CSU44062 – Computational Linguistics
Assignment 1

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Question 1

(i) implies (ii)

The definition of conditional probability is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

By re-writing (i) in terms of P(A) we can get:

$$P(A) = \frac{P(A \cap B)}{P(B)}$$

Therefore we can imply:

$$P(A|B) = P(A)$$

Q.E.D

(ii) implies (ii)

We can derive the chain rule by re-arranging the definition of conditional probability as stated above to give us:

$$P(A \cap B) = P(A|B)P(B)$$

As shown above we also know that:

$$P(A|B) = P(A)$$

Therefore, by substitution we can imply:

$$P(A \cap B) = P(A|B)P(B)$$

Question 2

(a)

The definition of conditional probability is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Therefore, for our problem we can write:

$$P(gw|ps) = \frac{P(gw \cap ps)}{P(ps)}$$

From the table we know the following:

- $P(gw \cap ps) = \frac{28}{200} = 0.14$
- $P(ps) = \frac{30}{200} = 0.15$

Plugging this into the above equation gives us:

$$P(gw|ps) = \frac{0.14}{0.15} = 0.9333..$$

(b)

The definition of conditional probability is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Therefore, for our problem we can write:

$$P(ps|gw) = \frac{P(ps \cap gw)}{P(gw)}$$

From the table we know the following:

- $P(ps \cap gw) = \frac{28}{200} = 0.14$
- $P(gw) = \frac{168}{200} = 0.84$

Plugging this into the above equation gives us:

$$P(ps|gw) = \frac{0.14}{0.84} = 0.1666..$$

Question 3

(a)

Letting *vmel* stand for Speaker = 'Victor Meldrew' and *dbi* stand for DBI = true

Given the following:

- $P(vmel) = 0.01$
- $P(dbi \mid vmel) = 0.95$
- $P(dbi \mid \neg vmel) = 0.01$

First, let us calculate $P(dbi)$. This can be done using the following formula:

$$P(dbi) = [P(dbi \mid vmel) * P(vmel)] + [P(dbi \mid \neg vmel) * P(\neg vmel)]$$

$$P(dbi) = [0.95 * 0.01] + [0.01 * 0.99]$$

$$P(dbi) = 0.0194$$

From this we can then calculate $P(vmel \mid dbi)$ using the definition of conditional probability as follows:

$$P(vmel \mid dbi) = \frac{P(vmel \cap dbi)}{P(dbi)}$$

$$P(vmel \mid dbi) = \frac{0.95 * 0.01}{0.0194} = 0.4897$$

Following the same structure we can also calculate $P(\neg vmel \mid dbi)$ as follows:

$$P(\neg vmel \mid dbi) = \frac{P(\neg vmel \cap dbi)}{P(dbi)}$$

$$P(\neg vmel \mid dbi) = \frac{0.01 * 0.99}{0.0194} = 0.5103$$

Therefore, from this it is clear that $P(\neg vmel)$ is more likely given *dbi*.

Question 3

(b)

Given the following:

- $P(vmel) = 0.15$
- $P(dbi \mid vmel) = 0.95$
- $P(dbi \mid \neg vmel) = 0.01$

First, let us calculate $P(dbi)$. This can be done using the following formula:

$$P(dbi) = [P(dbi \mid vmel) * P(vmel)] + [P(dbi \mid \neg vmel) * P(\neg vmel)]$$

$$P(dbi) = [0.95 * 0.15] + [0.01 * 0.85]$$

$$P(dbi) = 0.1510$$

From this we can then calculate $P(vmel \mid dbi)$ using the definition of conditional probability as follows:

$$P(vmel \mid dbi) = \frac{P(vmel \cap dbi)}{P(dbi)}$$

$$P(vmel \mid dbi) = \frac{0.95 * 0.15}{0.1510} = 0.9437$$

Following the same structure we can also calculate $P(\neg vmel \mid dbi)$ as follows:

$$P(\neg vmel \mid dbi) = \frac{P(\neg vmel \cap dbi)}{P(dbi)}$$

$$P(\neg vmel \mid dbi) = \frac{0.01 * 0.85}{0.1510} = 0.0563$$

Therefore, in this scenario it is clear that $P(vmel)$ is much more likely given dbi.

