COMP4901V HW2

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Q2.1.1.

Since the camera just translates along the X-axis, the rotational matrix $R = I$, and the translation matrix $T = [T_x \; 0 \; 0]$

then the essential matrix $E = [T_x]R$, and the epipolar line $l = Ex_r$, x_r is a 2D Points on Camera Coordinate system.

So, $l = Ex_r = [T_x]Rx_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T_x \\ 0 & +T_x & 0 \end{bmatrix} I x_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T_x \\ 0 & +T_x & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -T_x \\ T_x y_r \end{bmatrix}$, Hence the epipolar line is horizontal along the x-axis.

Q2.1.2.

Let x_1, x_2 be a 2D homogeneous points on left and right image, and X be a 3D Points on the world coordinate system.

Then,

$x_1 = K[R_1|t_1]X = K(R_1X + t_1) \Leftrightarrow X = R_1^T(K^{-1}x_1 - t_1)$

And by replacing X with $R^T(K^{-1}x_1 - t_1)$,

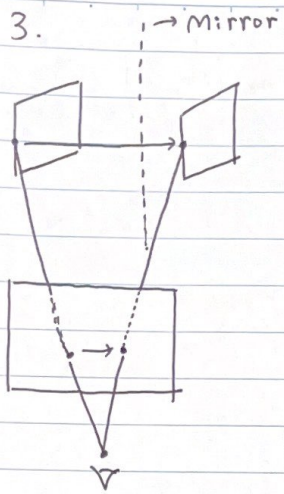
$$x_2 = K[R_2|t_2]X = K(R_2X + t_2) = K(R_2R_1^T(K^{-1}x_1 - t_1) + t_2)$$

$$= \underbrace{KR_2R_1^TK^{-1}}_{\downarrow R_{rel}} x_1 \underbrace{- KR_2R_1^Tt_1 + Kt_2}_{\downarrow t_{rel}}$$

Hence, $R_{rel} = KR_2R_1^TK^{-1}$, $t_{rel} = -KR_2R_1^Tt_1 + Kt_2$, and $E = [t_{rel,x}]R_{rel}$

$F = K^{-T}EK^{-1} = K^{-T}[t_{rel,x}]R_{rel}K^{-1}$

Q 2.1.3.



Since all points on the original object are of equal distance to the mirror, the reflected object is an image of the pure translation of the original ~~image~~ object.

Hence, Rotational matrix $R = I$ and translational matrix $t = [t_x \; t_y \; t_z]$, and $E = [t_x]R$

Then, $F = K^{-T}EK = K^{-T}\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} K^{-1}$

So, two images are related by a skew-symmetric fundamental matrix

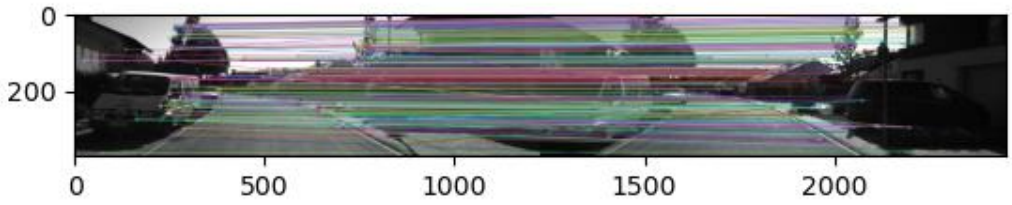


Fig 1. (Result of Q2.3_1)

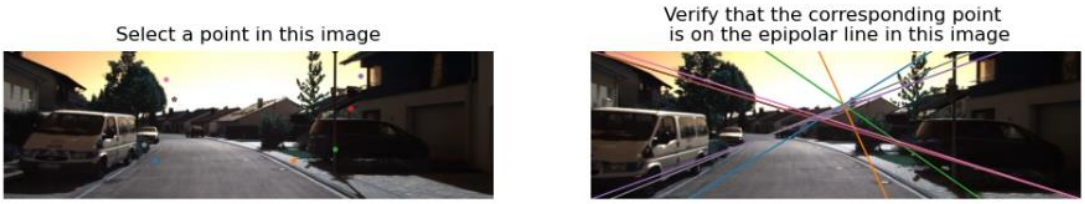


Fig 2. (Result of Q2.3_2)

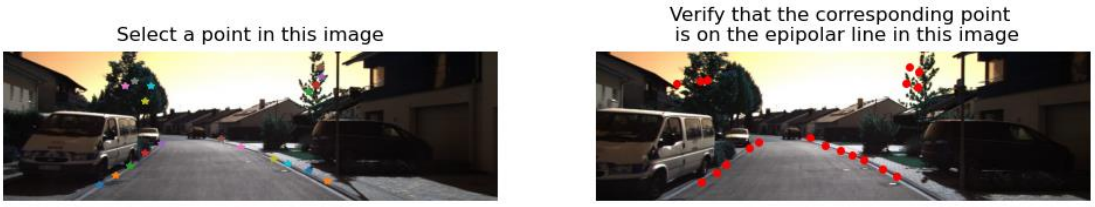


Fig 3. (Result of Q2.5_1)

