

# Ensemble Learning

**CSCI-P556 Applied Machine Learning**  
**Lecture 21**

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# Agenda and Learning Outcomes

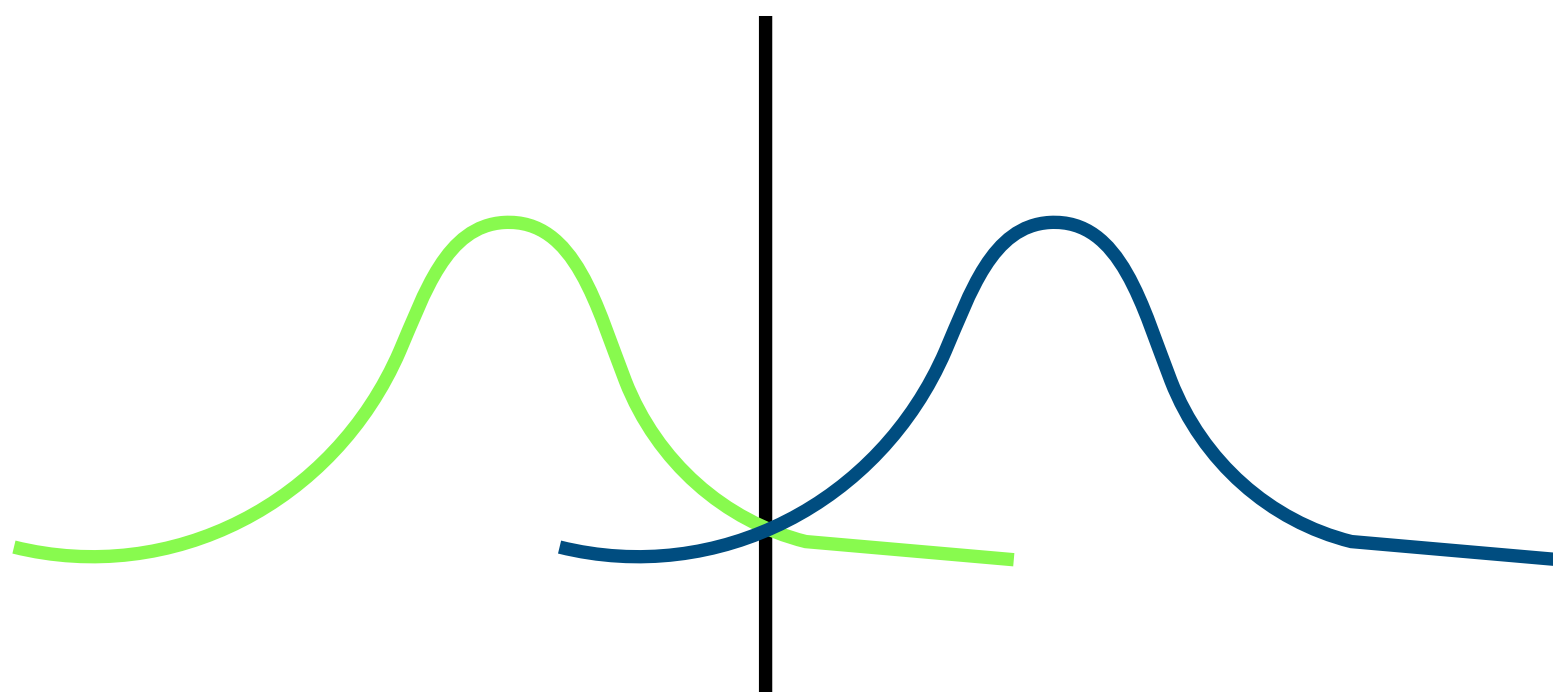
## Today's Topics

- **Topics:**
  - Quiz #2 review
  - Ensemble Learning
    - Bagging
    - Random Forests
    - Boosting

