Summary of previous lecture

- From analogue continuous signals to digital discrete signals:
  - Sampling to discretise time axes
  - Quantisation to discretise Amplitude axis

- Loss of information:
  - Sampling does not loose information if I sample at $f_s \geq 2 f_{\max}$
  - Quntisation looses information, but the higher the number of bits used the lower the noise created

- Data rate of a signal is obtained by multiplying sampling rate by number of bits per sample (i.e., quantisation bits).

- Difference between propagation speed and data rate of a signal
  - Propagation speed is fixed, data rate is proportional to the bandwidth of the signal

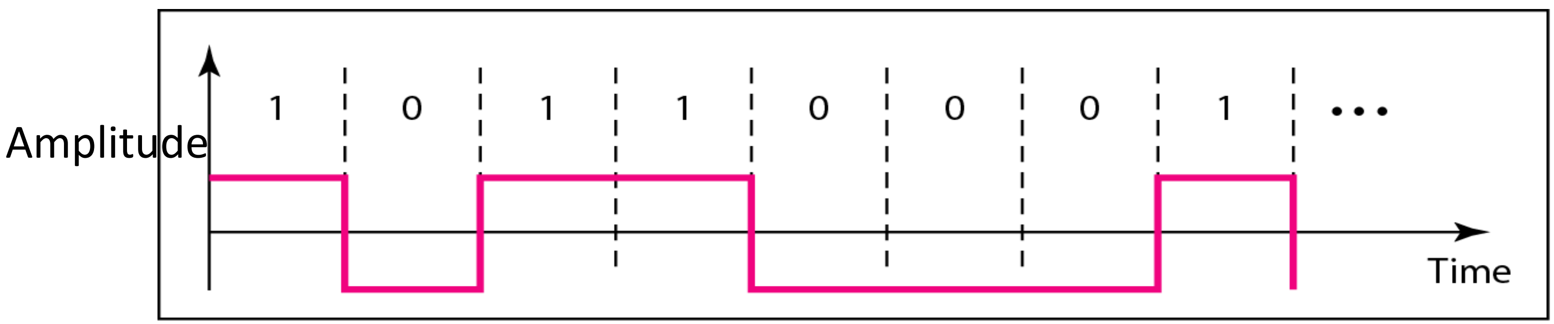
Modulation of digital signals

Transmission of digital signals

- We have shown how we can digitize a phone signal in the previous lecture.
- Once digitalized the signal will be a series of bits (here are grouped by 8-bit-quantised sample):

…10011101|00111010|10011100|10110011…

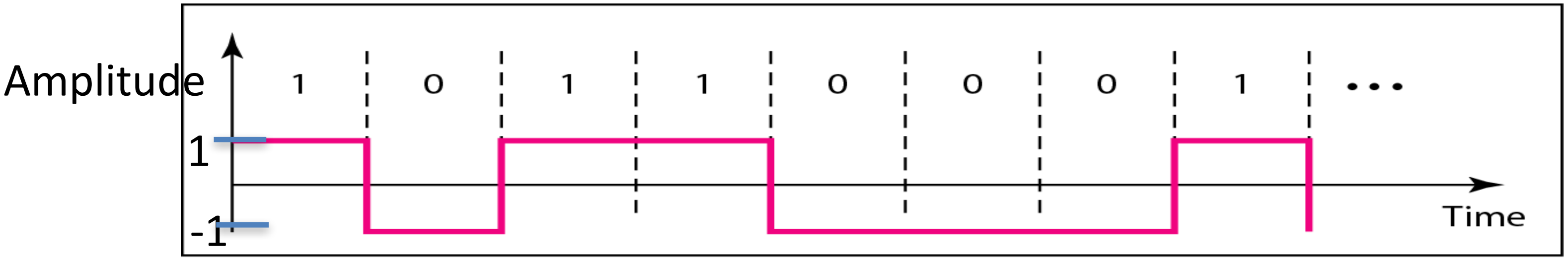
- If I want to visualise a series of bits in time I can do it by giving two different amplitude values to the 1 and 0:



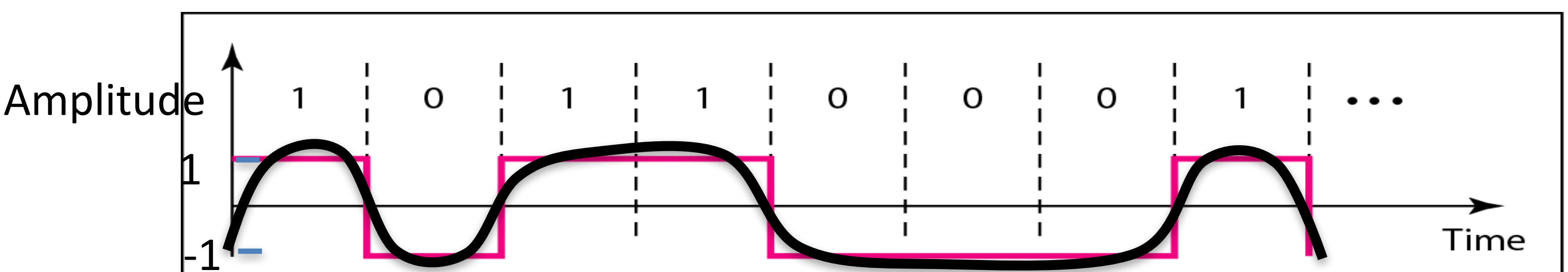
- I select a suitable time slot and transmit a bit on each time slot: for example a high amplitude value for a 1 and a low value for a 0

Transmission of digital signals

- This is what is called the baseband signal



- One option we have is to transmit is as it is, one bit at a time
- In practice, using real components, the signal will look smoother
  - Remember that a perfectly square signal has infinite frequency components from the Fourier series. In order to reduce its bandwidth we can use a smaller number of components, but the signal will have smoother edges



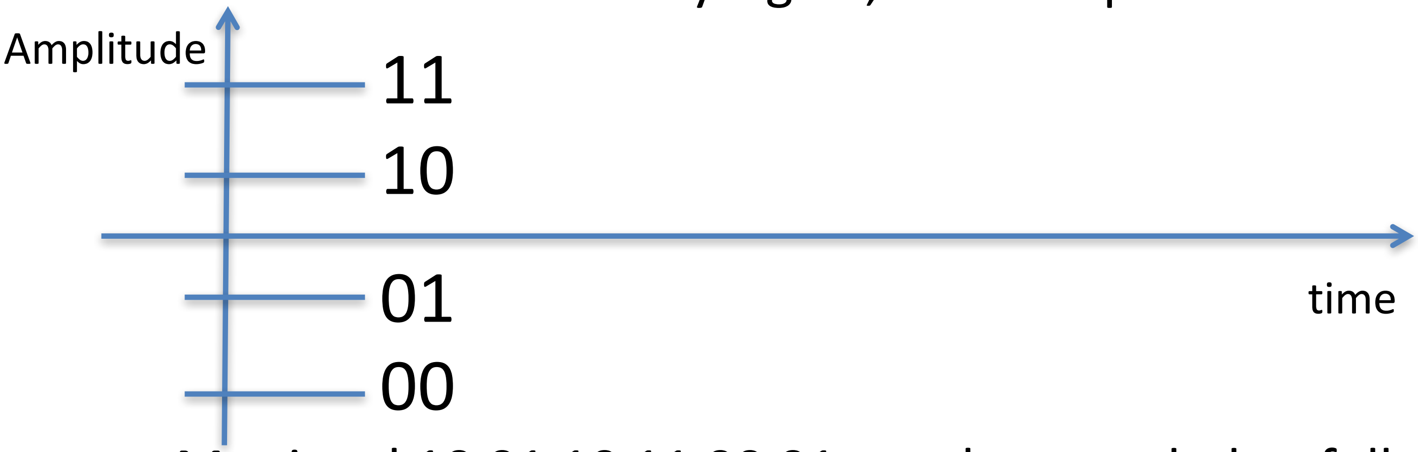
Multi level transmission

- Another option is to parallelize the signal and use a multi-level transmission format
- Suppose I have the sequence 1001101100
- Instead of seeing it as individual bits, I could see it as group of bits, for example groups of two bits: 10 01 10 11 00, then transmit them as 5 groups of two.

