# Section VII

# Electromagnetism

## *(i) Electric and Magnetic Fields, The Lorentz Force Law*

***The Electric Field***
Where there is an electrical potential difference between two points, an electric field is said to exist between them.

***Electric Field Strength (Intensity)*** *at a point is the force per unit positive charge at that point.*

*Mathematically:*

$$\vec{E} = \frac{\vec{F}}{Q}$$

Electric Field Strength is a vector quantity. Its unit is the N/C or V/m.

***Electric Flux***

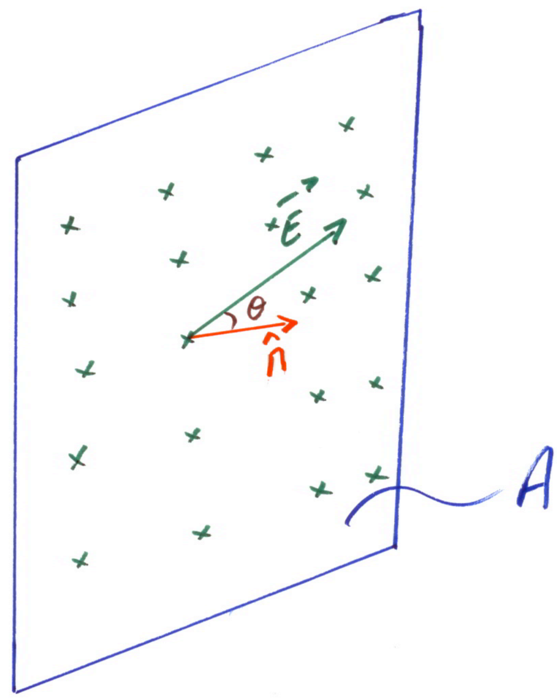
Where an electric field exists the electric flux through an area is defined as:

$$\psi_e = \hat{n} \cdot \vec{D} A$$

where:

- A is the area
- $\hat{n}$ is the unit vector normal (perpendicular) to the area in question.
- $\vec{D} = \varepsilon \vec{E}$, $\vec{D}$ is known as the Electric Flux Density vector.

Diagram:



***Electric Current***

When charge (or charged particles) move (flow) from one point to another, we have electrical current.

***Electric Current*** *is the rate of charge flow with respect to time.*

*Mathematically:*

$$\boxed{i(t) = \frac{dq(t)}{dt}}$$

Electric Current is a scalar quantity. Its unit is the Ampere.

***One Ampere*** *is one coulomb per second.*

***Ampere's Discovery***

Ampere discovered that when currents flow, they exert a force of attraction/repulsion on each other. This force can not be explained by Coulomb's Law. Currents attract if both are moving in the same direction and repel if moving in opposite directions. This phenomenon is used to define the unit of electrical current, the Ampere or 'Amp'.

***The Ampere*** *is that constant current which is maintained in two straight parallel conductors, placed 1m apart in a vacuum and of infinite length, causes each to exert a force of* $2x10^{-7}$*N.*

