CMPS/MATH 2170 Discrete Mathematics – Fall 23

10/3/23

Midterm

Name:

- Put your name on the exam.
- This exam is closed-book, closed-notes, and closed-calculators. You are allowed to use a helper sheet (one single-sided letter page).
- If you have a question, **stay seated** and raise your hand.
- Please try to write legibly – if we cannot read it you may not get credit.
- **Do not waste time** – if you cannot solve a question immediately, skip it and return to it later.

1) Logic		15
2) Rules of Inference		10
3) Functions		15
4) Proof By Contradiction		15
5) Counterexample		5
6) Sets		10
7) Functional Completeness		10
8) Summations		10
Bonus		10
Total		100

1 Logic (15 points)

1. **Equivalence (5 points)**
   Let p and q be propositions. Use a truth table to determine whether
   $(p \vee q) \to p$ is equivalent to $q \to p$.

p	q	
T	T	
T	F	
F	T	
F	F	

   Are $(p \vee q) \to p$ and $q \to p$ equivalent? Justify your answer.

2. **Negation (5 points)**
   Simplify the following Boolean formula such that it contains no negation operators $\neg$, but possibly $\leq$ symbols.

$$\neg\left((\exists p \in \mathbb{N})\ (\forall q \in \mathbb{Z}):\ p > q\right)$$

   (Hint: Use $a \leq b \equiv \neg(a > b)$.)

3. **Translation (5 points)**
   Express the following sentence as a logic formula with quantifiers:
   "For every positive integer, there is a different integer with the same square."

2 Rules of Inference (10 points)

Consider the following collection of premises:

> "If Dracula is out, then it is raining or it is night or the full moon shines."
> "Dracula is out."
> "It is not raining."
> "Whenever the full moon shines, it is night."

Use rules of inference to infer "It is night." from these premises.
You may use the rules in Zybooks table 3.8 and table 1.5.1.

3 Functions (15 points)

Let $f(x) = 7x - \frac{1}{2}$.

1. (10 points) Give a domain and a codomain for which f is bijective. Use the definitions of surjective, injective, and bijective to prove that f is bijective.

2. (5 points) Give a domain and a codomain for which f is injective but not surjective. Justify your answer shortly.

4 Proof By Contradiction (15 points)

Consider the theorem below.

Theorem: If x is irrational then $\frac{x}{2}$ is irrational.

Prove the theorem using a proof by contradiction.

5 Counterexample (5 points)

Disprove the following statement:

$$(\forall x, y \in \mathbb{Q}) : x^2 + y^2 + xy \geq x + y$$

6 Sets (10 points)

Let $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, 4\}$ and $C = \{1, 2, 3, 4, 5, 6, 7\}$. Please fill in the blanks below. (2 points each).

$A - C =$

$B \cap C =$

$A \cap B =$

$|A \times B| =$

$|2^{A \cap B}| =$

(Hint: In order to find $|2^{A \cap B}|$ and $|A \times B|$ you do not need to specify the power set.)

7 Functional Completeness (10 points)

Recall that the nor operator $\downarrow$ is defined by $p \downarrow q = \neg(p \vee q)$. Show that $\downarrow$ is functionally complete by expressing each operation in the set $\{\wedge, \neg\}$ in terms of $\downarrow$.

8 Summations (10 points)

1. (1 point) Express $5 + 15 + 25 + 35 + 45 + \cdots + 95 + 105$ in summation notation.

2. (8 points) Calculate the value of the expression from part 1.

3. (1 point) How many terms are in the series? (For example, the series $1 + 2 + 3$ has 3 terms.)