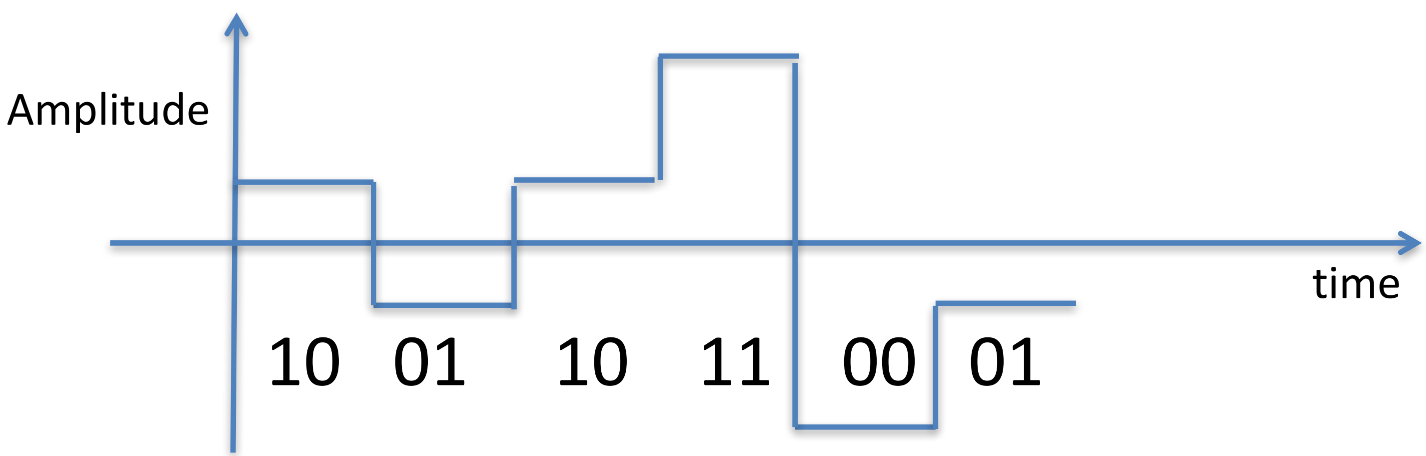
- The problem is that for each group I have 4 possibilities (or levels): 00, 01, 10, 11, so I need to reflect this in my signal

Multi level transmission

- A multi-level transmission uses more than two values in the amplitude axis.
- If I allow 4 levels to my signal, for example:

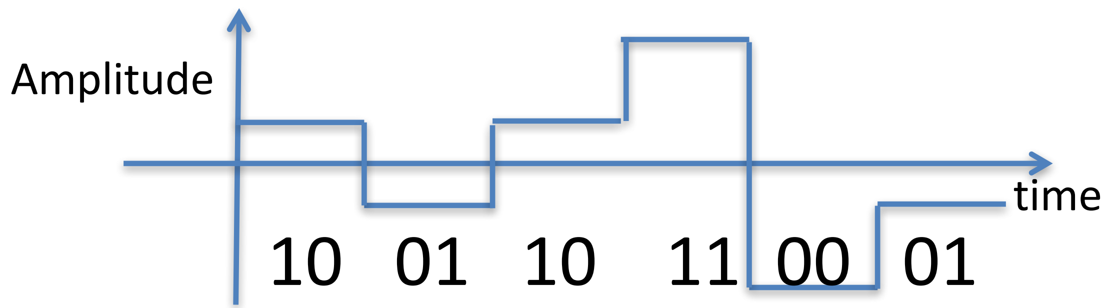


- My signal 10 01 10 11 00 01, can be encoded as follows:



Bit/s vs Baud

- Consider this signal. It has 6 time slots within 1 second of transmission time, but it transmits 12 bits in that time.



- What's the bit rate?
- Do I consider the number of slots (i.e. 6) or the number of bits (i.e. 12)?
- The **bit rate (R)** is 12 bit/s
- The **signal rate (S)** is 6 baud, this is the number of slots per second.
- Their relation is **S=R/n,** where n is the number of bits per slot

Quantisation vs. multi-level transmission

- Don’t confuse quantisation and multi-level transmission!
- They both use the same formula $L=2^n$, $n=\log_2 L$, <u>however</u>
  - Quantisation is used when we want to transform an analogue signal into a digital one. But it does not implies the transmission of the signal
  - Multi-level transmission concerns ways to transmit a signal which is already digital

Example

- You need to encode an analogue signal with phone quality and then transmit it using a 3-level baseband modulation. Find the bit rate and baud rate of the signal.

- First remember that a phone signal is encoded at 8KHz, quantised at 8 bits
  ➔ R=8*8=64Kb/s
  – What we do is have 8 bits for each sample: $\overbrace{10011010}^{\text{First sample}}|\overbrace{00110010}^{\text{Second sample}}|\ldots$
- Although we logically group them in groups of 8 when we want to transmit them we consider them as individual bits:
  – 1001101000110010…
- A 3-level modulation will group them again in group of 3 bits in order to transmit them:
  – 100|110|100|011|001|0…
  ➔ S=64/3 =21.3Kbaud

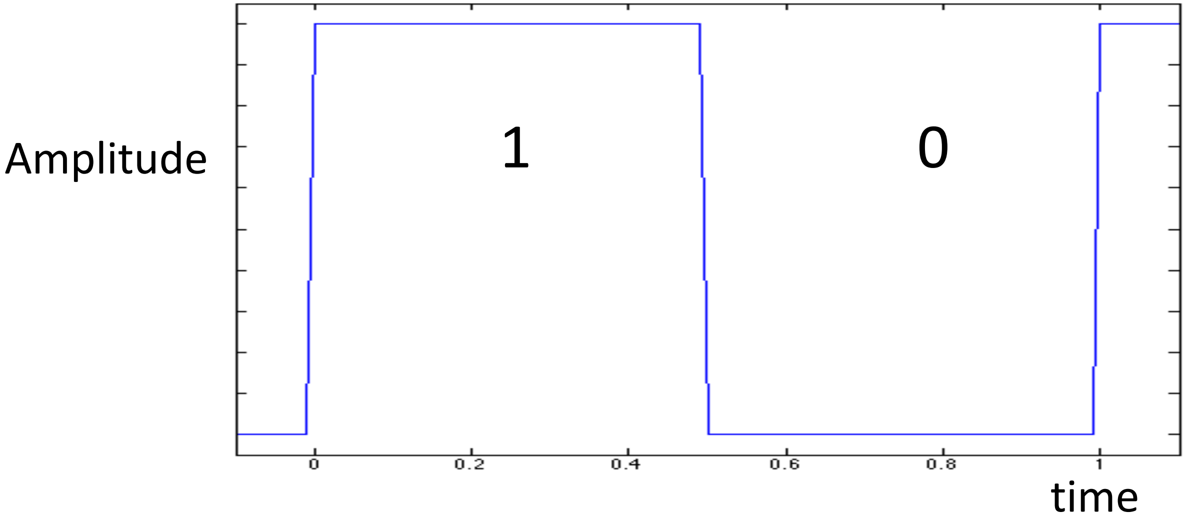
Bandwidth of a digital signal

The bandwidth of a digital signal is proportional to its bit rate

- However it also depends on other factors, such as type of encoding (we skip this part), number of levels of the encoding,…

