B505/I500: Applied Algorithms

HW1 (Due: **Sept 5, Saturday 11:59pm**)

1. (10 pts) Illustrate the operation of Insertion-sort algorithm on array
   A=<31,41,59,26,41,58>.

2. (20 pts) The input to the algorithm *Unknown* illustrated below is an array A of N
   numbers. (1) what is the output of the algorithm? (2) using big-O notation to show
   the running time of the algorithm.

Input: Array A of N numbers;
Unknown(A)
for j = 1 to N-1
     if A[N] < A[j]
         exchange A[j] and A[N]
Output A[N];

3. (15 pts) What is the run time for the following function? Justify your
   answer.

```
int foo(int n){
    int i,j,k=0;
    for(i = n/2; i <= n; i++){
        for(j = 2; j<= n; j = j * 2){ k +=
                        n/2;
        }
    }
    return k;
}
```

4. (10 pts) Show that $f(n) = 1/n \in O(1)$ using the formula.

5. (10 pts) You are given $f(n) = O(g(n))$ and $f(n) = O(h(n))$. Give an
   example where $g(n) = O(h(n))$ and where $g(n) \neq O(h(n))$. Justify your
   answer.

6. (20 pts) Compare the following pairs of functions, and show which one is big-
   O of the other one: $(n, log\ n)$, $(n^2, 2^n)$, $(2^n, 3^n)$, $(log\ n, log^2 n)$. Justify your
   answer.

7. (15 pts) Given an array of numbers as the input, devise an algorithm to generate a
   random permutation of the array, such that each number has equal probability to
   be placed in each position in the output array.