Eigen Values and Eigen Vectors

Eigen Values and Vectors

Up to now we have been concerned with solving equations of the form

$$A * x = c$$

where A is a given matrix $[a_{ij}]_{n \times n}$ and c is a given constant (column) vector.
In the equation $A * x = c$, we solve for the unknown vector, x.
We now consider an equation in two unknowns:

$$A * x = \lambda * x$$

where λ ('lambda') is an unknown scalar and x is an unknown vector. A trivial solution to this equation is $x = 0$, the origin.

Definition: Eigen Value, Eigen Vector

$A * x = \lambda * x$

A trivial solution is $x = 0$, the origin vector, as for any matrix, A, $A * 0 = 0$.
Also for any scalar, λ, $\lambda * 0 = 0$. Therefore $A * 0 = \lambda * 0$.

Definition

Eigen Value; Eigen Vector
A scalar, λ, is the **eigen value** of an $n \times n$ matrix A, if there is a non trivial solution, x, to the equation $A * x = \lambda * x$. Such a vector, x, is called the **eigen vector** corresponding to the eigen value λ.

Example: Eigen Value and Vector

Example

Let the matrix

$$A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$

Later, we show that $\lambda = 3$ is an eigen value of A and a corresponding eigen vector is $x = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$. We can check

$A * x = \lambda * x$

as: $\begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \\ 3 \end{bmatrix} = 3 * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

Geometric View

Geometrically, given the eigen vector, x ($\neq 0$), the matrix, A, scales the vector by a scalar λ i.e. for a particular vector (the eigen vector), x, the matrix, A, multiplies it by λ where $\lambda \in \mathbb{R}$.
An eigen vector cannot be 0, but an eigen value may be 0.

$A * x = 0 * x$

If $A * x = 0$, where $x \neq 0$, then the matrix A has no inverse.
Assume $x \neq 0$ and $A * x = 0$. If A has an inverse A^{-1} then

$$\begin{aligned} A * x &= 0 \\ A^{-1} * (A * x) &= A^{-1} * 0 \\ x &= 0 \end{aligned}$$

a contradiction

Generally, a matrix A has an inverse iff 0 is not an eigen value of A.

We can express $A * x = \lambda * x$ as:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \lambda * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

i.e. rewrite as linear system

$$\begin{aligned} a_{11} * x_1 + a_{12} * x_2 \dots a_{1n} * x_n &= \lambda * x_1 \\ a_{21} * x_1 + a_{22} * x_2 \dots a_{2n} * x_n &= \lambda * x_2 \\ \vdots \quad &\quad \vdots \\ a_{n1} * x_1 + a_{n2} * x_2 \dots a_{nn} * x_n &= \lambda * x_n \end{aligned}$$

i.e

Eigen Values and Vectors (Cont'd)

$$
\begin{aligned}
(a_{11} - \lambda) * x_1 + a_{12} * x_2 \ldots a_{1n} * x_n &= 0 \\
a_{21} * x_1 + (a_{22} - \lambda) * x_2 \ldots a_{2n} * x_n &= 0 \\
\vdots \quad &\quad \vdots \\
a_{n1} * x_1 + a_{n2} * x_2 \ldots (a_{nn} - \lambda) * x_n &= 0
\end{aligned}
$$

This is a **Homogeneous** equation. In Matrix form:

$$
\begin{bmatrix}
(a_{11} - \lambda) & a_{12} & \ldots & a_{1n} \\
a_{21} & (a_{22} - \lambda) & \ldots & a_{2n} \\
\vdots & \vdots & & \vdots \\
a_{n1} & a_{n2} & \ldots & (a_{nn} - \lambda)
\end{bmatrix}
*
\begin{bmatrix}
x_1 \\ x_2 \\ \vdots \\ x_n
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ \vdots \\ 0
\end{bmatrix}
$$

This can can written as: $(A - \lambda * Id) * x = 0$ where $(A - \lambda * Id)$ is the matrix of the coefficients. Recall that Id is the Identity Matrix.

Eigen Values and Vectors (Cont'd)

The matrix, $\lambda * Id$, can be written as $\begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix}$:

and $A - \lambda * Id$ can be written as:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} - \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11} - \lambda) & a_{12} & \dots & a_{1n} \\ a_{21} & (a_{22} - \lambda) & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & (a_{nn} - \lambda) \end{bmatrix}$$

Eigen Values and Vectors (Cont'd)

From before, a system of Homogeneous equations either has:

- Only the trivial solution, i.e. $x = 0$. This occurs if $|A - \lambda * Id)| \neq 0$.
- or Infinitely many non-trivial solutions, in addition to the trivial solution.