Bandwidth of a digital signal

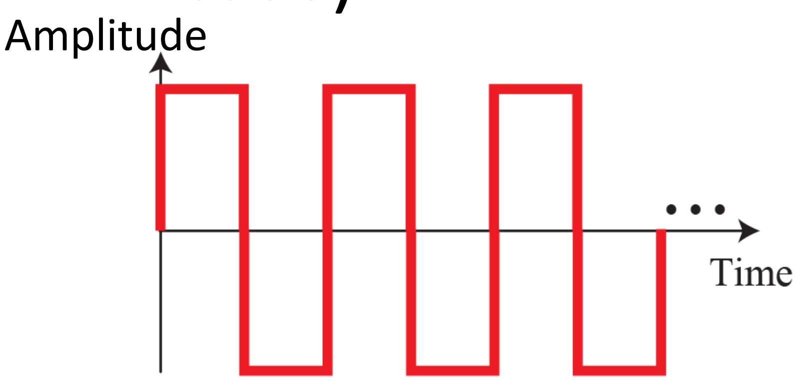
- If we look at a square wave, we can see that we can encode two bits for each period.

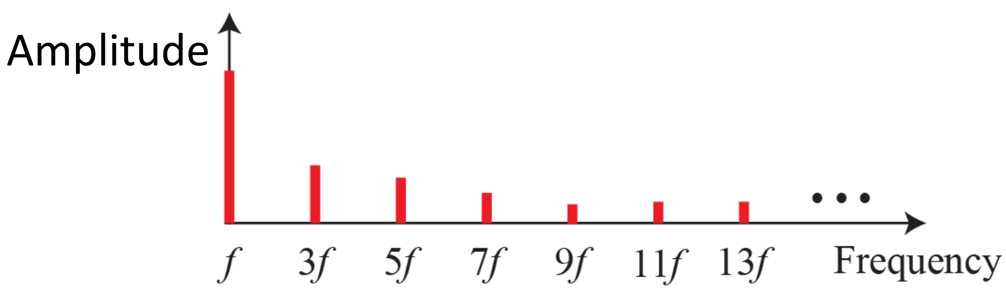


- So a **square wave** of frequency 1 Hz can encode 2 bits
- In general a square wave of frequency f can encode 2f bits

Bandwidth of a digital signal

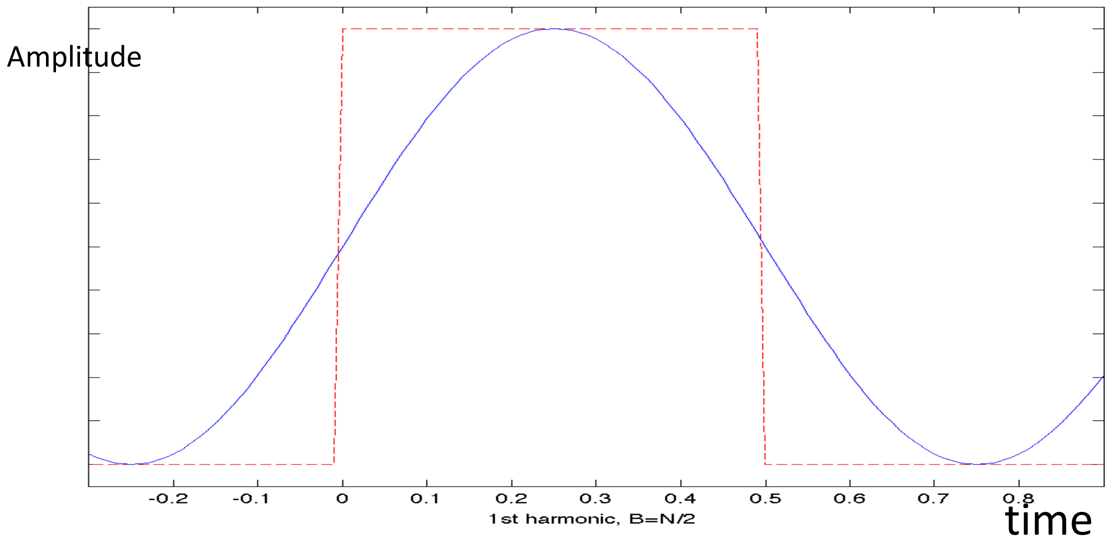
- However if we consider that the baseband signal is a square wave, then we would need a very large bandwidth to transmit information, because the frequency components (harmonics) are many (infinite in the ideal case)

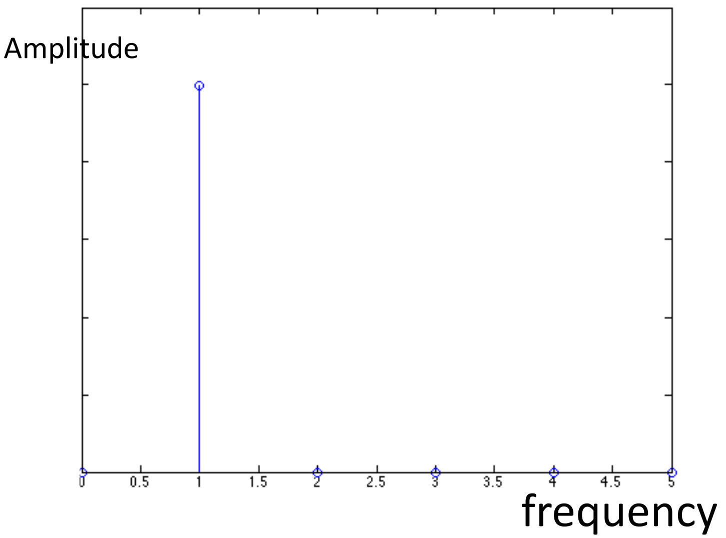




Bandwidth of a digital signal

- If we approximate the square wave with just the first harmonic:

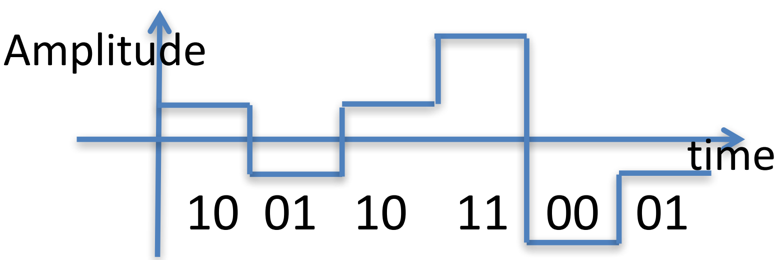




- The bandwidth will be 1 Hz
- So with a bandwidth of 1Hz, I can transmit 2 bits per second
- In general with a bandwidth of B Hz, I can transmit 2B bits per second

Bandwidth of a Multi-level signal

- If we use a multilevel coding system (or modulation), we could increase the bit rate of our transmission.
- **Since the bandwidth depends mostly on the baud rate**, if we pack more bits into the same period **we can increase the data rate, using the same bandwidth**



If I use a 2 level encoding I can pack 2 bits in each period. With a frequency of 1 Hz I can pack 2 bits

For a Multi-level signal the <u>maximum theoretical</u> bitrate I can achieve is: $R=2 \times B \times \log_2 L$ (Nyquist bit rate)

Example

- Given an analogue signal, what bandwidth do I need to transmit it over a digital system?

