1 Prove the following theorems algebraically:

(a) $X(X' + Y) = XY$ (b) $X + XY = X$

(c) $XY + XY' = X$ (d) $(A + B)(A + B') = A$

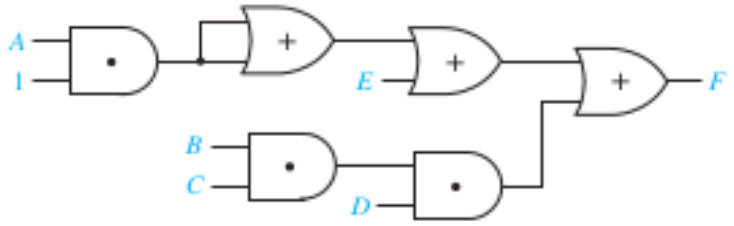
2 Simplify each of the following expressions by applying *one* of the theorems. State the theorem used

(a) $X'Y'Z + (X'Y'Z)'$ (b) $(AB' + CD)(B'E + CD)$

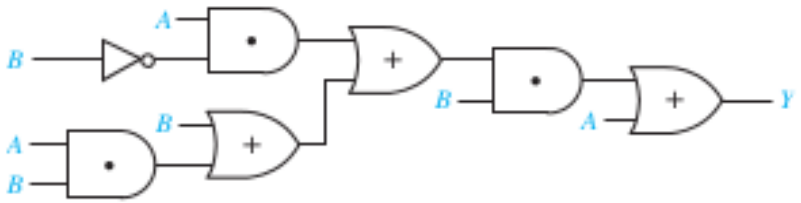
(c) $ACF + AC'F$ (d) $A(C + D'B) + A'$

(e) $(A'B + C + D)(A'B + D)$ (f) $(A + BC) + (DE + F)(A + BC)'$

3 For each of the following circuits, find the output and design a simpler circuit having the same output. (*Hint:* Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go.)



(a)



(b)

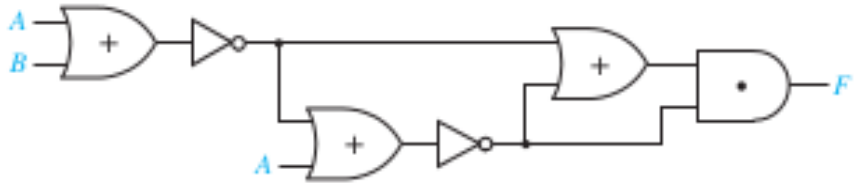
4 Multiply out and simplify to obtain a sum of products:
(a) $(A + B)(C + B)(D' + B)(ACD' + E)$
(b) $(A' + B + C')(A' + C' + D)(B' + D')$

5 Factor each of the following expressions to obtain a product of sums:
(a) $AB + C'D'$ (b) $WX + WY'X + ZYX$
(c) $A'BC + EF + DEF'$ (d) $XYZ + W'Z + XQ'Z$
(e) $ACD' + C'D' + A'C$ (f) $A + BC + DE$
(The answer to (f) should be the product of four terms, each a sum of three variables.)

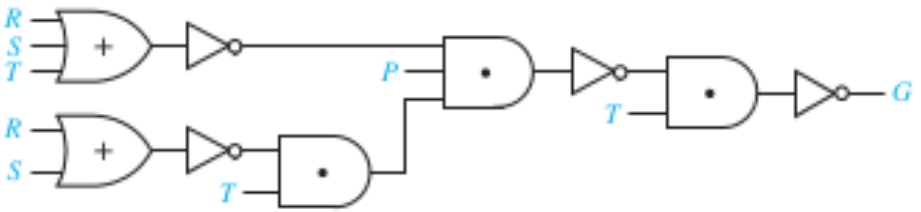
6 Draw a circuit that uses only one AND gate and one OR gate to realize each of the following functions:
(a) $(A + B + C + D)(A + B + C + E)(A + B + C + F)$
(b) $WXYZ + VXYZ + UXYZ$

7 Simplify the following expressions to a minimum sum of products.
(a) $[(AB)' + C'D]'$ (b) $[A + B(C' + D)]'$ (c) $((A + B')C)'(A + B)(C + A)'$

8 Find F and G and simplify:



(a)



(b)