<u>Section VI</u>

<u>Electric Circuit Components</u>

- <u>Capacitors and Inductors</u>

(i) The Capacitor

A capacitor is a two-terminal element that stores energy in an electric field. Its ability to do this is given numerically by its '<u>Capacitance</u>' value –'C'.

A capacitor is formed by two conductors (usually parallel conducting plates) separated by a distance. Oftentimes a dielectric material is placed between the conductors to provide a means of separating the plates and enhancing the capacitance.
When a voltage is applied across the capacitor, both conductors will possess charge equal in magnitude but opposite in sign. These opposite

charges result in an electric field between the conductors.
'Capacitance' is defined as the ratio between the charge on the (positive) plate and the applied voltage.

i.e.

$$C = \frac{q(t)}{v(t)}$$

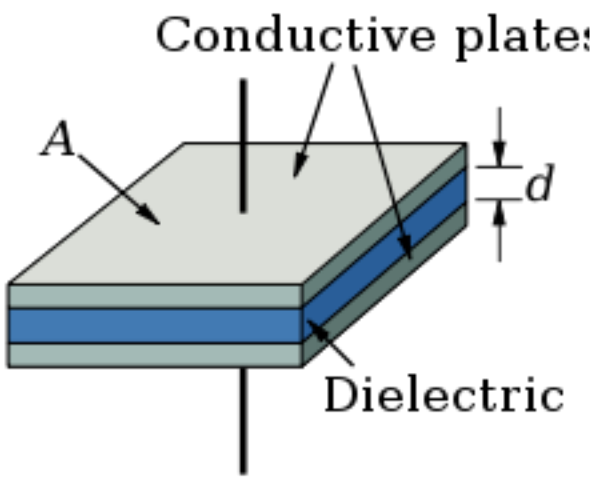
Although capacitance is the ratio of charge to voltage, it does not depend directly on either of these variables. Capacitance is determined only by the dimensions of the conductors, the separation distance between them and the electrical permittivity if the dielectric material between the conductors.

That is:

$$C = \frac{\varepsilon A}{d}$$

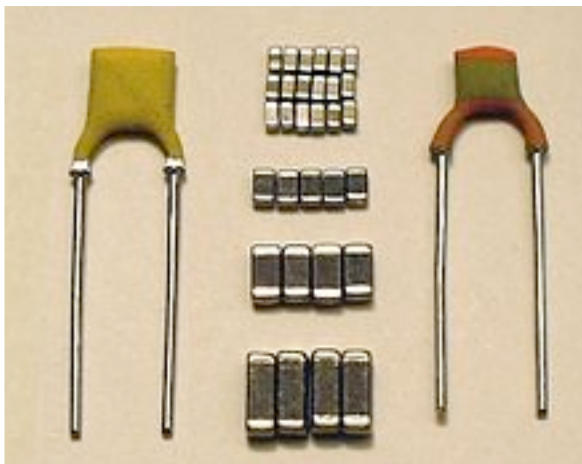
where:

- ‘ε’ is the electrical permittivity.
- ‘A’ is the common area between the conductors (or plates).
- ‘d’ is the separation distance between the conductors.



Fabrication methods for capacitors include parallel plates with ceramic or mica dielectric materials rolled with electrolytic paper or foil.

A **ceramic capacitor** is a fixed value capacitor in which ceramic material acts as the dielectric. It is constructed of two or more alternating layers of ceramic and a metal layer acting as the electrodes.



See Wikipedia

Electrolytic capacitors (e-caps) are polarized capacitors whose anode (+) is made of a particular metal on which an insulating oxide layer forms by anodization, acting as the dielectric of the electrolytic capacitor. A non-solid or solid electrolyte which covers the surface of the oxide layer in principle serves as the second electrode (cathode) (-) of the capacitor.

See Wikipedia

In order to obtain the relationship between the terminal voltage and the current in a capacitor, we differentiate:

$$q(t) = Cv(t) \Rightarrow$$
$$\frac{dq(t)}{dt} = c\frac{dv}{dt} \Rightarrow$$

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

From this equation it is clear that current will "pass through" a capacitor if and only if the terminal voltage is <u>changing</u> with respect to time. Hence a constant p.d. across a capacitor gives zero current!

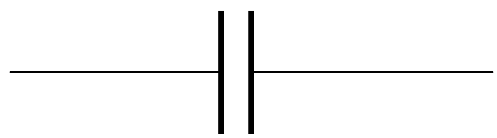
It is important to note that when current "passes through" a capacitor, <u>there is no movement of charge between the plates</u> (apart possibly from some small leakage current).