***The Coulomb*** *is that quantity of charge transferred when a current of 1 Ampere flows for 1 second.*

## *The Magnetic Field*

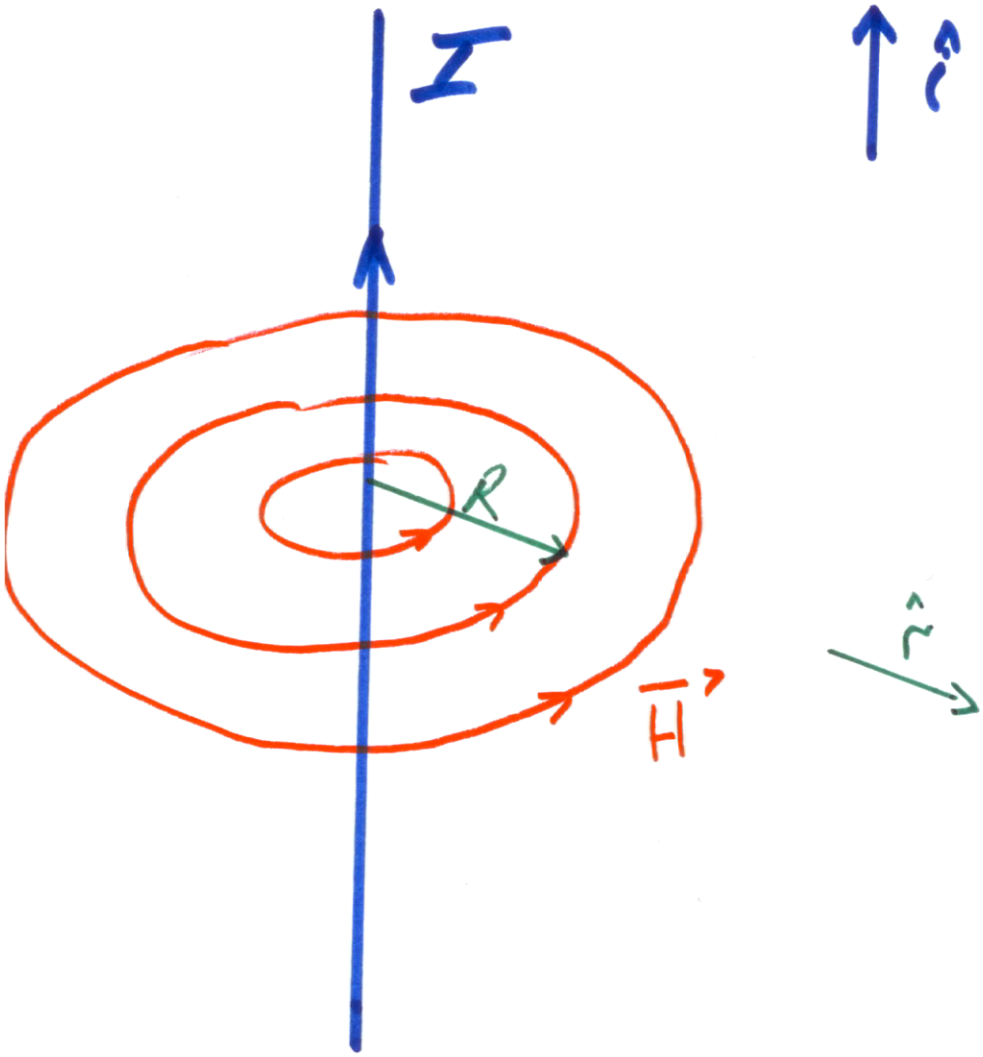
Ampere's discovery means that when charges move (i.e. when you have electric current) they exert forces on one another. Another way of looking at this phenomenon is to say that a moving charge (or electric current) has associated with it a field such that another charge moving in that field will experience a force. This field is referred to as the <u>magnetic field</u>.

From Ampere's discovery, we may therefore state that electrical current effects a magnetic field.

## *The 'Direction' of the Magnetic Field*

The direction of the magnetic field resulting from a moving charge is given by the 'Right Hand Rule'. The magnetic field will be in the sense of rotation of the fingers where the thumb is pointing in the direction of the current.

Diagram:



***Magnetic Pole (or Magnetic 'Charge')***

*If a pole of 1Wb (Weber) is moved around a current of 1A (Ampere) the work done is 1J (Joule).*

Note: there is no physical realization of a magnetic pole or charge. It is simply a concept to enable us to more easily understand magnetic field strength by making an analogy with the concept of electric field strength.

e.g. If a force of 1Wb is placed in a magnetic field of strength $1NWb^{-1}$ the force on the pole will be 1N.

***Magnetic Field Strength*** $\left(\vec{H}\right)$

*The magnetic field strength at a point is the force per unit positive magnetic pole (i.e. North (N)) at that point.*

Magnetic Field Strength is a vector quantity and its unit is the Ampere (A).

***Magnetic Flux Density*** $\left(\vec{B}\right)$

*The magnetic flux density at a point is the force per unit positive electric charge where the charge is moving with a velocity of 1m/s normal (perpendicular) to the direction of the magnetic field.*

Magnetic Flux density is a vector quantity and its unit is the Weber per metre squared $\left(Wbm^{-2}\right)$ or the Tesla (T).

***Magnetic Flux***

Where a magnetic field exists the magnetic flux through an area is defined as:

$$\psi_m = \hat{n} \cdot \vec{B}A$$

where:

- A is the area
- $\hat{n}$ is the unit vector normal (perpendicular) to the area in question.
- $\vec{B} = \mu\vec{H}$, where $\mu$ is a constant for the medium known as the magnetic permeability.

