrinos moving upward through the detector are created in the atmosphere at the opposite side of the Earth and travel thousands of kilometres before interacting. Apparently, muon-neutrinos disappear on the way whereas electron-neutrinos do not. Down-going muon-neutrinos, produced in the atmosphere directly above the detector, only travel a few dozen kilometres and are detected at the level expected. Since there is no indication of an increased electron-neutrino flux, the missing muon-neutrinos must have oscillated into tau-neutrinos.

The  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation interpretation of SK's zenith angle results and of further R measurements was strengthened by the observation of the expected sinusoidal behaviour of the  $\nu_\mu$  flux as a function of  $L/E$  – the ratio of the distance from the point of production reconstructed from the neutrino direction, and the neutrino energy – which displays a minimum at 500 km/GeV [53], (figure 6).

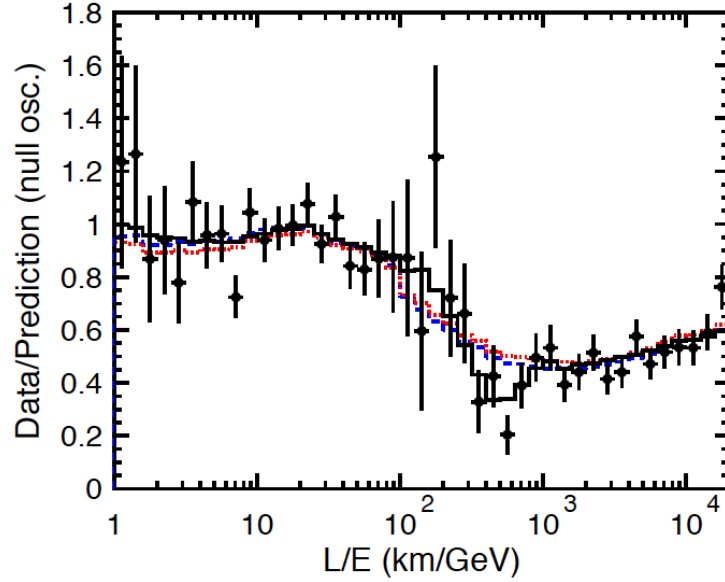


Figure 6: Ratio of data from Super-Kamiokande to Monte Carlo expectation assuming no oscillation, as a function of reconstructed  $L/E$  [53]. The black histogram is a fit to a two flavour oscillation hypothesis.

In summary, the SK observations support the conclusion that atmospheric muon-neutrinos are converted into tau-neutrinos and exclude alternative hypotheses like neutrino decay and neutrino decoherence at more than  $3\sigma$  (blue and red dashed lines above).

Recently, a statistical search was performed with SK data demonstrating not only muon-neutrino disappearance but also tau-neutrino appearance at almost  $4\sigma$  level [54].

Super-Kamiokande's oscillation results were confirmed by the detectors MACRO [55] and Soudan [56], by the long-baseline accelerator experiments K2K [57], MINOS [58] and T2K [59] and more recently also by the large neutrino telescopes ANTARES [60] and IceCube [61]. Appearance of tau-neutrinos in a muon-neutrino beam has been demonstrated on an event-by-event basis by the OPERA experiment in Gran Sasso, with a neutrino beam from CERN [62].

### Theory of neutrino oscillations

Neutrino flavour conversion is fundamentally a quantum mechanical effect. The discovery of neutrino oscillations implies that the neutrino flavour states are not mass eigenstates but superpositions of such states. A spectrum of mass eigenstates  $\nu_k$  could presumably contribute, with  $k = 1, 2, \dots, n$  where  $n$  in general could be greater than three – if, for instance, sterile neutrinos exist and mix with the known  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ .

A neutrino state with a well defined flavour,  $|\nu_\alpha\rangle$  with  $\alpha = e, \mu$  or  $\tau$ , can be described in terms of mass eigenstates  $|\nu_k\rangle$

$$|\nu_\alpha\rangle = \sum_{k=1}^3 U_{\alpha k}^* |\nu_k\rangle$$

assuming three contributing mass eigenstates.  $U$  is a unitary matrix called the lepton mixing matrix by analogy with the quark mixing matrix – or the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix to honor the pioneering work long predating the discovery of neutrino oscillations.  $U$  may in general be complex.