# Recap: Learning Algorithms

We've discussed several learning algorithms

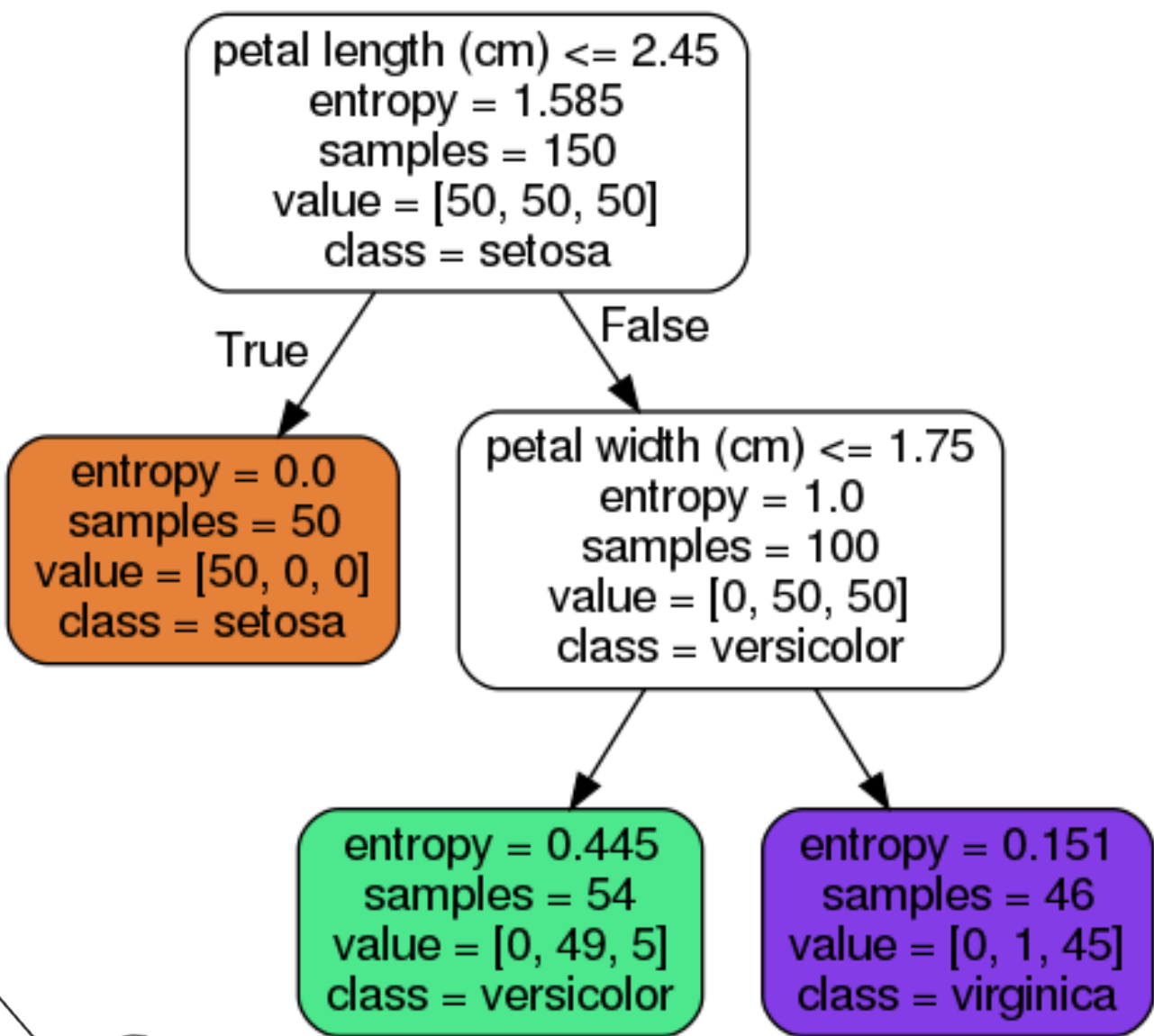
Naive Bayes



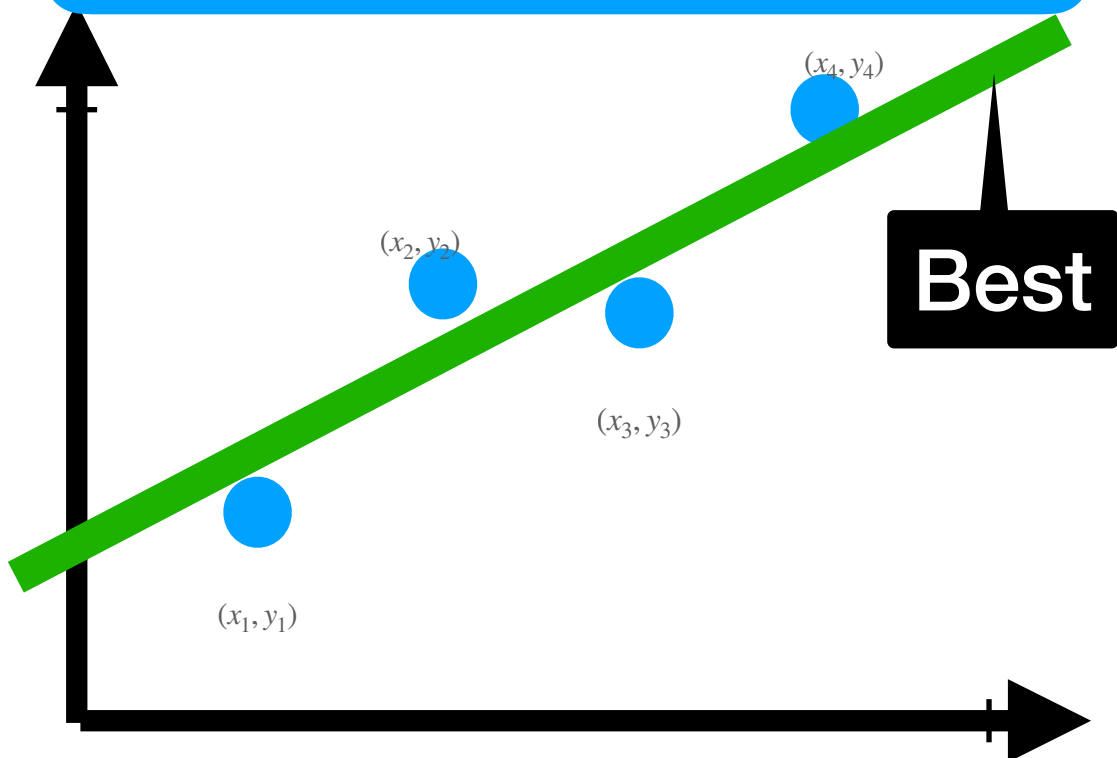
Support Vector Machines



Decision Trees

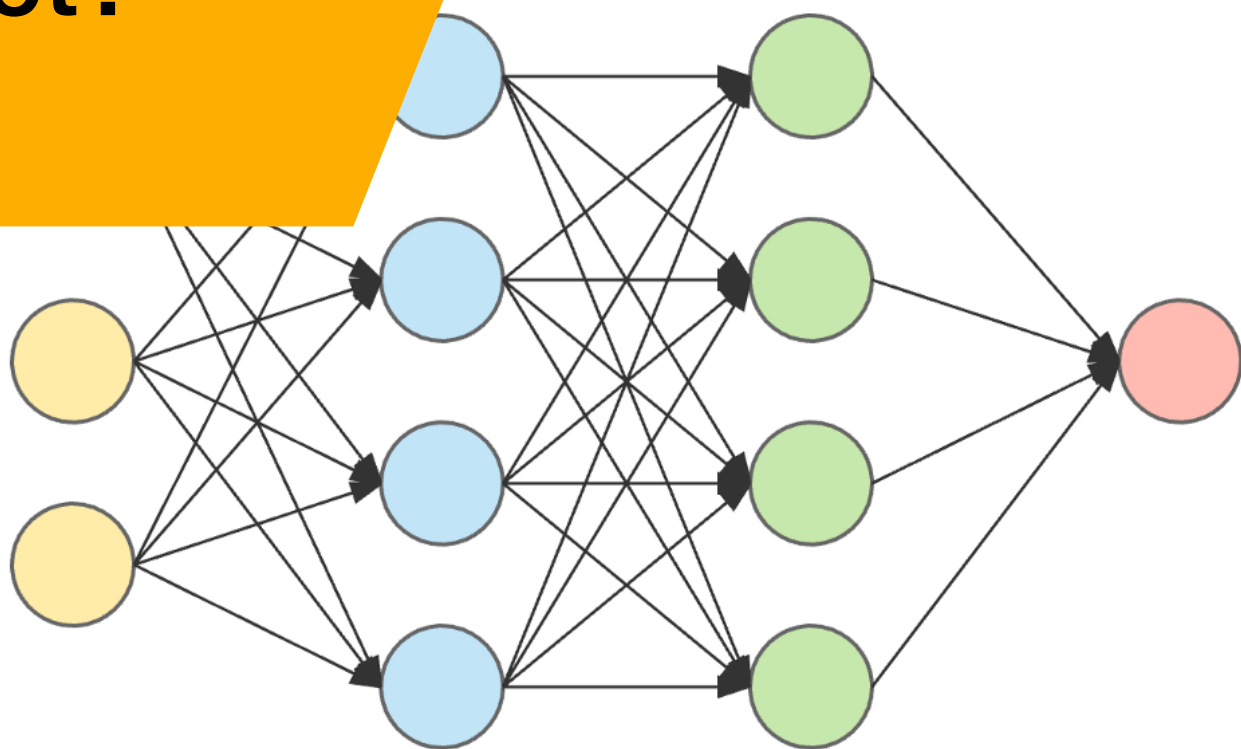


Linear Regression



How do we know which one to select?