Question 3

(c)

Given the following:

- $P(vmel) = 0.01$
- $P(dbi \mid vmel) = 0.95$
- $P(dbi \mid \neg vmel) = 0.001$

First, let us calculate $P(dbi)$. This can be done using the following formula:

$$P(dbi) = [P(dbi \mid vmel) * P(vmel)] + [P(dbi \mid \neg vmel) * P(\neg vmel)]$$

$$P(dbi) = [0.95 * 0.01] + [0.001 * 0.99]$$

$$P(dbi) = 0.0105$$

From this we can then calculate $P(vmel \mid dbi)$ using the definition of conditional probability as follows:

$$P(vmel \mid dbi) = \frac{P(vmel \cap dbi)}{P(dbi)}$$

$$P(vmel \mid dbi) = \frac{0.95 * 0.01}{0.0105} = 0.9056$$

Following the same structure we can also calculate $P(\neg vmel \mid dbi)$ as follows:

$$P(\neg vmel \mid dbi) = \frac{P(\neg vmel \cap dbi)}{P(dbi)}$$

$$P(\neg vmel \mid dbi) = \frac{0.001 * 0.99}{0.0105} = 0.0944$$

Therefore, in this scenario it is clear that $P(vmel)$ is much more likely given dbi.

Question 4

To calculate $P(cool : +)$ we can consider the scenarios where it is both cool AND noisy, and the scenarios where it is both cool AND NOT noisy as follows:

$$P(cool : +) = \frac{P(cool : +, noisy : +) + P(cool : +, noisy: -)}{500}$$

$$P(cool : +) = \frac{62 + 108}{500} = 0.34$$

To calculate $P(cool : +|\, noisy: +)$ we can use the definition of conditional independence similar to the previous questions as follows:

$$P(cool : +|\, noisy: +) = \frac{P(cool : +, noisy: +)}{P(noisy: +)}$$

From the table we can calculate $P(cool : +, noisy: +)$ quite simply as:

$$P(cool : +, noisy: +) = \frac{62}{500} = 0.124$$

Similarly, to calculate $P(noisy : +)$ we can consider the scenarios where it is both noisy AND cool, and the scenarios where it is both noisy AND NOT cool as follows:

$$P(noisy : +) = \frac{P(noisy : +, cool : +) + P(noisy : +, cool: -)}{500}$$

$$P(noisy : +) = \frac{62 + 38}{500} = 0.20$$

Plugging these values back into the formula for conditional independence above gives us:

$$P(cool : +|\, noisy: +) = \frac{0.124}{0.2} = 0.62$$

Two events are independent if $P(A \cap B) = P(A) * P(B)$, therefore for $cool : +$ to be independent of $noisy : +$ it must satisfy the following:

$$P(cool : +, noisy: +) = P(cool : +) * P\ (noisy : +)$$

$$0.124 = 0.34 * 0.20$$

$$0.124 \neq 0.068$$

Therefore, $cool : +$ is not independent of $noisy : +$.

<u>Question 5</u>

<u>(i) – Using table (2)</u>

First let us do:

$$P(cool: +|\, open: +) = \frac{P(cool: +, open: +)}{P(open: +)}$$

$$P(cool: +|\, open: +) = \frac{54 + 36}{100} = 0.90$$

Then find $P(noisy: +|\, open: +)$:

$$P(noisy: +|\, open: +) = \frac{60}{100} = 0.60$$

Then find $P(cool: +, noisy: +|\, open: +)$:

$$P(cool: +, noisy: +|\, open: +) = \frac{54}{100} = 0.54$$

Now, using these values we can calculate $P(cool: +\,|\, open: +, noisy: +)$ as follows:

$$P(cool: +\,|\, open: +, noisy: +) = \frac{P(cool: +, noisy: +\,|\, open: +)}{P(noisy: +\,|\, open: +)} = \frac{0.54}{0.60} = 0.90$$

Therefore, we can infer that $cool: +$ is conditionally independent of $noisy: +$ given $open: +$ since $P(cool: +\,|\, open: +, noisy: +) = P(cool: +|\, open: +)$.

ii) – Using table (3)

First let us do:

$$P(cool:\ +|\ open:\ -) = \frac{P(cool:\ +, open: -)}{P(open: +)}$$

$$P(cool:\ +|\ open:\ +) = \frac{8+72}{400} = 0.20$$

Then find $P(noisy:\ +|\ open:\ -)$:

$$P(noisy:\ +|\ open:\ -) = \frac{8+32}{400} = 0.10$$

Then find $P(cool:\ +, noisy:\ +|\ open:\ -)$:

$$P(cool:\ +, noisy:\ +|\ open:\ -) = \frac{8}{400} = 0.02$$

Now, using these values we can calculate $P(cool:\ +\ |\ open:\ +, noisy:\ +)$ as follows:

$$P(cool:\ +\ |\ open:\ -, noisy:\ +) = \frac{P(cool:\ +, noisy:\ +\ |\ open: -)}{P(noisy:\ +\ |\ open: -)} = \frac{0.02}{0.10} = 0.20$$

Therefore, we can infer that $cool:\ +$ is conditionally independent of $noisy:\ +$ given $open:\ -$ since $P(cool:\ +\ |\ open:\ -, noisy:\ +) = P(cool:\ +|\ open:\ -)$.