This effective ‘current’ without movement of charge is called displacement current – i.e. charge flow is effected over a displacement. This occurs because the changing electric field in the dielectric induces charge flow on either side of the plates.

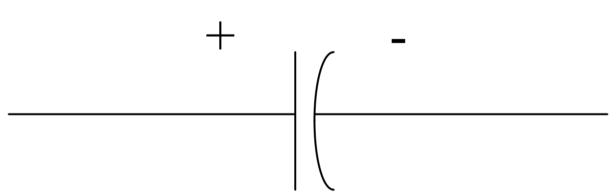
Units

The unit of Capacitance is the Farad (F) – defined as the Capacitance of a capacitor which contains one Coulomb (1C) of charge on its positive plate when a potential difference of 1 Volt is applied.

Electrical Symbol (for the Parallel Plate Capacitor)



Or (for the Electrolytic Capacitor)



Power Delivered to a Capacitor

The power delivered to the capacitor is:

$$p(t) = v(t)i(t)$$

But:

$$i(t) = C\frac{dv(t)}{dt} \Rightarrow$$

$$p(t) = Cv(t)\frac{dv(t)}{dt} \Rightarrow$$

$$\left[Since: \frac{du(x)v(x)}{dt} = \frac{v(x)du(x)}{dt} = \frac{u(x)dv(x)}{dt}\right] = \frac{C}{2}\frac{d(v^2(t))}{dt} \Rightarrow$$

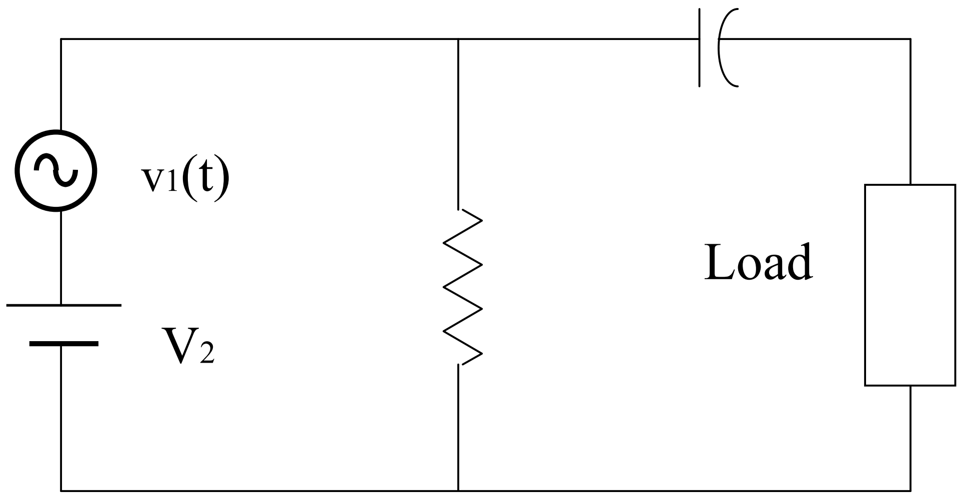
$$\frac{dw(t)}{dt} = \frac{C}{2}\frac{dv^2(t)}{dt} \Rightarrow$$

$$\boxed{w(t) = \frac{C}{2}v^2(t)}$$

i.e. the energy(work) stored in a capacitor.

Uses of Capacitors

Capacitors are widely used as filters. Consider the following setup:



Here we have an a.c. supply in series with a d.c. supply.

$$v_1(t) = V_1 \cos(\omega t)$$

$$v_2(t) = V_2$$

Then:

$$v_T(t) = v_1(t) + V_2$$

Since d.c. will not pass through a capacitor, the load receives or ‘sees’ only the a.c. signal – i.e. the d.c. component of the signal is filtered out.

Exercise

What would the load voltage look like if the resistor and the capacitor were interchanged in the above circuit?

