Theorem:
If A is a finite alphabet then the set of all words over A $A^* = \bigcup_{j=0}^{\infty} A^j$ is countably infinite.

Proof:
We showed A^j is a finite set for each j. In fact, $\#(A^j) = n^j$. $\bigcup_{j=0}^{\infty} A^j$ is therefore a countably infinite union of disjoint infinite sets.
Note that $A^j \cap A^k = \emptyset$ if $j \neq k$ as no words of length j can be of length k if $j \neq k$.
By Corollary 1 to the theorems that a countably infinite union of countably infinite sets is countably infinite,

$A^* = \bigcup_{j=0}^{\infty} A^j$ is countably infinite. (*q.e.d*)

Corollary 1: If A is a finite alphabet, the set of all languages over A is uncountably infinite.

Proof: Recall that a language L is any subset of words in the alphabet A, hence L is any subset of A*. Therefore the set of all languages over A is precisely P(A*), the power set of A*. We showed in the previous theorem that A* is countably infinite,
i.e. $A^* \sim N \Rightarrow P(A^*) \sim P(N)$, but we previously proved P(N) is uncountably infinite by putting it in one-to-one correspondence with the set of all sequences of 0s and 1s $\Rightarrow$ P(A*) is uncountably infinite. (*q.e.d*)

Corollary 2: The set of all programs in any programming language is countably infinite.

Proof: For any programming language, a program is a finite string over a finite alphabet, the set of characters allowable in that programming language. Let us call this finite alphabet A. Then the set of all programs in the given programming language is A*. Since A* ~ N as proven in the theorem, the set of all programs is countably infinite. (*q.e.d*)

Recall:
Theorem: A language over a finite alphabet is regular ⇔ it is given by a regular expression.

Recall the definition of a regular expression:
Definition: Let A be an alphabet.

1. Ø, ε, and all elements of A are regular expressions.
2. If w and w' are regular expressions then w∘w', w∪w' and w* are regular expressions.

Note that regular expressions sometimes have parentheses in order to change the priority of operations *, ∘ (concatenation) and ∪ (union). Therefore, any regular expression over the alphabet A is a string over the enlarged alphabet Ã.

Theorem: The set of all regular languages over a finite alphabet A is countably infinite.

Proof:
Since the alphabet A is finite, the enlarged alphabet Ã = A ∪ {"Ø", "ε", "*", "∘", "∪", "(", ")"} is also finite. By the theorem proven earler, Ã* is therefore countably infinite. A regular language then is given by a regular expression, which is a string over the enlarged alphabet Ã, hence an element of Ã*. Therefore, the set of all regular languages over the alphabet A is countably infinite. (*q.e.d*)

Moral of the Story:
Given a finite alphabet A, the set of regular languages (which is countably infinite) is much smaller than the set of all languages over A (which is uncountably infinite). Therefore, regular languages constitute a special category within the set of all languages over a given alphabet.