For neutrino oscillations in vacuum, the above relation allows simple derivation of the flavour change probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}(U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) + 2 \sum_{k>j} \text{Im}(U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*) \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

with  $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ . This assumes a flavour eigenstate  $\nu_\alpha$  produced at a neutrino source in a weak interaction, propagating as a superposition of mass eigenstates  $\nu_k$  over a distance  $L$  to the detector where  $\nu_\beta$  is observed;  $E$  is the common energy of all  $\nu_k$  components [63]. The expression remains valid if instead equal momenta are assumed [64]. Since neutrinos are extremely light, their momenta can be approximated by

$$p_k \approx E - \frac{m_k^2}{2E}$$

for all typical energies.

For the oscillation effects to be observable, the phase

$$\Delta m^2 \frac{L}{E}$$

must be of the order 1. This implies that the characteristic oscillation length  $L_{osc} \sim E / \Delta m^2$  must be similar to the distance between source and detector  $L$ . If  $L \ll L_{osc}$ , the oscillations have no time to develop. If  $L \gg L_{osc}$ , only the average effect on the probability is detectable.

If neutrinos are massless, all  $\Delta m^2 = 0$ , and  $P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta}$ . So observation of neutrino oscillations implies that at least one neutrino species has non-zero mass. The third term of the equation above can be rewritten in terms of the so-called Jarlskog invariant  $J$  [65]

$$\text{Im}(U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*) = s_{\alpha\beta kj} J$$

with the sign coefficient,  $s_{\alpha\beta kj} = +1$  or  $-1$  depending on the channel. This expression allows explicitly to quantify CP violation due to the Dirac phase in the neutrino sector. Discovery of  $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$  would imply violation of CP invariance.

In the case of three massive neutrinos, the matrix  $U$  can be parametrised in terms of three Euler angles (called mixing angles) and six phase parameters. If the neutrinos are Dirac fermions (and so have distinct anti-particles), only one of the phases is physical and gives rise to CP violation. If, however, neutrinos are Majorana particles (identical with their anti-particles) [66], additional CP violating phases are required. The PMNS matrix is often conveniently parametrised as [67]

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \text{diag}(1, e^{i\alpha_1/2}, e^{i\alpha_2/2})$$

where  $c_{ij} = \cos\theta_{ij}$  and  $s_{ij} = \sin\theta_{ij}$ .  $\delta$  is the CP violating Dirac phase, and  $\alpha_1$  and  $\alpha_2$  are the CP violating Majorana phases.

Often, the mixing matrix is decomposed in the form (neglecting possible Majorana phases)

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This combination is useful since it turns out that experimental data can be analysed, to a good approximation, in terms of oscillations between just *two* neutrino states. Solar (and reactor) data mainly measure  $\theta_{12}$ , while data on atmospheric neutrinos and neutrinos in accelerator experiments mainly determine  $\theta_{23}$ . Recently,  $\theta_{13}$  has been obtained by the dedicated reactor experiments Daya Bay in China [68], RENO in South Korea [69] and Double Chooz [70] in France.

For just two neutrino species, for instance  $\nu_\mu$  and  $\nu_\tau$  as is approximately the case for atmospheric neutrinos, the oscillation probability simplifies to

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \Rightarrow \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 c^3 L}{4\hbar E}\right)$$

with the Planck constant  $\hbar$  and the speed of light  $c$  reinserted in the latter expression. When  $\Delta m^2$  is measured in  $\text{eV}^2$ ,  $L$  in kilometres and  $E$  in GeV, this expression becomes

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right)$$

The mixing angles  $\theta_{23} \approx \theta_{\text{atm}}$  and  $\theta_{12} \approx \theta_{\text{sol}}$  are intriguingly large,  $42^\circ$  and  $34^\circ$ , respectively – much larger than the quark mixing angles. Why this is so, is currently not understood. The central matrix involves the small mixing angle  $\theta_{13} \approx 9^\circ$  and the Dirac phase  $\delta$  and is relevant for CP violation in the neutrino sector.

The above discussion applies to neutrino oscillations in vacuum. When neutrinos travel through matter, for instance in the Sun or in the Earth, the oscillation probabilities are modified due to the so-called Mikheev-Smirnov-Wolfenstein (MSW) effect [31, 32]. The MSW mechanism is a consequence of the fact that the weak interactions of electron-neutrinos

in matter differ from those of muon-neutrinos and tau-neutrinos, depending on the varying electron density (the number of electrons per unit volume) and the neutrino energy, and can give rise to large effects through resonant enhancement. In the simplest case of oscillations between two neutrino species and assuming a constant matter density, the modification of  $P(\nu_\alpha \rightarrow \nu_\beta)$  due to MSW can be described in terms of an effective mixing angle  $\theta_M$  and an effective mass difference squared

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 x}{4E}\right)$$

where

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}, \quad \Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

with the vacuum mixing angle  $\theta$  and  $x = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2}$  in terms of electron density  $N_e$ , neutrino energy  $E$ , and the mass difference squared in vacuum  $\Delta m^2$ .  $G_F$  is the Fermi constant. At resonance, i.e. when  $x = \cos 2\theta$ , the amplitude of the oscillations becomes 1 and total transitions between the two flavours can occur.