***The Lorentz Force Law***

When a charge ‘q’ moves with a velocity ‘$\vec{v}$’ through a magnetic field of magnetic flux density $\vec{B}$ then the force it experiences due to that field is:

$$\vec{F} = q\vec{v} \times \vec{B}$$

***Extension:***

If the charge is moving in a region where the is also an electric field then it follows from Coulomb’s Law that the total force it experiences is:

$$\vec{F}_{Total} = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B})$$

***Aside:***

Scalar/Dot Product:

$\vec{A}\,\vec{B} = |\vec{A}|\;|\vec{B}| \cos\theta$

Vector/Cross Product:

$\vec{A} \times \vec{B} = |\vec{A}|\;|\vec{B}| \sin\theta$

*where θ is the* <u>*acute*</u> *angle between* $\vec{A}$ *and* $\vec{B}$ *and normal to both as given by the Right Hand Rule*

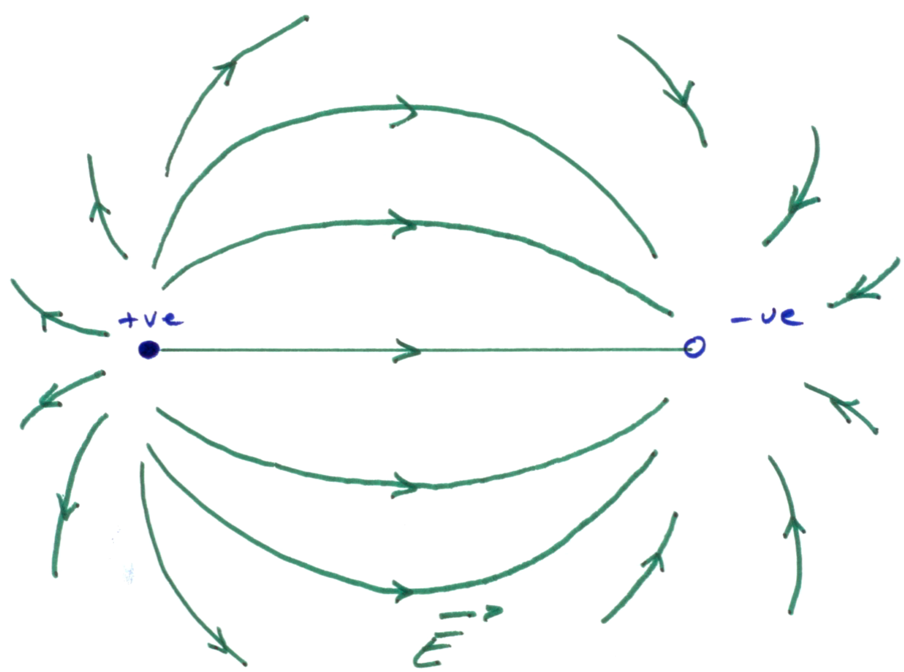
***The Right-Hand Rule:***

Close fist of right hand and extend thumb. Where your fingers are rotating in the order of the product A, B then your thumb gives the direction of the unit vector in the direction of the cross product.