  Additional info required:
  - Bandwidth of analogue signal → I need to have this information
  - Sampling frequency → determined by $2xf_{max}$. I know f_{max} once I know the frequency spectrum.
  - Quantisation bits → I need to have this information, let's say 8 bits
  - Transmission levels → I need to have this, let's say 16-level transmission.

Transmission of digital signals through modulation

- So far we have discussed baseband transmission. However like for analogue signals, using modulations has many advantages:
  - higher capacity, enable wireless transmission, smaller antennas, ability to multiplex signals
  - For example to transmit it over an 800 MHz GSM channel,…
- Digital signals are modulated using digital modulation formats

Modulation of digital signals

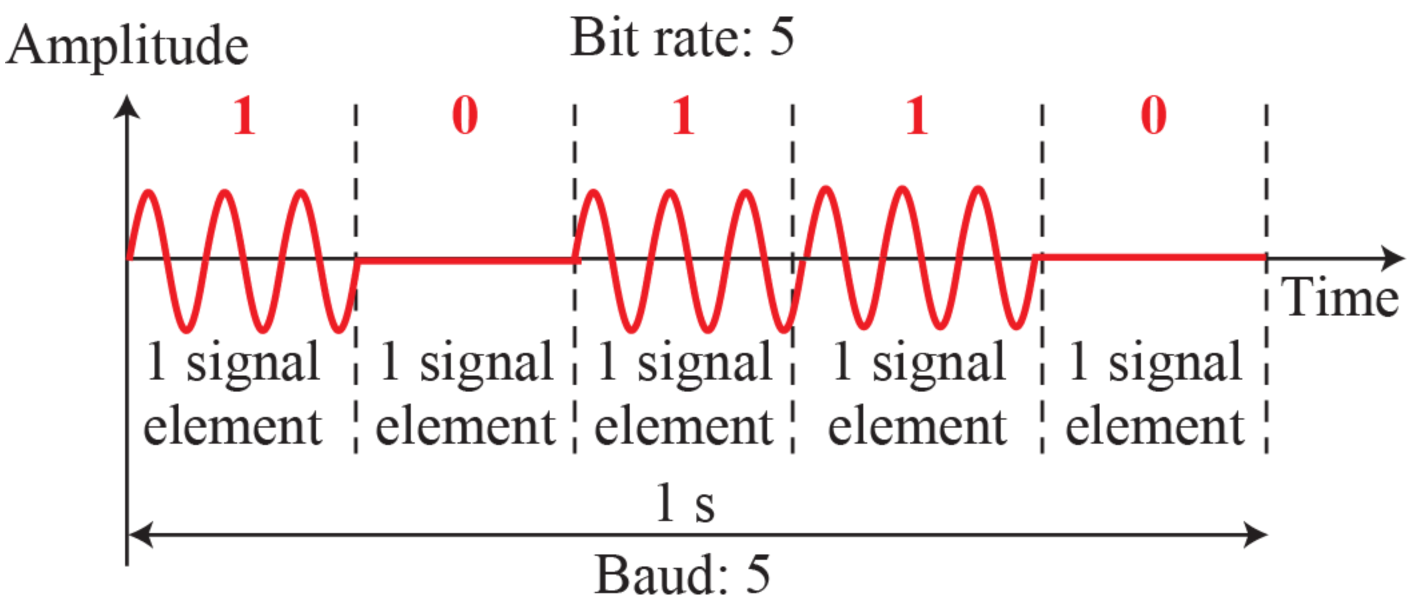
- Remember what is the meaning of modulation:
  - Take a carrier wave at a certain frequency (the carrier is the same, a sine wave)
  - Use your information to change some parameter of the carrier wave
  - The carrier wave will transport the information while it propagates

- In digital transmission we usually change the Amplitude, the frequency, or the phase of the carrier

- Differently from the analog case however, now the changes in amplitude, frequency or phase are <u>**DISCRETE, i.e. we can only use a certain number of different amplitudes, frequencies or phase shifts!**</u>

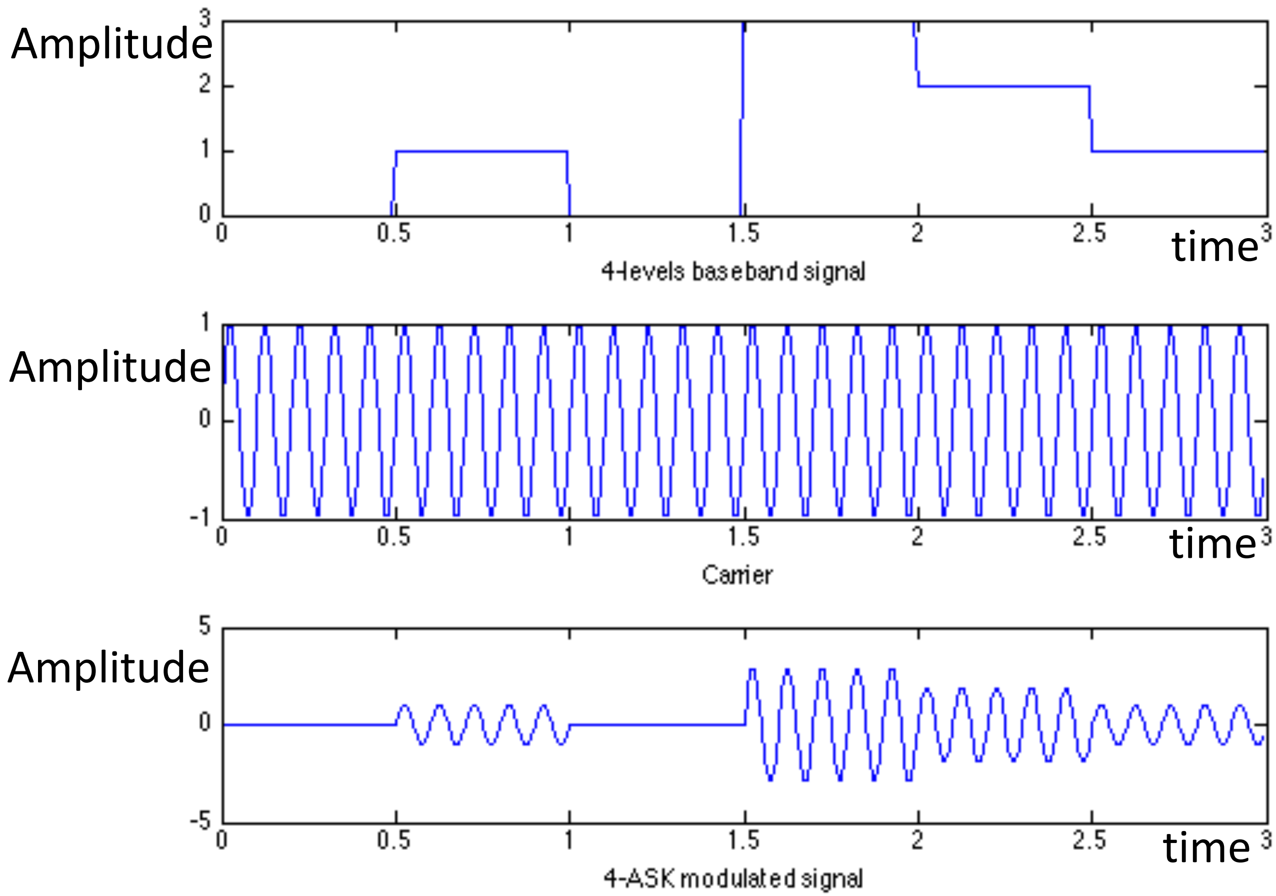
Amplitude Shift Keying - ASK

- The simplest ASK modulation is with only 2 levels, where I use a certain amplitude value A for a 1 bit and a **null amplitude** for a 0 bit
- This is also referred to as On-Off keying (OOK)



