Discrete Math Midterm: Practice Test 1

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1 Logic

1.1 Equivalence - Truth Tables

Show that $((\neg(q \vee r)) \vee (\neg q \wedge r)) \rightarrow \neg p$ is equivalent to $p \rightarrow q$ through a truth table.

1.2 Equivalence - Using Laws of Proposition Logic

Show that $((\neg(q \vee r)) \vee (\neg q \wedge r)) \rightarrow \neg p$ is equivalent to $p \rightarrow q$ using the laws of propositional logic.

1.3 Quantifiers - Translation

Express the following sentence as a logic formula with quantifiers:

It's possible for two negative integers to have an even sum that is at least -6.

This is a universal/existential statement (circle one).

1.4 Quantifiers - Negation

Negate the logic formula from the last question so that it does not contain any negation operators $\neg$.

2 Rules of Inference

Consider the following set of premises:

- My cat's name is Victor or Dipen.
- If my cat's name is Dipen, I also have a dog.
- I don't have a dog.

Use the rules of inference to infer "My cat's name is Victor." You may use the rules in Zybooks table 3.8 and table 1.5.1 (they will be projected on the board during the test).

3 Functions

Let $f(x) = \sin(\pi x)$.

3.1 Injectivity

Give a domain for which f is injective, and use the definition to state why it is injective on this domain.

3.2 Surjectivity

Give a codomain for which f is surjective, and use the definition to state why it is surjective on this codomain.

3.3 Bijectivity

Give a domain and codomain for which f is bijective, and use the definition to state why it is bijective on this range.

$$f : \qquad\qquad \to$$

4 Proofs

4.1 Proof by cases

Consider the theorem below:

> **Theorem:** For all real numbers x such that $x < -1$ or $x > 1$, $x^2 > 1$.

Prove the theorem using a proof by cases.

4.2 Proof by contrapositive

Consider the theorem below:

> **Theorem:** For all $mn \in \mathbb{R} - \mathbb{Q}$, m is irrational or n is irrational.

Prove the theorem using a proof by contrapositive.

5 Sets

Please use these sets to fill in the blanks below:

Let $X = \{\text{places that Tulane students live}\}$
Let $A = \{\text{Sharp, Monroe, Butler, Wall, Warren}\}$
Let $B = \{\text{Lake, River, Aron, DeLa, Weatherhead, Greenbaum}\}$
Let $C = \{\text{Butler, Weatherhead}\}$

- $(A \cup B) - C =$
- I live   if I live in $A \cap B$
- $|B - C| =$
- $|2^B \cup 2^C| =$

6 Functional Completeness

Define the logical operator $\not\to$ as the negation of the $\to$ operator. Show that $\{\to, \not\to\}$ is functionally complete.

7 Summations

Let $\sum_{i=2}^{8} 2i - 2$ be a finite sum.

7.1 Summation Notation

Rewrite the sum with the following indices:

$$\sum_{i=1}^{7}$$

7.2 Summation Value

Calculate the value of the expression.

7.3 Summation Expansion

Write out the summation with addition (eg. 1 + 2 + 3 + ... + 10 for $\sum_{i=1}^{10}$)