***Important Notes on Electric and Magnetic Fields:***

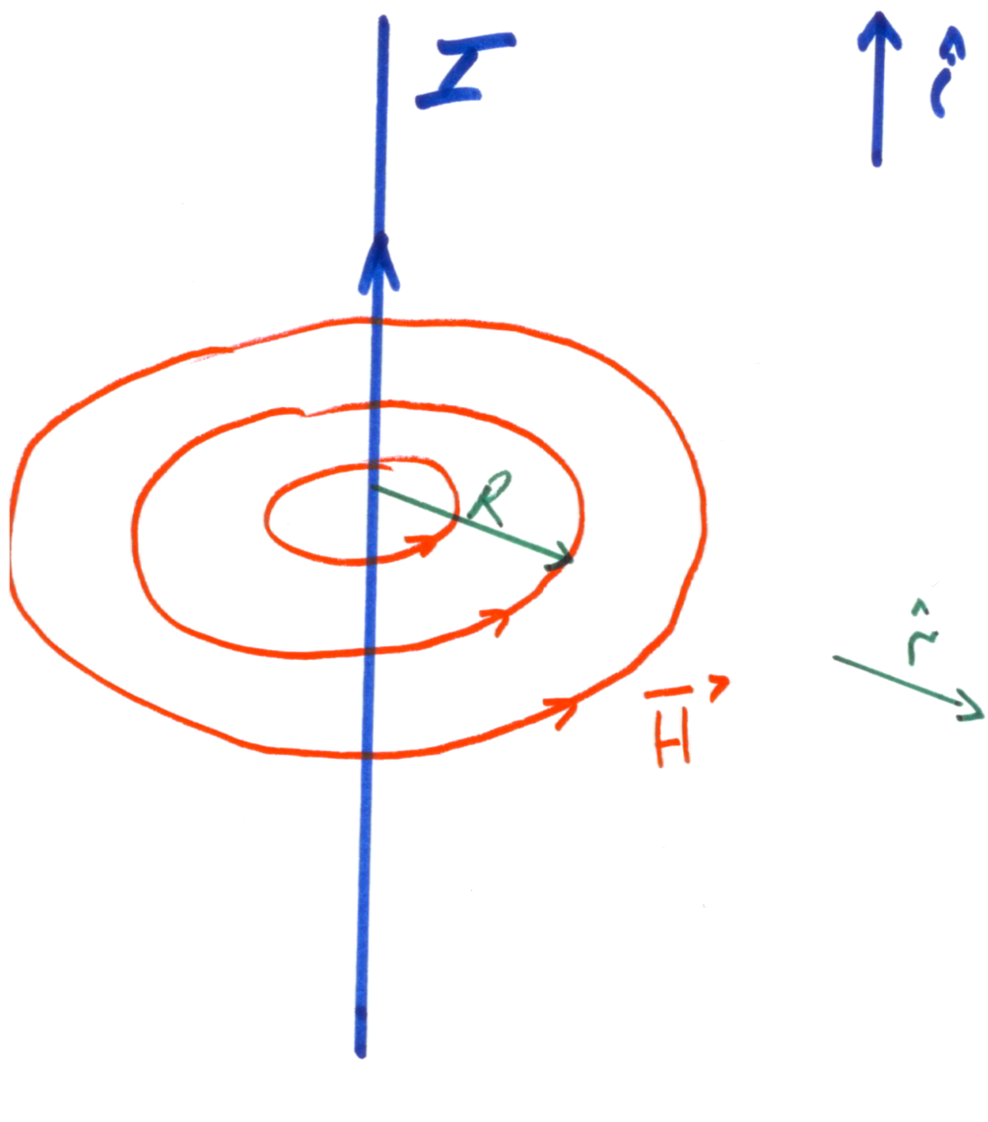
In the case of the electric field, the force exerted on a charge is <u>in the direction of the field</u>. An electric field therefore 'begins' at a positive charge and 'ends' at a negative charge.

Diagram:



In the case of the magnetic field, the force is exerted on a moving charge only and this force is <u>perpendicular</u> to the direction of the magnetic field – as given by the Lorentz Force Law. The magnetic field forms closed loops about the current.

Diagram:



## *The Vectors Used to Describe Electric Fields*

(i) $\vec{E}$ - The Electric Field Strength $\left(Vm^{-1}\right)$.

(ii) $\vec{D}$= The Electric Displacement Vector $\left(Cm^{-2}\right)$

The relationship between the two is:

$$\vec{D} = \varepsilon \vec{E}$$

Where ε, the electrical permittivity, is a constant for the medium in question. $\varepsilon_0$, the electrical permittivity of free space (or a vacuum) = 8.854 x $10^{-12}$ F/m (Farads per meter). Air is normally taken to have this permittivity. The relative permittivity of a medium is defined as $\varepsilon_r = \varepsilon / \varepsilon_0$.

***The Vectors Used to Describe Magnetic Fields***

(i) $\vec{H}$ - The Magnetic Field Strength $\left( A \right)$

(ii) $\vec{B}$ - The Magnetic Flux Density $\left( Wbm^{-2} \right)$

The relationship between the two is:

$$\boxed{\vec{B} = \mu \vec{H}}$$

Where $\mu$, the magnetic permeability, is a constant for the medium in question. $\mu_{0,}$ the magnetic permeability of free space, is $4\pi \times 10^{-7}$ H $m^{-1}$. The relative permeability of a medium is defined as $\mu_r = \mu / \mu_{0.}$

***The Significance of ε and μ:***

The electric and magnetic field strengths will vary depending on the medium. By contrast, the electric and magnetic flux does not!

When the electromagnetic field passes from Medium 1 to Medium 2, the following statements are true:

$\vec{B}_1 = \vec{B}_2$ $\qquad$ $\vec{D}_1 = \vec{D}_2$

$\vec{H}_1 \neq \vec{H}_2$ $\qquad$ $\vec{E}_1 \neq \vec{E}_2$

It is clear therefore, that ε and μ characterize the ability of a medium to 'sustain' an electric and magnetic field.

