Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics & Science

School of Computer Science and Statistics

Integrated Computer Science Programme **Trinity Term 2016**
BA (Mod) CSL
BA (Mod) Computer Science & Business
Year 1 Annual Examinations

Mathematics

Wednesday 4 May 2016 **RDS Main Hall** **9:30 - 12:30**

Dr. Hugh Gibbons

Instructions to Candidates:

- This exam paper has TWO SECTIONS.
- Answer TWO of the three questions in SECTION A.
- Answer TWO of the three questions in SECTION B.
- Use SEPARATE answer books for each SECTION.
- Each question is worth 25 marks.

Materials permitted for this examination:

- A Non-Programmable calculator is permitted for this examination. Please indicate the make and model of your calculator on each answer book used.
- An approved Formulae and Tables booklet is available from Invigilator.

Section A

Question 1

(a) Solve the following system of linear equations

$$2*x+6*y+2*z=2$$
$$-3*x-8*y=2$$
$$4*x+9*y+2*z=3$$

using

i. Gaussian Elimination
ii. Cramer's Rule.

[16 marks]

(b) Given the matrix A and constant vector b, consider the following technique for solving $A*x=b$.
If $A=L*U$ then we can solve $A*x=b$ as follows:
We rewrite $A*x=b$ as $(L*U)*x=b$
i.e. rewrite $A*x=b$ as $L*(U*x)=b$.
Let the vector $y=U*x$ and solve for y in $L*y=b$.
Using the solution for y and since $U*x=y$ we can then solve for x.
The matrix

$$A=\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

can be 'factored' as $L*U$ i.e. $A=L*U$ where
L is a lower triangular matrix and U is an upper triangular matrix ,
i.e. A = L*U where

$$L=\begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \quad \text{and} \quad U=\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Solve the following system of linear equations

$$\begin{aligned} 2*x_1+6*x_2+2*x_3 &= 2 \\ -3*x_1-8*x_2+0*x_3 &= 2 \\ 4*x_1+9*x_2+2*x_3 &= 3 \end{aligned}$$

by rewriting it the form $A*x=b$ and then using the above technique i.e. solve for y in $L*y=b$ and then solve for x in $U*x=y$. [9 marks]

Question 2

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

using

i. Gaussian Elimination

ii. the Cayley-Hamilton result that the matrix A satisfies its characteristic equation i.e.

$$A^3 - 6 * A^2 + 11 * A - 6 * Id = 0$$

where Id is the 3×3 identity matrix and 0 is the 3×3 zero matrix.

16 marks]

(b) Calculate the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

[9 marks]

Question 3

(a) Evaluate the following integrals

i. $\int \frac{\cos x}{\sqrt{1+\sin x}}\, dx$

ii. $\int_0^1 x * \sqrt{1-x^2}\, dx$

[8 marks]

(b) Using integration, find the area of a $\frac{1}{4}$ unit circle
i.e. evaluate

$$\int_0^1 \sqrt{1-x^2}\, dx$$

[6 marks]

(c) We can find a series for $\frac{1}{(x-1)^2}$ when $-1 < x < 1$ by finding the Taylor Series for $f(x) = \frac{1}{x^2}$ about the value -1. The Taylor Series about the value, a, is
$f(a+x) = f(a) + x * f'(a) + \frac{x^2}{2!} * f''(a) + \frac{x^3}{3!} * f'''(a) + \frac{x^4}{4!} * f^{(4)}(a) + ...$
(find the terms up to one in x^4).

[5 marks]

(d) Let a function $g(x) = ((x-6) * x + 11) * x - 6$. Using the Newton-Raphson method an approximation for a root of $g(x)$ can be found. The Newton-Raphson method generates terms, r_n, that approximate a root of $g(x)$ where:

$$\begin{aligned} r_0 &= c \\ r_{n+1} &= r_n - \frac{g(r_n)}{g'(r_n)} \end{aligned}$$

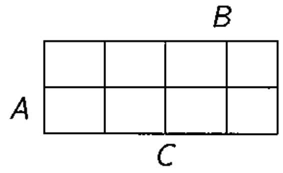
Find the term r_2 in this sequence when $r_0 = 0$.

