

Transmission of signals:
link transmission calculations

Optical link design

- When designing an optical link I usually have a BER requirement that needs to be satisfied
- In order to satisfy the BER, I need to consider the following constraints:
 - 1) Power budget:** $P_{\text{launch}} - \text{Loss (fibre, connectors,...)} - \text{Margin} \geq \text{receiver sensitivity}$
 - power margin about 3dB, considers component degradation and other unforeseen events
 - 2) OSNR:** if power budget is satisfied I also need to satisfy OSNR, so that the signal is good enough to achieve target BER
 - an OSNR margin should also be consider, say about 3 dB
 - 3) Dispersion** needs to be accounted for or compensated.
 - In practical exercises we consider modern coherent systems that carry out dispersion compensation digitally.

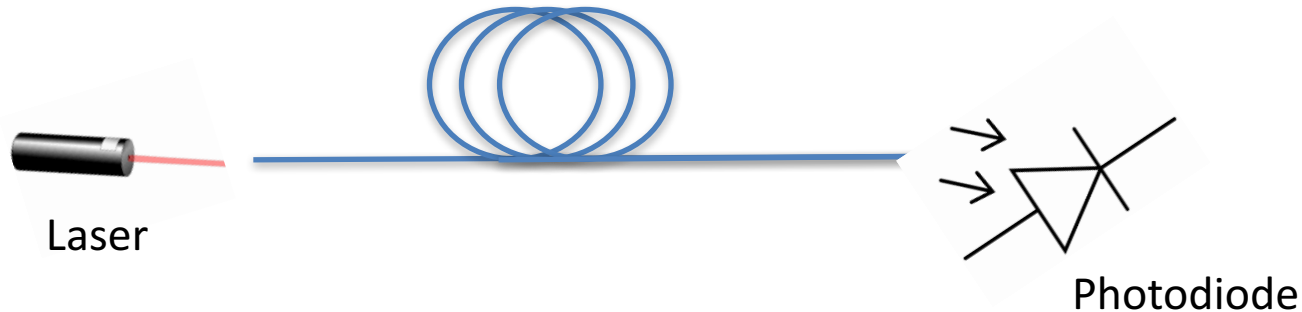
Noise in optical systems

- In optical systems there are three main types of noise
- **Amplified Spontaneous Emission (ASE):** due to optical amplifiers, which besides amplifying the signal carrying information also introduce additional noise.
- **Thermal noise:** due to the random thermal-driven motion of electrons at the receiver, which sums up to the signal carrying information
- **Shot noise:** due to the quantistic nature of an optical signal, the photons carrying the information arrive at the detector with random arrival times

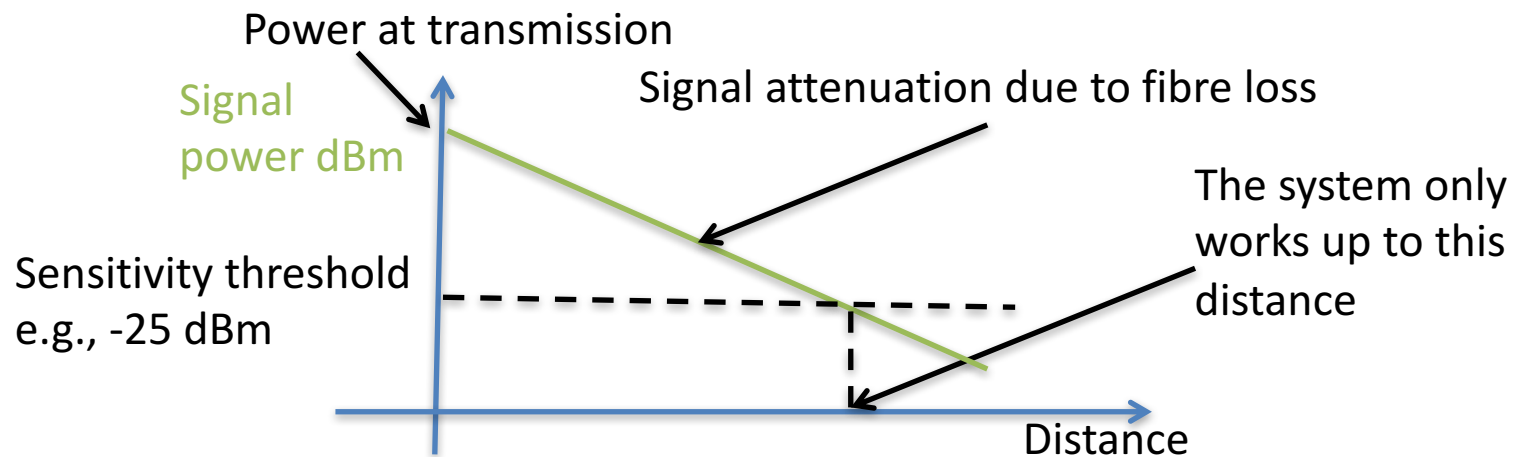
Signal transmission over a link

- We now want to see what happens when we try to transmit a signal over a fibre
- Unlike copper medium (e.g., twisted copper pair, CAT-5 cables, coaxial cables) which generates noise as the temperature makes electrons move randomly in the cable

Optical fibre does not generate noise, it only attenuates the signal.