The MSW effect has to be taken into account when analysing solar neutrino data. This applies both to flavour transitions during propagation through solar matter and to possible  $\nu_e$  regeneration in the Earth resulting in a day/night effect. It also plays a role for the analysis of atmospheric neutrino data [36, 64].

### Neutrino oscillation parameters

The experimental results are used to obtain the mass differences  $\Delta m^2$  and the mixing angles  $\theta$ . This is done using global fits including all available experimental data – observations of solar and atmospheric neutrinos, and neutrinos studied in reactor and accelerator experiments. The MSW effect is taken into account. Recent values of the parameters of the PMNS matrix based on a global analysis of all available oscillation data assuming a three-neutrino mixing scheme can be found in [36]. All entries, except  $U_{e3}$ , turn out to be large which is very different from the quark mixing matrix.

### Why neutrino oscillations matter

Observation of the quantum mechanical phenomenon of neutrino oscillations implies that at least two neutrino species have non-zero mass. The mechanism which generates neutrino masses is still unknown, and the Standard Model must be extended to include this new physical reality.

Although mass differences between neutrino flavours have been determined with precision, no one has yet succeeded in actually measuring the neutrino mass itself. The best upper limits derived from laboratory experiments give



$m_{\nu_e} < 2$  eV (from tritium decay), while limits on the mass of the muon-neutrino and the tau-neutrino (from pion and tauon decays) are considerably higher. Planned experiments should reach a sensitivity of  $\sim 0.20$  eV.

Meanwhile, the universe itself is a laboratory providing constraints on the neutrino mass. Relic neutrinos from the early universe are almost as abundant as microwave background photons, with about 330 neutrinos per  $\text{cm}^3$  (adding neutrinos and anti-neutrinos of all flavours) as compared to about 410 photons per  $\text{cm}^3$ . Non-zero neutrino masses therefore contribute non-negligibly to the dark matter fraction of the cosmological energy density,  $\Omega$ . Since large numbers of neutrinos streaming through the early universe would also influence large-scale structure formation, an upper limit on the sum of neutrino masses can be obtained by combining cosmic microwave background data with galaxy surveys and data on baryon acoustic oscillations [71]

$$\sum_i m_i < 0.23 \text{ eV}$$

Such small masses, more than a million times smaller than the mass of the electron, are indicative of the existence of a new fundamental mass scale, not easily explained in the Standard Model.

Inclusion of the tiny neutrino masses in extensions of the Standard Model requires “new physics”. This might be Majorana fermions (i.e. spin  $\frac{1}{2}$  particles that are their own anti-particles), heavy sterile neutrinos or additional Higgs particles – none of which has so far been observed. The “see-saw” mechanism [72] is one idea implying that physical neutrinos come in pairs: a heavy neutrino with mass  $M$ , possibly of the order of  $\sim 10^{15}$  GeV, which would not yet have been observed – and a light one with mass  $m \approx m_D^2/M$  where  $m_D$  is a mass similar to that of the charged lepton of the same generation. These particles would be Majorana fermions. However – nature may, of course, have surprises in store.

Understanding the nature of the neutrino is today of prime importance – not only for elementary particle physics but also for astrophysics and cosmology. The best way to investigate if neutrinos are indeed Majorana particles is believed to be neutrino-less double beta decay. These processes are forbidden in the Standard Model but could in principle occur for the handful of naturally occurring isotopes that normally decay through emission of two electrons (positrons) and two neutrinos. Many experiments search for neutrino-less double beta decay, so far without success. Many other experiments attempt to determine the neutrino mass ordering, search for sterile neutrinos or measure the CP violating effects in the neutrino sector. The leptonic CP violation effects might have played a role for the baryon-antibaryon asymmetry in the universe through a mechanism called leptogenesis [73]. Hence, the discovery of neutrino oscillations has opened a door towards a more comprehensive understanding of the universe we live in.

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