ST3009 Mid-Term Test 2018

Attempt **all** questions. Time: 1 hour 30 mins.

1.

(i)

(a) Solve the equation -5x+20=25 (find the value of x) [1 mark]
(b) Solve the equation 5(2x+1)+3=10 (find the value of x) [1 mark]
(c) Suppose x-y=0 and x+y=1. What are the values of x and y? [1 mark]
(d)) Is x/2+y/2+z/2 = (x+y+z)/2 ? Briefly explain. [1 mark]
(e) Simplify (xy+xz)/x [1 mark]

(ii) Define the term "random variable" and give an example. [5 marks]

(iii)What is the probability mass function of a discrete random variable? Give an example. [5 marks]

Let X and Y be independent random variables that take values in set {-1,0,+1}. Assume that X and Y are uniformly distributed on {-1, 0, 1} i.e. the probability of each value occurring is the same. Let V =2X+2Y.

(iv) Calculate E[X] and E[V] [5 marks]

(v) Define what it means for two random variables to be independent. [5 marks]

(vi) Are V and X independent? Explain with respect to the definition of independence. [5 marks]

2.

(i) Define the conditional probability of a random event and state Bayes Theorem. [5 marks]

(ii) Suppose two websites A and B take hotel bookings. Site A takes 60% of all bookings and site B takes 40%. However, only 75% of the bookings made on site A result in positive reviews after the hotel stay, while on site B it is 90%. Given that a booking received a positive review, what is the probability that booking was made on site B ? Hint: use Bayes Theorem. [10 marks]

3. Data is stored in encoded form across 10 disks to provide some protection against disk failures. To read a file data needs to be successfully read from any 3 of the 10 disks.

(i) Suppose a server selects 3 disks independently and uniformly at random to read from. What is the probability that disk 1 is read ? Hint: think of drawing balls from a bag without replacement. [10 points]

(ii) Suppose now that disks 1 and 2 cannot be read together (the set of disks that can be read includes disk 1 or disk 2 or neither, but not both). What is the probability that disk 1 is read now ? [10 points]

(iii) Each disk fails independently with probability 0.01. Remember 3 disks need to be read successfully to reconstruct a file. When the server reads 3 disks what is the probability that the file fails to be reconstructed ? [5 points]

(iv) With the same setup as in (iii) what is the probability when the server now reads 4 disks ? [5 points]