The photodiode has a certain sensitivity: if the power of the light is too low it will not work



Signal transmission over a non-amplified link

- 1) The signal starts with a given power (dBm)
- 2) The loss in the fibre reduces the **signal power**
- 3) **At the receiver I need to make sure that:
The signal power is at least equal to the receiver sensitivity**

P.S. The dispersion also should be considered in principle, but here we assume that coherent transmission systems can remove it at the end

Example I

- A non-amplified optical communication system has a receiver sensitivity (Rx_{sens}) of -25dBm. A signal leaves the transmitter with a power (P_{tx}) of 0dBm. The transmission line introduces loss of 0.25 dB/km (Loss_coeff). If the line is 80Km long (l), and you include a margin of 3 dB, can the communication be achieved effectively?
- You need to satisfy the equations:
 1. $P_{tx} - \text{Loss} - M \geq Rx_{sens}$, (Receiver sensitivity constraint)
where $\text{Loss} = \text{lenth}_{[km]} \times \text{Loss_coeff} (\alpha_{[dB/km]})$

The exercise is about checking that this conditions is satisfied!

0 dBm – 0.25 x 80 -3 = -23 dBm > -25, so the system works

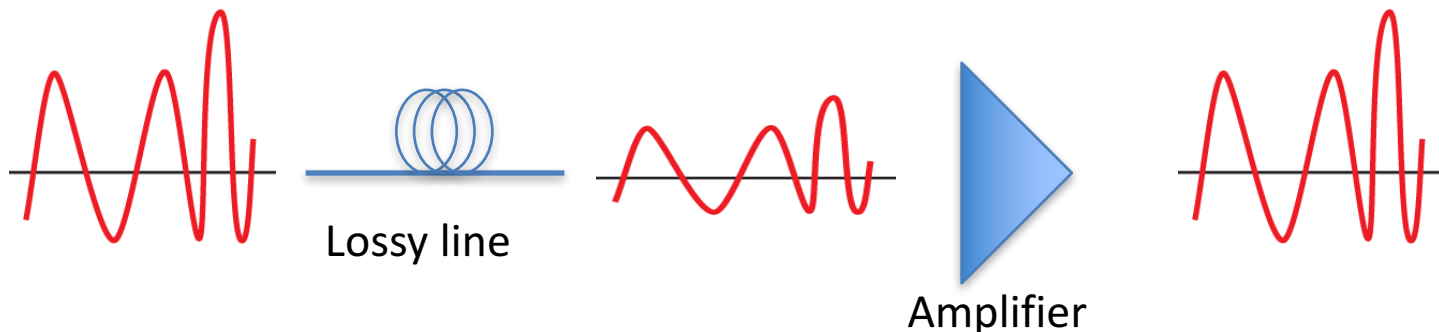
Example II

Same problem as before, but now the distance is 120Km.

0 dBm – $0.25 \times 120 \times -3 = -33$ dBm < -25, so the system does not work

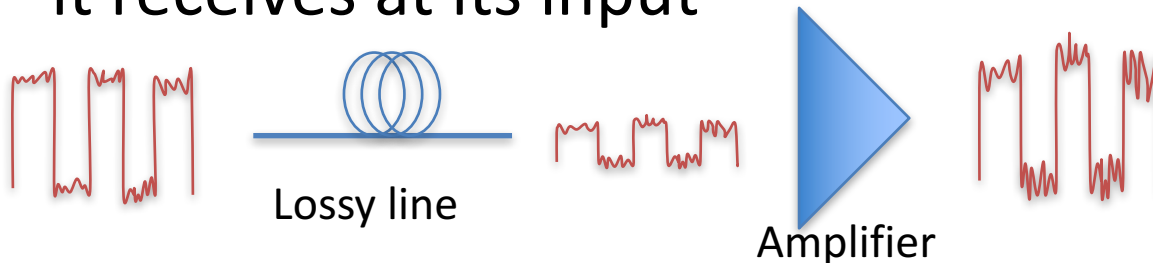
Counteracting impairments

- What do we do then? Can communication not be achieved over distances above 100km?
- One thing we can do is amplify the signal.
- An ideal amplifier would amplify the signal without adding noise.

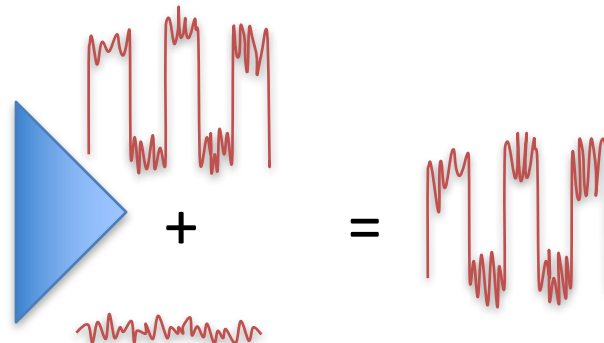


Signal amplification

- A real amplifier:
 1. It amplifies both the signal and the noise power it receives at its input



