CMPS/MATH 2170 Discrete Mathematics – Spring 24

3/12/24

Midterm

Name: ______________________________

Student ID: ______________________________

- Put your name on the exam.
- This exam is closed-book, closed-notes, and closed-calculators. You are allowed to use a helper sheet (one single-sided letter page).
- If you have a question, **stay seated** and raise your hand.
- Please try to write legibly – if we cannot read it you may not get credit.
- **Do not waste time** – if you cannot solve a question immediately, skip it and return to it later.

1) Logic		15
2) Rules of Inference		10
3) Proof by Contradiction		15
4) Counterexample		5
5) Sets		10
6) Functions		15
7) Sequences		10
8) Induction		10
Bonus		10
Total		100

1 Logic (15 points)

1. **Equivalence (10 points)**
   Show that $(p \to r) \lor (q \to r)$ and $(p \land q) \to r$ are logically equivalent by using the laws of propositional logic (Zybooks table 1.5.1). Show each step.

2. **Translation (5 points)**
   Let the domain be the members of a chess club. The predicate $B(x, y)$ means that person x has beaten person y at some point in time. Give a logical expression equivalent to the following English statement.

   "Everyone has won at least one game."

2 Rules of Inference (10 points)

Consider the following collection of premises:

> "I ate dinner or I ate lunch."
> "If I ate lunch, then I neither ate dinner nor breakfast."
> "If I ate dinner, then I ate breakfast."
> "I ate breakfast."

Use rules of inference to infer "I ate dinner." from these premises.
You may use the rules in Zybooks table 3.8 and table 1.5.1.

3 Proof by Contradiction (15 points)

Use a proof by contradiction to show that for any five real numbers $a_1, a_2, ..., a_5$, at least one of them is greater than or equal to the average of these numbers. Recall that the average of the five numbers is given by $(a_1 + a_2 + \cdots + a_5)/5$.

4 Counterexample (5 points)

Disprove the following statement:

$$\forall x, y \in \mathbb{R}^+, x^2 + y^2 \geq 2\min(x, y)$$

5 Sets (10 points)

Let $A = \{1, 3, 5, 7\}$, $B = \{5, 6, 7\}$, and $C = \{1, 2, 3, 4, 5, 6, 7\}$. Please fill in the blanks below. (2 points each).

$A - C =$

$B \cap C =$

$A \cup B =$

$\{1\} \times A =$

$|P(A \cap B)| =$

6 Functions (15 points)

Let $f : (1, \infty) \to (0, \infty)$ be defined by $f(x) = \sqrt{x-1}$.

1. (10 points) Use the definitions of surjective, injective, and bijective to prove that f is bijective on this domain and codomain.

2. (5 points) Give a domain and a codomain for which f is injective but not surjective. Justify your answer shortly.

7 Sequences (10 points)

1. (4 points) Give an example of a geometric sequence that is neither non-increasing nor non-decreasing.

2. (2 point) Express $5 + 15 + 25 + 35 + 45 + \cdots + 95 + 105$ in summation notation

3. (4 points) Calculate the value of the expression from (2).

8 Induction (10 points)

Show by mathematical induction that for all $n \in \mathbb{N}$ satisfying $n \geq 3$, the following holds.

$$n^2 \geq 2n + 1.$$