Deep Neural Networks

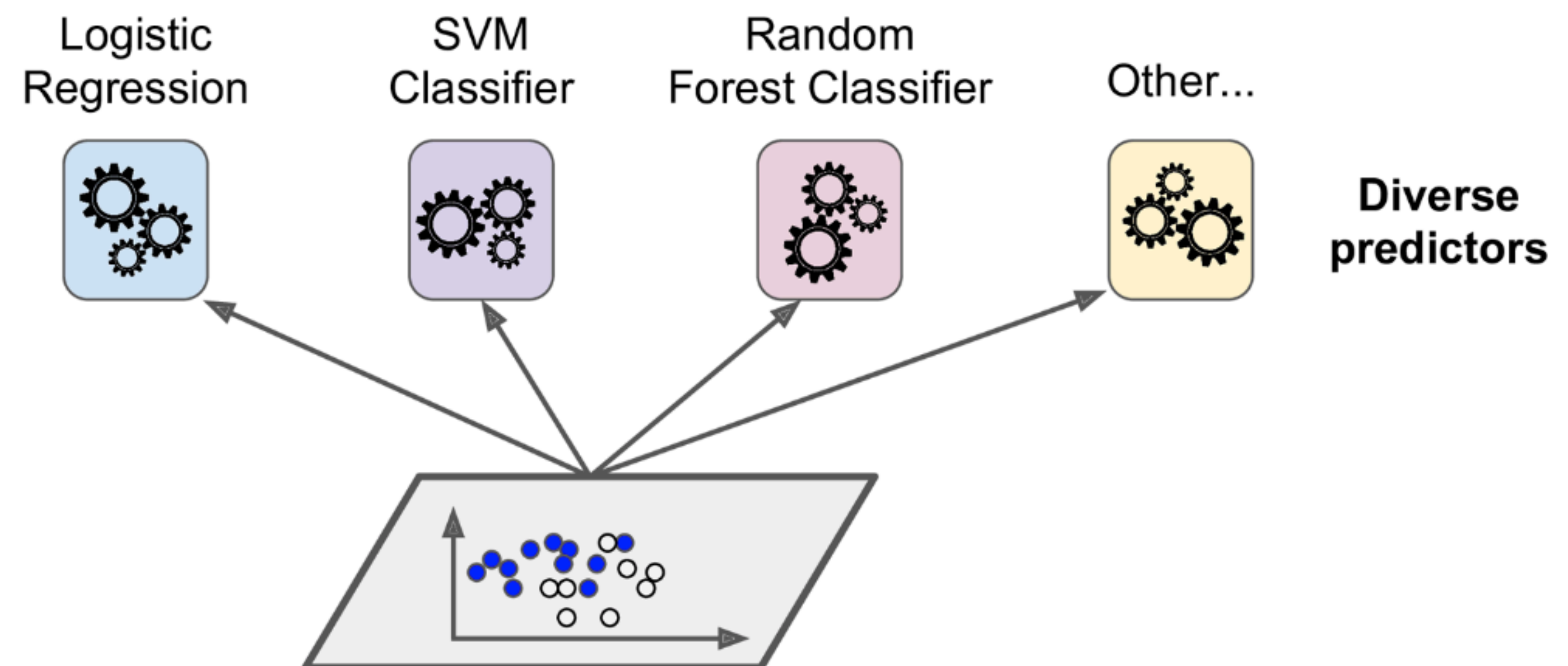


input layer      hidden layer 1      hidden layer 2      output layer

# Which Learning Algorithm Do We Choose?

## Ensemble Learning

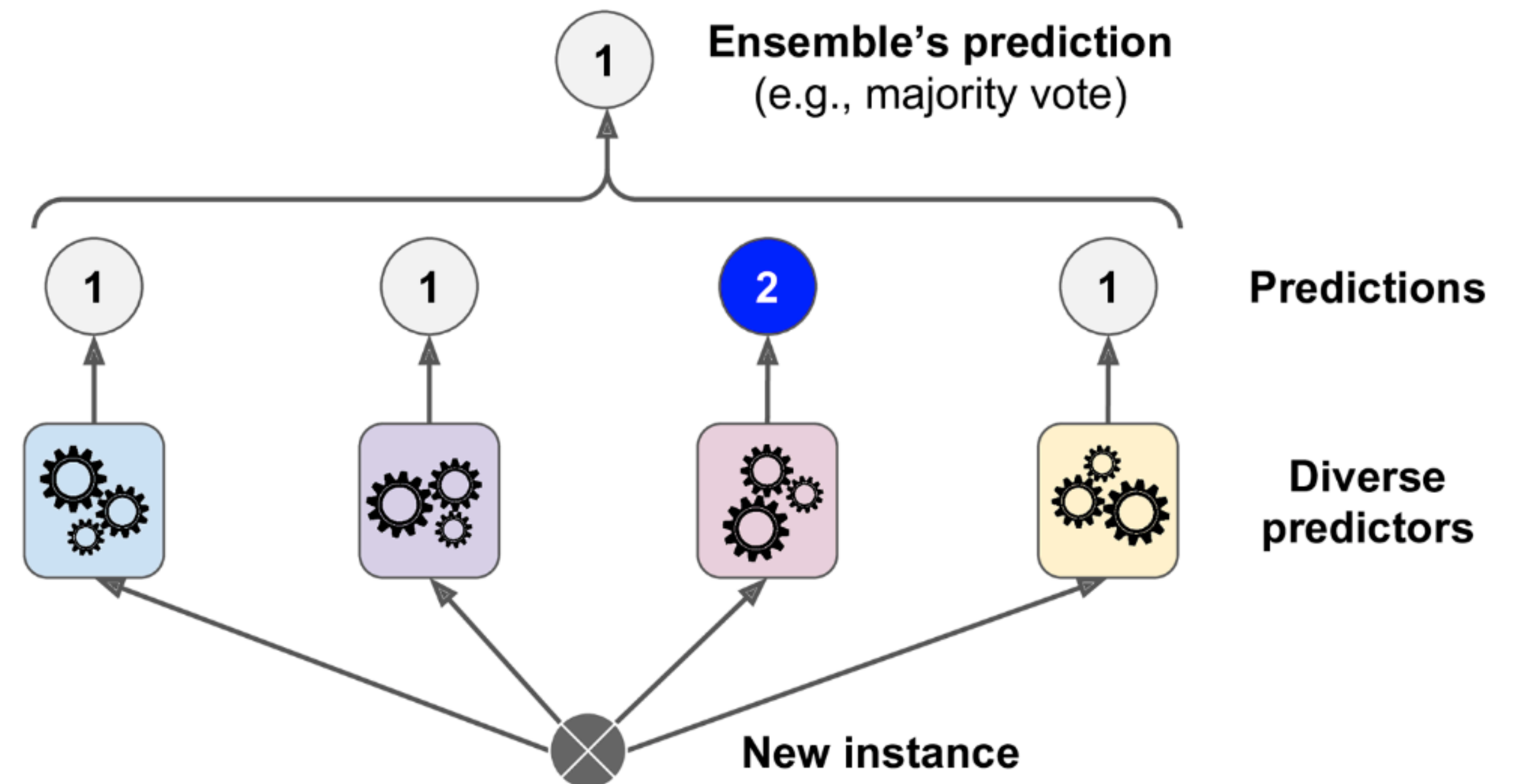
- This is not always clear and obvious
- Hence, we select ALL (or many of them)!
- The idea of considering multiple learning algorithms is known as **Ensemble Learning**.
- It is based on the idea that more opinions is better than one.



# Ensemble Learning: Voting or Averaging

## The Aggregate Experience

- The simplest ensemble learning strategy involves:
  - Training multiple algorithms using the same data
  - Generating predictions for each of the algorithms
  - *Aggregate the predictions of each algorithm to form a single prediction*
    - **Classification:** hard-voting approach (majority wins). Soft voting (pick class with highest probability, averaged across)
    - **Regression:** average the predictions





# Ensemble Learning for Classification

## A Python Example: Hard Voting

- **Consider three learning algorithms:** logistic regression, Random Forest (more on this later), and Support Vector Machine
- Use these to train and test an ensemble voting classifier.

```
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import VotingClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC

log_clf = LogisticRegression()
rnd_clf = RandomForestClassifier()
svm_clf = SVC()

voting_clf = VotingClassifier(
    estimators=[('lr', log_clf), ('rf', rnd_clf), ('svc', svm_clf)],
    voting='hard'
)
voting_clf.fit(X_train, y_train)
```

### Individual Accuracy

- Logistic Regression: 86.4%
- Random Forest: 87.2%
- Support Vector Machine: 88.8%

**Ensemble Accuracy: 89.6%**

# Ensemble Learning for Classification

## A Python Example: Soft Voting

- **Consider three learning algorithms:** logistic regression, Random Forest (more on this later), and Support Vector Machine
- Use these to train and test an ensemble voting classifier.

```
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import VotingClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC

log_clf = LogisticRegression(solver="liblinear", random_state=42)
rnd_clf = RandomForestClassifier(n_estimators=10, random_state=42)
svm_clf = SVC(gamma="auto", random_state=42, probability=True)

voting_clf = VotingClassifier(
    estimators=[('lr', log_clf), ('rf', rnd_clf), ('svc', svm_clf)],
    voting='soft')
```