2. It also introduces its own noise



Signal amplification

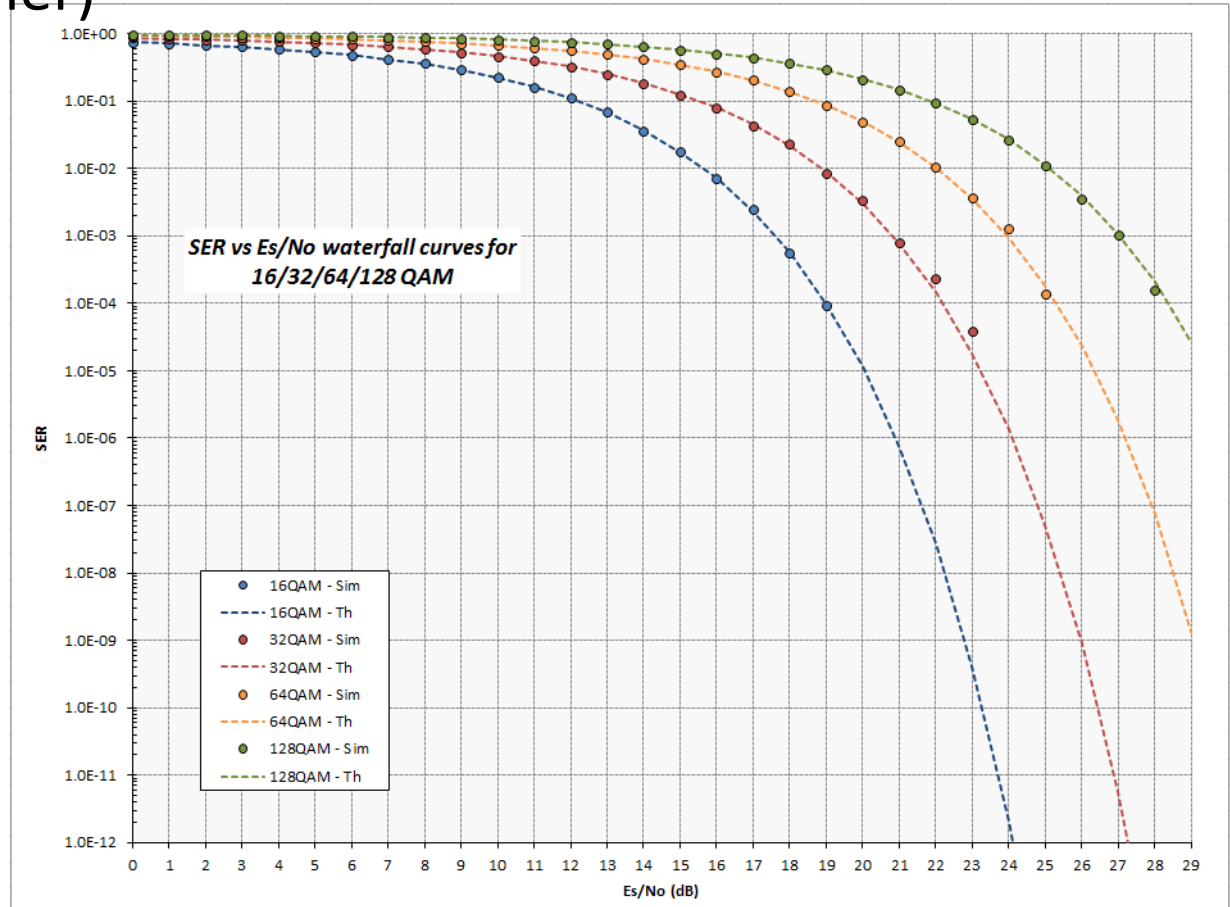
1. The amplifier will amplify the signal by a certain amount called Gain, expressed in dB
 2. The SNR can never be improved!
 - An **ideal amplifier** amplifies both signal and noise, keeping the same SNR.
 3. The SNR actually deteriorates!
 - A **real amplifier** further deteriorates the SNR, by adding more noise
 - The additional noise is expressed as noise figure (NF) of the amplifier, so that:
$$\text{OSNR}_{\text{out}} = \text{OSNR}_{\text{in}} - \text{NF}$$
- This means that we cannot just keep amplifying the signal, because at each amplification the noise will increase!

Systems with one amplifier

- When we put an amplifier into the system we will introduce noise (Amplified Spontaneous Emission – ASE).
 - Now besides making sure that there is enough power at the receiver we also need to check the Optical Signal to Noise Ratio (OSNR)
1. $P_{\text{launch}} - \text{Loss} + \text{Gain} - M \geq R_{x_{\text{sens.}}}$ (Rx Sensitivity constraint)
 2. $\frac{\text{OSNR}_{\text{recv}}}{\text{OSNR}_{\text{threshold}}} = \frac{P_{\text{launch}}[\text{dBm}] - \alpha_{[\text{dB/km}] \times L_{\text{span}}[\text{km}] - \text{NF}_{\text{ampl}}[\text{dB}] + 58_{[\text{dB}]}}{M} \geq 1$ (OSNR constraint)
- The OSNR constraints can be determined by the BER curves

BER/OSNR curves

- These curves are used to understand how the Bit Error Rate (BER) on the Signal to noise ratio (SNR)
- Higher order modulations are more affected by noise (as explained earlier)



Example II, with amplifier

We try and see if we can now solve the problem, by adding an amplifier with Gain= 20 dB, and NF = 6 dB.

