COMPUTER NETWORKS HOMEWORK 1

VICTOR BANKSTON

Problem 1. Consider the circuit switched network shown in figure 1. There are 4 circuits on each link. Label the four switches A, B, C and D going in the clockwise direction.

a) What is the maximum number of simultaneous connections that can be in progrsess in the network at any time?

b) Suppose that all connections are between switches A and C. What is the maximum number of simultaneous connections that can be in progress?

c) Suppose we want to make four connections between switches A and C, and another four connections between switches B and D. Can we route these calls through the four links to accommodate all eight connections?

a) We can connect (A, B) 4 times, (B, C) 4 times, and (C, D) 4 times, and (D, A) 4 times for a total of 16 connections. There cannot be more, because each each switch only has 8 connections to the network and this scheme uses all 8 connections for each host.

b) In this case, we can only have 8 connections, 4 of which go through switch D, and 4 of which go through switch B. We cannot have more connections between A and C because both A and C use all 8 of their connections to the network.

c) Yes. We can connect (A, C) twice through the circuit A, B, C, and twice through the circuit A, D, C. We can connect (B, D) twice through (B, C, D) and twice through (B, A, D).

Problem 2. Review the caravan analogy discussed in class. Assume that toll booths are 100km apart, and the cars propagate at $100\frac{km}{hr}$. A toll booth services a car at a rate of one car every 12 seconds. Suppose the caravan (with 10 cars) begins in front of one toll booth, passing through a second tollbooth, and finishes just after a third toll booth. What is the end-to-end delay? (As we did in class, suppose that whenever the first car of the caravan arrives at a toll booth, it waits at the entrance until the other nine cars have arrived and lined up behind it.)

First note that it takes $1hr = 60min$ for a car to travel from tollbooth to tollbooth (the propagation delay). It will take $12\frac{car}{sec} \cdot 10sec = 120sec = 2min$ for the caravan to be serviced by the tollbooth. It will therefore take 62 minutes for the last car to reach the second gate. The caravan will be in the same scenario as the start of the problem, but with one fewer tollbooth and one fewer road to traverse. Applying the same logic again demonstrates that it will take $2 \cdot 62min = 124min$ for the cars to line up at the third toll booth, and 2 more minutes for the third tollbooth to service all 10 cars. The total end to end delay is therefore $126min$.

Alternatively, we can see that there will be 3 tollbooths with delay $2min$ and 2 roads each with delay $60min$ which the caravan must travel across. The total delay is then $3 \cdot 2min + 2 \cdot 60min = 126min$.

Problem 3. We would like to transfer a $20KB$ file (1 Byte = 8 bits) across a network from node A to node F. The file is broken into 20 packets of length $1KB$ each (neglecting the packet header). The path from node A to node F passes through 5 links and 4 routers. Each of the links is a 10km optical fiber with a transmission rate of 10 Mbps. Assume that the speed of light in optical fiber is $2.0 \times 10^8 \frac{m}{s}$. The 4 routers are store-and-forward devices, and each of them must perform a $50\mu s$ ($1\mu s = 10^{-6}s$) routing table look up after receiving a packet before it can begin sending it on the outgoing link. How long does it take to send the entire file across the network?

Observe that except at node B, there will be no queueing delay. This is because each of the 4 routers transmits information at the same rate. For example, the the end of the second packet will reach node C $50\mu s + 100s$ (the processing delay plus the transmission delay) after the end of the first packet. This means that C will finish the forwarding lookup and finish transmitting the first packet as soon as the second packet fully arrives, so there will be no queueing delay.

However, as I understand the problem, we are not assuming that node A needs to perform a $50\mu s$ lookup, so there will be some queueing delay at node B. To calculate how much, note that node A will finish transmitting the last packet will arrive at the queue for node B $19 \cdot 100s = 1900s$ after the first packet. It will take node B a total of $19 \cdot (50\mu s + 100)$ to process the 19 other packets. Hence, the last packet must wait at node B for $950\mu s$.

