CS1026 – Digital Logic Design

A slow introduction to the Quine-McCluskey Algorithm I

Alistair Morris [1]

[1]Distributed Systems Group
Trinity College Dublin

February 9, 2016

Today's Overview

Quine-McCluskey [Majumder et al., 2015] I

- An exact algorithm which finds a minimum-cost sum-of-products implementation of a Boolean function
- This lecture partially introduces the method

Quine-McCluskey [Majumder et al., 2015] II

Steps of the Quine-McCluskey algorithm:

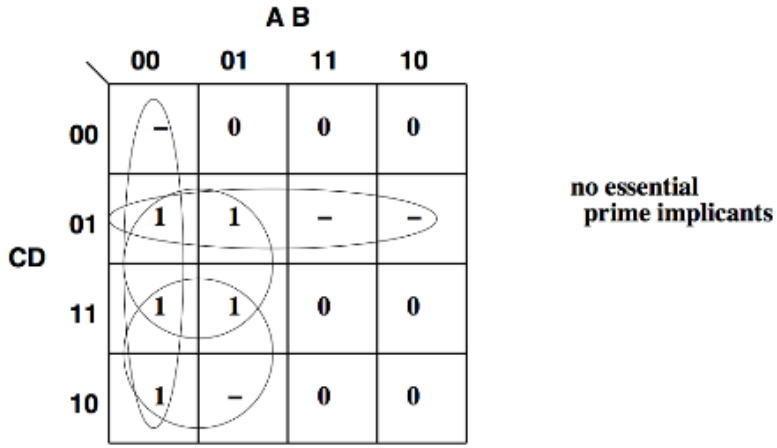
1. Generate Prime Implicants
2. Construct Prime Implicant Table
3. Reduce Prime Implicant Table
    1. Remove Essential Prime Implicants
    2. Row Dominance
    3. *Column Dominance*
4. Solve Prime Implicant Table

Row Dominance I

Consider the following Karnaugh map of a 4-input Boolean function: $F(A, B, C, D) = \sum m(1, 2, 3, 5, 7) + \sum d(0, 6, 9, 13)$

Row Dominance II

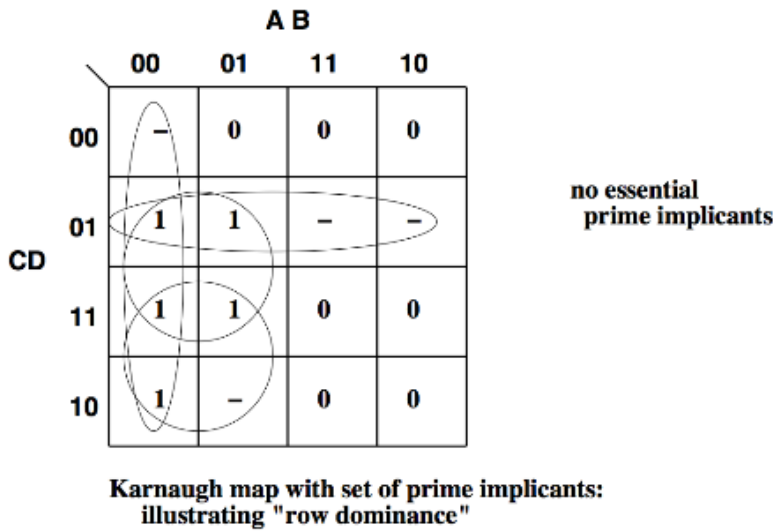
The Karnaugh-Map!