Electric and Magnetic fields are functions of location and time i.e. $\vec{E}(s,t)$, $\vec{H}(s,t)$ etc. where 's' indicates location.

## *(ii) The Laws of Faraday, Lenz and Ampere*

***Faraday's Law***

*When a changing magnetic flux threads ('cuts') a conductor, an emf is induced according to the equation:*

$$\boxed{E = -\frac{d\psi_m}{dt}}$$

where:

- E is the emf (electromotive force)
- $\psi_m$ is the magnetic flux

***Note:*** This effect also occurs when a conductor is moving in a magnetic field.

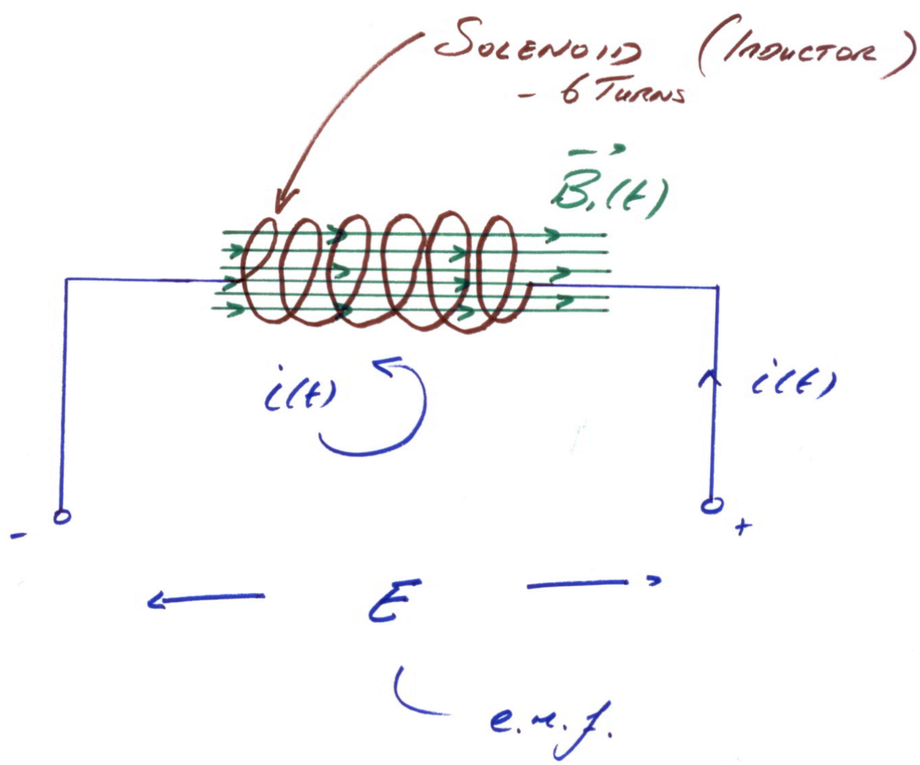
We may expect Faraday's Law as a result of the Lorentz Force Law.

## *Lenz's Law*

If the conductor is a solenoid (i.e. spring shaped) with 'n' loops or turns (i.e. n=6 when there are 6 loops in the solenoid), then the equation becomes:

$$E = -n\frac{d\psi_m}{dt}$$

Diagram:



The induced voltage ‘E’ will have a polarity such that it will oppose the charge in the flux.
i.e. the induced emf will cause an (induced) current to flow in the conductor (or solenoid). This current in turn will have its own magnetic field (or flux) which will oppose the original charge in flux of B(t)

***Ampere's Law***

- for an infinite straight wire:

$$\vec{B} = \frac{\mu I}{2\pi R}\hat{i} \times \hat{r}$$

- relates the magnetic field to the current for an infinite straight wire. Hence the magnetic field is said to decrease 'like 1/R' where R is the radial distance from the wire.

where:

- I is the current
- μ is the permittivity of the medium
- $\hat{i}$ is the unit vector in the direction of the current
- $\hat{r}$ is the unit vector in the direction of the radial distance

***Examples (The Lorentz Force Law)***

Calculate the force on a straight 0.5m long conductor carrying 1A and placed perpendicular to a magnetic field of 2T.

