CS1026 – Digital Logic Design

A slow Introduction to the Quine-McCluskey Algorithm I

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January 22, 2017

Today's Overview

Quine-McCluskey [Majumder et al., 2015] I

- An exact algorithm which finds a minimum-cost sum-of-products implementation of a Boolean function
- This lecture introduces the method

Quine-McCluskey [Majumder et al., 2015] II

- Why do we care???
- Systematic approach for "optimisation" of boolean function.
- Saves us from the hell of N variable Karnaugh maps (for N>5)
- Can be programmed and implemented in a computer (Automation is life)

Quine-McCluskey [Majumder et al., 2015] III

Quine defined a PRIME IMPLICANT of F to be an implicant that is minimal - that is, the removal of any literal from P results in a non-implicant for F. Essential prime implicants are prime implicants that cover an output of the function that no combination of other prime implicants is able to cover.

Quine-McCluskey [Majumder et al., 2015] IV

PRIME IMPLICANT is a product term that cannot be combined with another term to eliminate a variable
i.e. X.Y'.Z could be a prime implicant

Quine-McCluskey [Majumder et al., 2015] V

ESSENTIAL PRIME IMPLICATS are prime implicants that cover an output of the function that no combination of other prime implicants is able to cover

Quine-McCluskey [Majumder et al., 2015] VI

Steps of the Quine-McCluskey algorithm:

1. Generate Prime Implicants
2. Construct Prime Implicant Table
3. Reduce Prime Implicant Table
    1. Remove Essential Prime Implicants
    2. Row Dominance
    3. Column Dominance
4. Solve Prime Implicant Table

Quine-McCluskey [Majumder et al., 2015] VII

In Step 1, the prime implicants of a function are generated using an iterative procedure

- You can use a Karnaugh-Map

Reminder

- Any single 1 or group of 1's that can be combined together on a Karnaugh map of the function F represents an *Implicant*
- *Prime Implicant* denotes a product term that we cannot combine with another term to eliminate a variable

Quine-McCluskey [Majumder et al., 2015] VIII

In Step 2, a prime implicant table is constructed

- The columns of the table are the prime implicants
- The rows are min-terms of where the function is 1

Our Goal

To cover all the rows using a minimum-cost cover

- Of all prime implicants

Quine-McCluskey [Majumder et al., 2015] IX

This goals needs more explanation..

- *Minimum-cost* means to have fewest prime implicants (i.e. AND gates) in the final solution.

However, the algorithm has been extended to consider more complex cost functions:

- E.g. Minimising the total number of gate inputs, power optimisation, and so on.

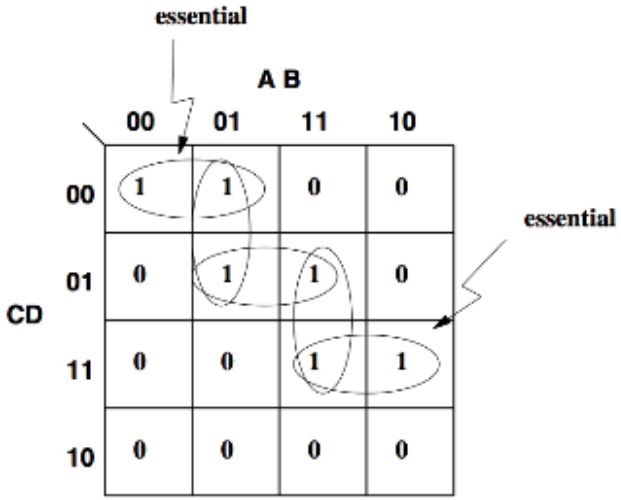
Column Dominance I

A column P_1 whose 1-entries are a superset of another column P2 is said to dominate P_2.

P_1	P_2
1	0
0	0
1	1
0	0
1	1

P_1 dominates P_2.

