CS1026 – Digital Logic Design

Boolean Algebra II

Alistair Morris [1]

[1]Distributed Systems Group
Trinity College Dublin

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Today's Overview

1 The Lab this week

2 More minimisation..

3 Sum of Products (SOP) Example

4 But what about POS?

Lab 1 [Brown, 2012]

- Design an XOR using only NAND gates
- $F(A, B) = A \oplus B$
- Questions?

Before minimisation can occur..

We must use one of the two standard forms:

- Standard Sum of Products (SOP)
- Standard Product of Sums (POS)

Standard Sum of Products (SOP) [Ciletti, 2003] I

In standard or canonical SOP form:

- All the variables present in each product term.
- E.g $F(A, B) = A + B$

Standard Sum of Products (SOP) [Ciletti, 2003] II

Step 1

- Write the Truth table to see all the possible values

Input		Output
A	B	F(A,B)=A+B
0	0	0
0	1	1
1	0	1
1	1	1

Table: $F(A, B)$

Standard Sum of Products (SOP) [Ciletti, 2003] III

Step 2

- Write the full product term for all the possible combinations

$$
\begin{aligned}
F(A,B) &= F(0,0).A.B + F(0,1).AB + F(1,0).A.B + F(1,1).A.B \\
&= 0.A.B + 1.AB + 1.A.B + 1.A.B \\
&= 1.AB + 1.A.B + 1.A.B \implies \text{Canonical or Standard Form}
\end{aligned} \tag{1}
$$

Standard Sum of Products (SOP) [Ciletti, 2003] IV

Some hints:

- A standard product or "min-term" denotes a product of all independent input variables for a function
- This corresponds to a row of the truth table with output of 1
- E.g. $A.B$ denotes a min term in the above example.

Truth table to min-terms example I

Step 1 – Understand the problem

- Write out an expression for the function that is true, when 2 out of 3 inputs are true.
- Output is false for all other input combinations.

Truth table to min-terms example II

Step 2 – Develop a truth table for the function

X	Y	Z	Mid-terms	Mid-term Designators	F
0	0	0	$X'.Y'.Z'$	m_0	$F(0,0,0) = F_0 = 0$
0	0	1	$X'.Y'.Z$	m_1	$F(0,0,1) = F_1 = 0$
0	1	0	$X'.Y.Z'$	m_2	$F(0,1,0) = F_2 = 0$
0	1	1	$X'.Y.Z$	m_3	$F(0,1,1) = F_3 = 1$
1	0	0	$X.Y'.Z'$	m_4	$F(1,0,0) = F_4 = 0$
1	0	1	$X.Y'.Z$	m_5	$F(1,0,1) = F_5 = 1$
1	1	0	$X.Y.Z'$	m_6	$F(1,1,0) = F_6 = 1$
1	1	1	$X.Y.Z$	m_7	$F(1,1,1) = F_7 = 0$

Truth table to min-terms example III

By the way:

- The min-term subscript corresponds to the binary of the input.
- All three independent input variables present in min-term.
- When input is 1, the variable appears in the Min-term
    - Otherwise the variable is complemented in the min-term

Truth table to min-terms example IV

Step 3 – Write the algebraic function equivalent to the truth table

- If the output function (F) is 1 for the min-term
  - Then the value appears in the algebraic form of the expression

$$\begin{aligned} F(X,Y,Z) &= F_0.m_0 + F_1.m_1 + F_2.m_2 + F_3.m_3 + F_4.m_4 + F_5.m_5 + F_6.m_6 + F_7.m_7 \\ &= \sum_{i=0}^{7}(F_i.m_i) \\ &= 0.m_0 + 0.m_1 + 0.m_2 + 1.m_3 + 0.m_4 + 1.m_5 + 1.m_6 + 0.m_7 \\ &= m_3 + m_5 + m_6 \end{aligned} \tag{2}$$

Truth table to min-terms example V

Using $\sum$ we can say:

- $F(X, Y, Z) = \sum m(3, 5, 6)$
  - Explicit Compact Min-term form
- $F(X, Y, Z) = \sum(3, 5, 6)$
  - Implicit Compact Min-term form

Also we find the complement of F:

- $F(X, Y, Z) = \sum m(0, 1, 2, 4, 7)$
  - Explicit Compact Min-term form
- $F(X, Y, Z) = \sum(0, 1, 2, 4, 7)$
  - Implicit Compact Min-term form

Obtaining the Standard Products of Sum (POS) I

POS is not used as much

- However the POS form is more efficient than SOP

Note

- All three independent variables are present..
  - .. in either complemented or uncomplemented form.

Obtaining the Standard Products of Sum (POS) II

For each pattern..

- If the independent variable value is 0, it is un-complemented
- If 1, it is complemented in the max-term which is the OR of all independent variables.

Example

- $X = 1, Y = 1, Z = 0$
- $M_6 = X' + Y' + Z'$

Each max-term will result in the output for that term being zero.

Obtaining the Standard Products of Sum (POS) III

Another exciting example..

.. *In the tutorial next week ;-)*

That's it (for now)

Thanks.. Any Questions?

You can ask later at:

morrisa5@scss.tcd.ie

Useful links

- Notes/Slides: bitbucket.com/morrisa5/DLD
- LinkedIn: ie.linkedin.com/in/alistair-morris-9712b247

References (Homework) I

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