Karnaugh map with set of prime implicants:
illustrating "row dominance"

$$F(A, B, C, D) = \sum m(1, 2, 3, 5, 7) + \sum d(0, 6, 9, 13)$$

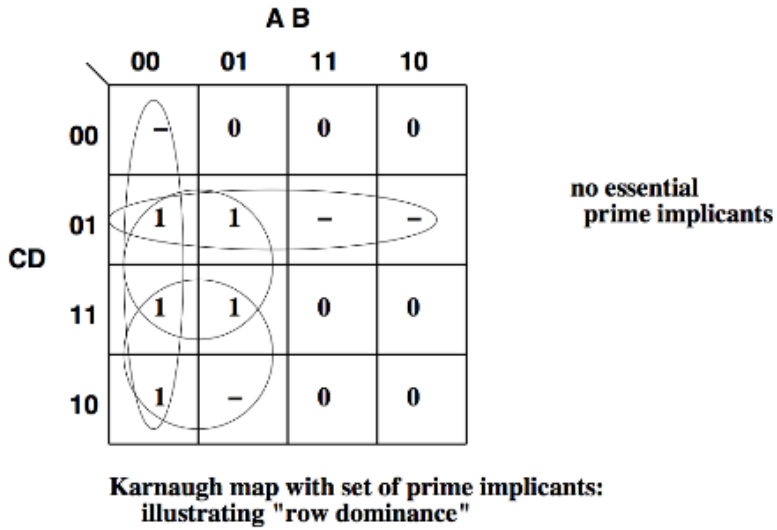
Row Dominance III



Karnaugh map with set of prime implicants: illustrating "row dominance"

- We have 4 prime implicants: A'B', C'D, A'D and A'C.
- None denotes an essential prime implicant

Row Dominance IV



Karnaugh map with set of prime implicants: illustrating "row dominance"

Our task:

- Pick a minimum subset of these implicants..
- ..to cover the 5 ON-set min-terms

Row Dominance V

The prime implicant table for the Karnaugh map:

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

- 4 prime implicants listed as columns
- 5 ON-set min-terms are listed as rows

Row Dominance VI

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

- Row 2 covered by two of these three columns: A'B' and A'C
- Row 7 also covered by two of these three columns: A'D and A'C

Row Dominance VII

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

- Any prime implicant which contains row 2 also contains row 3
- Any prime implicant which contains row 7 also contains row 3

We can ignore the covering of row 3: *we cover row 2 or row 7!*

Row Dominance VIII

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

- Row 3 row dominates row 2
- Row 3 row dominates row 7

The situation is now the reverse of column dominance: *we cross out the dominating (larger) row*

Row Dominance IX

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

In this case..

- We can cross row 3 crossed out
- We no longer need to worry about it

Row Dominance X

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

But also..

- Row 1 dominates row 5
- We can cross row 1 out!

Row Dominance XI

	A'B' (1,2,3)	C'D (1,5)	A'D (1,3,5,7)	A'C (2,3,7)
1	X	X	X	
2	X			X
3	X		X	X
5		X	X	
7			X	X

But why?

- Row 1 covered by row 5

Does this really replace Karnaugh Maps?

So far we have just used Karnaugh Maps to help us find prime implicants

- K-Maps good at this for functions with less than 5 Boolean values

However.. let's minimise:

$F(A, B, C, D, E, F, G, H, I, J) =$
$\sum m(61, 63, 125, 127, 450, 593, 595, 625, 627, 721, 723, 753, 755, 874, 875, 878, 879, 957, 959, 1021, 1023)$

emmm.. help! :'(

We need a better method..

.. To find prime implicants

Abstract Method

1. List Min-terms (sorted by number of 1's)
2. Combine Pairs of Min-terms
3. Combine Pairs of Products

Putting it all together

Next time.. time for more tips about the lab! (zzzzz...)

Any Problems?

- Ask!
- E-Mail: *morrisa5@scss.tcd.ie*

References (Homework) I

Majumder, A., Chowdhury, B., Mondai, A. J., and Jain, K. (2015).
Investigation on quine mccluskey method: A decimal manipulation based novel approach for the minimization of boolean function.
In *Electronic Design, Computer Networks & Automated Verification (EDCAV), 2015 International Conference on*, pages 18–22. IEEE.