2. The formulaic/set builder method: give a formula that generates all elements of the set.
   $A = \{x \in \mathbb{N} \mid 0 \leq x \wedge x \leq 5\} = \{0, 1, 2, 3, 4, 5\} = \{x \in \mathbb{N} : 0 \leq x \wedge x \leq 5\}$

Using $\mathbb{N}$ and the set-builder method, we can define:
$\mathbb{Z} = \{m - n \mid \forall m, n \in \mathbb{N}\}$
$n = 0$ and m any natural number $\Rightarrow$ we generate all of $\mathbb{N}$
$m = 0$ and n any natural number $\Rightarrow$ we generate all negative integers
$\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z} \wedge q \neq 0\}$

Definition: A set A is called finite if it has a finite number of elements; otherwise, it is called infinite.

3.2 Set Operations

Task: Understand how to represent sets by Venn diagrams. Understand set union, intersection, complement, and difference.

Definition: Let A, B be sets. A is a subset of B if all elements of A are elements of B, **i.e.** $\forall x(x \in A \to x \in B)$. We denote that A is a subset of B by $A \subseteq B$

Example: $\mathbb{N} \subseteq \mathbb{Z}$

Definition: Let A, B be sets. A is a proper subset of B if $A \subseteq B \wedge A \neq B$, **i.e.** $A \subseteq B \wedge \exists x \in B \;\; s.t. \;\; x \notin A$.

Notation: $A \subset B$

Example: $\mathbb{N} \subset \mathbb{Z}$ since $\exists (-1) \in \mathbb{N}$

NB: $\forall A$ a set, $\emptyset \subseteq A$

Recall: $B \subseteq C$ means $\forall x(x \in B \to x \in C)$, but $\emptyset$ has no elements, so in $\emptyset \subseteq A$ the quantifier $\forall$ operates on a domain with no elements. Clearly, we need to give meaning to $\exists$ and $\forall$ on empty sets.

$$\left.\begin{array}{l} \underline{\text{Boolean Convention}} \\ \forall \text{ is true on the empty set} \\ \exists \text{ is false on the empty set} \end{array}\right\} \text{Consistent with common sense}$$

Definition: Let A, B be two sets. The union $A \cup B = \{x \mid x \in A \vee x \in B\}$

Definition: Let A, B be two sets. The intersection $A \cap B = \{x \mid x \in A \wedge x \in B\}$

Definition: Let A, B be sets. A and B are called disjoint if $A \cap B = \emptyset$

Definition Let A, B be two sets. $A - B = A \backslash B = \{a \mid x \in A \;\wedge\; x \notin B\}$

Examples:
$$\begin{array}{ll} A = \{1, 2, 5\} & B = \{1, 3, 6\} \\ A \cup B = \{1, 2, 3, 5, 6\} & A \cap B = \{1\} \\ A \backslash B = \{2, 5\} & B \backslash A = \{3, 6\} \end{array}$$

Definition: Let A, U be sets s.t. $A \subseteq U$. The complement of A in U = $U \backslash A = A^C = \{x \mid x \in U \wedge x \notin A\}$

Remark: The notation A^C is unambiguous only if the universe U is clearly defined or understood.