# CS1026 – Digital Logic Design Quine-McCluskey Algorithm Example II

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### Today's Overview

- 1 Introduction
- 2 4 Variables
- 3 Don't cares
- 4 Petrick's method
- 5 Problems?



## Quine-McCluskey Overview I

So a few more examples where we fins:

 A minimum-cost sum-of-products implementation [Majumder et al., 2015]

for a Boolean function.



# Quine-McCluskey Overview II

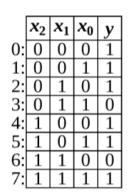
Main steps in the Quine-McCluskey algorithm:

- 1 Generate Prime Implicants
- 2 Construct Prime Implicant Table
- 3 Reduce Prime Implicant Table
  - 1 Remove Essential Prime Implicants
  - 2 Row Dominance
  - 3 Column Dominance
- 4 Solve Prime Implicant Table



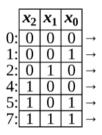
## Simple Example I

$$F(x_0.x_1,x_2) = \sum m(0,1,2,4,5,7)$$



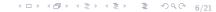
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#### Simple Example II



#### Iteration 0:

- ✓ Prime Implicants
- $\blacksquare$   $\rightarrow$  Non Prime Implicants



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### Simple Example III

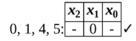
	$x_2$	$x_1$	$x_0$	
0, 1:	0	0	-	$\rightarrow$
0, 2:	0	-	0	1
0, 4:	-	0	0	$\rightarrow$
1, 5:	ı	0	1	$\rightarrow$
4, 5:	1	0	ı	$\rightarrow$
5, 7:	1	-	1	1

#### Iteration 1:

- ✓ Prime Implicants
- lacksquare ightarrow Non Prime Implicants



#### Simple Example IV



#### Iteration 2:

- ✓ Prime Implicants
- lacksquare ightarrow Non Prime Implicants



## Simple Example V

#### Prime Implicant Table:

	<i>x</i> <sub>2</sub>	<i>x</i> <sub>1</sub>	$x_0$	0	1	2	4	5	7
0, 1, 4, 5:	-	0	-	0	•		•	0	
0, 2:	0	-	0	0		•			
5, 7:	1	-	1					0	•

We have..

- x<sub>1</sub>′
- $x_2' x_0'$
- $X_2X_0$

## Simple Example VI

#### Prime Implicant Table:

	<i>x</i> <sub>2</sub>	<i>x</i> <sub>1</sub>	$x_0$	0	1	2	4	5	7
0, 1, 4, 5:	-	0	-	0	•		•	0	
0, 2:	0	-	0	0		•			
5, 7:	1	-	1					0	•

So we get the SOP:

$$x_1' + x_2'x_0' + x_2x_0$$

#### Don't Care Example I

	$x_2$	$x_1$	$x_0$	y
0:	0	0	0	0
1:	0	0	1	×
2:	0	1	0	0
3:	0	1	1	×
4:	1	0	0	1
5:	1	0	1	0
6:	1	1	0	0
7:	1	1	1	0

	<i>x</i> <sub>2</sub>	$x_1$	$x_0$	
1:	0	0	1	$\rightarrow$
3:	0	1	1	$\rightarrow$
4:	1	0	0	1

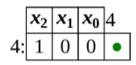
$$x_2 | x_1 | x_0$$
  
1, 3: 0 - 1 (×)

$$F(x_0, x_1, x_2.x_3) = \sum m(4) + \sum d(1,3)$$

x denotes Don't Cares



## Don't Care Example II



Nice and easy:

$$y = x_2 x_1' x_0'$$

#### Solving the Table I

	<i>x</i> <sub>2</sub>	<i>x</i> <sub>1</sub>	$x_0$	y
0:	0	0	0	0
1:	0	0	1	1
2:	0	1	0	×
3:	0	1	1	×
4:	1	0	0	1
5:	1	0	1	1
6:	1	1	0	×
7:	1	1	1	0

