CS1003-3

TRINITY COLLEGE DUBLIN
THE UNIVERSITY OF DUBLIN

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

Integrated Computer Science Programme
BA (Mod) Computer Science and Business
BA (Mod) CSL
Year 1 Annual Examinations

Trinity Term 2015

Mathematics

Monday 27 April 2015 **RDS Main Hall** **09:30 – 12:30**

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Instructions to Candidates:

Answer TWO questions from SECTION A and TWO questions from SECTION B; FOUR questions in total to be answered.

All questions carry equal marks. Each question is scored out of a total of 25 marks.

Answer each SECTION in a separate answer book.

The Formulae & Tables Booklet and graph paper are available from the invigilators, if required.

You may not start this examination until you are instructed to do so by the Invigilator.

Materials permitted for this examination:

Non-programmable calculators are permitted for this examination — please indicate the make and model of your calculator on each answer book used.

SECTION A

1. (a) Solve, if possible, each of the following systems of linear equations using Gaussian Elimination:

(i)

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 2, \\ 2x_1 + x_2 + x_3 + 3x_4 &= 1, \\ 3x_1 + 5x_2 - 2x_3 + 7x_4 &= 3, \\ 2x_1 + 6x_2 - 4x_3 + 9x_4 &= 8. \end{aligned}$$

(ii)

$$\begin{aligned} 3x + 2y + z &= 3, \\ 6x + 3y + 3z &= 0, \\ 6x + 2y + 4z &= 6. \end{aligned}$$

[18 marks]

(b) Find x and y if

$$\begin{pmatrix} 18 & -2 & -7 \\ -17 & 8 & 2 \\ 10 & -5 & -6 \end{pmatrix} \begin{pmatrix} -14 & -8 & -10 \\ 3 & 9 & 19 \\ x & -11 & 20 \end{pmatrix} = \begin{pmatrix} -132 & -85 & -358 \\ 226 & y & 362 \\ -47 & -59 & -315 \end{pmatrix}.$$

[7 marks]

[25 marks]

2. (a) Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}.$$

Show that $\det(A) = (b-a)(c-a)(c-b)$.

[11 marks]

(b) Calculate the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

[14 marks]

[25 marks]

3. (a) Evauate $\cos\left(\frac{\pi}{10}\right)$ to four decimal paces by using appropriate Taylor polynomials.

[6 marks]

(b) (i) Find the Taylor series about 0 for the function

$$f(x) = (1+x)^{-\frac{2}{3}},$$

giving all terms up to the one in x^4. You may make use of the standard Taylor series below. Give each coefficient as a single integer or in exact, fractional form. State the interval of validity for this series.

(ii) Use the series in (b)(i) and the standard Taylor series for $\cos x$ to find the cubic Taylor polynomial about 0 for the function

$$f(x) = \frac{\cos x}{(1+x)^{\frac{2}{3}}}.$$

[10 marks]

(c) (i) Give the first three non-zero terms of the Taylor series about 0 for the function

$$f(x) = \sin(2x) - 2xe^{-x}.$$

(ii) Use the Taylor series found in (c)(i) to show that $f(x)$ has a stationary point at $x = 0$, and deduce whether this point is a local maximum or local minimum of $f(x)$.

[9 marks]

Note: Standard Taylor Series about 0 are provided on page 5.

[25 marks]

Standard Taylor Series about 0

$$
\begin{aligned}
\sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 + \dots, \quad \text{for } x \in \mathbb{R} \\
\cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \dots, \quad \text{for } x \in \mathbb{R} \\
e^x &= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots, \quad \text{for } x \in \mathbb{R} \\
\ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots, \quad \text{for } -1 < x < 1 \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \dots, \quad \text{for } -1 < x < 1 \\
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots,
\end{aligned}
$$

for $-1 < x < 1$ and for any $\alpha \in \mathbb{R}$.

SECTION B

4. (a) Let the set operator, $\uparrow$, be defined so that

$$X \uparrow Y = X \cup \overline{Y}$$

where $\overline{Y}$ is the complement of Y.
Determine using Veitch diagrams whether:

(i) $(A \uparrow B) \uparrow C = A \uparrow (B \uparrow C)$,

(ii) $(A \uparrow B) \uparrow C = A \uparrow (B \cap C)$,

(iii) $A \uparrow (B \cup C) = (A \uparrow B) \cap (A \uparrow C)$.

[9 marks]

(b) Let

$$\begin{aligned} A &= \{x | x = 4 * a + 1 \wedge a \in \mathbb{N}\} \\ B &= \{y | y = 3 * b + 5 \wedge b \in \mathbb{N}\} \end{aligned}$$

where $\mathbb{N}$ is the set of Natural numbers, $\{0, 1, 2, 3, \ldots\}$.

