Calculating the inverse using row operations

v. 1.24

PROBLEM

Find (if possible) the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$

SOLUTION

- Step 1: Adjoin the identity matrix to the given matrix
- Step 2: Transform the matrix to the reduced row echelon form
- Step 3: Interpret the reduced row echelon form
- Comments

Step 1: Adjoin the identity matrix to the given matrix

Adjoining I_2 to the given matrix, we obtain the 2x4 matrix: $\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$

Step 2: Transform the matrix to the reduced row echelon form (Hide details)

Row Operation 1: $\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$ multiply the 1st row by **1/2** $\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$

Row Operation 2: $\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$ add **-1** times the 1st row to the 2nd row $\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \frac{-5}{2} & \frac{-1}{2} & 1 \end{array}\right]$

Row Operation 3: $\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \frac{-5}{\,} & \frac{-1}{\,} & 1 \end{array}\right.$ multiply the 2nd row by **-2/5** $\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{\,} & \frac{-2}{\,} \end{array}\right.$

$$\begin{bmatrix} & & 2 & 2 \end{bmatrix} \qquad \begin{bmatrix} & & 5 & 5 \end{bmatrix}$$

Row Operation 4:

$$\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{-2}{5} \end{array}\right]$$

add **-3/2** times the 2nd row to the 1st row

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{3}{5} \\ 0 & 1 & \frac{1}{5} & \frac{-2}{5} \end{array}\right]$$

Step 3: Interpret the reduced row echelon form

The matrix in reduced row echelon form obtained above can be interpreted as [C | D], where both C and D are 2x2 matrices.

Since C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ equals I_2

then D = $\begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$ is the **inverse** of the original matrix.

Comments

- **Shortcut:** To decide whether the matrix is invertible (without actually determining its inverse), stop the row operations as soon as the pattern of leading entries is established. If every column of the left half has a leading entry, the matrix is invertible; otherwise, it is singular.
- **Reference:** Kolman *Introductory Linear Algebra with Applications*, 6th Ed., p.73 - A Practical Method for Finding A^{-1}.

This concludes the solution of the problem. Do you want to

- solve another problem of the same type, or
- go to the main Toolkit page?