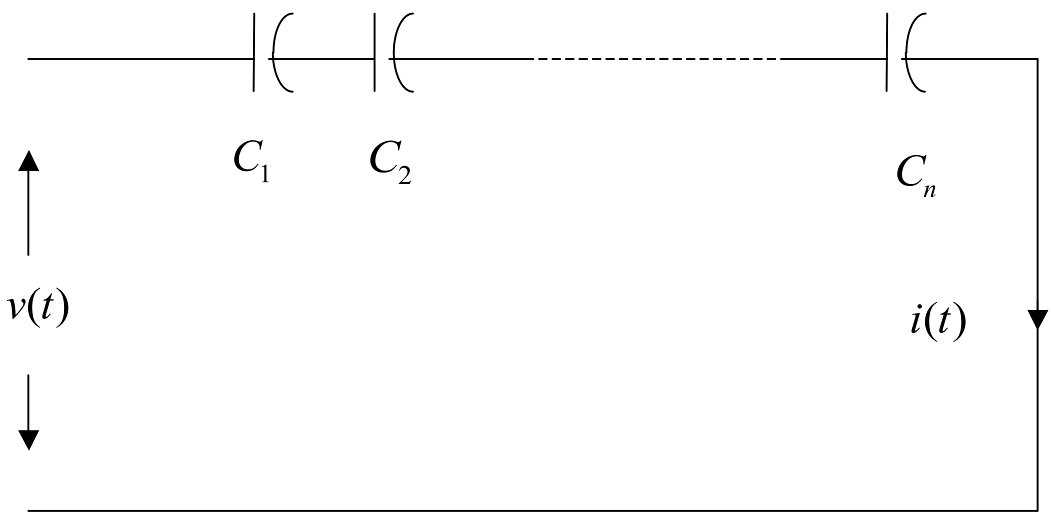
Example:

What is the capacitance of a system of two circular discs of radius 1cm separated by a distance of 1mm – comprised of plastic (ε_r = 6) ?

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_r \varepsilon_o \pi . r}{d} = 16.68 pF$$

Capacitors in Series

Consider the following setup:



$v_1(t)$, $v_2(t)$, ….. , $v_n(t)$ are the voltages across capacitors C_1, C_2, …. , C_n.

The charge residing on each capacitor is the same – regardless of the value of its capacitance (why?) i.e.

$$q(t) = q_1(t) = q_2(t) = ... = q_n(t)$$

and

$$v(t) = v_1(t) + v_2(t) + ... + v_n(t)$$

The total capacitance of the system is:

$$C_{Total} = \frac{q(t)}{v(t)}$$

Hence:

$$\frac{1}{C_{Total}} = \frac{v(t)}{q(t)} = \frac{v_1(t)}{q(t)} + \frac{v_2(t)}{q(t)} + \ldots.. + \frac{v_n(t)}{q(t)} \Rightarrow$$

$$\boxed{\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}}$$

- for capacitors in series

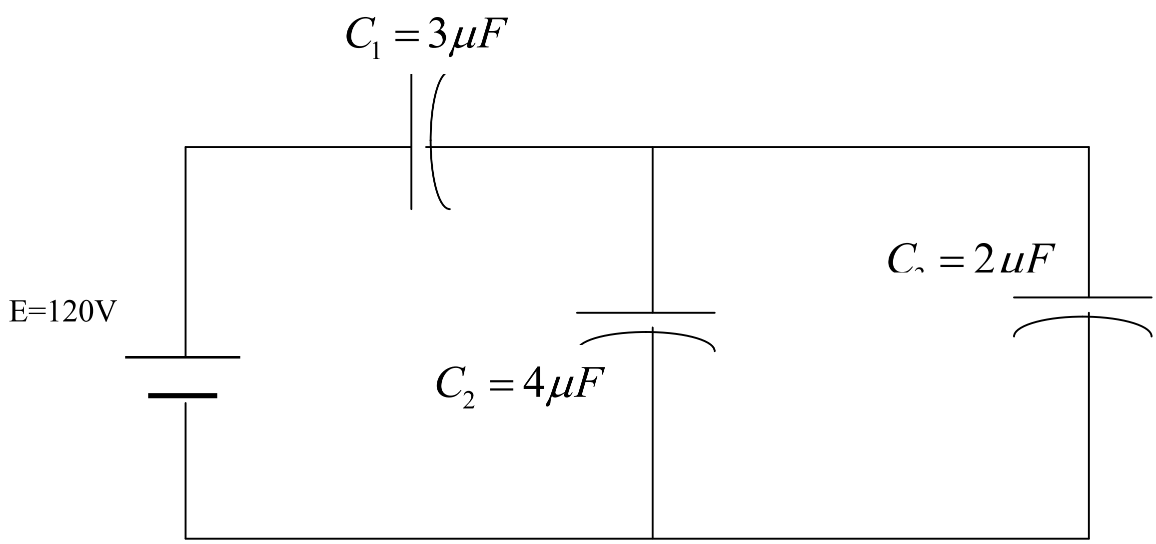
Capacitors in Parallel

$$\boxed{C_{Total} = C_1 + C_2 + \ldots.. + C_n}$$

Exercise: Prove this.