New equations with amplifier included

1. $\text{OSNR} = P_{\text{tx}} - \text{Loss}_{\text{span}} - \text{NF}_{\text{ampl}} + 58 - M \geq \text{OSNR}_{\text{threshold}}$ (OSNR Constraint)
2. $P_{\text{tx}} - \text{Loss} + \text{Gain} - M \geq \text{Rx}_{\text{sens}}$, (Receiver sensitivity constraint)

1. Assuming a 16-QAM and 10^{-5} BER, the minimum OSNR is 20 dB and $P_{\text{recv}} = -25$ dBm

2. The amplifier is located half way: $\text{Loss}_{\text{span}} = 0.25 \text{ dB/km} \times 60 \text{ km} = 15 \text{ dB}$,

3. What are the OSNR and signal power after the line and receiver?

The amplifier now boosts the signal by 20 dB, and reduces SNR by 6 dB

$$\rightarrow \text{OSNR} = 0 - 15 - 6 + 58 - 3 = 34 \text{ dB} > 20 \text{ dB target SNR}$$

$$\rightarrow P = 0 - 30 + 20 - 3 = -13 \text{ dBm} > -25 \text{ dBm receiver sensitivity}$$

4. Answer: Yes! The communication can be achieved!

\rightarrow The amplifier has boosted the signal power. Although the SNR is worse than before it is still > 20 dB target SNR

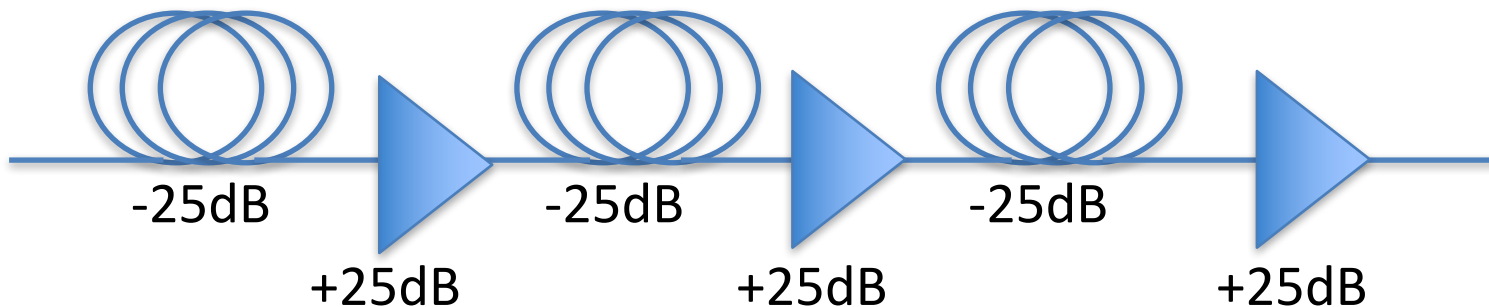
Example III

Same problem as Example II, but now the distance is 1000km.

1. Minimum OSNR and signal power are the same as before:
OSNR=20dB, $P_{\text{recv}} = -25 \text{ dBm}$
2. Loss = $0.25\text{dB/km} \times 1000\text{km} = 250\text{dB}$,
3. If we only put one amplifier the receive power would be $P_{\text{tx}} - \text{Loss} + \text{Gain} = 0 - 250 + 20 - 3 = -233 \ll R_{\text{x}_{\text{sens}}}$
4. The solution is to put more amplifiers: a chain of amplifiers spaced by a certain distance.

Chain of amplifiers

- For long distances, multiple optical amplifiers are used in cascade
- Each amplifier needs to provide enough gain to overcome the loss of its previous span



- High gain in the amplifiers increase the ASE noise. Distance between amplifiers can be reduced to decrease the amount of ASE in the system
- Since each amplifier adds noise, the OSNR deteriorates.
- In long-haul systems, before the OSNR becomes too low, the signal needs to be regenerated.