### Individual Accuracy

- Logistic Regression: 86.4%
- Random Forest: 87.2%
- Support Vector Machine: 88.8%

**Ensemble Accuracy: 91.2%**

# Why Ensembles?

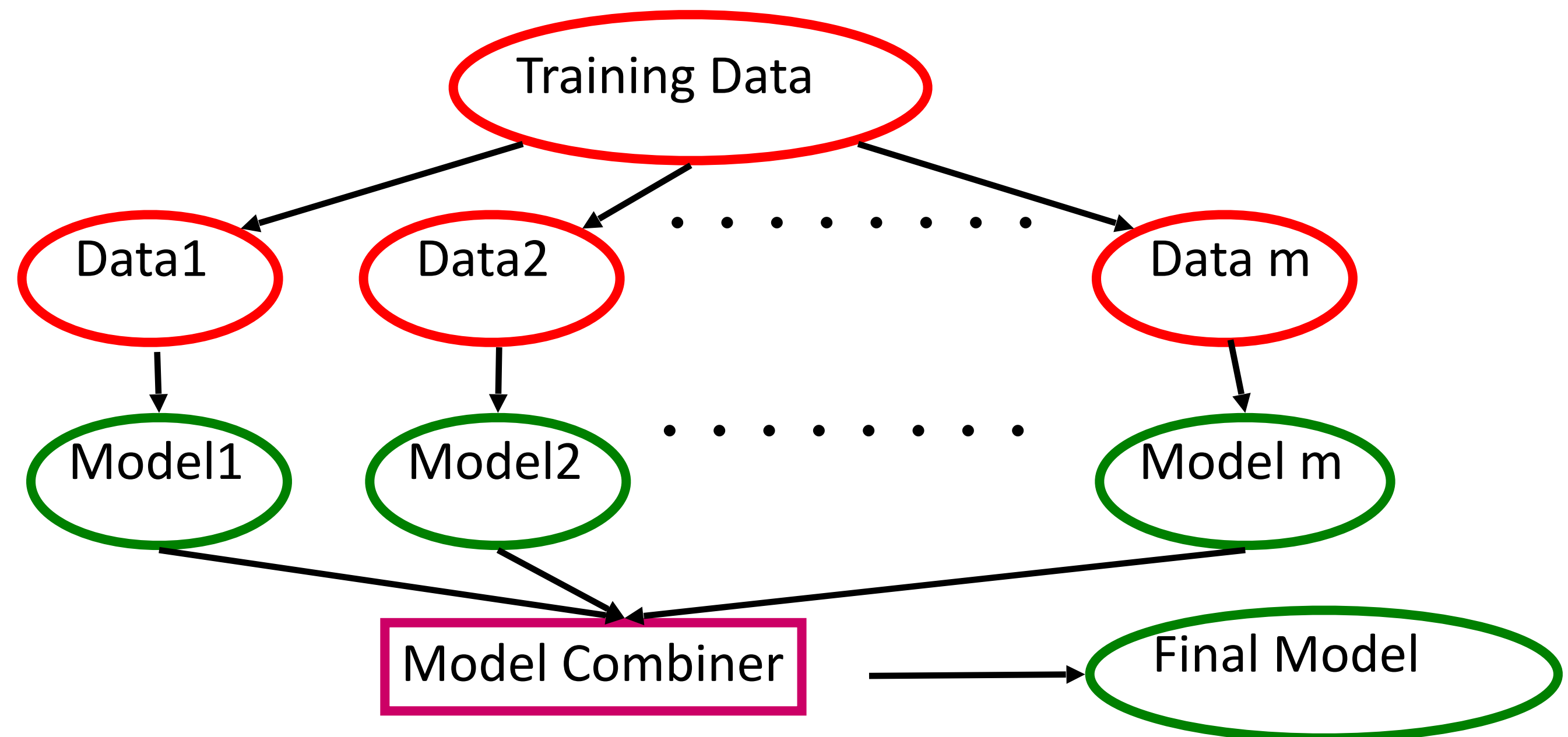
- When combining multiple ***independent*** and ***diverse*** decisions each of which is at least more accurate than random guessing, ***random errors cancel each other out, correct decisions are reinforced.***
- Human ensembles are demonstrably better
  - How many jelly beans in the jar?: Individual estimates vs. group average.
  - Who Wants to be a Millionaire: Expert friend vs. audience vote.
- **Theoretically:** They serve to reduce bias and/or variance



# Learning Ensembles

## Two approaches

- Perform learning using *different training data* or *different learning algorithms*.
- Combine decisions of multiple definitions, e.g. using weighted voting.
- **When the data varies**, these ensemble learners is either based on **(1) bagging (bootstrap aggregation)** or **(2) pasting**
- **Key Feature:** They take a single learning algorithm and generate multiple variations (ensembles)



# Bagging and Pasting

## Only a Subtle Difference

- **Key Feature:** They take a **single** learning algorithm and generate multiple variations (ensembles)
- With **Bagging**, training samples are randomly selected for each variation, but **sampling is performed with replacement**. Hence, there may be data replicates within/across the variations.
- **Pasting** performs sampling without replacement.

