Tutorial Exercises Week 9

Qs. 1 (Matrix Inverse via Cayley-Hamilton Theorem)

Let the matrix

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

1. Show that the characteristic polynomial, $p(t)$, of A is $-t^3 + 2 * t^2 + 3 * t - 6$
2. Determine A^{-1} using the Cayley-Hamilton Theorem.

Qs. 2 (Matrix Inverse by Matrix of Co-Factors)

Find the inverse of the following matrix, A, using the **matrix of co-factors** method.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$$

Recall for a Matrix $A = [a_{ij}]_{n \times n}$, the Minor M_{ij} is the determinant of the submatrix of A obtained by deleting the i^{th} row and the j^{th} column. The **matrix of co-factors** is the matrix $[C_{ij}]$ where $C_{ij} = (-1)^{i+j} M_{ij}$ and M_{ij} is the minor of the matrix entry a_{ij}. The inverse, A^{-1}, is obtained by:

$$A^{-1} = \frac{1}{|A|} * [C_{ij}]^T$$