Power budget calculation for amplified systems

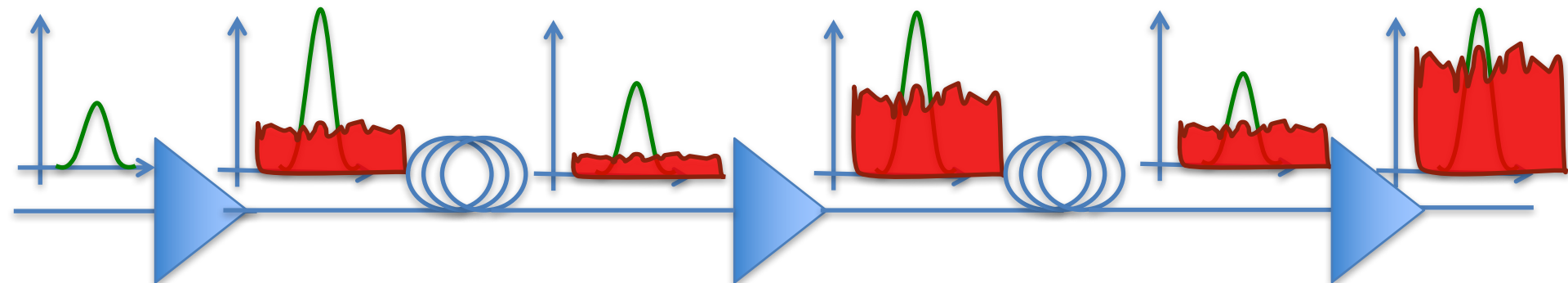
- This is the power budget for an amplified system
- The objective is still to make sure that the received power is \geq **receiver sensitivity**
- I sum up amplification gains and subtract losses on a link span and the system margin (M)

$$P_{\text{recv}} = P_{\text{launch}} - L_1 + G_1 - L_2 + G_2 \dots - L_n + G_n + M > \text{receiver_sensitivity}$$

- L_i are the losses of each span between two amplifiers. They will include fibre loss, and loss due to additional equipment (e.g., multiplexers, connectors, dispersion compensating fibre, optical switches...)
- Usually spans are all the same length, the amplifier gains G_i tend to be all equal and they should compensate for the loss L_i preceding the amplifier (again, this will include fibre loss, connectors loss, DCF fibre loss, and other components that might be in the span)

OSNR for a chain of amplifiers

- The noise figure (NF) is a measure of the noise introduced in the system by an amplifier
- It's measured in dB and depends on the amplifier type, its gain,...
- An average working value for EDFAs can be $NF=5-6\text{dB}$
- **Typically amplifiers are spaced 80Km apart, but this can change**
- **Reducing their spacing increases the system OSNR**
- In a chain of amplifiers, each additional amplifier amplifies both incoming signal and incoming noise, plus it will add its own noise
- After each amplifier the OSNR decreases and **this is permanent, it cannot be recovered!**



OSNR calculation

- For a **chain** of n amplifiers the OSNR is:

$$\text{OSNR}_{\text{recv}} = P_{\text{launch[dBm]}} - \alpha_{\text{[dB/km]}} L_{\text{span[km]}} - NF_{\text{ampl[dB]}} - 10\log_{10}(n) + 58_{\text{[dB]}} - M$$

- You can see from this that reducing the amplifiers spacing increases the OSNR
- If I express the NF_{chain} as $NF_{\text{chain}} = 10\log_{10}(n) + NF_{\text{ampl}} + \alpha_{\text{[dB/Km]}} L_{\text{span[Km]}}$
- The above expression becomes:

$$\text{OSNR}_{\text{recv}} = P_{\text{launch[dBm]}} - NF_{\text{chain[dB]}} + 58_{\text{[dB]}} - M$$

Example III continued

1. Assuming 80 km spacing, we need $\lceil 500/80 \rceil = \lceil 6.25 \rceil = 7$ amplifiers.
2. The NF of the chain is: $10 \log_{10}(7) + 5 + 0.25 * 80 = 8.5 + 5 + 20 = 33.5$.
3. The OSNR of the amplifier chain is: $0 - 33.5 + 58 - 3 = 22$. This is still above the OSNR threshold of 20 dB, so it's OK. In fact I can work with a NF_{chain} of up to 35.5
4. For the received power we have: $0 - 250 + 20 * 13 = 10$, which is OK. *In practice the amplifiers will be used at lower gain so that the received power is not too high, say about -10 dBm.*

Example IV

Now let's increase the distance to 2000 km:

1. Assuming an 80 km spacing, we need $\lceil 2000/80 \rceil = \lceil 25 \rceil = 25$ amplifiers.
2. The NF of the chain is: $10 \log_{10}(25) + 5 + 0.25 * 80 = 14 + 5 + 20 = 39$.
3. The OSNR of the amplifier chain is: $0 - 39 + 58 - 3 = 16$. This is below the OSNR threshold of 20 dB, so it won't work.

Tentative solution:

1. We can try with a lower span distance: 50 km
2. We have $2000/50=40$ amplifiers but the span distance is shorter
3. The NF of the chain is $10 \log_{10}(40) + 5 + 0.25 * 50 = 16 + 5 + 12.5 = 33.5$, which is good
4. The power received will be $0 - 0.25 * 2000 + 40 * 20 - 3 = -500 + 800 - 3 = \underline{297} * >> -25$

*Notice that amplifiers, beside a gain value also have a maximum output, typically around 20-25 dB. So in practice anywhere in the chain the power never goes above this value.

Example V

What if we increase it to 5000 km?

1. Assuming a 50 km spacing, we need $\lceil 5000/50 \rceil = \lceil 100 \rceil = 100$ amplifiers.
2. The NF of the chain is: $10 \log_{10}(100) + 5 + 0.25 * 50 = 20 + 5 + 12.5 = 37.5$.
3. The OSNR of the amplifier chain is: $0 - 37.5 + 58 - 3 = 17.5$. This is below the OSNR threshold of 20 dB, so it won't work.