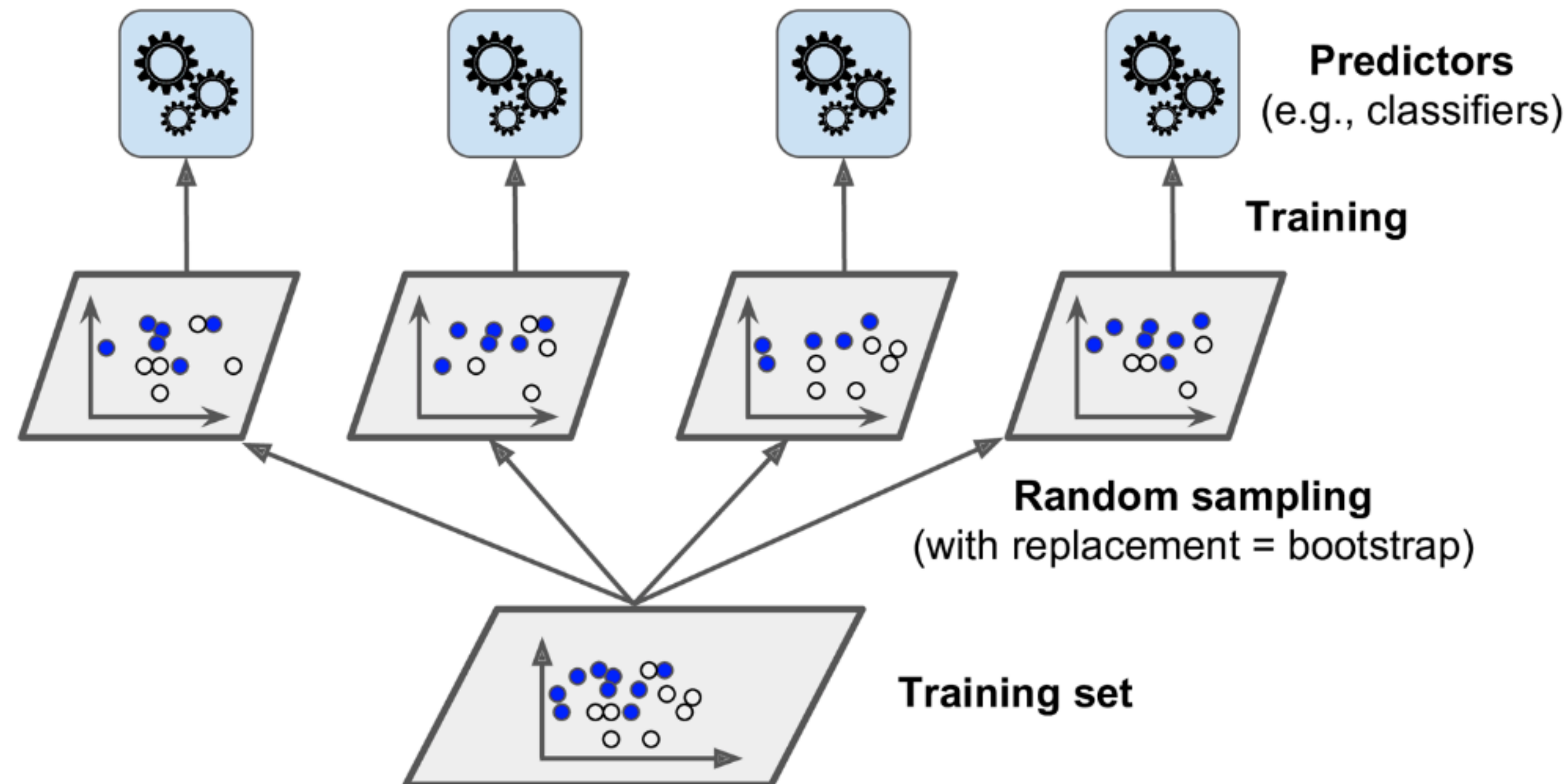
# Bagging Algorithm

Given training set  $S$ , bagging works as follows

- Given a training set  $S$  and  $m$  different predictors, bagging works as follows:
  1. Randomly select  $n$  ***bootstrap samples*** from  $S$  with replacement,  $m$  different times
  2. Train the predictor using its corresponding data
  3. Aggregate the results from the  $m$  resulting models (averaging or hard vote)
- On average each bootstrap sample will contain 63.2% of the unique training examples, the rest (36.8%) are replicas.

# A Depiction of Bagging

- With **Bagging**, training samples are randomly selected for each variation, but **sampling is performed with replacement**. Hence, there may or may not be data overlap across the variations





# Benefits of Bagging

- Training and predictions may be done in parallel
- **Individual predictors have a higher bias** (e.g. underfit the data), but the **aggregation reduces the variance** (e.g. less overfitting).
  - Models that fit the data poorly have high bias
  - Models that can fit the data very well have low bias but high variance
- Bagging tends to outperform Pasting

# Bagging Classifier in Python

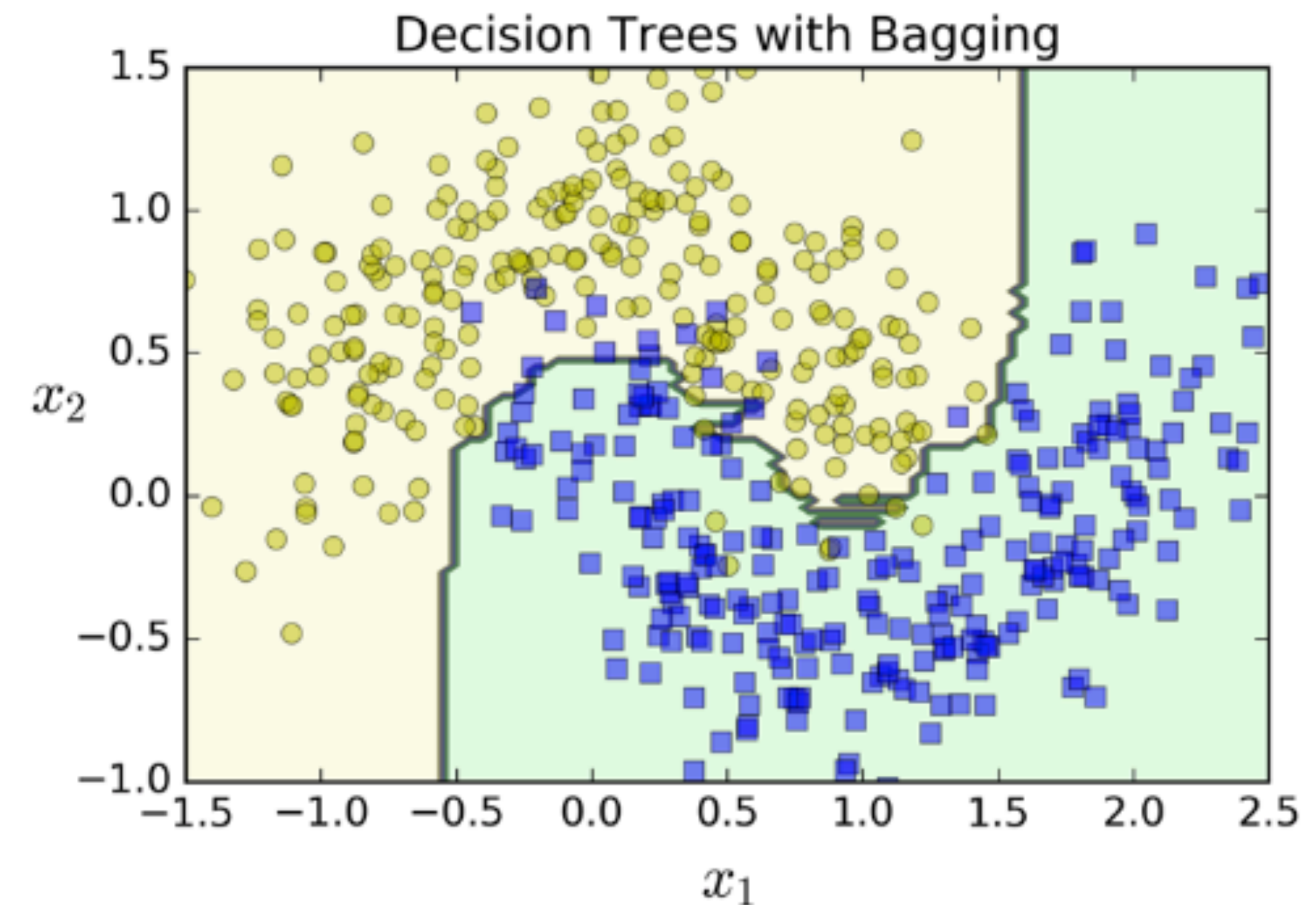
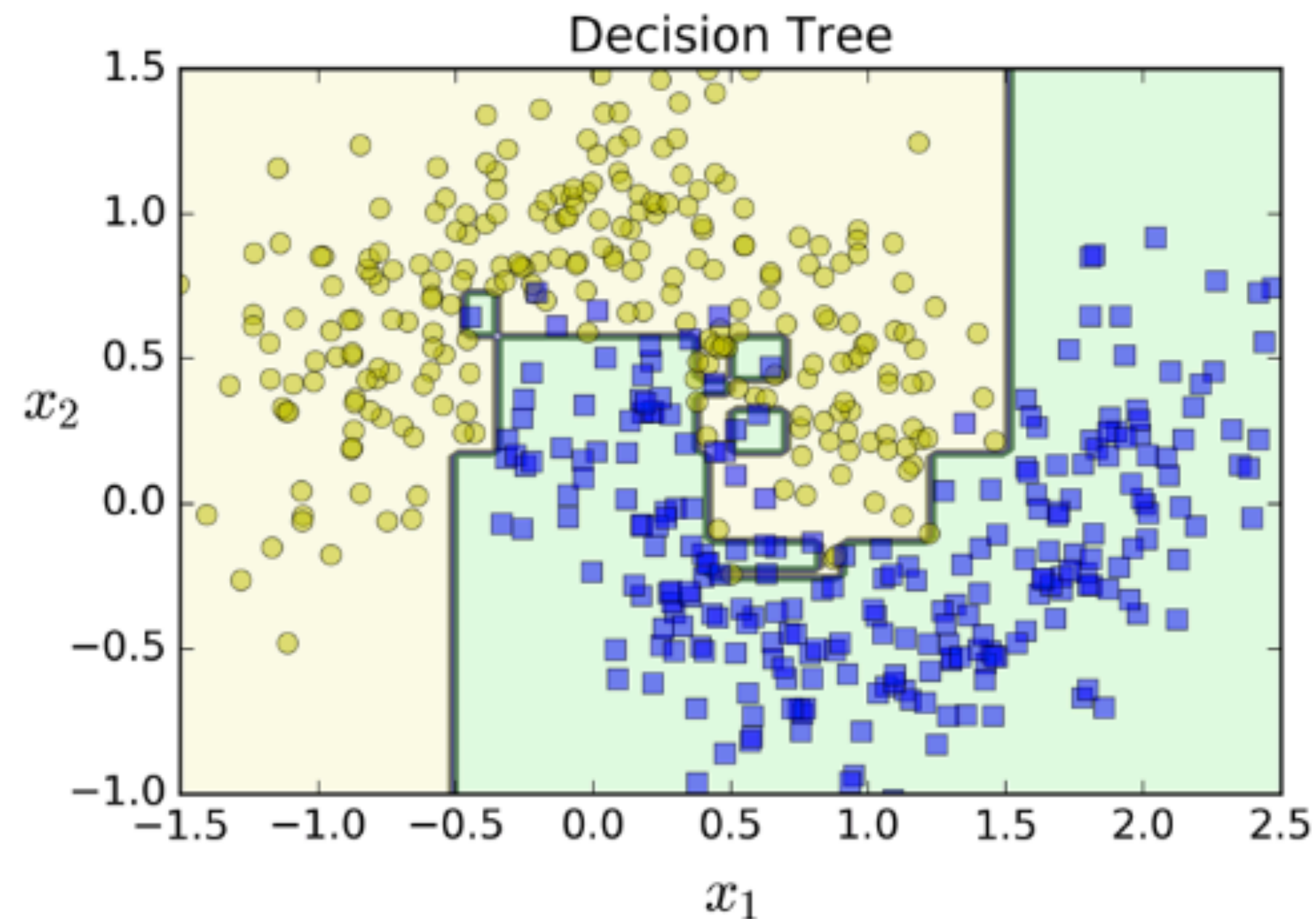
- Train 500 Decision Trees, each trained on 100 training instances using bagging

```
from sklearn.ensemble import BaggingClassifier
from sklearn.tree import DecisionTreeClassifier

bag_clf = BaggingClassifier(
    DecisionTreeClassifier(random_state=42), n_estimators=500,
    max_samples=100, bootstrap=True, n_jobs=-1, random_state=42)
bag_clf.fit(X_train, y_train)
y_pred = bag_clf.predict(X_test)
```