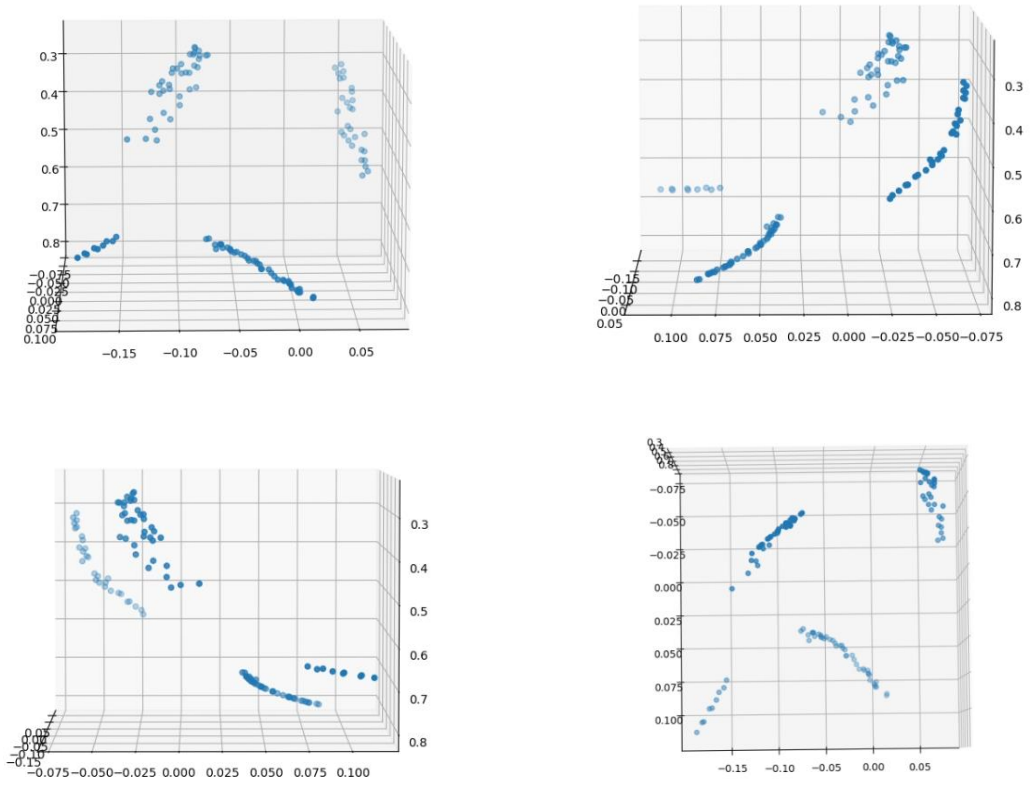


Fig 4. (Result of Q2.5_2)

(Front, Right, Left, Top)

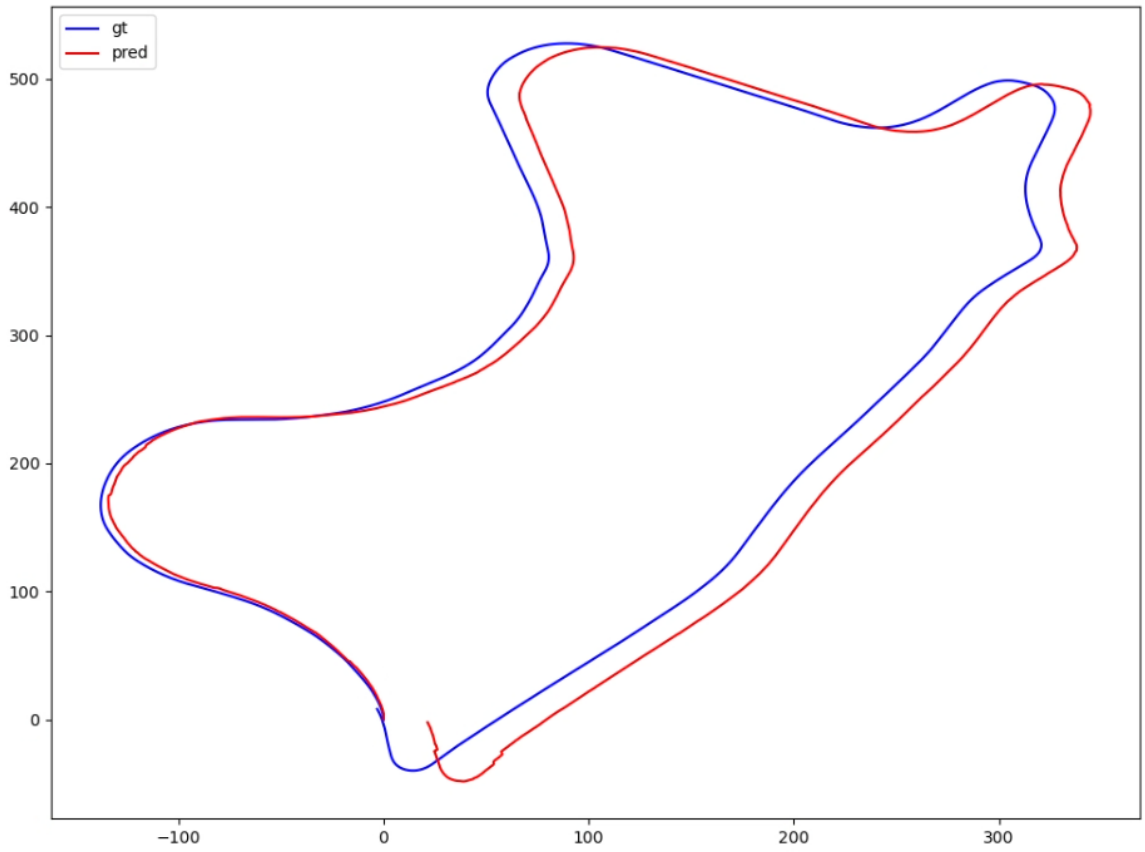


Fig 5. (Result of Q3_2)