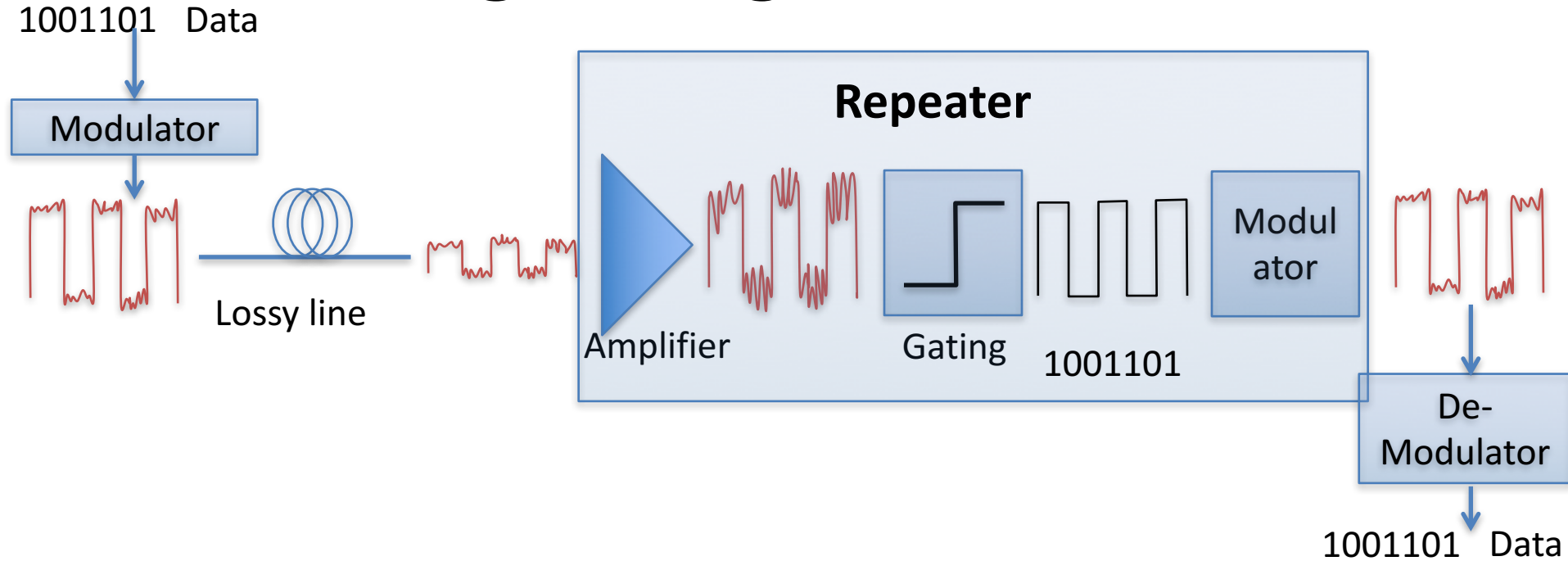
Solutions?

1. I can try a lower span of 40 km... $21 + 5 + 10 = 36$ but not enough
2. I can try a 20 km span... $22 + 5 + 7.5 = 34.5 \rightarrow$ it would work but I need 67 more amplifiers (this is a lot!)
3. Or else...?

Signal regeneration

- Digital signal can be regenerated!
- Contrarily to an analogue signal, a digital signal can only assume 1 or 0 values, so I can:
 - Intercept the signal
 - Demodulate it and retrieve the bit values*
 - Transmit the bits again

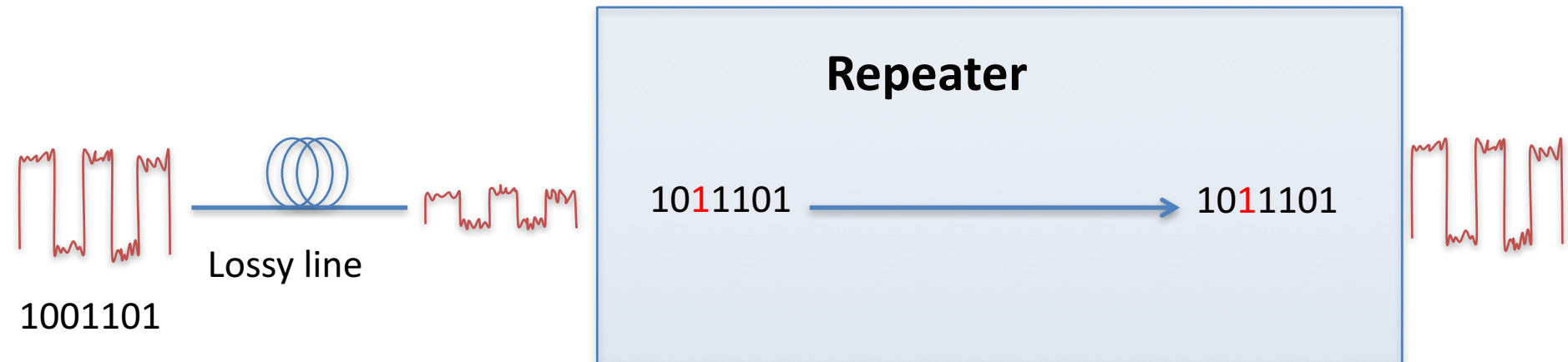
Signal regeneration



- The repeater (or regenerator) recreates the signal completely, so that at the output the SNR is back to the original value.