# Bagging Classifier in Python

- Train 500 Decision Trees, each trained on 100 training instances using bagging



# Random Forest

## Bagging Decision Trees

- The previous example use Bagging for an ensemble of Decision Trees. This is called a **Random Forest**. They are helpful for feature selection/importance.
- Random Forests can be used for classification or regression.
- When forming trees, it often ***searches for the best feature among a random subset of features*** (e.g. it does not look at all features).
  - This provides better diversity amongst trees
  - Produces a lower variance (but with higher bias).
- Use the *RandomForestClassifier* or *RandomForestRegressor* in Scikit-Learn



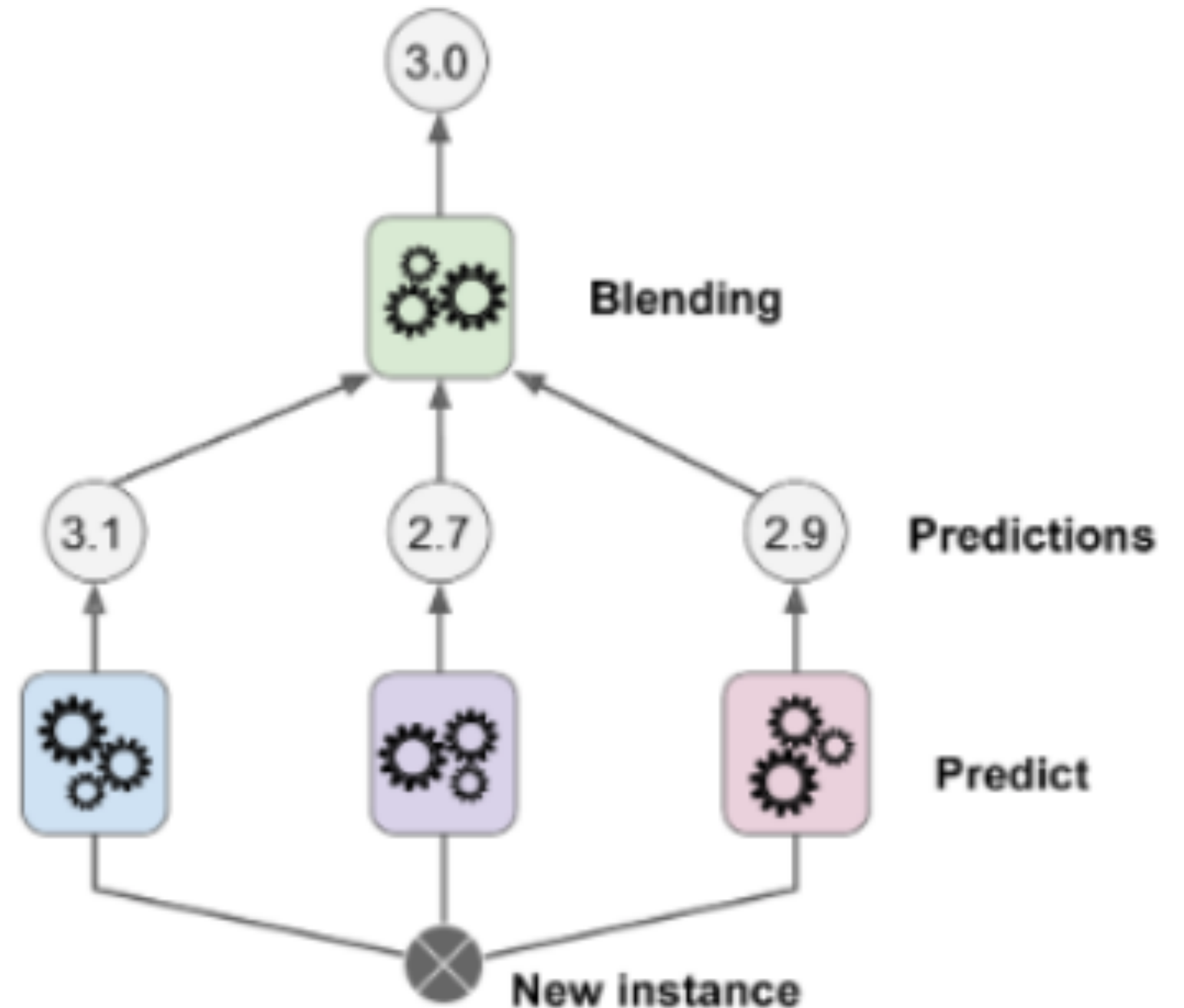
# Other Ensemble Learnings

- **Boosting** - sequentially train learning algorithms, where subsequent predictors correct mistakes made by the predecessor. **Two popular approaches are**
  - *AdaBoost* - Based on sample misclassification/error
  - *Gradient Boost* - Based on learners error
- **Stacking** (stacked generalization) - train a model to perform the aggregation between multiple learners.