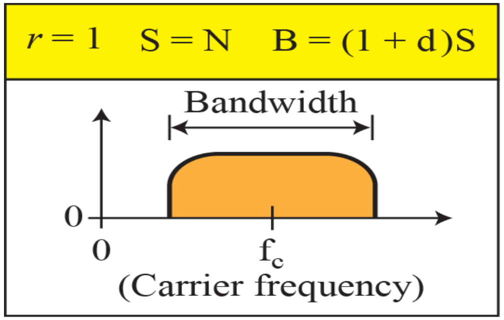
M-ASK

- ASK can also be modulated by a multi-level signal, again the implementation is easy, just a multiplication between carrier and base-band
- Remember that M is the number of levels, not the number of bits!



M-ASK bandwidth occupation

- We have seen that for analog modulation the bandwidth of a modulated signal is twice the bandwidth of the baseband signal, $B_{mod}=2B_b$.
- We have also seen that the rate of a digital signal is related to its baud rate (S) and is $B_b=S/2$.
- Ideally for an ASK modulation I would have a $B_{mod} = 2S/2 = S$. So the bandwidth of the modulated signal is in theory equal to the baud rate.



- More in general, because of non-ideal effects: the bandwidth of an ASK is $\mathbf{B_{mod} = (1+d) \times S}$, where d is between 0 and 1.

Example

We have an available bandwidth of 100 kHz which spans from 200 KHz to 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using 8-ASK with d = 0?

Solution

The middle of the bandwidth is located at 250 kHz. This means that our carrier frequency can be at fc = 250 kHz. We can use the formula for bandwidth to find the bit rate (with d = 0).

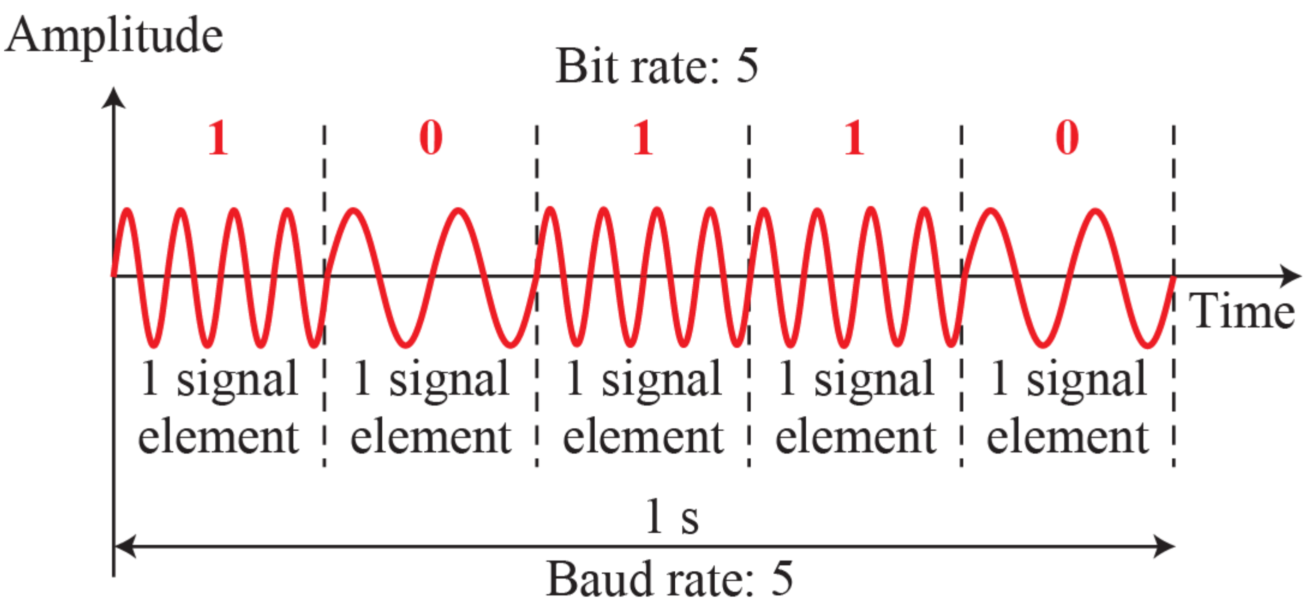
$$B_{mod} = (1+d) \times S = (0+1) \times S = S = 100KHz$$