Example

Find the voltage across and charge on each capacitor in the following network:



Solution:

$$C' = C_2 + C_3 = 4 + 2 = 6\mu F$$

$$C_T = \frac{C_1 C'}{C_1 + C'} = \frac{3.6}{3+6} = 2\mu F$$

$$Q_T = C_T E = 2.10^{-6}.120 = 240\mu C = Q_1 = Q' \Rightarrow$$

$$V_1 = \frac{Q_1}{C_1} = \frac{240.10^{-6}}{3.10^{-6}} = 80V$$

$$V' = \frac{Q'}{C'} = \frac{240.10^{-6}}{6.10^{-6}} = 40V$$

or

$$V' = E(=120V) - 80V = 40V$$

$Then:$

$$Q_2 = C_2 V' = 4.10^{-6}.40 = 160\mu C$$

$$Q_3 = C_3 V' = 2.10^{-6}.40 = 80\mu C$$

(ii) The Inductor

The inductor is a two-terminal element which stores energy in its magnetic field. Its ability to do this is given by its '<u>Inductance</u>' value.

An inductor is normally a solenoid (i.e. a coil of wire, like a spring).

In order to enhance the inductance of the device, sometimes a ferrite core (high magnetic permeability) is inserted.

Inductance – denoted 'L' – is by definition the <u>total</u> magnetic flux linking (or threading) the inductor turns to the current flowing through the device.

i.e.

$$\boxed{L = \frac{\psi(t)}{i(t)}}$$

The inductance does not depend directly on either of these variables, however, but on the design of the inductor itself.

To a good approximation:

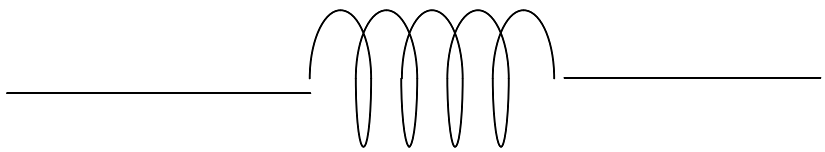
$$L \approx \frac{N^2 \mu A}{l}$$

- l – is the length of inductor
- A – is its cross sectional area
- N – number of turns
- μ - the magnetic permeability of the core

Units

The unit of inductance is the Henry (H) – defined as the inductance of an inductor which has 1A of current flowing through it and 1Wb flux in its core.

Electrical Symbol



From Faraday's Law:

$$v_L(t) = \frac{d\psi(t)}{dt}$$

Hence:

$$\boxed{v_L(t) = L\frac{di(t)}{dt}}$$

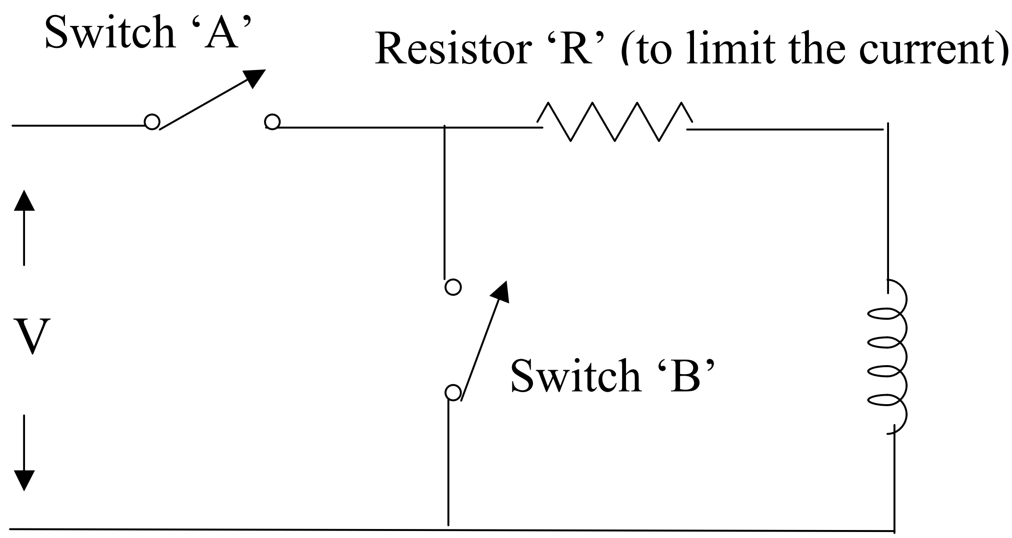
i.e. the relationship between the terminal voltage and the current in an inductor.

The polarity of this voltage is such as to <u>oppose</u> the change in current which induced it.

Example

Hence if I switch in the voltage supply below (by closing switch ‘A’) a p.d. will be induced across the inductor opposing the supply voltage.