We consider the non-trivial solutions and for non-trivial solutions the determinant $|A - \lambda * Id)| = 0$.
The determinant, $|A - \lambda * Id|$ is called the **Characteristic Determinant** of A and this gives rise to the **Characteristic Polynomial** of A. The equation $|A - \lambda * Id| = 0$ is called the **Characteristic Equation**. The solutions to the Characteristic Equation are the **eigen values** of the matrix, A.

Trace of Matrix = Sum of Eigen Values

The Trace of Matrix $M = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$ is the sum along the main diagonal i.e.

$Tr(M) = a_{11} + a_{22} + \cdots + a_{nn} = \Sigma_{i=1}^{n} a_{ii}$

Theorem

$Tr(M)$ = sum of the eigen values of M

Also

The product of the eigen values $= |M|$, the determinant of M.

Eigen Values and Vectors (Cont'd)

If A is a $n \times n$ matrix then $|A - \lambda * Id|$ gives rise to order n polynomial. The roots of this polynomial are the eigen values of A.

Example

Find the eigen values of the matrix, $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

The characteristic equation is: $\begin{vmatrix} 4-\lambda & 1 \\ 3 & 2-\lambda \end{vmatrix} = 0 \therefore$

$(4-\lambda) * (2-\lambda) - 3 = 0 \therefore$

$\lambda^2 - 6 * \lambda + 5 = 0 \therefore$ factorising we get,

Cont'd

Formula:Use the formula for the roots of $a * \lambda^2 + b * \lambda + c$

$$\begin{aligned} \lambda &= \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2 * a} \\ &= \frac{6 \pm \sqrt{36 - 20}}{2} = 3 \pm 2 \end{aligned}$$

either $\lambda = 1$ or $\lambda = 5$.
$\lambda^2 - 6 * \lambda + 5 = (\lambda - 1) * (\lambda - 5)$.
This results in two eigen values: $\lambda_1 = 1$ and $\lambda_2 = 5$.

Trace of Matrix is the sum along the main diagonal.
Let $M = [a_{ij}]_{n\times n}$ then $Tr(M) = a_{11} + a_{22} + \cdots + a_{nn}$
From Theorem: above;
If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigen values of M then
$\lambda_1 + \lambda_2 + \ldots, +\lambda_n = Tr(M)$.

Example:
From above, $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and the
eigen values are $\lambda_1 = 1$ and $\lambda_2 = 5$.
$Tr(A) = 4 + 2 = 6$ and also $Tr(A) = \lambda_1 + \lambda_2 = 1 + 5 = 6$.
Also,
$\lambda_1 * \lambda_2 * \ldots, *\lambda_n = |A|$
$|A| = 8 - 3 = 5$, Also $\lambda_1 * \lambda_2 = 1 * 5 = 5$ and so $|A| = \lambda_1 * \lambda_2$.

Eigen Vector Example

Example

Find the eigen vectors of the matrix, $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ corresponding to the eigen values $\lambda_1 = 1$ and $\lambda_2 = 5$.

For $\lambda_1 = 1$ the equation $A * x = \lambda_1 * x$ becomes:

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ i.e}$$

$$\begin{bmatrix} 4-1 & 1 \\ 3 & 2-1 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ i.e.}$$

$$\begin{bmatrix} 3 & 1 \\ 3 & 1 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ $\therefore$ this represents the linear equation:

$3 * x_1 + x_2 = 0$ i.e. $x_1 = -\frac{1}{3} * x_2$

Eigen Vector Example (Cont'd)

The eigen vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ has the form $\begin{bmatrix} -\frac{1}{3} * t \\ t \end{bmatrix}$ where t is a parameter for x_2. We can write $\begin{bmatrix} -\frac{1}{3} * t \\ t \end{bmatrix}$ as $t * \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix}$ and with $t = -3$, we can consider $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ as an eigen vector for eigen value $\lambda_1 = 1$.

Eigen Vector Example (Cont'd)

Check:

Show that $A * x = \lambda_1 * x$ when

$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, $\lambda_1 = 1$ and $x = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 \\ 3-6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= 1 * \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Eigen Vector Example (Cont'd)

Find the eigen vectors of the matrix, $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ corresponding to the eigen value $\lambda_2 = 5$.

