8.7 Applications of Formal Languages and Grammars as well as Automata Theory

1. Compiler architecture uses context-free grammars

2. Parsers - recognise if commands comply with the syntax of a language

3. Pattern matching and data mining - guess the language from a given set of words (applied in CS, genetics, etc.)

4. Natural language processing - example in David Wilkins' notes pp.40-44

5. Checking proofs by computers/automatic theorem proving - simpler example of this kind in David Wilkins' notes pp.45-57 that pertains to propositional logic

6. The theory of regular expressions enables

   (a) grep/awk/sed in Unix

   (b) More efficient coding (avoiding unnecessary detours in your code)

7. Biology - John Conway's game of life is a cellular automaton

8. Modelling of AI characters in games uses the finite state automation idea. Our character can choose among different behaviours based on stimuli - like a finite state automation reacting to input

9. Strategy and tactics in games - teach the opposition to recognise certain patterns, then suddenly change them to gain an advantage and score - used in football, fencing, etc.

10. Learning a sport/a numerical instrument/a new field or subject - split the information into blocks and learn how to combine them into meaningful patterns - uses notions from context-sensitive grammars.

11. Finite state automata and probability - chaos theory, financial mathematics.

etc...

9 Graph Theory

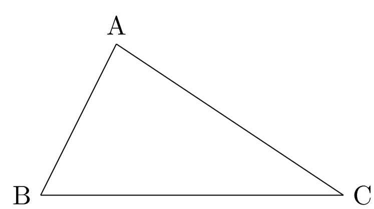
Task: Introduce terminology related to graphs; understand different types of graphs; learn how to put together arguments involving graphs.
An undirected graph consists of:

1. A finite set of points V called vertices
2. A finite set E of edges joining two distinct vertices of the graph.

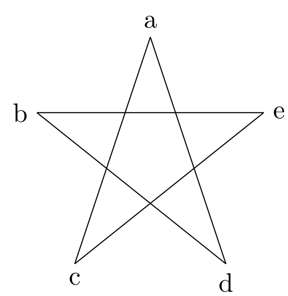
Understand the meaning of an edge better: Let V be the set of vertices. Consider $P(V)$, the power set of V. Let $V_2 \subseteq P(V)$ consist of all subsets of V containing exactly 2 points, **i.e.** $V_2 = \{A \in P(V) \mid \#(A) = 2\}$
Identify each element in V_2 with the edge joining the two points. In other words, if $\{a, b\} \in V_2$, then we can let ab be the edge corresponding to $\{1, b\}$.

Examples:

1. A triangle is an undirected graph.
   $V = \{A, B, C\}$

   3 possible 2 element subsets of V: $\{A, B\} \to AB$
   $\{A, C\} \to AC$
   $\{B, C\} \to BC$
   $E = \{AB, AC, BC\}$
2. A pentagram is an example of an undirected graph.
   $V = a, b, c, d, e$
   $E = \{ac, ad, be, ce, bd\}$



Convention: The set of vertices cannot be empty, **i.e.** $V \neq 0$.

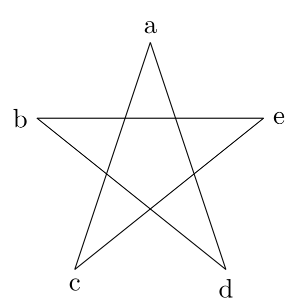
Q: If $V \neq \emptyset$, what is the simplest possible undirected graph?

A: A graph consisting of a single point, **i.e.** with one vertex and zero edges.

Definition: A graph is called trivial if it consists of one vertex and zero edges.
Next, study how vertices and edges relate to each other.

Definition: If v is a vertex of some graph, if e is an edge of that graph, and it $e = vv'$ for v' another vertex, then the vertex v is called incident to the edge e and the edge e is called incident to the vertex v.

Example:
b is incident to edges be and bd
be is incident to vertices b and e



Definition: Let (V, E) be an undirected graph. Two vertices $A, B \in V \quad A \neq B$ are called adjacent if $\exists$ edge $AB \in E$.
We represent the incidence relations among the vertices V and edges E of an undirected graph via:

1. An incidence table
2. An incidence matrix

Legend:

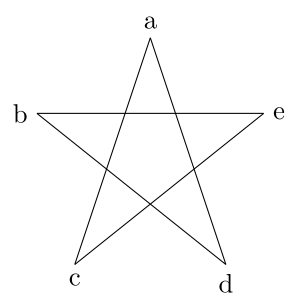
1 an incidence relation holds

0 an incidence relation does not hold

From the pentagram:

$V = \{a, b, c, d, e\}$

$E = \{ac, ad, be, bd, ce\}$



The incidence table is:

	ac	ad	be	bd	ce
a	1	1	0	0	0
b	0	0	1	1	0
c	1	0	0	0	1
d	0	1	0	1	0
e	0	0	1	0	1

Correspondingly, the incidence matrix is:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Note that for the incidence matrix to make sense, we need to know that vertices were considered in the order $\{a, b, c, d, e\}$ and edges in the order $\{ac, ad, be, bd, ce\}$. If we shuffle either set, the incidence matrix changes. With this in mind, we can now rigorously define the incidence matrix:

Definition: Let (V, E) be an undirected graph with m vertices and n edges. Let vertices be ordered as $v_1, v_2, ..., v_m$, and let the edges be ordered

$e_1, e_2, ..., e_n$. The incidence matrix for such a graph is given by $\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$,
where the entry a_{ij} in row i and column j has the value 1 if the i^{th} vertex is incident to the j^{th} edge and has value 0 otherwise.
Similarly, we can define the adjacency table and the adjacency matrix of a graph: