Notation Issues

In Trigonometry, for example,
$cos^2\theta$ is traditionally used for $(cos\,\theta)^2$.
For example, we write

$$cos^2\theta + sin^2\theta = 1$$

instead of

$$(cos\,\theta)^2 + (sin\,\theta)^2 = 1$$

Note:
For trigonometric functions and the logaritmn functions, the brackets around a single argument is sometimes dropped. For example, $ln\,x$ may be used instead of $ln(x)$.But if the argument is a complex expression then brackets are used, for example
$ln(x * y) = ln\,x + ln\,y$.
If in doubt, brackets are used to clarify the argument to a function.

Difficulty with Inverse Notation

In Arithmetic, $\frac{1}{x} = x^{-1}$, where $x \in \mathbb{R}$ i.e. where x is Real number.
When functions are used in mathematics;
the notation f^{-1} is used for the inverse of the function, f , i.e.
$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

A conflict arises in using notation such as $cos^{-1}x$.
While $cos^2\theta = (cos\,\theta)^2$ **but** $cos^{-1}x \neq (cos\,x)^{-1}$ i.e.

$$cos^{-1}x \neq \frac{1}{cos\,x}$$

Note: $\frac{1}{cos\,x} = sec\,x$.

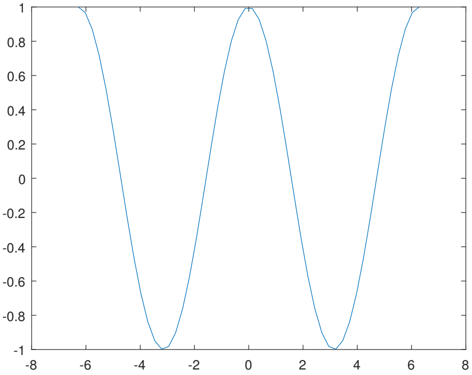
cos^{-1} and $arccos$

In Trigonometry, cos^{-1} is the **inverse function** of cos i.e.
for $0 \leq \theta \leq \pi$, $cos^{-1}(cos\,\theta) = \theta$.
For example, $cos\,\frac{\pi}{3} = \frac{1}{2} \therefore cos^{-1}(cos\,\frac{\pi}{3}) = cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$.
Also, $cos\left(cos^{-1}\left(\frac{1}{2}\right)\right) = cos\,\frac{\pi}{3} = \frac{1}{2}$

In some textbooks, $arccos$ is used instead of cos^{-1}, i.e. $arccos(x)$ is the angle, or 'arc' ,θ, whose cos is x i.e. $cos\,\theta = x$ i.e.
if $arccos(x) = \theta$ then $cos\,\theta = x$ where $-1 \leq x \leq 1$ and $0 \leq \theta \leq \pi$
$\therefore$
for $0 \leq \theta \leq \pi$, $arccos(cos\,\theta) = \theta$ and
for $-1 \leq x \leq 1$ $cos(arccos(x)) = x$.

Graph Cosine

Graph of $\cos x$, $-2\pi \leq x \leq 2\pi$



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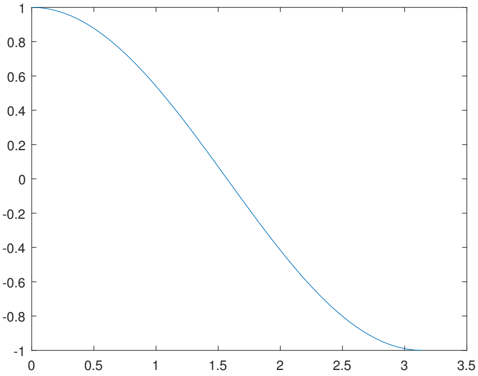
Graph of $cos\, x$, from $0 \leq x \leq \pi$ to $-1 \leq x \leq 1$ $\therefore$
cos is a bijective function so its inverse cos^{-1} exists.
cos^{-1} is a function from $-1 \leq x \leq 1$ to $0 \leq x \leq \pi$



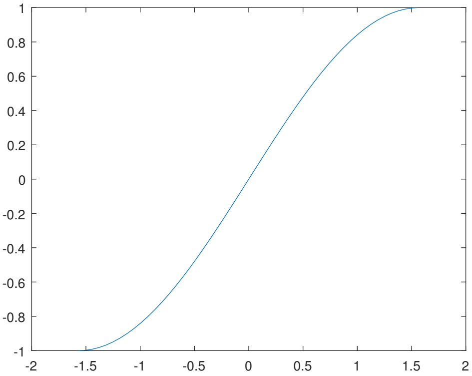
sin^{-1} and $arcsin$

$arcsin$ is used for sin^{-1} i.e.
if $arcsin(x) = \theta$ then $sin\,\theta = x$ where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
i.e.
for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $arcsin(sin\,\theta) = \theta$ and
for $-1 \leq x \leq 1$ $sin(arcsin(x)) = x$

tan^{-1} and $arctan$

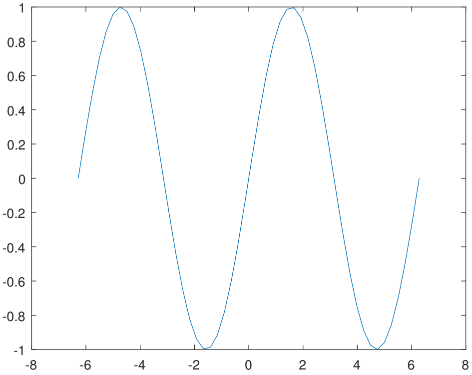
$arctan$ is used for tan^{-1} i.e.
for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $arctan(tan\,\theta) = \theta$ and
for $x \in \mathbb{R}$, $tan(arctan(x)) = x$

Graph of $sin\,\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$,



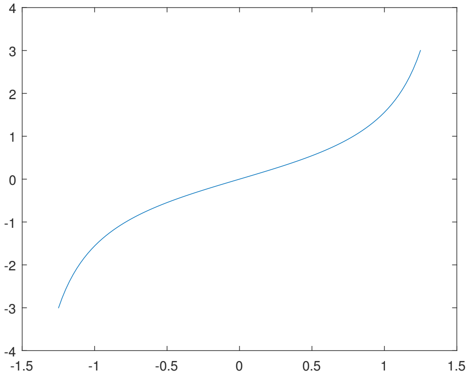
Graph Sine

Graph of $sin\,\theta$, $-2\pi \leq \theta \leq 2\pi$,



Graph Tan

Graph of $tan(x)$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$



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Graph of $tan(x)$, $-6 \leq \theta \leq 6$