$$\vec{F} = Q\vec{V} \times \vec{B}$$

$$I = \frac{dQ}{dt} \Rightarrow$$

$$Q = \int_0^t I dt \Rightarrow$$

$$\vec{F} = \int_0^t I dt \frac{d\vec{s}}{dt} \times \vec{B} \Rightarrow$$

$$\vec{F} = \int_0^l I d\vec{s} \times \vec{B} \Rightarrow$$

$$\vec{F} = BIl\hat{s} \times \hat{b}$$

$$=> \vec{F} = 2.1.\frac{1}{2} = 1N\hat{s} \times \hat{b}$$

Example:

If a force of 1N applied in a direction perpendicular to $\vec{v}$ and perpendicular to $\vec{B}$ to a 0.5m length conductor which is itself perpendicular to a magnetic field of 2T – what is the current induced in the conductor?

Answer: 1A

## *Notes on the Lorentz Force Law*

The fact that a current is induced in a conductor moving through a magnetic field follows from the principle of Conservation of Energy.

Since a current is induced in a conductor moving through a magnetic field, it follows that an emf has been induced in the conductor. This emf is given by Faraday's Law.

***Examples (Faraday's Law)***

Example:

A magnetic flux of 400 $\mu Wb$ passing through a coil of 1200 turns is reversed in 0.1 s.
Calculate the average value of the emf induced in the coil.

$$E = -n\frac{d\psi_m}{dt}$$

$$\Delta\psi_m = -800\mu Wb$$

$$E = \frac{(1200)(800.10^6)}{0.1} = 9.6V$$

Example:

Calculate the emf generated in the axle of a car traveling at 80kmph where the length of the axle is 2m and the vertical component of the earth's magnetic field is 40μT.

$$800Kmph = \frac{(80)(1000)}{3600} = 22.2ms^{-1}$$

$$\vec{B}_{\perp} = 40x10^{-6}T$$

Flux cut by the axle in one second: (i.e. $E = \frac{d\psi_m}{dt}$)

$= 40 \times 10^{-6} \times 2 \times 22.2$

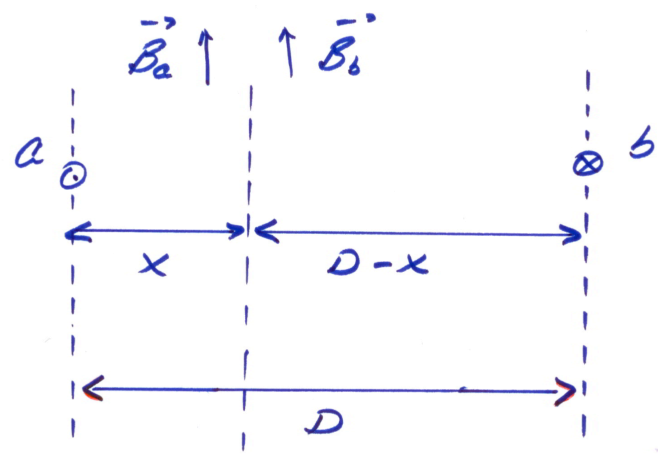
$= 1776\ \mu V$

***Example (Ampere's Law)***

Example:

Two very long, straight, thin copper wires, placed Dm apart are carrying equal currents I in opposite directions.

If the wires are placed perpendicular to the plane of the page, find an expression for the magnetic in terms of distance 'x' from the wire 'a' as shown in the diagram:




Since the wires are carrying opposite currents, the $\vec{B}_a$ and $\vec{B}_b$ act in the same direction between the wires from Ampere's Law.

Then:

$$\vec{B}_a = \frac{\mu I}{2\pi x}\hat{z}$$

$$\vec{B}_b = \frac{\mu I}{2\pi(D-x)}\hat{z}$$

$$\vec{B}_{Total} = \vec{B}_a + \vec{B}_b = \frac{\mu I}{2\pi}\left\{\frac{D}{x(D-x)}\right\}\hat{z}$$

## *Electrostatics Vs Electrodynamics*

In this section, we have examined the phenomena of Electrostatics and Magnetostatics i.e. Electric and Magnetic fields which are independent of time – i.e. static fields.

Dynamic fields on the other hand are fields that are time dependant – i.e. time-varying.

We will not examine these fields in this course but they are deserving of some discussion given their importance – especially in telecommunications.

Electrodynamic fields are very important in nature and to engineers.The reason is that energy is transferred (radiated) via these fields - hence the heat of the sun, wireless communications, etc.