[6 marks]

Section B

Question 4

(a) Using the Veitch diagram



determine whether:

$$(A \cup B) \cap (A \cup C) \cap (B \cup C) = (A \cap B) \cup (A \cap C) \cup (B \cap C)$$

[5 Marks]

(b) Given that $|X \cup Y| = |X| + |Y| - |X \cap Y|$ show that

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

[5 Marks]

(c) In a group of 200 students, each of whom studied at least one of the subjects, Computing, Physics or Mathematics it is found that

- 90 studied Computing
- 60 studied Physics
- 110 studied Mathematics
- 20 studied Computing and Mathematics
- 20 studied Computing and Physics
- 30 studied Mathematics and Physics

How many students, if any, studied all 3 subjects.

[6 Marks]

(d) Show by induction that

$$(+k \,|\, 1 \leq k \leq n \,:\, k^2) = \frac{n * (n+1) * (2 * n + 1)}{6}$$

Note: the factors of $(2 * n^2 + 7 * n + 6) = (2 * n + 3) * (n + 2)$

[9 marks]

Question 5

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \equiv Q$
F	F	F	F	T	T
F	T	F	T	T	F
T	F	F	T	F	F
T	T	T	T	T	T

(a) Determine using truth tables, whether the following are Tautologies
(Note: $\equiv$ is associative, i.e. $P \equiv (Q \equiv R) = (P \equiv Q) \equiv R$

(a) $(P \rightarrow Q) \vee (Q \rightarrow P)$

(b) $P \wedge Q \equiv P \equiv Q \equiv P \vee Q$

(c) $P \wedge (Q \equiv R) \equiv (P \wedge Q) \equiv (P \wedge R)$

[9 marks]

(b) Determine by Truth Table or otherwise whether the following argument is valid:

The system will crash if the programmer does not act quickly.
If the system crashes then the programmer is over worked.
If the programmer is over worked then the programmer does not act quickly
$\therefore$
If the programmer is over worked then the system crashes.

Abbreviate:
C: The system will crash..
P: The programmer acts quickly.
W: The programmer is over worked.

[6 marks]

(continued next page)

(c) **The Tardy Bus Problem**

The following three statements are given as premises:

If Bill takes the bus then, Bill misses his appointment if the bus is late.

Bill does not go home, if Bill misses his appointment and Bill feels downcast.

If Bill does not get the job then Bill feels downcast and Bill goes home.

Determine, by the use of KE Deduction, whether each of the following 2 conjectures can be inferred from the premises.

- *If the bus is late and Bill does not gets the job then Bill does not take the bus.*
- *If Bill goes home and Bill does not get the job then Bill does not miss his appointment.*

Abbreviations:

TB:	*Bill Takes the Bus.*
MA:	*Bill Misses his Appointment*
BL:	*The Bus is Late.*
GH:	*Bill Goes Home.*
FD:	*Bill Feels Downcast.*
GJ:	*Bill Gets the Job.*

Translations of Premises:

P1: $TB \rightarrow BL \rightarrow MA$

P2: $MA \wedge FD \rightarrow \neg GH$

P3: $\neg GJ \rightarrow FD \wedge GH$

Translation of Conjectures.

C1. $BL \wedge \neg GJ \rightarrow \neg TB$

C2. $GH \wedge \neg GJ \rightarrow \neg MA$

KE_Deduction Rules

	α – *rules*			
Premise	$\neg\neg P$	$P \wedge Q$	$\neg(P \vee Q)$	$\neg(P \rightarrow Q)$
Conclusion	P	P	$\neg P$	P
Conclusion		Q	$\neg Q$	$\neg Q$

	β – *rules*			
Premise	$P \vee Q$	$\neg(P \wedge Q)$	$P \rightarrow Q$	$P \rightarrow Q$
Premise	$\neg P$	P	P	$\neg Q$
Conclusion	Q	$\neg Q$	Q	$\neg P$

Branching Rule

$$\begin{array}{cc} & \wedge \\ P & \neg P \end{array}$$

[10 Marks]

Question 6

(a) Calculate the following expressions:

i. $(+k \mid 1 \leq k \leq 4 \,:\, k^2)$

ii. $(+i,j \mid 1 \leq i \leq j \leq 4 \,:\, j)$ [6 marks]

(b) Express the following numbers in powers of primes i.e. express in the form $2^{p1} * 3^{p2} * 5^{p3} * \ldots$ where $p1 \geq 0, p2 \geq 0, \ldots$

i. 1812.

ii. 1572 [6 marks]

(c) i. Find the multiplicative inverse of 5 *mod* 16

ii. Find integers x and y such that

$$1812 * x + 1572 * y = gcd((1812, 1572)$$

[13 marks]