# Stacking

## Another Ensemble approach

- Train a model to perform the aggregation between ensemble learners, instead of using average or majority vote.



# AdaBoost

## A Sequential Approach

- **Idea:** Give weights to training instances (or samples)
  - Increase weight along the sequence (after classification/regression) for misclassified samples
  - Train subsequent learner where data is weighted
  - Add more learners and adjust sample weighing each time
- Finally, weigh predictions of each learner in the sequence, based on a learner weight
- Choose class that majority of weighted votes

# AdaBoost

## A Depiction

Assess Training Performance and compute learner weight,  $\alpha_1$

Compute learner weight,  $\alpha_2$ . This learner performs better on bad sample

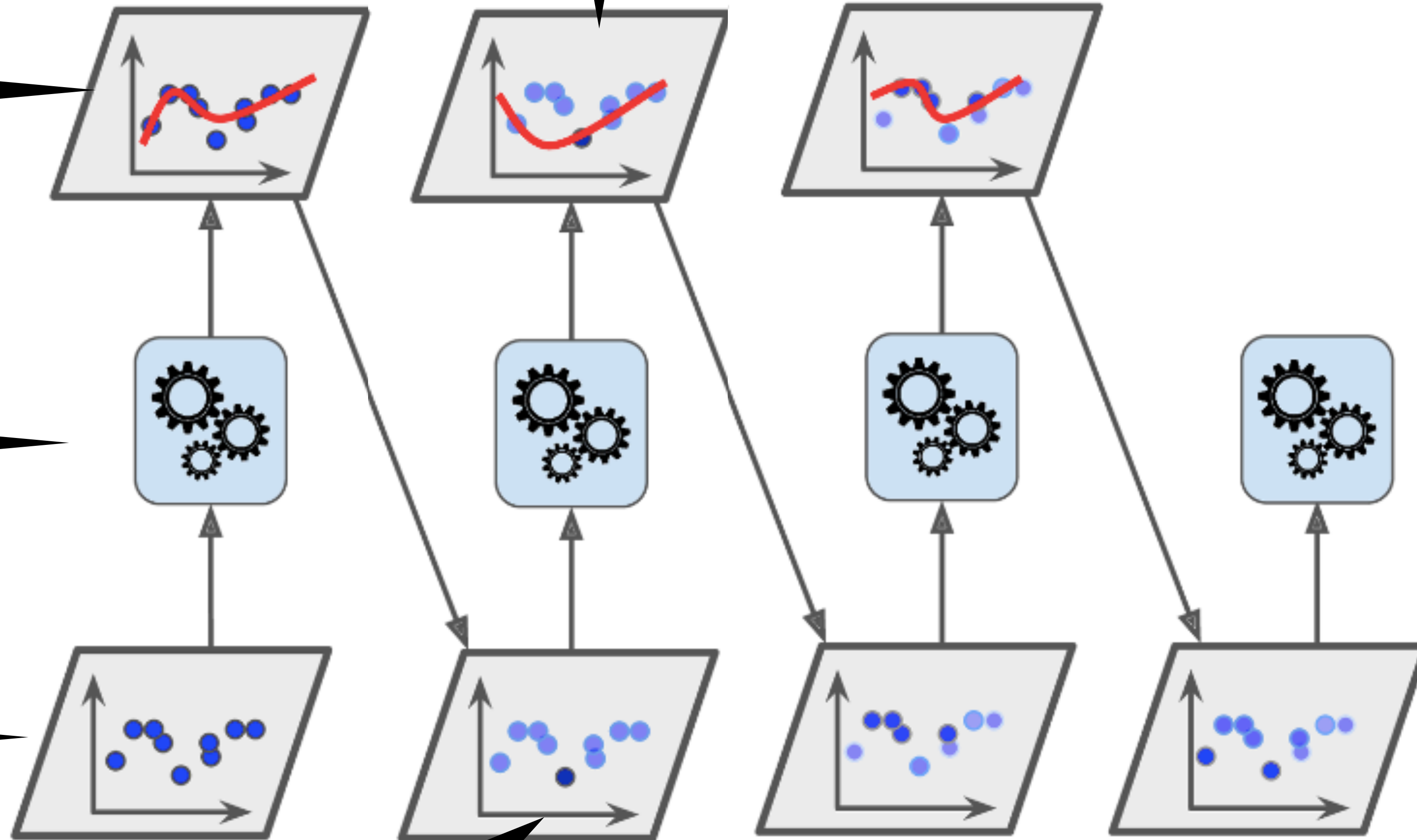
Compute learner weight,  $\alpha_3$ . This learner performs better on bad sample

Train Learner

Assign initial weight to each sample,  $w^{(i)} = \frac{1}{m}$

Update sample weights based on performance,  $w^{(i)}$

Update sample weights based on performance,  $w^{(i)}$





# AdaBoost

## Weight Updates

- **Sample-weight update rule** (assuming  $m$  samples)

$$\text{for } i = 1, 2, \dots, m$$
$$w^{(i)} \leftarrow \begin{cases} w^{(i)} & \text{if } \hat{y}_j^{(i)} = y^{(i)} \\ w^{(i)} \exp(\alpha_j) & \text{if } \hat{y}_j^{(i)} \neq y^{(i)} \end{cases}$$

- **Learner-weight calculation** for the  $j$ th learner in the sequence

$$\alpha_j = \eta \log \frac{1 - r_j}{r_j}$$

- **Weighted error rate of  $j$ th learner**

$$r_j = \frac{\sum_{i=1, \hat{y}_j^{(i)} \neq y_j^{(i)}}^m w^{(i)}}{\sum_{i=1}^m w^{(i)}}$$

# Gradient Boosting

## Fit new learners to residual errors from predescors

- Train first learner:

```
from sklearn.tree import DecisionTreeRegressor  
  
tree_reg1 = DecisionTreeRegressor(max_depth=2, random_state=42)  
tree_reg1.fit(X, y)
```

- Compute residual error then train second learner to predict them

```
y2 = y - tree_reg1.predict(X)  
tree_reg2 = DecisionTreeRegressor(max_depth=2, random_state=42)  
tree_reg2.fit(X, y2)
```