$\Rightarrow$ $S=B_{mod} = 100KHz = 100$ Kbaud

$\Rightarrow$ $S=R/3$ $\Rightarrow$ $R=s*3 = 300$ Kb/s

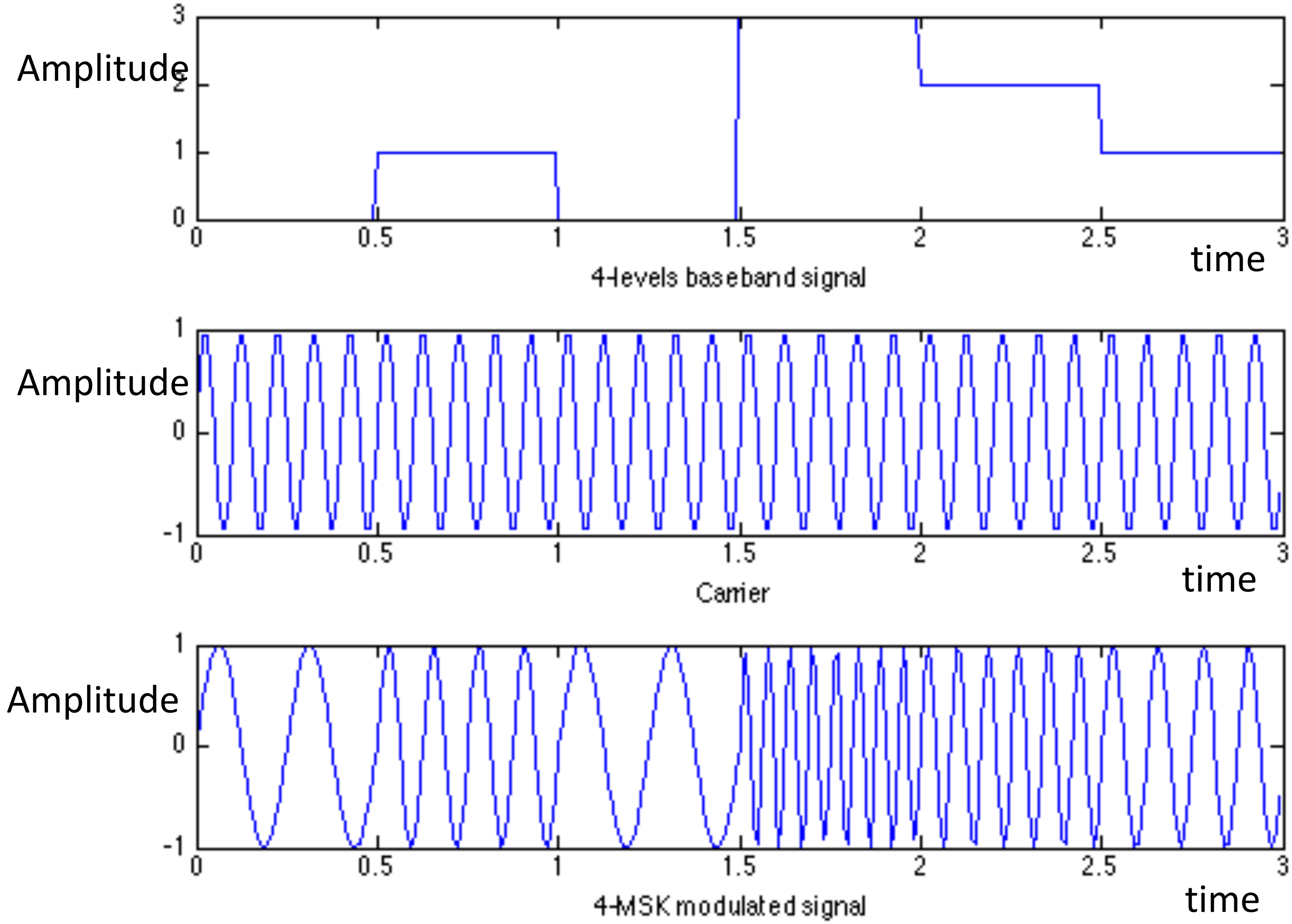
Frequency Shift Keying - FSK

- FSK modifies the frequency of the carrier to encode the different bits. In a 2-FSK we use a frequency f_1 to represent a 1 bit, and another frequency f_2 to represent a 0 bit.



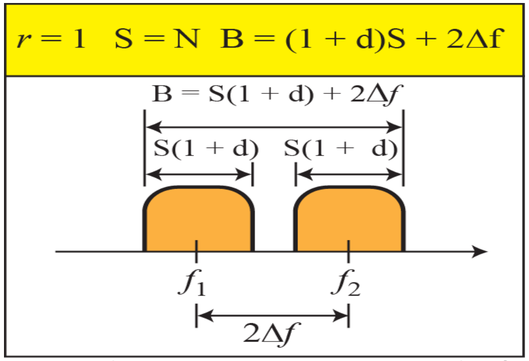
M-FSK

- FSK can also be modulated by a multi-level signal.



M-FSK bandwidth occupancy

- If we think of 2-FSK as two OOK signals, one at frequency f_1 and the other at frequency f_2, we get **B_{mod}=(1+d) x S + 2Δf,** where $2\Delta f=|f_1-f_2|$
- 2Δf needs to be at least equal to S to be demodulated



- For an M-FSK signal I need one OOK modulated signal for each level:
  ➔ **B_{mod}=(1+d) x S + (L-1)x2Δf**

- Since however the difference 2Δf needs to be at least equal to S, this becomes B_{mod}=(1+d) x S + (L-1)xS= S+Sd+SL-S
  ➔ **Bmod= S x (L+d)**, with d between 0 and 1

- In the ideal case, with d=0, B_{mod}=S x L

Example

We have an available bandwidth of 100 kHz which spans from 200 KHz to 300 kHz. What should be the carrier frequency and the bit rate if we modulated our data by using 2-FSK with d = 1?

Solution

This problem is similar to the one for ASK, but we are modulating by using FSK. The midpoint of the band is at 250 kHz.

$$B_{mod} = (L+d) \times S = (1+2) \times S = 100KHz$$

$$\rightarrow 3S=100KHz \rightarrow S=33.3Kbaud \rightarrow R=s*1 = 33.3Kb/s$$

Example

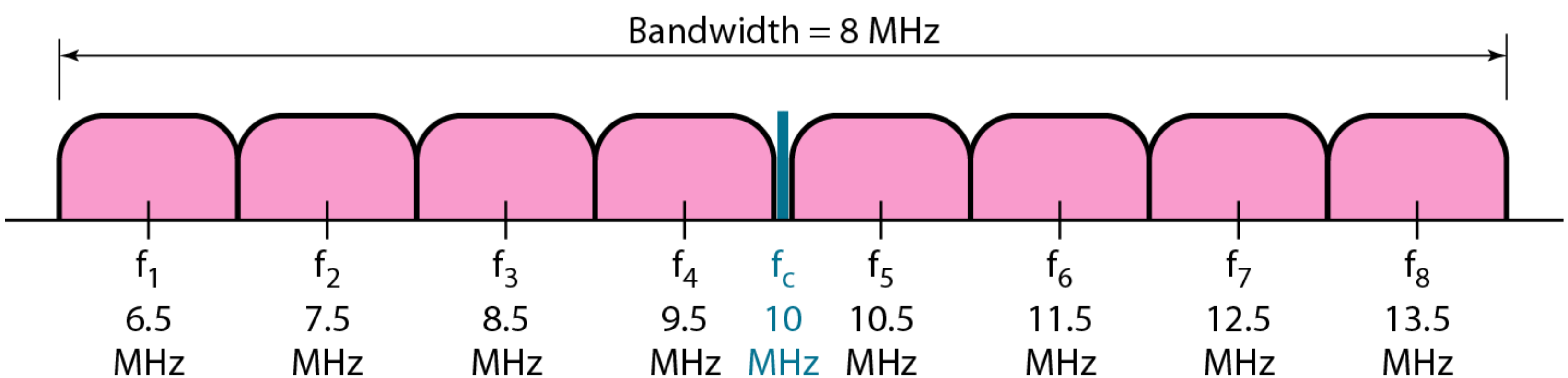
We need to send data 3 bits at a time at a bit rate of 3 Mbps, using M-FSK. The carrier frequency is 10 MHz. Calculate the number of levels (different frequencies), the baud rate, and the bandwidth, if d=0.

Solution

We can have $L = 2^3 = 8$.