	<i>x</i> <sub>2</sub>	$x_1$	$x_0$				<i>x</i> <sub>2</sub>	$x_1$	$x_0$	
1:	0	0	1	$\rightarrow$	1,	3:	0	-	1	<b>/</b>
2:	0	1	0	$\rightarrow$	1,	5:	-	0	1	<b>/</b>
3:	0	1	1	$\rightarrow$	2,	3:	0	1	-	(×)
4:	1	0	0	$\rightarrow$	2,	6:	-	1	0	(×)
5:	1	0	1	$\rightarrow$	4,	5:	1	0	-	1
6:	1	1	0	$\rightarrow$	4,	6:	1	-	0	✓

$$F(x_0, x_1, x_2.x_3) = \sum m(1, 4, 5) + \sum d(2, 3, 6)$$

x denotes Don't Cares

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## Solving the Table II

	$x_2$	$x_1$	$x_0$	1	4	5
1, 3:	0	-	1	0		
1, 5:	-	0	1	0		0
4, 5:	1	0	-		0	0
4, 6:	1	-	0		0	

Whoops.. What to do?



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## Solving the Table III

	$x_2$	<i>x</i> <sub>1</sub>	$x_0$	1	4	5
1, 3:	0	ı	1	0		
1, 5:	-	0	1	0		0
4, 5:	1	0	-		0	0
4, 6:	1	ı	0		0	

#### Use Petrick's method

■ Determine all minimum Sum-Of-Products (SOP) solutions



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#### Solving the Table IV

- Label the rows of the reduced prime implicant chart  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$ , etc.
- 2 Form a logical function *P* which is true when all the columns are covered.
- Reduce P to a minimum sum of products by multiplying out and applying X + XY = X.
- 4 Each term in the result represents a solution, that is, a set of rows which covers all of the minterms in the table. To determine the minimum solutions, first find those terms which contain a minimum number of prime implicants.
- Next, for each of the terms found in step five, count the number of literals



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# Solving the Table V

6 Choose the term or terms composed of the minimum total number of literals, and write out the corresponding sums of prime implicants.

#### Remember

- P consists of a product of sums where each sum term has the form  $(P_{i0} + P_{i1} + \cdots + P_{iN})$ , where each  $P_{ij}$  represents a row covering column i.
- You saw this process with SAR



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# Solving the Table VI

	$x_2$	$x_1$	$x_0$	1	4	5
1, 3:	0	-	1	0		
1, 5:	-	0	1	0		0
4, 5:	1	0	-		0	0
4, 6:	1	ı	0		0	

#### We have the implicants:

$$x_2'x_0 \equiv p_0$$

$$x_1'x_0 \equiv p_1$$

$$x_2x_1' \equiv p_2$$

$$x_2x_0' \equiv p_3$$

## Solving the Table VII

$$(p_0 + p_1)(p_2 + p_3)(p_1 + p_2) \equiv (p_0p_2 + p_0p_3 + p_1p_2)(p_1p_2)$$
  

$$\equiv (p_0p_2 + p_1p_2 + p_1p_3)(p_1p_2)$$
  

$$\equiv (p_0p_1p_2 + p_0p_2 + p_0p_2p_3 + p_1p_2 + p_1p_3 + p_1p_2p_3)$$
  

$$\equiv (p_0p_2 + p_1p_2 + p_1p_3)$$

#### Minimal boolean Expression

$$y = (x_2'x_0) + (x_2x_1')$$

Painful.. a computer does not mind (if you have a lot of time)!



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#### Any now relax

Next time.. Flip Flops and Latches

■ No more simiplication! zzz

#### Any Problems?

- Ask!
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# References (Homework) I



Majumder, A., Chowdhury, B., Mondai, A. J., and Jain, K. (2015).

Investigation on quine mccluskey method: A decimal manipulation based novel approach for the minimization of boolean function.

In Electronic Design, Computer Networks & Automated Verification (EDCAV), 2015 International Conference on, pages 18–22. IEEE.