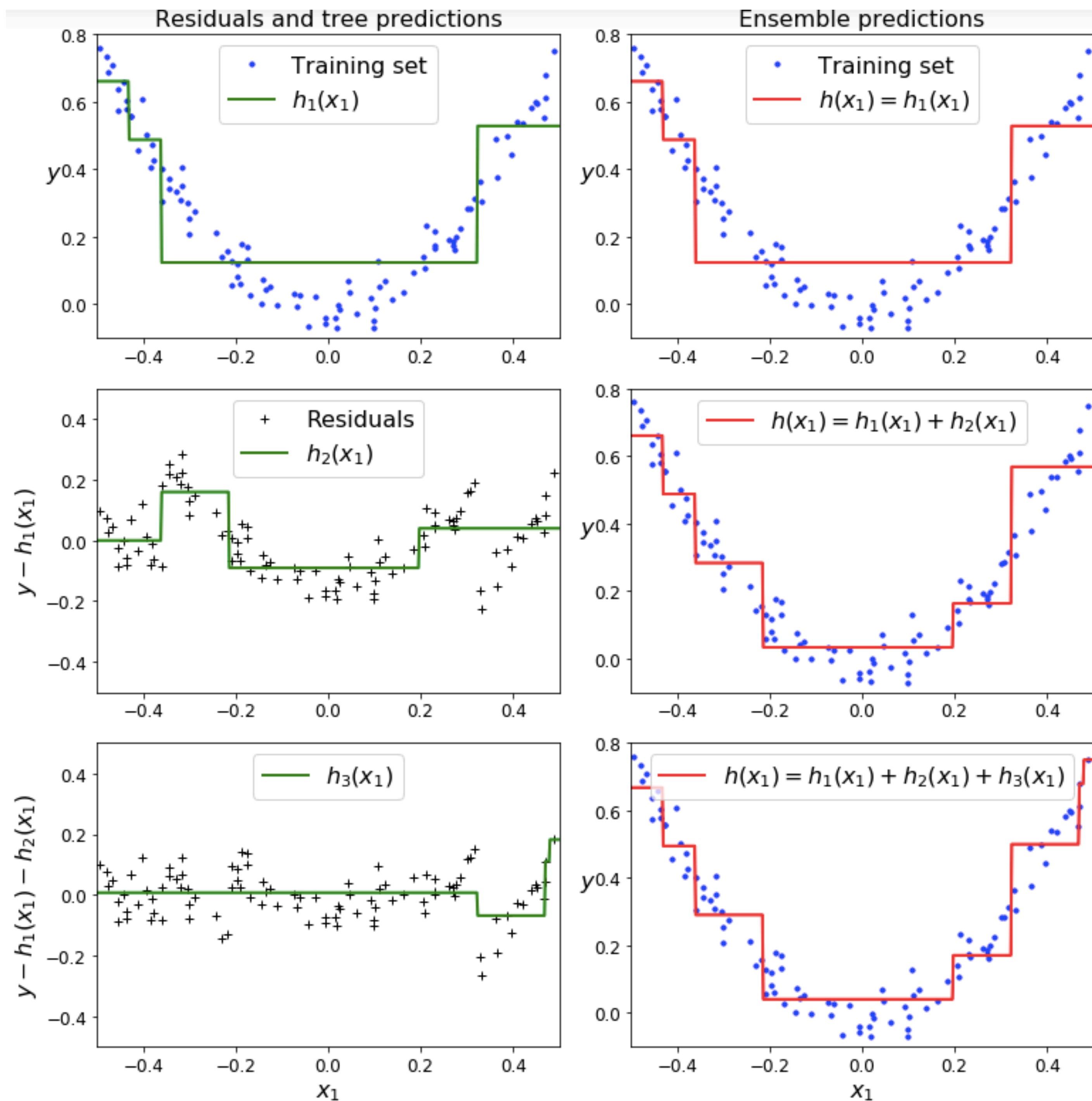
- Compute residual error from second learner and train third learner

```
y3 = y2 - tree_reg2.predict(X)  
tree_reg3 = DecisionTreeRegressor(max_depth=2, random_state=42)  
tree_reg3.fit(X, y3)
```

# Gradient Boosting

- Make predictions by adding predictions from each learner

```
y_pred = sum(tree.predict(X_new) for tree in (tree_reg1, tree_reg2, tree_reg3))
```

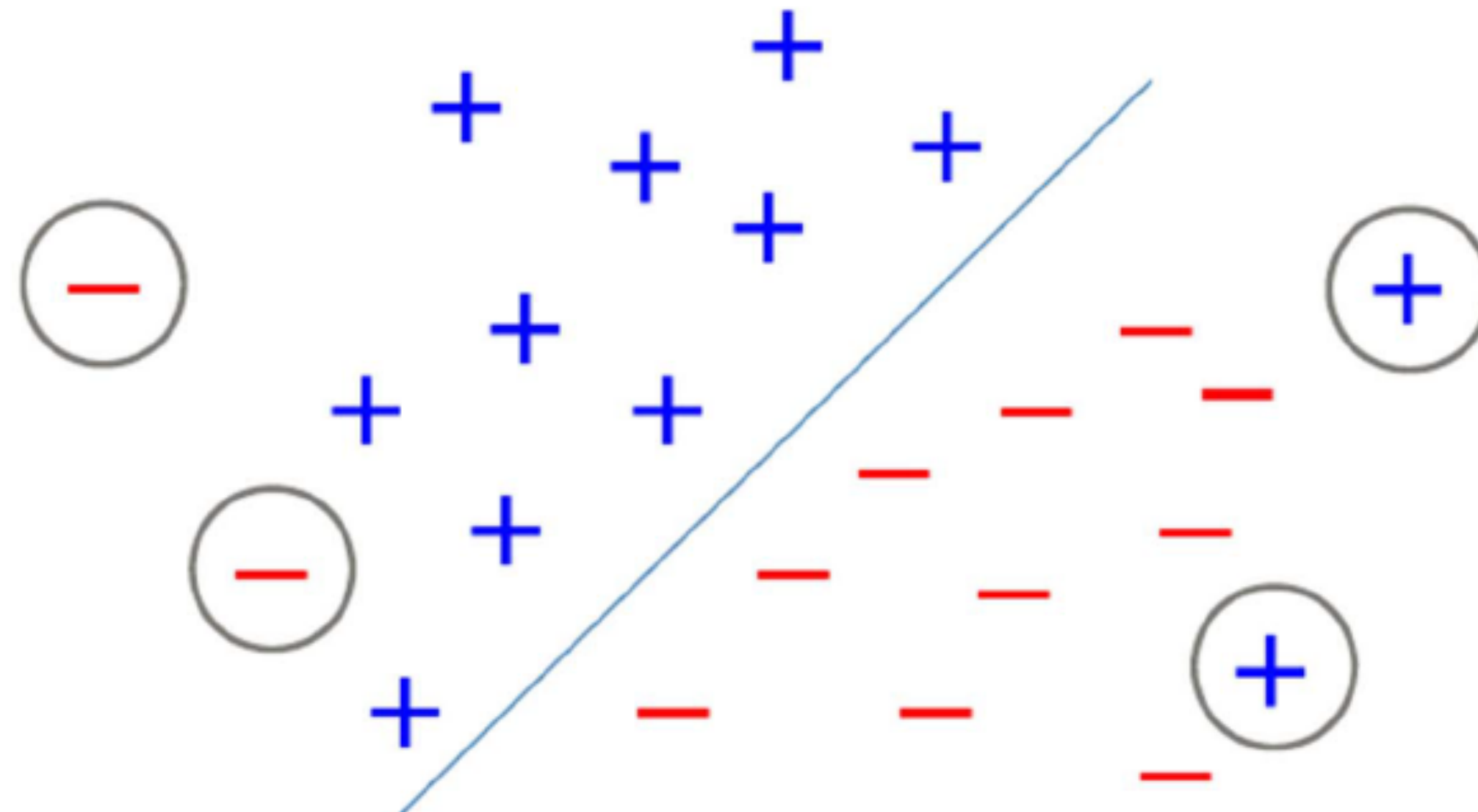




# Pitfall of Boosting

## Sensitive to noise and outliers

- **The Good:** Can identify outliers since focuses on examples that are hard to categorize
- **The Bad:** Too many outliers can degrade classification performance dramatically increasing time to convergence



# Summary: Ensemble Learning

## Boosting and Bagging

- Bagging
  - Resample data points
  - Weight of each classifier is the same
  - Only variance reduction
  - Robust to noise and outliers
- Boosting
  - Reweight data points (modify data distribution)
  - Weight of classifier vary depending on accuracy
  - Reduces both bias and variance
  - Can hurt performance with noise and outliers

# Next Class