This is equivalent to assuming that node A does perform a forwarding lookup. In this case, the last node waits its extra $950\mu s$ at the beginning rather than at node B.

If there were just 1 packet, it would take $5 \cdot 10km/2.0 \times 10^5 \frac{km}{s} + 4 \cdot 10^{-6}s + 4 \cdot 1KB/10Mbs \approx 400s$

The total time for all 20 packets is therefore the amount of time it takes for the last packet to be sent across the network. This is $1900s + 950\mu s$ more than the value above, or about $2300s$. That's a long time, and comes almost entirely from the low transmission rate.

Problem 4. Consider a link with transmission rate Rbps. Suppose every T seconds, N packets arrive simultaneously to the link where each packet is of length L bits. Assume $T \geq LN/R$. What is the average queueing delay of a packet?

The assumption that $T \geq LN/R$ implies that the queue will be cleared before the next burst arrives. The kth packet in a burst will have to wait for $k-1$ packets, or $(k-1)\,L/R$ seconds. The average queueing delay per packet is then

$$\frac{1}{N}\sum_{k=1}^{N}(k-1)\,\frac{L}{R} = \frac{(N-1)\,L}{2R}$$

This is the packet-average. Given that a packet is likely to be sent according to the given distribution, it can expect to wait this long.

We should also find the time average, or the expected wait time of a packet which is sent at time chosen uniformly at random. This time can be described as some number $t \in [0, T]$. The packet cannot be serviced until time $\frac{NL}{R}$, which is the time it takes for the burst to be handled. The wait time is then 0 for $t > \frac{NL}{R}$, and

$\frac{NL}{R} - t$ otherwise, so the expected wait time is

$$\frac{1}{T}\int_{t=0}^{\frac{NL}{R}} \left(\frac{NL}{R} - t\right) dt = \frac{1}{T}\frac{1}{2}\left(\frac{NL}{R}\right)^2$$

Problem 5. Consider sending a large file of F bits from Host A to Host B. There are three links and two switches between A and B, and the links are uncongested (that is, no queueing delays). Host A segments the file into segments of S bits each and adds 80 bits of header to each segment, forming packets of $L = 80 + S$ bits. Each link has a transmission rate of Rbps. Disregard processing delay and propagation delay.

a) What is the delay of moving the file from Host A to Host B?

b) Find the value of S that minimizes the above delay.

a) Disregarding processing delay and propagation delay, we have only to contend with transmission delay, which is $(N-1)\frac{L}{R} + 3 \cdot \frac{L}{R}$, where $N = \frac{F}{S}$ is the number of packets which must be an integer. The factor of 3 reflects that there are two switches plus the host which must push the packets onto the wire. The delay, in terms of S is then

$$D = \frac{F}{S} \cdot \frac{80+S}{R} + 2\frac{80+S}{R}$$

with the understanding that $\frac{F}{S} \in \mathbb{N}$.

b) To minimize this quantity, we should find the critical points, where $\frac{\partial D}{\partial S} = 0$.

$$\frac{\partial D}{\partial S} = -\frac{80F}{S^2 R} + \frac{2}{R} = 0$$

gives

$$S = \sqrt{40F}$$

Of course, we need to pick a natural number for S so that $\frac{F}{S} \in \mathbb{N}$. If F is not large enough, then $S > F$, so our answer doesn't make sense. This reflects that the packet header becomes large in proportion to the packets, so it becomes more efficient to send the whole file as a single packet.

Problem 6. Suppose there are M paths between a server and a client, where no two paths share any link. Path k ($k = 1, \ldots, M$) consists of N links with transmission rates $R_1^k, R_2^k, \ldots R_N^k$.

a) If the server can only use one path to send data to the client, what is the maximum throughput that the server can achieve?

b) If the server can use all M paths to send data, what is the maximum throughput that the server can achieve?

a) The throughput of path k will be determined by its slowest link, so the maximum throughput for the server is $\max_k \min_n R_n^k$.

b) If the server can use all paths, then we can just add the throughput of each path. $\sum_{k=1}^{M} \min_n \left(R_n^k\right)$