

Information Principles and the Quantum Set

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Models for Distributed Computation

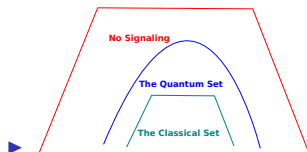
- ▶ Message passing:
 - ▶ Limited by the speed of light
 - ▶ No obvious way to coordinate processes otherwise
- ▶ Non-Locality
 - ▶ Features of the universe can depend on other distant “non-local” parts of the universe.
 - ▶ Counterintuitive in light of relativity
 - ▶ Can be achieved via entanglement
 - ▶ A potential means of coordination without communication

A metaphor for non-locality

- ▶ We have Boxes, which abstract physics
- ▶ Alice (Bob) chooses an input $\{0, 1\}$ and receives probabilistically an output $\{0, 1\}$.
 - ▶ A priori, Alice's output may depend on not only her but also Bob's input and output, as well as random variables.
 - ▶ This allows for correlations between their outputs.
- ▶ We will impose various constraints on these boxes.
 - ▶ Most generally, they cannot transfer information.

Box Distributions

- ▶ Boxes are characterized by their joint probability distribution:
 $Pr(a_1 a_2 a_3 \mid x_1 x_2 x_3)$.
- ▶ Which joint probability distributions can be achieved?
 - ▶ By choosing randomly between two distributions, we can always form convex combinations.



- ▶ We would like an information principle which characterizes the quantum set.

Bell Inequalities

- ▶ A mathematical definition of correlation.
- ▶ Mathematical Motiff: Study a space by studying functions on that space.
 - ▶ These bodies may be studied by optimizing linear functions over them.
 - ▶ A convex set is defined by the set of hyperplanes tangent to it.
 - ▶ Each hyperplane represents an inequality which that set satisfies.

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 - ▶ Each hyperplane represents an inequality which that set satisfies.
- ▶ Example: For 2 parties, the CHSH inequality:
$$Pr(00 | 00) + Pr(11 | 00) + Pr(00 | 01) + Pr(11 | 01) + Pr(00 | 10) + Pr(11 | 10) - Pr(00 | 11) - Pr(11 | 11)$$
 - ▶ Classically, maximized at 2.
 - ▶ Can achieve $2\sqrt{2}$ using the quantum set (Tsirelson's bound)
 - ▶ \therefore the quantum set is strictly larger than the classical set.

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- ▶ Example: For 3 parties, Guess Your Neighbor's Input

$$Pr(000 | 000) + Pr(110 | 001) + Pr(011 | 101) + Pr(101 | 110) \leq 1$$

- ▶ Same bound for both quantum and classical correlations.

Some Information Principles

- ▶ No Signalling

- ▶ The statistics of a person's outcome should only depend on his input.
- ▶ (But the outcome itself may can depend on non-local events!)
- ▶ $\forall \tilde{x}_2 \in \{0, 1\} Pr(a_1 | x_1) = \sum_{a_2 \in \{0, 1\}} Pr(a_1 a_2 | x_1 \tilde{x}_2)$
 - ▶ (i.e. both summations on the right have the same value, so $Pr(a_1 | x_1)$ is well-defined.)
- ▶ Defines a larger set of correlations than the Quantum Set.

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► Information Causality

- If Alice sends Bob m bits, then Bob cannot learn more than m bits about Alice, even using his correlations.
- When $m = 0$, we recover no-signalling.
- Compatible with Quantum Mechanics.
- Has been proposed as a candidate to characterize the quantum set.
- Naturally stated as a bipartite principle.

Main Argument:

- ▶ To create multipartite principles from bipartite ones, apply the bipartite principle over every bipartition.
- ▶ However, there is a tripartite distribution which is classical over any bipartition, but is not in the Quantum Set!
- ▶ Since it is classical over any bipartition, it will satisfy any information principle based on bipartite concepts.
- ▶ To characterize the quantum set, we will need an inherently tripartite information principle.

Classical Probabilities

- ▶ Classically, we assume the collection of Boxes is in some state, λ , which determines the probability distributions of all the boxes for each input. (There may be additional, local sources of randomness)
 - ▶ The classical assumption is that for any state, the Boxes behave like pairwise independent random variables.
 - ▶ $Pr(a_1 a_2 a_3 \mid x_1 x_2 x_3 \lambda) = \sum_{\lambda} p(\lambda) Pr(a_1 \mid x_1 \lambda) Pr(a_2 \mid x_2 \lambda) Pr(a_3 \mid x_3 \lambda)$
- ▶ We would like to define what it means to be classical over some bipartition of the parties, such as 1 – 23.

“Classical over a bipartition”

- ▶ Naive Definition (Bilocal distributions):

- ▶ $P(a_1 a_2 a_3 \mid x_1 x_2 x_3) = \sum_{\lambda} p(\lambda) P_1(a_1 \mid x_1 \lambda) P_{23}(a_2 a_3 \mid x_2 x_3 \lambda)$
- ▶ Allows arbitrary correlations between 2 and 3
 - ▶ Even those where a_2 depends on x_3 and a_3 depends on x_2 .
 - ▶ In reality, one of 2 and 3 must measure first.
- ▶ If 2 feeds his output into 3's input, the resulting probability distribution does not factor.
- ▶ We can create non-locality with 1 through local (between 2 and 3) means!
- ▶ “Classical over a bipartition” should be preserved under any protocol 2 and 3 implement.

