Determinants, 2x2

Simultaneous Equations, again

Consider

$$\begin{aligned} a_1 * x + a_2 * y &= k_1 \\ b_1 * x + b_2 * y &= k_2 \end{aligned}$$

We found that:

$$x = \frac{k_1 * b_2 - k_2 * a_2}{a_1 * b_2 - b_1 * a_2}$$

and

$$y = \frac{a_1 * k_2 - b_1 * k_1}{a_1 * b_2 - b_1 * a_2}$$

Double Indexing

In the equations

$$\begin{aligned} a_1 * x + a_2 * y &= k_1 \\ b_1 * x + b_2 * y &= k_2 \end{aligned}$$

write a as a_1 and b as a_2
so that $a_1$becomes $(a_1)_1$ which we write as a_{11}.
Similarly, we write $(a_1)_2$ as a_{12}
Also we write $b_1$as $(a_2)_1$ and rewrite this as a_{21}.
Similarly, we write b_2 as a_{22}.
We can rewrite the simultaneous equations as:

$$\begin{aligned} a_{11} * x + a_{12} * y &= k_1 \\ a_{21} * x + a_{22} * y &= k_2 \end{aligned}$$

Determinant Notation

In Determinant notation the solution

$$x = \frac{k_1 * a_{22} - k_2 * a_{12}}{a_{11} * a_{22} - a_{21} * a_{12}}$$

can be written as:

$$x = \frac{\begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

Cont'd

Similarly, in Determinant notation the solution:

$$y = \frac{a_1 * k_2 - b_1 * k_1}{a_1 * b_2 - b_1 * a_2}$$

can be rewritten as

$$y = \frac{\begin{vmatrix} a_{11} & k_1 \\ a_{22} & k_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

There is no solution if the Determinant

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$$

Calculating a 2x2 Determinant

The Determinant

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

is calculated as

$$a_{11} * a_{22} - a_{21} * a_{12}$$

i.e. 'cross multiply and subtract'

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} * a_{22} - a_{21} * a_{12} \ .$$

Example: Using 2x2 Determinants

Solve the equations:

$$\begin{aligned} 5 * x + 8 * y &= \mathbf{18} \\ 3 * x + 5 * y &= \mathbf{11} \end{aligned}$$

The Determinant of the co-efficients is:

$$\begin{vmatrix} 5 & 8 \\ 3 & 5 \end{vmatrix}$$

The solutions in Determinant form are:

$$x = \frac{\begin{vmatrix} \mathbf{18} & 8 \\ \mathbf{11} & 5 \end{vmatrix}}{\begin{vmatrix} 5 & 8 \\ 3 & 5 \end{vmatrix}} \text{ and } y = \frac{\begin{vmatrix} 5 & \mathbf{18} \\ 3 & \mathbf{11} \end{vmatrix}}{\begin{vmatrix} 5 & 8 \\ 3 & 5 \end{vmatrix}}$$

The Determinant

$$D = \begin{vmatrix} 5 & 8 \\ 3 & 5 \end{vmatrix} = 5 * 5 - 3 * 8 = 1$$

and

$$D_x = \begin{vmatrix} \mathbf{18} & 8 \\ \mathbf{11} & 5 \end{vmatrix} = 18 * 5 - 11 * 8 = 90 - 88 = 2$$

and

$$D_y = \begin{vmatrix} 5 & \mathbf{18} \\ 3 & \mathbf{11} \end{vmatrix} = 5 * 11 - 3 * 18 = 55 - 54 = 1$$

$\therefore x = \frac{D_x}{D} = \frac{2}{1}$ and $y = \frac{D_y}{D} = \frac{1}{1}$ i.e. $x = 2$ and $y = 1$.