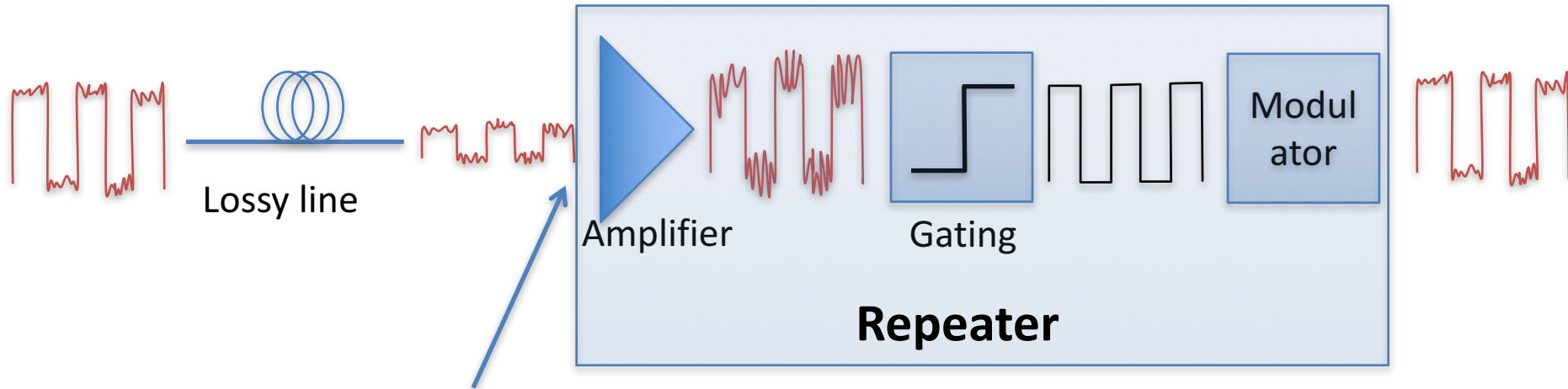
Signal regeneration

- The truth is that when we regenerate a signal we actually put a receiver in there. It's as if we terminated the communication link.
- So, as we terminate the communication link, we'll have a certain SNR, which will give me a BER value.
- ➔ When I terminate the signal to regenerate it I will collect a number of errors. These errors will be carried over till the destination
- ➔ **These errors represent loss of information!**



- The number of errors introduced will depend on the SNR at the receiver.

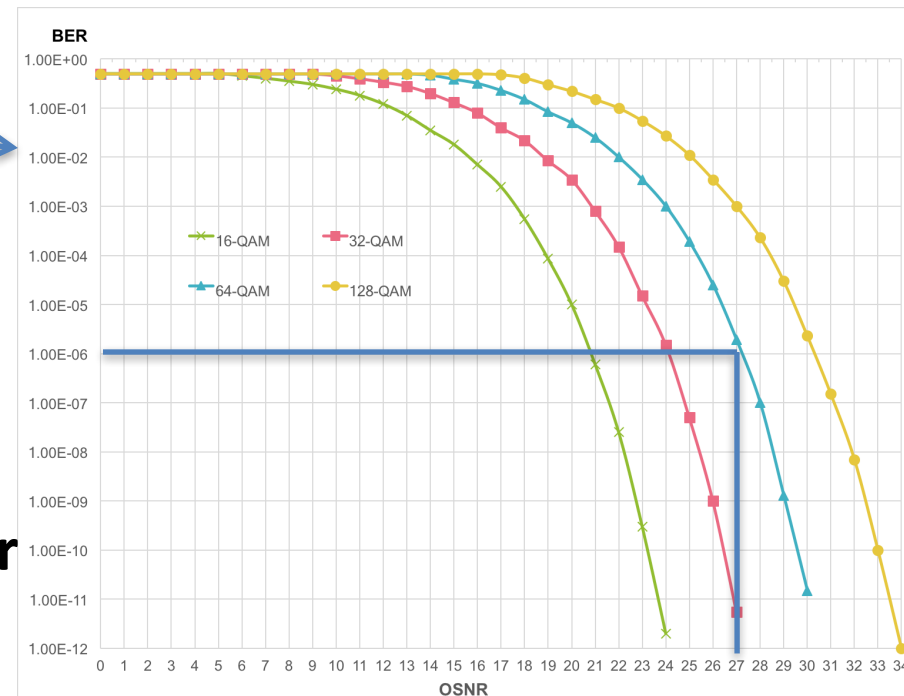
Signal regeneration



I calculate my OSNR here and from that derive my BER

If I'm using a 64-QAM, and the SNR is 27 dB, my BER is $\approx 10^{-5}$

These bit errors cannot be recovered, and I need to account for them at the end of the link



Example V- revisited

The distance was 5000 km, but with a regenerator in between it becomes two links of 2500km.