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Classical over a bipartition

- ▶ Updated Definition (Time Ordered Bilocal Distributions):

- ▶ $P(a_1 a_2 a_3 \mid x_1 x_2 x_3) =$
$$\sum_{\lambda} p(\lambda) P(a_1 \mid x_1) P_{2 \rightarrow 3}(a_2 a_3 \mid x_2 x_3, \lambda) =$$
$$\sum_{\lambda} p(\lambda) P(a_1 \mid x_1) P_{2 \leftarrow 3}(a_2 a_3 \mid x_2 x_3, \lambda)$$

- ▶ where $P_{2 \rightarrow 3}$ satisfies

$$P_{2 \rightarrow 3}(a_2 \mid x_2, \lambda) = \sum_{a_3} P_{2 \rightarrow 3}(a_2 a_3 \mid x_2 x_3, \lambda) \text{ and}$$

- ▶ $P_{2 \leftarrow 3}$ satisfies $P_{2 \leftarrow 3}(a_2 \mid x_2, \lambda) = \sum_{a_3} P_{2 \leftarrow 3}(a_2 a_3 \mid x_2 x_3, \lambda)$
 - ▶ Essentially, 2 or 3 must be able to measure first.

- ▶ Being able to think of 2 or 3 as measuring first gives a local model for any protocol 2 and 3 enact.

The Counterexample:

	000	001	101	011	100	101	110	111
000	$\frac{2}{3}$	0	0	0	0	0	0	$\frac{1}{3}$
001	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
101	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$
▶ 011	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0
100	$\frac{1}{3}$	0	0	$\frac{1}{6}$	$\frac{1}{3}$	0	0	$\frac{1}{6}$
101	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0
110	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0
111	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0

- ▶ Invariant with respect to renaming parties
- ▶ Guess Your Neighbor's Input is $\frac{7}{6} > 1$: It's not in the quantum set!
- ▶ Found via linear program maximizing GYNI over TOBL

Decompositions

► Decomposition: $\sum_{\lambda} p(\lambda) P(a_1 \mid x_1) P_{2 \rightarrow 3}(a_2 a_3 \mid x_2 x_3, \lambda)$

λ	p_{λ}	a_0	a_1	b_0	b_1	c_{00}	c_{01}	c_{10}	c_{11}
1	$\frac{1}{12}$	0	0	0	1	0	1	0	1
2	$\frac{1}{12}$	0	0	0	0	0	1	0	1
3	$\frac{1}{12}$	0	0	0	0	0	0	0	1
4	$\frac{1}{12}$	0	0	0	1	0	0	0	1
5	$\frac{1}{12}$	0	1	0	1	0	0	0	0
6	$\frac{1}{12}$	0	1	0	0	0	1	0	0
7	$\frac{1}{12}$	0	1	0	0	0	0	0	0
8	$\frac{1}{12}$	0	1	0	1	0	1	0	0
9	$\frac{1}{6}$	1	0	1	1	1	1	1	0
10	$\frac{1}{6}$	1	1	1	0	1	0	1	1

Decompositions

► Decomposition

$$p(abc \mid x_1 x_2 x_3) = \sum_{\lambda} p(\lambda) P(a \mid x_1) P_{2 \leftarrow 3}(bc \mid x_2 x_3, \lambda)$$

λ	p_{λ}	a_0	a_1	b_{00}	b_{01}	b_{10}	b_{11}	c_0	c_1
1	$\frac{1}{12}$	0	0	0	0	0	1	0	0
2	$\frac{1}{12}$	0	0	0	0	0	1	0	1
3	$\frac{1}{12}$	0	0	0	0	1	1	0	0
4	$\frac{1}{12}$	0	0	0	0	1	1	0	1
5	$\frac{1}{12}$	0	1	0	0	0	0	0	0
6	$\frac{1}{12}$	0	1	0	0	0	0	0	1
7	$\frac{1}{12}$	0	1	0	0	1	0	0	0
8	$\frac{1}{12}$	0	1	0	0	1	0	0	1
9	$\frac{1}{6}$	1	0	1	1	1	0	1	1
10	$\frac{1}{6}$	1	1	1	1	0	1	1	0

Conclusion:

- ▶ Information Causality cannot characterize the quantum set.
- ▶ Information Causality for multiple parties is defined based on a bipartite principle
- ▶ The counter-example distribution will obey information causality,
 - ▶ The distribution behaves classically over any bipartite set.
- ▶ The distribution is not in the quantum set.
 - ▶ Its value for GYNI is $\frac{7}{6} > 1$
- ▶ We will need a multipartite principle to characterize the quantum set.
 - ▶ Or we need to reconsider how to extend a bipartite principle to a multipartite one.