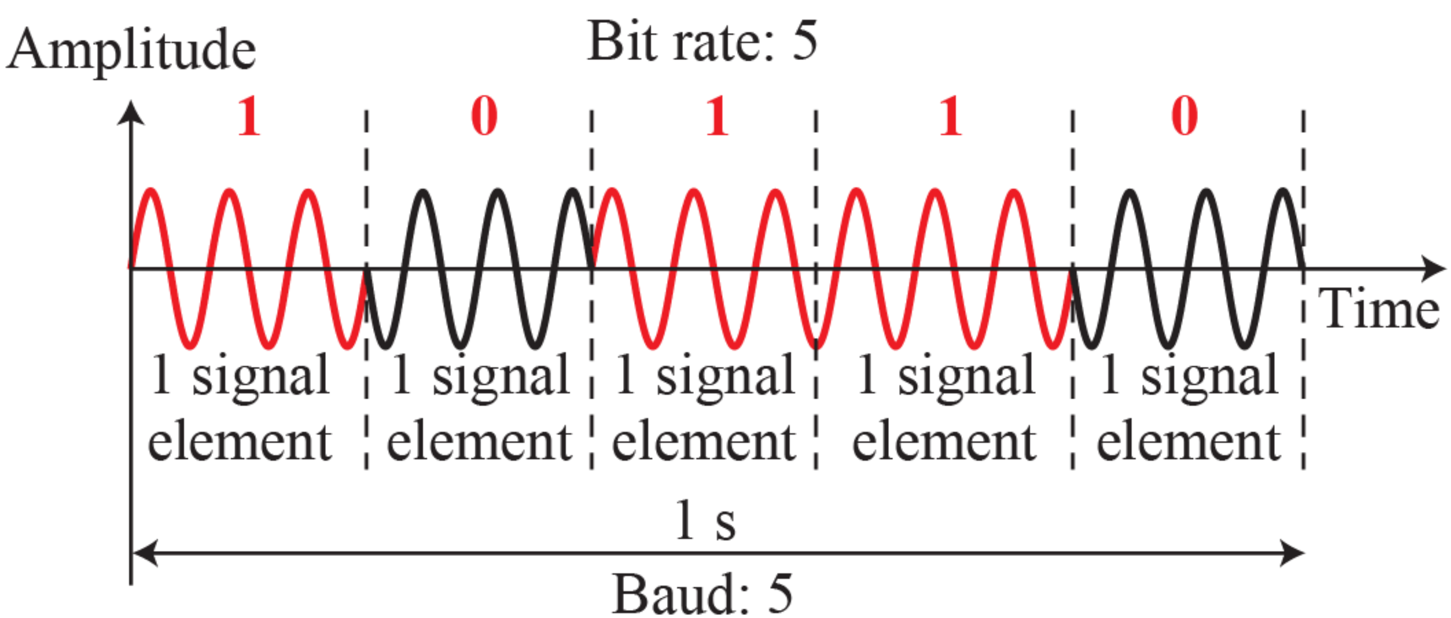
The baud rate is $S = 3$ Mb/s $/3 = 1$ Mbaud.

$B_{mod}=(L+d) \times S=L \times S = 8 \times 1 = 8$MHz



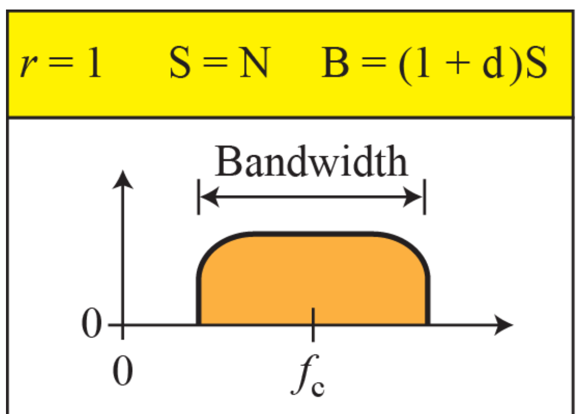
Phase Shift Keying - PSK

- PSK modifies the phase of the carrier to encode the different bits. In a 2-PSK we use a phase p_1 to represent a 1 bit, and another frequency p_2 to represent a 0 bit.



M-PSK bandwidth occupancy

- The bandwidth occupancy of PSK is the same as that of ASK, as we only use one carrier frequency.
- So for a PSK we have $\mathbf{B_{mod} = (1+d) \times S}$, where d is between 0 and 1.



Example

Find the bandwidth of a signal transmitting at 12 Mbps for QPSK (4-PSK). The value of d = 0.

Solution

For QPSK, 2 bits are carried by one signal element. This means that r = 2. So the signal rate (baud rate) is:

$$B_{mod} = (1+d) \times S$$

R= 12 Mbps, but we transmit 2 bits per slot, so S=R/2= 6 Mbaud.

With a value of d = 0, we have B = S = 6 MHz.

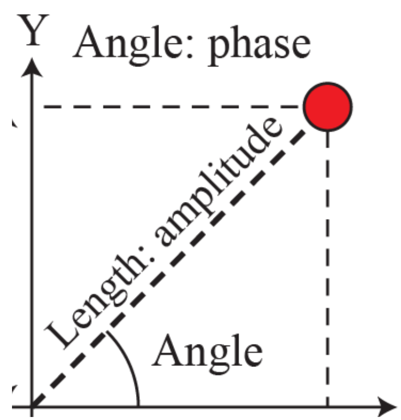
M-PSK good and bad

- PSK is good as it doesn’t change the amplitude and is less susceptible to non linearity and noise that affect the signal amplitude
- However the higher the M level, the less the phase difference between the M levels. So it becomes difficult to distinguish them

- There is a fourth modulation type called Quadrature Amplitude Modulation (QAM) which merges the benefits of ASK and PSK

Concept of a constellation diagram

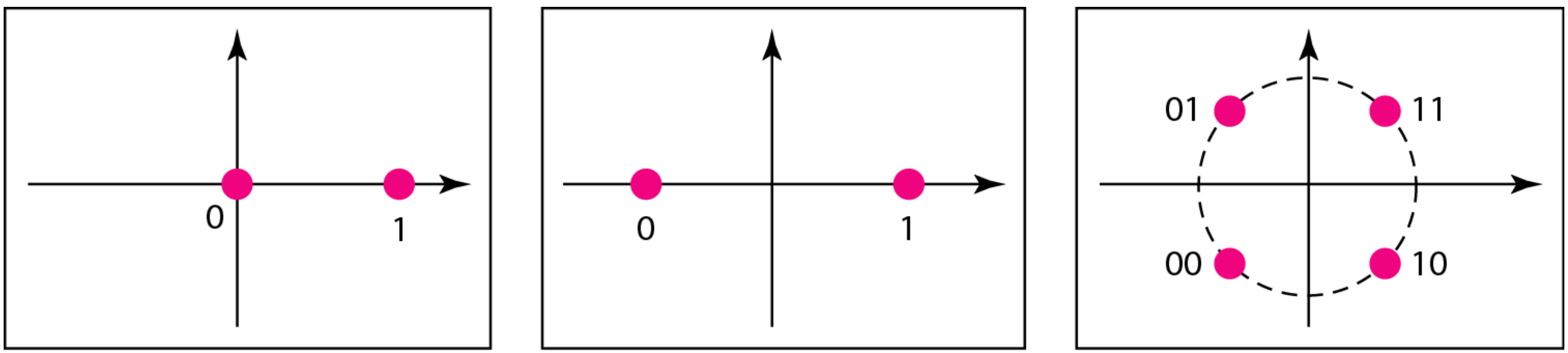
- Constellation are used to give a graphical representation of digitally modulated signals
- They are used to represent ASK, PSK, and a mix of the two, called QAM
- The distance of the point from the origin is the amplitude, and the angle is the phase
- For amplitude modulation the phase is always null, so we only show the x axis



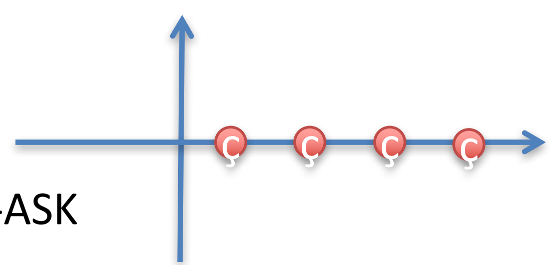
<u>Notice that both axes are now amplitudes</u>

Example

- Show the constellation diagrams for ASK (OOK), BPSK, and QPSK signals.