This induced p.d. (across the inductor) is transient. It is often referred to as a ‘Back EMF’. From the above equation it is clear that the back-emf, because it depends only on the rate of change of magnetic flux, can easily exceed the supply voltage – albeit for a short period only. This phenomenon has useful applications, the most common is spark plugs in cars. The spark plug is essentially an inductor which when connected to the car battery (12V) will have a voltage induced across it large enough (~300V) to produce a spark between connected terminals and so ignite the fuel.



If very shortly having closed switch 'A', I simultaneously open switch 'A' and close switch 'B' a p.d. will be set up (induced) across the inductor with a polarity opposite to that of the previous p.d. !

Why?

Power Delivered to the Inductor

$$p(t) = v(t).i(t) = L.i(t).\frac{di(t)}{dt} \Rightarrow$$

$$p(t) = \frac{L}{2}.\frac{di^2(t)}{dt} = \frac{dw(t)}{dt}$$

Hence the energy stored in the inductor at any instant in time is:

$$\boxed{w(t) = \frac{L.i^2(t)}{2}}$$

Uses of inductors:

Inductors are used to generate sparks (arcing) for ignition.

They are also used in resonant circuits (tuning) and to model inductive effects in particular in electrical machines (which contain many loops of conductor in their motors/generators).

Inductors are also used to model inductive effects in transmission cables, etc.

Inductors in Series and Parallel

For inductors in series:

$$L_{Total} = L_1 + L_2 + ... + L_n$$

For inductors in parallel:

$$\frac{1}{L_{Total}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$$

It is left to the student to work these out.

Example

The current through a 10mH inductor is given by:

$$i(t) = 2t^2 e^{\frac{-t}{10}}; t > 0$$

Find:

a) The p.d. across the inductor
b) The power delivered to the inductor
c) The energy stored in the inductor

Solution:

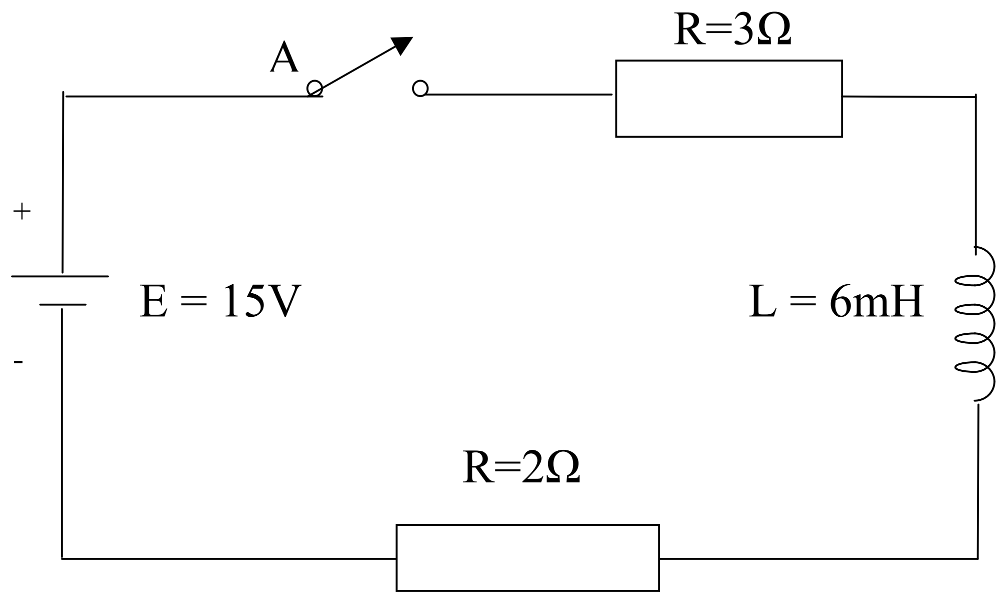
a) $$v(t) = L\frac{di(t)}{dt} = 0.04te^{\frac{t}{10}}(1 - \frac{t}{20})V$$

b) $$p(t) = v(t)i(t) = 0.8t^3 e^{\frac{-t}{5}}(1 - \frac{t}{20})W$$

c) $$w(t) = L\frac{i^2(t)}{2} = 0.02t^4 e^{\frac{-t}{5}} J$$

Example

Find the energy stored in the inductor in the following circuit when the current has stabilized having closed switch A.



When the current has stabilized i.e. E has been switched in long enough such that $\frac{di(t)}{dt} = 0$ then:

$$V = IR \Rightarrow$$

$$I = \frac{V}{R} = \frac{15}{3+2} = 3A \Rightarrow$$

$$W_{stored} = L\frac{I^2}{2} = \left(6x10^{-3}\right)\frac{(3)^2}{2} = 2.7x10^{-4}J = 27mJ$$