CS3081 – Computational Mathematics
Assignment 1

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Question 3.2 – *Determine the root of $f(x) = x - 2e^{-x}$ by:*

(a) *Using the **bisection method**. Start with a = 0 and b = 1 and carry out the first three iterations.*

First we must solve for $f(a)$ and $f(b)$ and inspect their signs:

$$f(a) = 0 - 2 = -2$$

$$f(b) = 1 - 0.74 = 0.26$$

From this we can see that **f(a) < 0** and **f(b) > 0**.

We must then estimate a numerical solution by finding the midpoint of a and b (bisection).

$$x_{NS1} = \frac{a+b}{2}$$

$$x_{NS1} = \frac{0+1}{2}$$

$$x_{NS1} = 0.5$$

Using this numerical solution estimate, evaluate the value of the function at this midpoint and inspect the sign.

$$f(x_{NS1}) = 0.5 - 2e^{-0.5} = -0.71$$

Since the result of $f(x_{NS1})$ is negative, for the next iteration set a = x_{NS1} = 0.5 **since f(a) must be less than 0**.

We then calculate the second iteration now with a = 0.5 and b = 1. First, estimate a numerical solution by finding the midpoint of our new a and b.

$$x_{NS2} = \frac{a+b}{2}$$

$$x_{NS2} = \frac{0.5+1}{2}$$

$$x_{NS3} = 0.75$$

Using this numerical solution estimate, evaluate the value of the function at this midpoint and inspect the sign.

$$f(x_{NS2}) = 0.75 - 2e^{-0.75} = -0.19$$

Since the result of $f(x_{NS2})$ is negative, for the next iteration set a = x_{NS2} = 0.75 **since f(a) must be less than 0**.

We then calculate the third iteration now with a = 0.75 and b = 1. First, estimate a numerical solution by finding the midpoint of our new a and b.

$$x_{NS3} = \frac{a+b}{2}$$

$$x_{NS3} = \frac{0.75+1}{2}$$

$$x_{NS3} = 0.875$$

Using this numerical solution estimate, evaluate the value of the function at this midpoint and inspect the sign.

$$f(x_{NS3}) = 0.875 - 2e^{-0.875} = 0.04$$

Since the result of $f(x_{NS3})$ is positive, for the next iteration we would have set b = x_{NS3} = 0.875 **since f(b) must be more than 0**.

From our three iterations we can estimate that the root of $f(x) \approx 0.875$

(b) Using the ***<u>secant method</u>****. Start with the two points $x_1 = 0$ and $x_2 = 1$, and carry out the first three iterations.*

Since we are given the two starting points, evaluate the value of $f(x)$ at both of these points.

$$f(x_1) = 0 - 2 = -2$$

$$f(x_2) = 1 - 0.74 = 0.2642$$

Now, using these points and the identity for a secant containing two points, solve for x_3.

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{f(x_2) - 0}{x_2 - x_3}$$

$$x_3 = x_2 - \frac{f(x_2)(x_1 - x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = 1 - \frac{0.2642(0 - 1)}{-2 - 0.2642}$$

$$x_3 = 0.8833$$

Using this, we then calculate the second iteration using the points $(x_2, f(x_2))$ and $(x_3, f(x_3))$ to solve for x_4

$$x_4 = x_3 - \frac{f(x_3)(x_2 - x_3)}{f(x_2) - f(x_3)}$$

$$x_4 = 0.8833 - \frac{0.0565(1 - 0.8833)}{0.2642 - 0.0565}$$

$$x_4 = 0.8516$$

Using this, we then calculate the third iteration using the points $(x_3, f(x_3))$ and $(x_4, f(x_4))$ to solve for x_5

$$x_5 = x_4 - \frac{f(x_4)(x_3 - x_4)}{f(x_3) - f(x_4)}$$

$$x_5 = 0.8516 - \frac{-1.8632 \times 10^{-3}(0.8833 - 0.8516)}{0.0565 - (-1.8632 \times 10^{-3})}$$

$$x_5 = 0.8526$$

From our three iterations we can estimate that the root of $f(x) \approx 0.85$

*(c) Using the **Newtons method**. Start with x = 1 and carry out the first three iterations.*

First we must determine $f'(x)$, the derivative of f with respect to x.

$$f(x) = x - 2e^{-x}$$

$$f'^{(x)} = 1 + 2e^{-x}$$

Since we are our starting point, evaluate the value of $f(x)$ at this point.

$$f(x_1) = 1 - 0.74 = 0.2642$$

Now, using the identity for the slope of a tangent at the point $(x_1, f(x_1))$, solve for x_2.

$$f'(x_1) = \frac{f(x_1) - 0}{x_1 - x_2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \frac{0.2642}{1.7358}$$

$$x_2 = 0.8478$$

Using the new point $(x_2, f(x_2))$, solve for x_3.

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.8478 - \frac{-8.9126 \times 10^{-3}}{1.7045}$$

$$x_3 = 0.8530$$

Using the new point $(x_3, f(x_3))$, solve for x_4.

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 0.8530 - \frac{7.3078 \times 10^{-4}}{1.7053}$$

$$x_3 = 0.8526$$

From our three iterations we can estimate that the root of $f(x) \approx 0.85$