

## Course Work Assignment #1

1. Let the set operator,  $\approx$ , be defined so that

$$X \approx Y = (\overline{X} \cup Y) \cap (X \cup \overline{Y})$$

where  $\overline{X}$  is the complement of  $X$ .

Determine by Veitch diagram whether:

- (a)  $A \approx B = \overline{A} \approx \overline{B}$
- (b)  $(A \cap B) \approx A = B \approx (A \cup B)$
- (c)  $A \cap (B \approx C) = (A \cap B) \approx (A \cap C)$

2.

- (a) Each student in a group of 139 spoke either French, German or Spanish.  
63 spoke French, 91 spoke German, 44 spoke Spanish.  
25 spoke French and German.  
23 spoke French and Spanish.  
21 spoke German and Spanish.

How many spoke all 3 languages?

- (b) Let the Universal set,  $U = \{1..30\}$ .  
For a subset  $X$  of  $U$ , let  $\overline{X} = U - X$   
Define subsets  $M_2, M_3$  and  $M_5$  of  $U$  such that  
 $M_2 = \{n \in U \mid n \text{ is a multiple of } 2\}$   
 $M_3 = \{n \in U \mid n \text{ is a multiple of } 3\}$   
 $M_5 = \{n \in U \mid n \text{ is a multiple of } 5\}$   
Determine
  - i.  $M_2 \cap M_3 \cap M_5$
  - ii.  $\overline{M_2} \cap M_3 \cap M_5$
  - iii.  $M_2 \cap \overline{M_3} \cap M_5$
  - iv.  $M_2 \cap M_3 \cap \overline{M_5}$
  - v.  $\overline{M_2} \cap \overline{M_3} \cap \overline{M_5}$

3.

(a) Determine using truth tables, whether the following are Tautologies

(Note:

The operator  $\equiv$  has lower precedence than  $\rightarrow$

i.e. read  $p \rightarrow q \equiv r$  as  $(p \rightarrow q) \equiv r$ .

The operator  $\rightarrow$  has lower precedence than both  $\wedge$  and  $\vee$

i.e. read  $p \wedge q \rightarrow r$  as  $(p \wedge q) \rightarrow r$

and read  $p \vee q \rightarrow r$  as  $(p \vee q) \rightarrow r$ .

The operator  $\rightarrow$  is right associative,

i.e. read  $p \rightarrow q \rightarrow r$  as  $p \rightarrow (q \rightarrow r)$

i.  $p \rightarrow q \equiv q \rightarrow p$

ii.  $p \rightarrow q \rightarrow p \wedge q$

iii.  $(p \rightarrow q) \rightarrow (p \rightarrow \neg q) \rightarrow \neg p$

iv.  $(p \rightarrow r) \rightarrow (q \rightarrow r) \rightarrow p \vee q \rightarrow r$

(b) Determine by Truth Tables whether the following argument is valid

*The world will end, if the eclipse occurs.*

*Either the eclipse occurs or the Sun is hidden*

therefore

*If the Sun is hidden then the world will end.*

In formalising the argument, use the abbreviations:

W: *The world will end*

E: *The eclipse occurs.*

S: *The Sun is hidden.*