For $\lambda_2 = 5$ the equation $A * x = \lambda_2 * x$ becomes:

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 5 * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ i.e}$$

$$\begin{bmatrix} 4-5 & 1 \\ 3 & 2-5 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ i.e.}$$

$$\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \therefore \text{ this represents the linear equation:}$$

$-x_1 + x_2 = 0$

$3 * x_1 - 3 * x_2 = 0$

From these equations: we have $x_1 = x_2$.

Eigen Vector Example (Cont'd)

The eigen vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ has the form $\begin{bmatrix} t \\ t \end{bmatrix}$ where t is a parameter for x_2. We can write $\begin{bmatrix} t \\ t \end{bmatrix}$ as $t * \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and with $t = 1$ we can consider $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as an eigen vector for eigen value $\lambda_2 = 5$.

Eigen Vector Example (Cont'd)

Check:

Show that $A * x = \lambda_2 * x$ when
$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, $\lambda_2 = 5$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 \\ 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$= 5 * \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigen Values and Vectors, 3×3 Matrices.

Cont'd

To find the solutions to the Equation:

$$A * x = \lambda * x$$

we first find the values for λ(there may be more than one solution for λ).
For a given value of λ, the eigen value, we find the corresponding eigen vector and so a solution for the vector, x.

Find the Eigen values for the matrix:

$$A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$

The Characteristic Equation is $|A - \lambda * Id| = 0$ i.e.

$$\begin{vmatrix} 5-\lambda & 6 & 2 \\ 0 & -1-\lambda & -8 \\ 1 & 0 & -2-\lambda \end{vmatrix} = 0$$

Cont'd

Calculating the determinant: (Expand along colum 1)

$$(5-\lambda) * \begin{vmatrix} (-1-\lambda) & -8 \\ 0 & (-2-\lambda) \end{vmatrix}$$

$$-0 * \begin{vmatrix} 6 & 2 \\ 0 & (-2-\lambda) \end{vmatrix}$$

$$+1 * \begin{vmatrix} 6 & 2 \\ (-1-\lambda) & -8 \end{vmatrix}$$

$$= (5-\lambda) * ((-1-\lambda) * (-2-\lambda)) - 0 + (6 * (-8) - (-1-\lambda) * 2))$$
$$= (5-\lambda) * (2 + \lambda + 2 * \lambda + \lambda^2) + (-48 + 2 + 2 * \lambda)$$
$$= (5-\lambda) * (\lambda^2 + 3 * \lambda + 2) + (2 * \lambda - 46)$$
$$= 5 * \lambda^2 + 15 * \lambda + 10 - \lambda^3 - 3 * \lambda^2 - 2 * \lambda + 2 * \lambda - 46$$
$$= -\lambda^3 + 2 * \lambda^2 + 15 * \lambda - 36$$

Cont'd

Solve for λ in the equation:

$$-\lambda^3 + 2 * \lambda^2 + 15 * \lambda - 36 = 0$$

Multiply across by 1

$$\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36 = 0$$

From properties of polynomials:
The roots of the polynomial $(\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36)$ divide 36.
Assuming integers: the factors of 36 are:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36.$$

By a process of elimination, 3 is a root as

$$3^3 - 2 * 3^2 - 15 * 3 + 36 = 0$$

Cont'd

Since 3 is a root of $(\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36)$ then $\lambda - 3$ is a factor:

Dividing $(\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36)$ by $\lambda - 3$ we get $(\lambda^2 + \lambda - 12)$
$\therefore$
$\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36 = (\lambda - 3) * (\lambda^2 + \lambda - 12)$.

Cont'd

$$
\begin{array}{l|l}
 & \lambda^2 + \lambda - 12 \\
\hline
\lambda - 3 & \lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36 \\
 & \lambda^3 - 3 * \lambda^2 \\
\hline
\text{subtract} & \lambda^2 - 15 * \lambda + 36 \\
 & \lambda^2 - 3 * \lambda \\
\hline
\text{subtract} & -12 * \lambda - 36 \\
 & -12 * \lambda - 36 \\
\hline
\text{subtract} & 0
\end{array}
$$

Cont'd

Factorise $\lambda^2 + \lambda - 12$,
Use the formula for the roots of $a * \lambda^2 + b * \lambda + c$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2 * a}$$

Cont'd

$$\lambda = \frac{-1 \pm \sqrt{1^2 - 4 * 1 * (-12)}}{2}$$
$$= \frac{-1 \pm 7}{2}$$

$\lambda = -4$ or $\lambda = 3$
From above we can factorise $\lambda^3 - 2 * \lambda^2 - 15 * \lambda + 36$ as $(\lambda - 3) * (\lambda + 4) * (\lambda - 3)$
The number, 3, is a repeated root of the characteristic equation.
Note:
The roots of $\lambda^2 + \lambda - 12$ may be obtained by inspection as $(-4) * 3 = -12 \therefore \lambda^2 + \lambda - 12 = (\lambda + 4) * (\lambda - 3)$.