(i) Determine the first 5 elements of $\mathbb{N}$ that are in $A \cap B$.

(ii) Determine whether 101 is an element of $A \cap B$.

(iii) Determine the first 5 elements of $\mathbb{N}$ that are in $A \oplus B$ where

$$X \oplus Y = (X - Y) \cup (Y - X)$$

[9 marks]

(c) From a survey of sports fans,
30% follow Soccer, 44% follow Gaelic, 42% follow Rugby,
10% follow only Soccer, 20% follow only Gaelic and 16% follow only Rugby,
12% follow both Soccer and Gaelic,
14% follow both Soccer and Rugby,
18% follow both Gaelic and Rugby.

(i) What percentage of fans follow all three sports?

(ii) What percentage of fans don't follow any of the three sports?

[7 marks]

[25 marks]

5. (a) A logic operator, $\#$, is defined such that

$$p\#q = \neg(p \wedge q)$$

(i) Show by Truth Table that

$$\bullet \quad \neg p = p\#p$$
$$\bullet \quad p \vee q = (p\#p)\#(q\#q)$$

(ii) Define $p \rightarrow q$ in terms of $\#$.

[5 marks]

(b) Consider the argument:

`There is no evil in the world if Superman is benevolent.`
`If Superman exists then he is benevolent.`
`Superman is benevolent and there is evil in the world.`

Therefore

`Superman does not exist.`

Abbreviate:

B: `Superman is benevolent.`
E: `There is evil in the world.`
S: `Superman exists.`

(i) Determine, by Truth Table or otherwise, whether the argument is valid.

(ii) Is the argument valid if instead the conclusion was:

`Superman exists.`

[8 marks]

(c) **The Tardy Bus Problem**

The following three statements are given as premises:

P1: If Bill takes the bus then, he misses his appointment if the bus is late.
P2: Bill does not go home, if he misses his appointment and he feels downcast.
P3: If Bill does not get the job then, he feels downcast and he goes home.

Determine, by the use of KE Deduction, whether each of the following two conjectures can be inferred from the premises.

C1: If Bill goes home then, he either feel downcast or he gets the job.
C2: If Bill takes the bus and he does not go home then, he gets the job if he does not miss his appointment.

Note: The KE-Deduction Rules are provided on page 9.

Abbreviations:

TB: Bill Takes the Bus.
MA: Bill Misses his Appointment.
BL: The Bus is Late.
GH: Bill Goes Home.
FD: Bill Feels Downcast.
GJ: Bill Gets the Job.

Translations

Premises

P1: $TB \rightarrow (BL \rightarrow MA)$
P2: $MA \land FD \rightarrow \neg GH$
P3: $\neg GJ \rightarrow FD \land GH$

Conjectures

C1: $GH \rightarrow (FD \lor GJ)$
C2: $TB \land \neg GH \rightarrow (\neg MA \rightarrow GJ)$

[12 marks]
[25 marks]

KE Deduction Rules

Branch Rules:

$$\begin{array}{ccc} & \wedge & \\ P & & \neg P \end{array}$$

α–Rules

Premise	$\neg\neg P$	$P \wedge Q$	$\neg(P \vee Q)$	$\neg(P \rightarrow Q)$
Conclusion	P	P	$\neg P$	P
Conclusion		Q	$\neg Q$	$\neg Q$

β–Rules

Premise	$P \vee Q$	$\neg(P \wedge Q)$	$(P \rightarrow Q)$	$(P \rightarrow Q)$
Premise	$\neg P$	P	P	$\neg Q$
Conclusion	Q	$\neg Q$	Q	$\neg P$

6. (a) In modular arithmetic, where a and b are Integers and $b > 0$, *(a div b)* and (a *mod* b) are defined so that

$$a = b * (a \text{ } div \text{ } b) + (a \text{ } mod \text{ } b) \quad \text{and } 0 \leq (a \text{ } mod \text{ } b) < b$$

Determine

(i) 101 *div* 11 and 101 *mod* 11

(ii) (-101) *div* 11 and (-101) *mod* 11

[7 marks]

(b) Calculate 11$*$ gcd(10101, 11011) where gcd(a,b) is the greatest common divisor of a and b.

[8 marks]

(c) If $a * x + b * y = \text{gcd}(a, b)$ then $a * (k * x) + b * (k * y) = k * \text{gcd}(a, b)$. Find integers x and y such that

$$10101 * x + 11011 * y = 1001$$

[10 marks]

[25 marks]