# MA2C03 Assignment 1
# Remarks

**General remarks:**

- When attempting to show that two expressions are equal, it isn't enough to assume they're equal and manipulate the expressions until both sides of the equality are obviously equal. For example, many proofs for the induction step in Question 5 began with the statement

$$1^3 + 2^3 + \cdots n^3 + (n+1)^3 = (1 + 2 + \cdots + n + (n+1))^2$$

  and proceeded to manipulate the expressions until both sides were the same. However, by writing the above, we've already assumed what we are trying to show, making the proof invalid. An example of how this style of argument might fail is as follows:

$$\begin{aligned} 1 &= -1 \\ 1^2 &= (-1)^2 \\ 1 &= 1, \text{ true} \\ \therefore 1 &= -1. \end{aligned}$$

- There was frequent use of many logical connectives like $\wedge$ and $\vee$ to say things that would be more naturally expressed in normal language.
- If I've written "why?" somewhere on your script, I mean that you've made an assertion without justifying it properly.

**Question 2:**

- It isn't true that if $x - y \in \mathbb{Q}$ then either $x = y$ or $x, y \in \mathbb{Q}$. For a counterexample, take $x = \sqrt{2}$, $y = \sqrt{2} + 1$. Their difference is rational, but $x \neq y$ and neither are rational.
- To show that the relation is not anti-symmetric, some said that we have already shown it is symmetric and therefore it can't be anti-symmetric. However, a relation can be symmetric and anti-symmetric. If we look at the relation '=', we know that it is symmetric and we see that $x = y$ and $y = x$ implies $x = y$.

**Question 3:**

- Many tried to show that $R$ is an equivalence relation without using the fact that $\mathcal{A}$ is a partition. However, in showing reflexivity and transitivity, it's necessary to make use of the properties of a partition. For example, in showing reflexivity we need the fact that $x \in A_\alpha$ for some $A_\alpha \in \mathcal{A}$.
- Some people wrote $(x, y) \in A_\alpha$ to mean that both $x$ and $y$ are in $A_\alpha$. But $(x, y)$ refers to a pair of elements of $A$, which will be in the set $A^2$. The correct way to express this is $x, y \in A_\alpha$.
- Some tried to show that $R$ is an equivalence relation by using properties of equivalence classes. We can't use these properties until we know that $R$ is an equivalence relation.
- It's already been shown in the lecture notes that the set of equivalence classes forms a partition, which was called $A/R$. Showing that the partition we get from $R$ is the same as $\mathcal{A}$ means showing $A/R = \mathcal{A}$. It isn't enough to show that the set of equivalence classes forms some partition.

**Question 4:**

- In the inductive step, we write $k+1 = i+j$. This doesn't mean that we set $i = k$ and $j = 1$, merely that we choose numbers $i$ and $j$ such that their sum is $k+1$.
- This proof would be valid if we could apply the inductive hypothesis to $i$ and $j$. To do this, we need to know that $i, j \leq k$. The proof fails because when $k = 0$, we can't find $i$ and $j$ such that $1 = i + j$ and $i, j \leq 0$.

**Question 6:**

- Drawing a graph of a function and appealing the the vertical line test doesn't constitute a proof of injectivity or surjectivity. To show injectivity we can show that the function is decreasing, or prove directly that $f(x) = f(y) \implies x = y$. To show surjectivity, we can use the fact that the function is decreasing to say $f(0) \leq f(x) \leq f(-2)$ for all $x \in [-2, 0]$.

**Question 7:**

- For '+' to be a binary operation on $A$, we need to know that the sum of two elements of $A$ is again in $A$. If we don't check this, we can't say that '+' is a binary operation on $A$, and so we can't say that $(A, +)$ is a semigroup.
- It's essential to check that the identity we find is contained in $A$. We also need to check that the inverses we get are still in $A$.
- A few people tried to show that $(0, 0)$ is the identity by saying

$$\begin{aligned} & (x, y) + (a, b) = (x, y) \\ \implies & a = 0, b = 0 \\ \implies & (a, b) = (0, 0). \end{aligned}$$

  However, this only shows that if we assume there is an identity, then it must be $(0, 0)$. It hasn't shown that $(0, 0)$ is an identity.