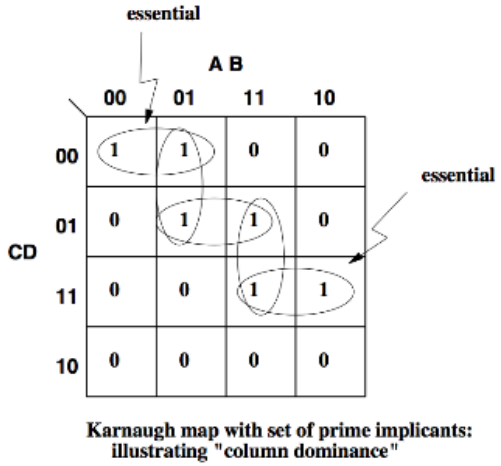
Column Dominance II



Karnaugh map with set of prime implicants: illustrating "column dominance"

$$f(A, B, C, D) = \sum m(0, 4, 5, 11, 13, 15)$$

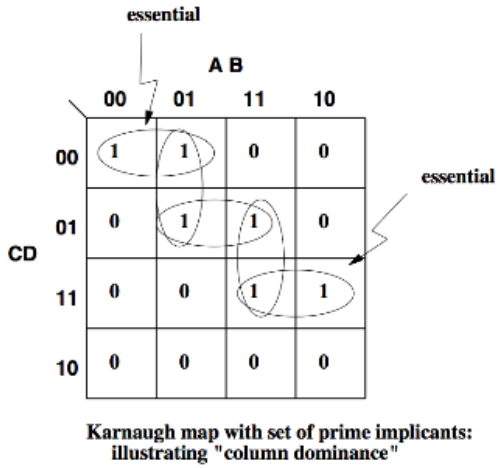
Column Dominance III



Karnaugh map with set of prime implicants: illustrating "column dominance"

- There are 5 prime implicants
  - Each covers 2 ON-set min-terms
- First, we note that two implicants are essential prime implicants: $A'C'D'$ and ACD

Column Dominance IV



Karnaugh map with set of prime implicants: illustrating "column dominance"

- These implicants must be added to the final cover
- There are 3 remaining prime implicants
- We must pick a minimum subset of these to cover the uncovered ON-set min-terms

Column Dominance V

	A'C'D' (0,4)	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)
0	X				
4	X	X			
5		X	X		
11					X
13			X	X	
15				X	X

- The prime implicant table for the Karnaugh map
- The 5 prime implicants are listed as columns
- The 6 ON-set min-terms are listed as rows

Column Dominance VI

	A'C'D' (0,4)*	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)*
0	X				
4	X	X			
5		X	X		
11					X
13			X	X	
15				X	X

- We cross out columns A'C'D' and ACD and mark them with *

(These indicate essential implicants)

Column Dominance VII

	A'C'D' (0,4)*	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)*
+	X				
+	X	X			
5		X	X		
+					X
13			X	X	
+				X	X

- Cross out each row intersected by one of these columns
  - Because that min-term is now covered
- At this point, prime implicant BC'D covers 2 remaining ON-set min-terms (5 and 13)

Column Dominance VIII

	A'C'D' (0,4)*	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)*
+	X				
+	X	X			
5		X	X		
+					X
13			X	X	
+				X	X

- However, prime implicant A'BC' covers only one of these (namely, 5)
  - As does ABD (namely, 13).

Column Dominance IX

	A'C'D' (0,4)*	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)*
+	X				
+	X	X			
5		X	X		
+					X
13			X	X	
+				X	X

- We can use BC'D instead of either A'BC' or ABD
  - Since it covers the same min-terms
  - That is, BC'D column-dominates A'BC, and BC'D column-dominates ABD

Column Dominance X

	A'C'D' (0,4)*	A'BC' (4,5)	BC'D (5,13)	ABD (13,15)	ACD (11,15)*
0	X				
4	X	X			
5		X	X		
11					X
13			X	X	
15				X	X

The dominated prime implicants can be crossed out, and only column BC'D remains

Bored Yet?

We will look at *Row Dominance* next time (zzzzz...)

Any Problems?

- Ask!
- E-Mail: *sheehas1@scss.tcd.ie*

References (Homework) I

Majumder, A., Chowdhury, B., Mondai, A. J., and Jain, K. (2015).
Investigation on quine mccluskey method: A decimal manipulation based novel approach for the minimization of boolean function.
In *Electronic Design, Computer Networks & Automated Verification (EDCAV), 2015 International Conference on*, pages 18–22. IEEE.