a. ASK (OOK) b. BPSK c. QPSK



4-ASK

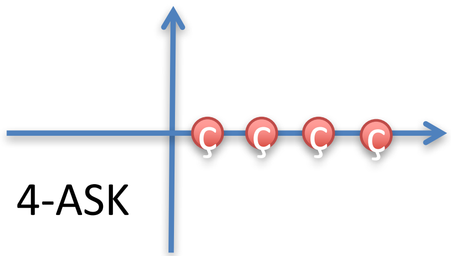
Quadrature Amplitude Modulation - QAM

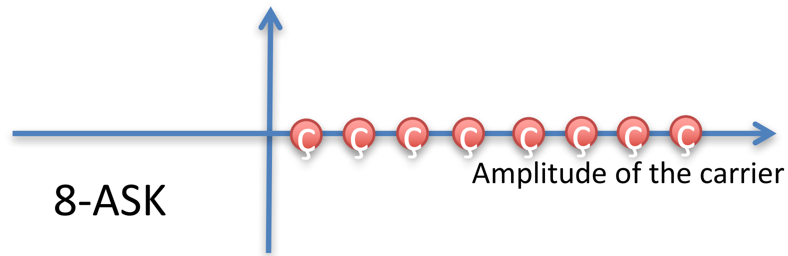
- The idea behind QAM is to use two carriers, one in-phase and the other in quadrature (i.e. 90 degrees, or $\pi/2$ apart) , with different amplitude levels for each carrier.

- QAM is one of the most used modulation both in wireless and electrical wired transmission (e.g., old modems, DSL).

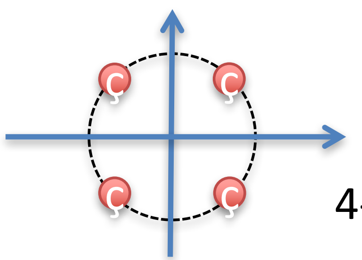
M-ASK, M-PSK constellation diagrams

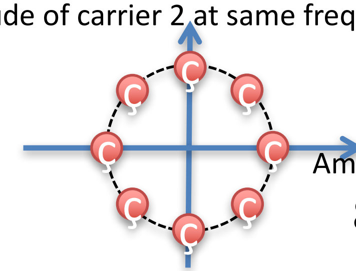
- For M-ASK I need to fit all levels into one axis





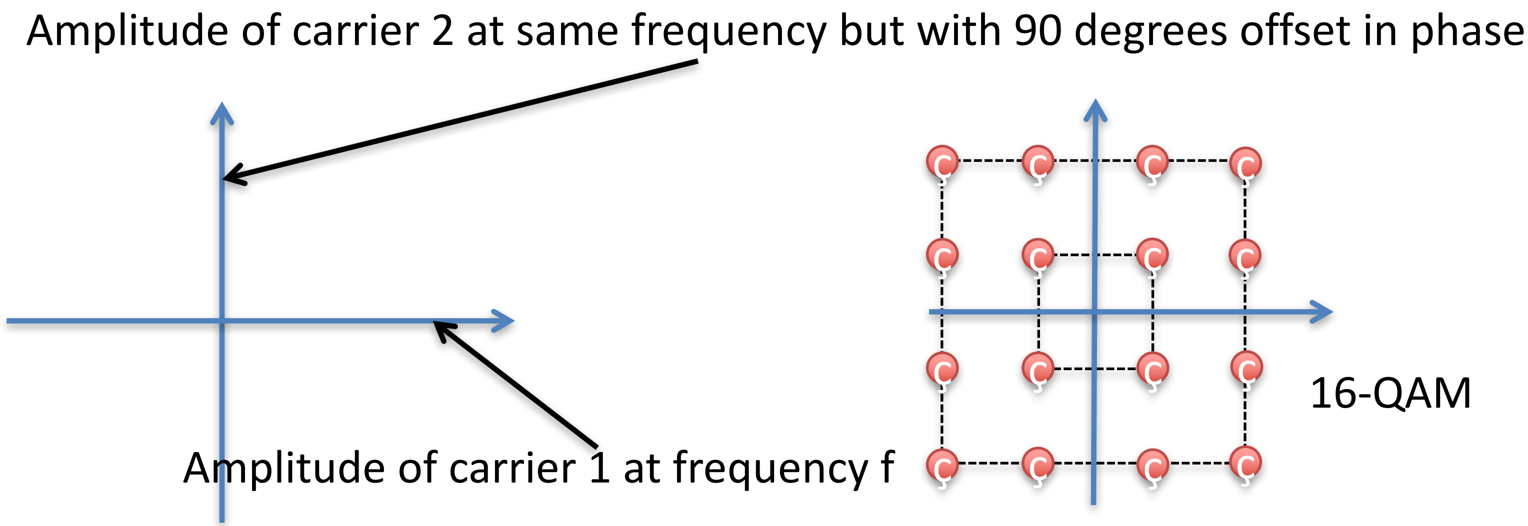
- For PSK I can use 2 axis because I can use phase difference, but since I only use the phase (i.e., amplitude is always 1), I need to fit into a circle





- The more levels I use, the more my points become squeezed, the more likely it will be to have a transmission error. For example PSK is only used up to 8 levels, i.e. 8-PSK, after which we use QAM.

QAM constellation diagram



- QAM uses the two axis, but since it allows different amplitudes it can use the entire 2-dimensional space, instead of constraining levels to be all in one axis (e.g., for M-ASK) or within a circle (e.g., for PSK)
- Each point is determined by two values, one for each axis: one value for the carrier 1 and another for the carrier 2
- QAM is used commercially up to 1024 levels (i.e., each symbol encodes a group of 10 bits)

Bandwidth occupancy of QAM

- The bandwidth occupancy for QAM is

  $\mathbf{B_{mod} = (1+d) \times S}$, with d between 0 and 1.
- Remember that S is the baud rate (or symbol rate), not the bit rate.
- So the higher the level of modulation I use the higher the data rate I can fit into a given bandwidth!

Example

- What is the Bandwidth occupancy of a 1Mbps signal modulated with QAM?
  - An M-QAM carries $\log_2 M$ bits per symbol.
  - The bit rate $R = S \times \log_2 M$ and $B_{mod} = (1+d) \times S = (1+d) \times R / \log_2 M$

  ➔ for d=1, $B_{mod} = 2 \times R / \log_2 M$
- If I use 16-QAM for example $B_{mod} = 2 \times 1\text{Mbps} / 4 = 500\text{KHz}$
- If I use 256-QAM, $B_{mod} = 2 \times 1\text{Mbps} / 8 = 250\text{KHz}$
- If I use 1024-QAM, $B_{mod} = 2 \times 1\text{Mbps} / 10 = 200\text{KHz}$

Example

- The other way around, what is the bit rate I can achieve if I have to fit a modulated QAM signal within a 1 MHz bandwidth?
  - An M-QAM carries $\log_2 M$ bits per symbol.
  - The bit rate $R = S \times \log_2 M$ and $B_{mod} = (1+d) \times S = (1+d) \times R / \log_2 M$

  ➔ for d=1, $R = B_{mod} \times \log_2 M / 2$
- If I use 16-QAM for example R= 1MHz x 4 / 2 = 2Mb/s
- If I use 256-QAM, R= 1MHz x 8 / 2 = 4Mb/s
- If I use 1024-QAM, R= 1MHz x 10 / 2 = 5Mb/s