Finding Eigen Vectors

For each eigen value we have a corresponding eigen vector, i.e. find a vector, x, such that for $\lambda = -4$:

$$\begin{aligned} A * x &= -4 * x \\ (A + 4 * Id) * x &= 0 \end{aligned}$$

i.e. Homogeneous equation with Augmented Matrix

$$\begin{bmatrix} 9 & 6 & 2 & 0 \\ 0 & 3 & -8 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

Reduce to Reduced Row Echelon Form.

Eigen Vector corresponding to $\lambda = -4$

Augmented matrix

$$\begin{bmatrix} 9 & 6 & 2 & 0 \\ 0 & 3 & -8 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

swap $R1$ and $R3$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -8 & 0 \\ 9 & 6 & 2 & 0 \end{bmatrix}$$

$R3 := R3 - 9 * R1$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -8 & 0 \\ 0 & 6 & -16 & 0 \end{bmatrix}$$

$R2 := \frac{R2}{3}$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -\frac{8}{3} & 0 \\ 0 & 6 & -16 & 0 \end{bmatrix}$$

Cont'd

$$R3 := R3 - 6 * R2$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -\frac{8}{3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rewrite this augmented matrix back as a system of linear equations:

$$\begin{aligned} x_1 + 2 * x_3 &= 0 \\ x_2 - \frac{8}{3} * x_3 &= 0 \end{aligned}$$

$\therefore$

$$\begin{aligned} x_1 &= -2 * x_3 \\ x_2 &= \frac{8}{3} * x_3 \end{aligned}$$

Cont'd

Thus the eigen vector corresponding to the eigen value -4:

$$x = \begin{bmatrix} -2 * x_3 \\ \frac{8}{3} * x_3 \\ x_3 \end{bmatrix} \text{ i.e. } x = x_3 * \begin{bmatrix} -2 \\ \frac{8}{3} \\ 1 \end{bmatrix}$$

(Use parameter s for x_3.)

$$x = s * \begin{bmatrix} -2 \\ \frac{8}{3} \\ 1 \end{bmatrix}$$

With $s = 3$ we have the eigen vector $\begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix}$

Cont'd

Check that $A * \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix} = -4 * \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} * \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 24 \\ -32 \\ -12 \end{bmatrix} = -4 * \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix}$$

Eigen Vector corresponding to $\lambda = 3$

With $A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$ and $\lambda = 3$ we seek vectors x such that

$$(A - 3 * Id) * x = 0.$$

i.e.

$$\begin{bmatrix} 2 & 6 & 2 \\ 0 & -4 & -8 \\ 1 & 0 & -5 \end{bmatrix} * x = 0$$

Cont'd

Augmented Matrix

$$\begin{bmatrix} 2 & 6 & 2 & 0 \\ 0 & -4 & -8 & 0 \\ 1 & 0 & -5 & 0 \end{bmatrix}$$

swap $R1$ and $R3$

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & -4 & -8 & 0 \\ 2 & 6 & 2 & 0 \end{bmatrix}$$

$R3 := R3 - 2 * R1$

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 6 & 12 & 0 \end{bmatrix}$$

$R2 := \frac{R2}{-4}$

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 6 & 12 & 0 \end{bmatrix}$$

Cont'd

$$R3 := R3 - 6 * R2$$

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rewrite this augmented matrix back as a system of linear equations:

$$\begin{aligned} x_1 - 5 * x_3 &= 0 \\ x_2 + 2 * x_3 &= 0 \end{aligned}$$

$\therefore$

$$\begin{aligned} x_1 &= 5 * x_3 \\ x_2 &= -2 * x_3 \end{aligned}$$

Cont'd

Thus the eigen vectors corresponding to the eigen value 3:

$$x = \begin{bmatrix} 5 * x_3 \\ -2 * x_3 \\ x_3 \end{bmatrix} = x_3 * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

Using parameter, s, for x_3 we have eigen vectors:

$x = s * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$, where s is a scalar.

With $s = 1$, we have an eigen vector: $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

Cont'd

Check

$$\begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ -6 \\ 3 \end{bmatrix}$$

$$= 3 * \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$