Using a 50 km span:

$$\lceil 2500/50 \rceil = 50 \rightarrow 17+5+0.25*50=34.5$$

So this works fine, and I use a total of 100 amplifiers (instead of 167).

Analogue vs digital transmission

- We can finally see now why digital transmission is better than analogue transmission:
 - Both can be amplified
 - **Only digital signals however can be regenerated!!!**
- If I regenerate the digital signal early, when the OSNR is still high, then the BER will be very small, and the number of errors will be negligible!

Capacity of a real communication channel

Capacity of a real communication channel

- In the lecture on digital modulations we said that theoretically, in a noiseless channel, if we have a bandwidth B , we can transmit:
 - $2 \times B$ bit/s in single-level baseband
 - B bit/s in single-level modulation (e.g., 2-ASK, 2-PSK)
 - $2 \times B \times \log_2 L$ bit/s in baseband with a multi-level coding of L levels
 - $B \times \log_2 L / (1+d)$ bit/s with a modulation of L levels
- From these formulas in principle by increasing L , I can increase the bit rate indefinitely
- However we now know that real channels introduce noise.
- Does this change the maximum capacity of a channel?

Shannon capacity in a noisy channel

- In 1944, Claude Shannon introduced a formula, (known now as Shannon-Hartley theorem), to determine the theoretical highest data rate for a noisy channel:

$$\text{Capacity} = \text{bandwidth} \times \log_2(1 + \text{SNR})$$

Where capacity is measured in bit/s, the bandwidth in Hz, and the SNR is a linear value (i.e., not in dB)

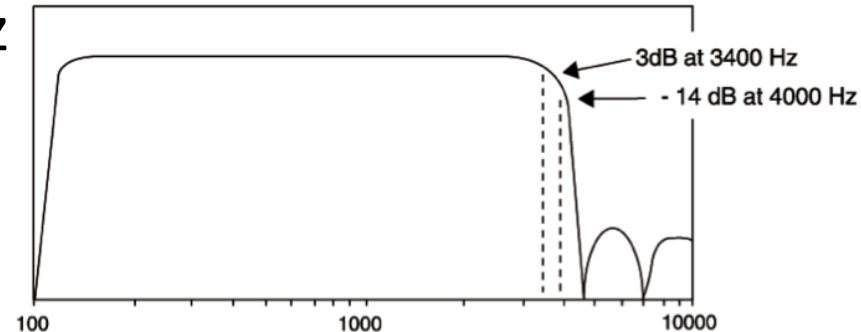
Example I

- A transmission channel has a bandwidth of 2 MHz. The signal-to-noise ratio is 36 **dB**.
- What's the maximum capacity?
- $C = B \times \log_2(1 + \text{SNR})$ (remember that SNR is linear here)
 - $\text{SNR} = 36 \text{ dB} \rightarrow \text{SNR}_{\text{lin}} = 10^{\text{SNR}_{\text{dB}}/10} = 10^{36/10} = 3981$
 - $C = 2,000,000 \times \log_2(3982) = 2,000,000 \times 11.96 = 23,92 \text{ Mb/s}$

Example II

- What is the maximum theoretical capacity of a telephone line?

- The nominal bandwidth is 4 KHz, but the usable bandwidth is about 3.4 KHz
- The SNR is about 30 dB



- $C = B \times \log_2(1 + \text{SNR})$

- $\text{SNR} = 30 \text{ dB} \rightarrow \text{SNR}_{\text{lin}} = 10^{\text{SNR}_{\text{dB}}/10} = 10^{30/10} = 1000$

- $\rightarrow C = 3400 \times \log_2(1000) = 3400 \times 9.966 = 33.9 \text{ Kb/s}$

- In the 90s, it was thought that the maximum rate achievable over a modem was 33.6 Kb/s

From Shannon-Hartley theorem it is possible to transmit information when the noise is higher than the signal ($\text{SNR} < 1$)... how is it possible?!?

$$\text{Capacity} = \text{bandwidth} \times \log_2(1 + \text{SNR})$$

- A. It is possible and such systems do exist
- B. The formula only holds for $\text{SNR} > 1$
- C. It's only possible if noise is not random but predictable
- D. ... again, this theorem it's just a theory...

Example III

- Consider a noisy channel, where the noise is at the same level of the signal. Is it possible to have a communication in this channel?
 - $C = B \times \log_2(1 + \text{SNR})$
 - $\text{SNR}=1$
 - ➔ $C=B$, yes it is possible and the maximum capacity is equal to the bandwidth
-
- Is it possible to have communication then if the noise is stronger than the signal??
 - $C = B \times \log_2(1 + \text{SNR})$
 - $\text{SNR}=0.1$
 - ➔ $C= B \times 0.137$, yes it is possible but I need larger bandwidth!

There are indeed transmission techniques called spread spectrum that use this result (e.g., frequency hopping, direct sequence)